Rocket Propulsion

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Lecture No. 02

Motion in Space

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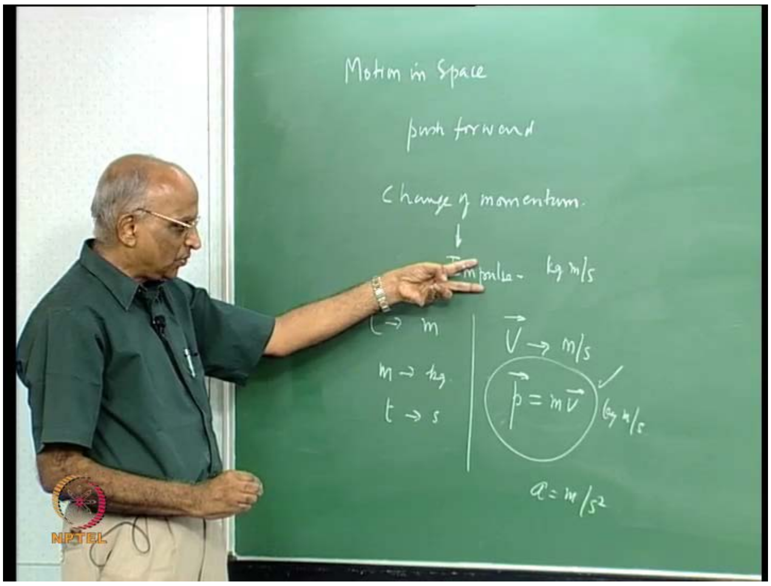

Well good morning. Well you know, in the last class we considered motion in space. We told ourselves we are pushing ourselves in space. Therefore, we are trying to push forward for which we have to give or provide a change of momentum. You know, when say change of momentum; change of momentum is what we call as impulse and it has units of same as momentum namely kilogram meter per second. We also considered what are the parameters by which we quantify motion. We told ourselves length is in terms of meter based on a standard, but, ever since 1982 the standard is based on physical constant which is the velocity of light in vacuum. We also told ourselves well, mass is in kilograms which is still based on the standard.

Now, current research effects are on. How to define a physical constant rather than an object by which I can define mass? It is still not done when we say time. We say it is a period and that period is second. When we say time is second how do I define the direction of time? The direction of time comes from the second law thermodynamics which says that it time progresses in the direction in which the entropy increases. May be we will try to take a look at it in the subsequent classes because thermodynamics forms the basis of the entire rocket propulsion. Having defined these three quantities; we tell ourselves for change of momentum, I need a velocity and velocity is defined as meter per second. Distance is a vector, velocity is also therefore, a vector. Therefore, momentum p is equal to mass into velocity.

Therefore, momentum p is also a vector unit being kilogram meter per second. The question immediately arises why is momentum when I can use velocity. See, the problem is momentum is a more fundamental quantity compared to the velocity. The reason being you known if I have an iron ball which travels at one meter per second and hit's me and have a feather which travels at the same thing and hits me; feather does lead to anything only whereas, iron ball leads to this. Therefore, in all subjects in which we deal with motion of molecules or classical mechanics, we deal with a quantity momentum rather than directly the velocity. The other quantity as we saw is acceleration which is meter per second square. Therefore, to be able to push and change the momentum rather provide an impulse; we are talking of a change of momentum and let us say whether we can define something properly because we still have to reconcile ourselves that we are used to talking in terms of forces.

If we talk in terms of forces why do we have to talk in terms of momentum velocity and all that? There may be some rational relation between them and therefore, we should we should go into this, but, before I get into this I want to bring out one example.

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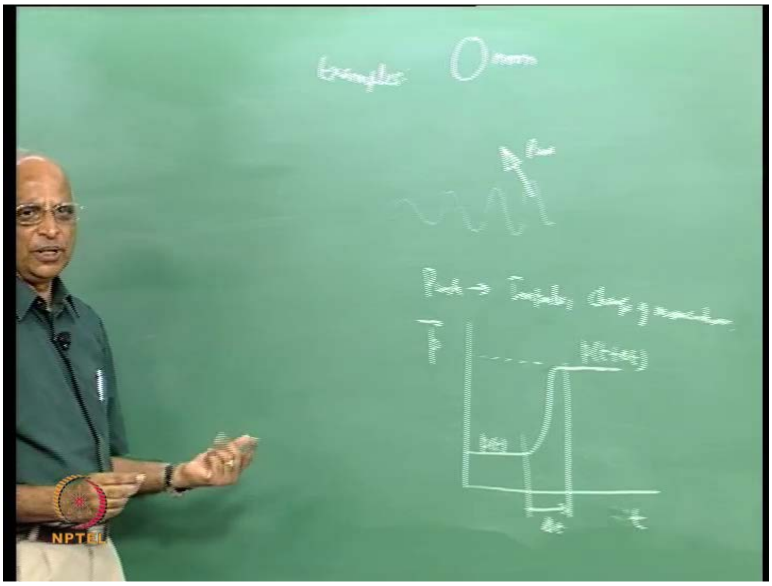

And these examples are important because when people talked in terms of going to the moon; may be some 2000 years back what they had in mind was may be the sea is very rough, you have lot of wave motion on the sea and if the in the sea there is a storm and there is a boat which is sealing it gets caught in the storm and because of the storm it is pushed forward; that means, an large push is given then perhaps from earth I can go to the moon. This was the first scientific article or first friction science friction article we say how I go from the earth to the moon. The idea of being pushed up by storm in the sea namely when you have huge waves all over the sea, that is huge title waves such as when I storm occurs over sea and a ship being pushed upwards towards the moon was purposed by Lucian, he was a Greek philosopher who live in the period let say around 40BC or so. Therefore, we are talking of something like push. Can I quantify push in terms of impulse that is change of momentum? Or rather collect in terms of change of momentum itself.

Therefore, we tell ourselves well I am looking at may be the momentum which is a vector, may be as a function of time. I have something like a body traveling at constant momentum and may be after over a short duration of time, I change the momentum to a slightly a larger value. the How do I change it? May be I change it gradually here. It as go to the constant value and therefore, it as to go like this the period of change what I have here is a period delta t; that means, I plot momentum as a function of time and here

I have the momentum changing from at time t it was p t at time t plus delta t it is p t plus delta t is what is the momentum change.

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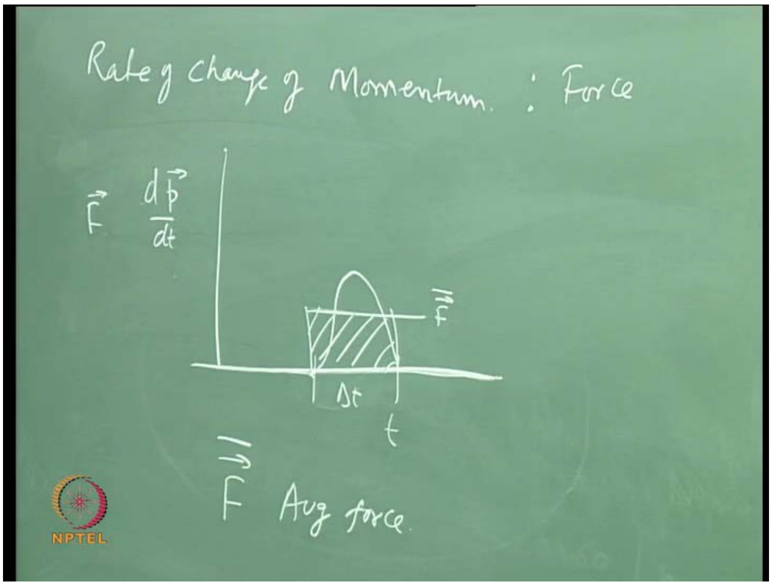

Therefore what is the impulse associated with the change? The impulse is equal to the value of momentum at t plus delta t minus p t so much kilo gram into meter per second.

Therefore, let us be very clear about the definitions impulse is just a change of momentum you give an impulse to a body; that means, you change its momentum. And now, if I were to ask myself what is the rate of change of momentum? I want to plot the rate for this particular figure. How will the rate look like? Let us make a plot of this. We find that I have d t divided by d t now. I plot as a function of time. I find that the momentum remains p t right from zero to this particular time. In other words, p t is a constant therefore, d p by d t is zero up to this particular point. Thereafter it increases reaches a maximum goes back to zero here. In other words over a period of delta t it increases, reaches maximum and comes back over here and then after this momentum is a constant and this is the signature what I should get for d p by d t as a function of time is it all right?

Now, I tell myself I call rate of change of momentum as force. In other words, here I say this is my force. Then the force due to change of momentum is not something which is a constant, but, keeps varying. Therefore, whenever I change the momentum and I ask myself what is a force? It is difficult for me to say how this force is going to vary with time. Therefore, what we say is well the force is continually varying during the change of momentum and best for me to do is to take an average value and say this is my average value, force is a vector. This is my average value and then I say this is my average force and whenever we talk of force, we mean something like an average value is what we call as force.

Therefore, force is really a derived unit. It is not something fundamental like momentum and we must keep in mind that the force during a particular change of momentum continually varies.

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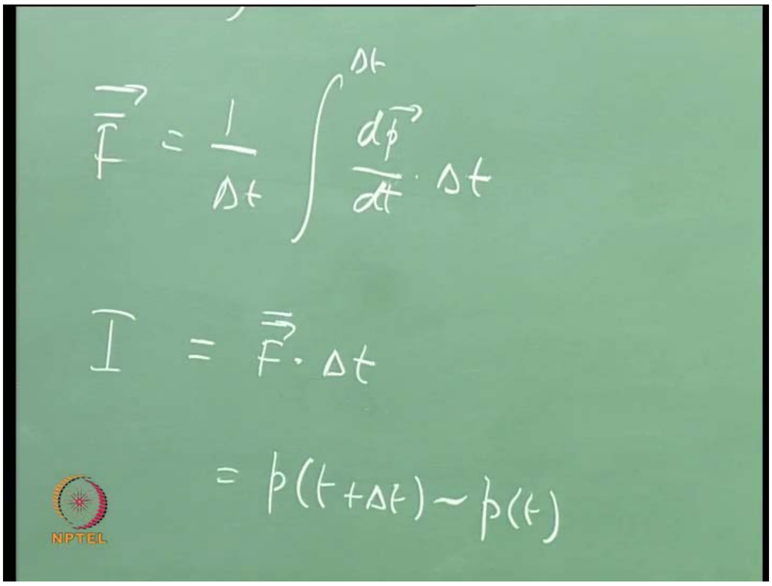

But I take the average value and what will this average value of be? Let us put it down f bar and we say it is a vector again have reverse my thinking anything will do here. Therefore, its equal to I talk in terms of may be d p by d t is a momentary value. May be I take it over the time d t or delta t. I integrate it over the time may be from here to here over delta t is what will be my my my my my average value. That means, I must also multiple over here by delta t. This must be that is rate change of momentum, average over a period of delta t is what is force or rather f by is equal to 1 over delta t of the integral what have written here delta t of.

Therefore, you really see force is something which is not that that good a unit compared to what we should say this and if I have to write it in terms of impulse; how I should write it? This I can write it again as equal to or impulse, I can write I is equal to force into delta t, average value into delta t. This into delta t is the change of momentum and therefore, impulse and this is equal to I have p t plus delta t minus p t over here. Is it all right? Because we find force is equal to this divided by delta t or f into delta t is this and the change of momentum is impulse and therefore, the they correct connection between impulse force and change of momentum comes from this particular equation.

If we are very clear about it; having seen the parameters we tell ourselves momentum is more primary than than force. May be its time for us to get into this universal law of gravitation and what we did tell ourselves in the last class?

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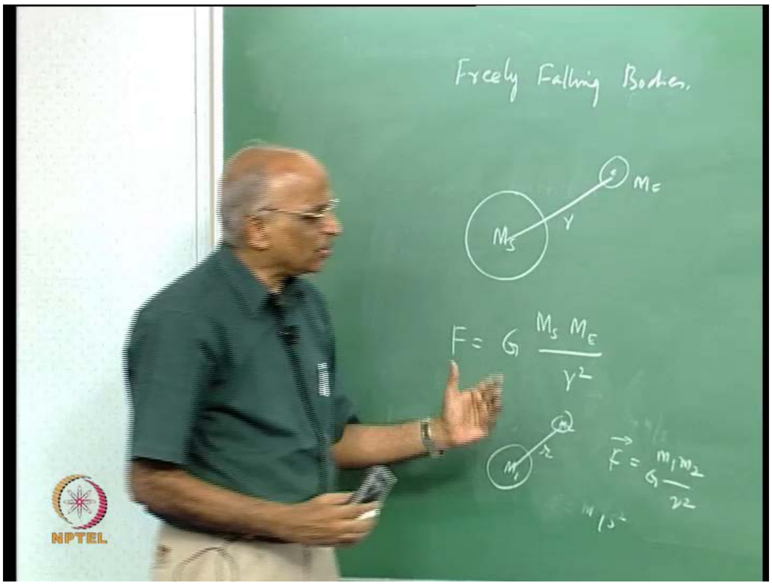

We told ourselves well, all planets as they go around the sun are all freely falling bodies right. We told ourselves a planet may be earth is continually falling on to the sun. So, also an apple is falling down and this is the observation which Newton has and he says a heavy body like the earth attracts a body which is up in space or may be sun is attracting the earth. And if I say this is the mass of the sun and this is mass of the earth; he said well the force with which it attracts is given by a constant multiplied by the attracting body, the body which is attracted divided by r square. In other words why it should be the sun and the earth? It could be any heavy body. Let us say m one attracting a small body. Let us say m two at a distance, let us say r the distance between them is r ovary the distance is a r over here. Therefore, the force is equal to m 1 m 2 divided by r square into a constant.

And this becomes the universal law for gravitation and g becomes the universal gravitational constant. You know this this law is important as we shall see in moment r two. Therefore, let let us rewrite it you know this the the way have written is cannot be correct because force is a vector r is a vector m and m are masses.

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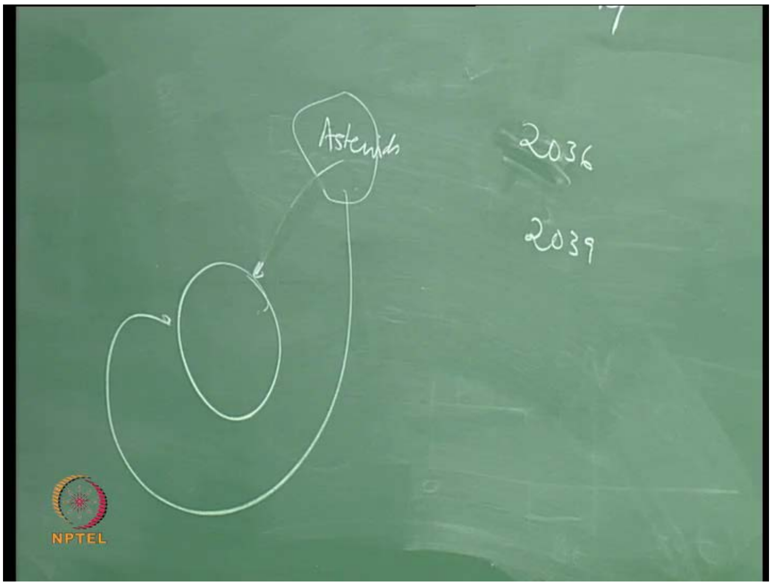

Therefore, I should have really written this as the force is equal to may be a body one attracted towards two or let us say the body two attracted towards one. Therefore, I have the body two which is a light body being attracted by a heavy body m goes as a gravitational constant into r square. Therefore, I put it as r is a vector to the power 3 mod of this vector into r bar; that means, m one m two by r square into g and it is being attracted. Therefore, it is a negative sign.

This is the universal law for gravitational. G is the gravitational constant and therefore, the unit for g should be what? It should have the units of force divided by kilogram square multiplied by radius square right or meter square. What is the unit for force then? We should be clear about it. What did we tell ourselves there? We told ourselves force is the rate of change of momentum, force is equal to d p by d t. Therefore, we are telling the units for force is equal to 1 over time divided by kilogram meter per second which is equal to kilogram meter per second square. And this particular thing kilogram meter per second square is what we call as Newton.

Therefore, the force has unit Newton which is nothing, but, kilogram meter per second square. It comes from rate of change of momentum and therefore, we have the units of g as Newton meter square by kilogram square and the value is something like 6.671 into ten to the power minus eleven Newton meter square by kilogram square. This is the constant in the universal law for gravitation. I think this is very important. Why I say this important? l[et] let Let us take a physical example you know I I have been telling you that may be in our solar system we had the eight planets going around the sun you also have some lose objects like asteroids which are also going around, but they do not have a well defined path or an elliptical orbit like what the planets have.

You know it is said that one of these asteroids is lightly to hit the earth may be in the year 1936 I am sorry 2036 and I if does not hit may be it come back and again hit it in 1930 2039. What is it we are talking of? May be some of these asteroids may come and collide with the earth. You know we have been talking of these asteroids. These asteroids when they enter the earth atmosphere they rapidly burn out. Therefore, the question is how do I make sure that an asteroids, let us say it is going around, it is going to come and hit it. How do I make sure that the asteroid is not going to hit it? What is the type of propulsion system or how would I design my propulsion module such that I prevent the asteroid from hitting the earth? Can we think of it from the universal law for force over here? People talk of difference strategies how to how to prevent some of these things happening and let us take let us try to solve this problem it helps us in doing something with the universal law for gravitation.

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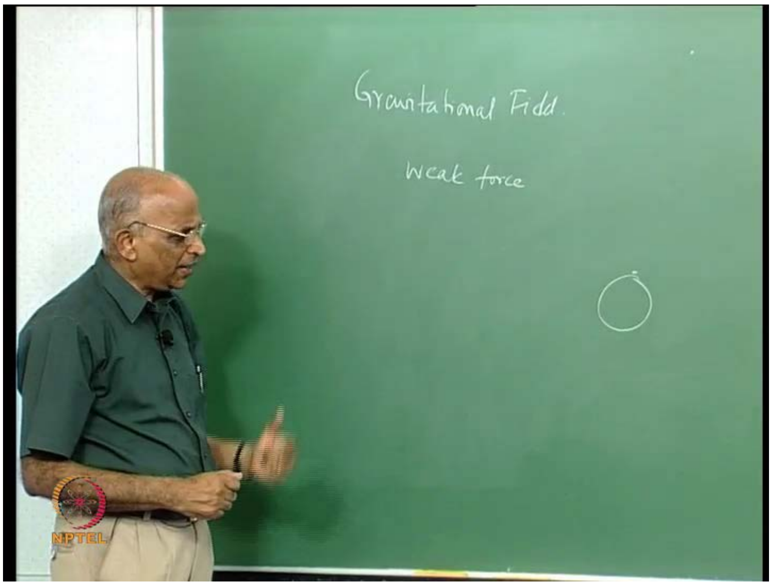

Let us say one of the thinking is may be this our earth. May be I I launch a rocket on to space and I keep on accumulating satellite over here; I make a heavy mass over here and when the asteroid comes over here this mass being heavy compare to the mass of the asteroid over here, will attract it towards this and may be instead of the asteroid going in this particular direction or in some particular direction, this change the direction and it will miss the earth. This is known as a gravity tractor. That means, I put a mass in space and make sure that this mass is near to the asteroid and the distance is small, it gets attracted towards it and the asteroid instead of coming like this can get deflected away from this.

You know these are all possible right. That means, you know the the law is not only for let say conventionally doing problems in mechanics; but, can be applied for changing the trajectories and changing the trajectories is as good as giving some propulsion element to it. The gravitational force what we are talking of or rather the gravitational field is a weak force and it purses over a very long distance. Like let us say I have the earth here. May be a mass above the surface of the earth is attracted with a higher field than something which is may be very far away because the field decreases as the distance from the earth the attractive field from the earth decreases as we progress away from the earth as it where.

Let us do one problem to be able to access the gravitational field and I take a model problem again of an asteroid and let us calculate what is the force exerted may be by this asteroid on something which is moving near it. I think this will tell us what is the magnitude of the gravitational field for some space related problems.

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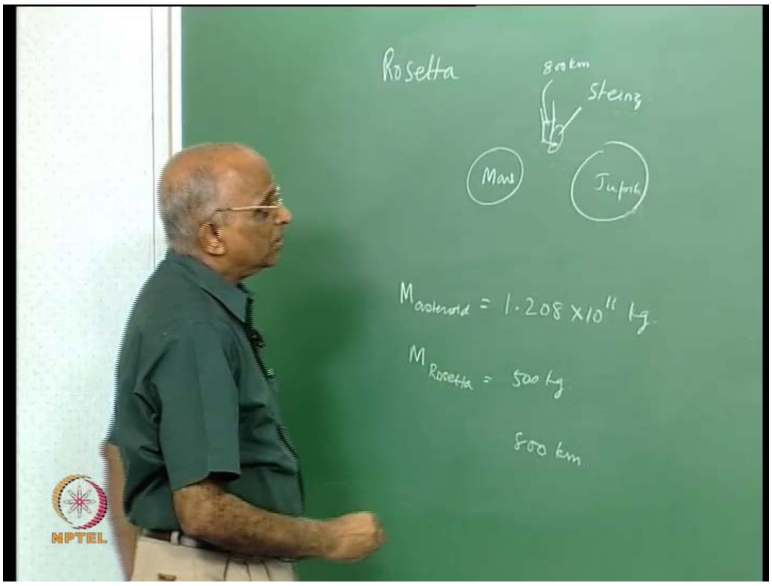

Let us do this problem. I make note of it. You know sometime back a space probe by name Rosetta was launched to study the asteroids in the space between let us say Mars and Jupiter. Jupiter is a large planet and this space between mars and Jupiter had a number of asteroids and therefore, this particular space capsule Rosetta was used to study a particular asteroid by name Steinz.

Now, Steinz had a has let say as a mass. Mass of the asteroid Steinz is around 1.208 into 10 to the power eleven kg. You know the asteroids are somewhat lose material and they do not have a particular fixed path in space and therefore, they wonder up and down and the interest was to bring this space capsule Rosetta as near to Steinz as possible and it the nearest distance it came near to this asteroids Steinz was something like 800 kilometers. Therefore, we would like to know when this space capsule Rosetta is 800 kilometers away from this asteroids Steinz what is the attraction attractive force exerted by this asteroid on this Rosetta? Therefore, mass of this space capsule namely Rosetta is around I think it is around 500 kg. You take it as 500 kg. The nearest distance between the space capsule and the asteroid is 800 kilometers.

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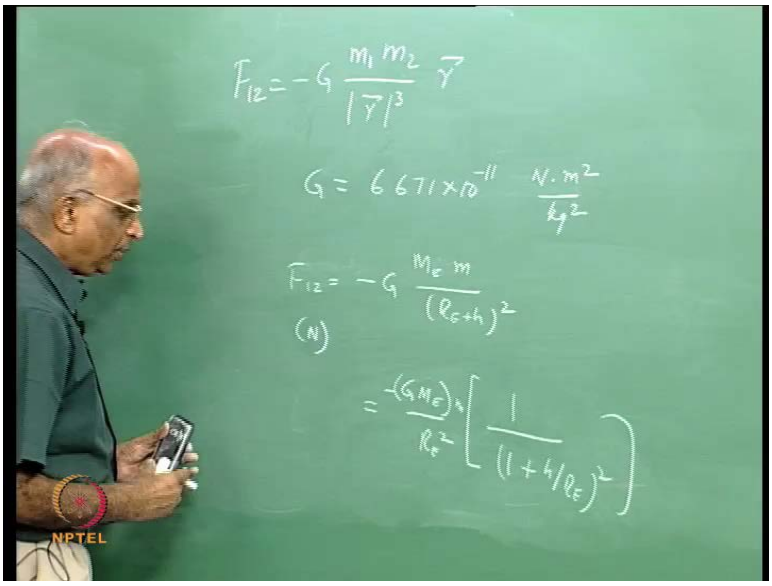

Therefore, what is the force which the asteroid exerts or pulls the space capsule? We say f is equal to g mass of the asteroid into mass of this space capsule namely here in Rosetta divided by r square which is equal to the the gravitational constant is 6.67 into ten to the power minus eleven. What is the unit? Newton meter square by kilogram square into the mass of the asteroid which is 1.208 into 10 to the power eleven. Mass of the space capsule 500 kg and this is divided by the distance square. The distance between the two is let us say nearest position is 800 kilometers and the diameter of the asteroid can be assume to be 1200 kilometers.

Therefore, that total distance of the from the center of the space capsule to the center of the asteroid to the center of the space capsule is 600 plus 800 into this is kilometers into ten to the power three square is meter square and therefore, we are getting a force of the order of 6.671 670 let us say into 10 to the power minus eleven into 1.208 into 10 to the power 11 kg over here and into 500 divided by 1400 into 1000 square and let us take look at the units. It is Newton meter square by kilogram square into kilogram into this was again kilogram over here, kilogram square. Denominator is meter square and therefore, we have so much Newton's as attractive force.

When I look at it even without solving I find well I have large number here and therefore, the type of force what I get is of the order of 10 to the power minus 6 of a Newton which is something like a micro Newton. That means, the attractive force exerted by this asteroid on this space capsule is something like a micro Newton, but, in space even small forces are of interest and therefore, we find that weak gravitational forces attract the space capsule. However, if we had something like an asteroid over here and we could have massive massive, very massive satellite satellite like what I said is gravity tractor then in that case if it is really massive, I could I could pull it with much larger force. Having said that let lets know see what is the gravitation. What do you mean by gravity? You say acceleration due to gravity. What is acceleration due to gravity? How would you define? How would you define acceleration? What do you mean by gravity and what is acceleration due to gravity? How how how to define it? Let us say whenever we do some problems, we say force is equal to m g. What is g here? How will you define it because all of us know well g as unit.

So, meter per second square we call it as a acceleration, but, how can how can earth give something an acceleration? Acceleration is rate of change of velocity therefore, you know small g how will I define m? You know we go back to the universal law for gravitational and then we write over here F 12 is equal to let us say I have the earth here somebody is falling on the earth let the mass of this body b m let the mass of the earth be M E and therefore, the force which this body is pull towards the earth as per the universal law for gravitational is minus g into mass of this m 1 is the heavy mass M E and I have m 2 which the mass which is being attracted over here divided by I forget about the vector part of it I say r square and r square is the radius of the earth plus the height about the body is a force which is the attracting and what is this force? The force is so much Newtons. Now I want to simplify it. Therefore, I write this is as equal to minus g M E by r e square and then I write it within the bracket now as m I put it here into 1by one plus h by r e square; take it to the top it becomes one plus h e r e to the power minus 2. I expand it out and I know that the height above the earth is very much smaller than r e.

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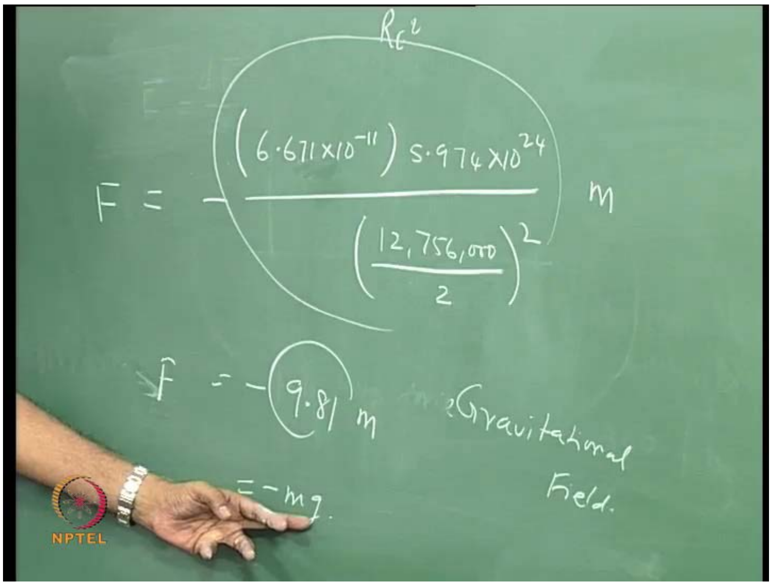

And therefore, I can write this expression as force is equal to minus g M E by r e square into mass of the object into 1 minus 2 h upon r e higher values. Anyway h is smaller than this e 1drop it out as a first approximation.

And therefore, I get the value of force is equal to minus g M E by r e square into the mass. We just now told ourselves that the mass of the earth is how much? We we give some number ten to the power 23 5.974 74 into the 10 to the power 24 kg and the value of the g was equal to 6.671 10 to the power minus eleven and the value of the radius of the earth was equal to 12756 kilometers 1000 divided by 2 radius square into m minus which is the value of the force.

Now, I simplify this. I find out the value of this and this will come out to be minus 9.81 of m which is equal to f and therefore, this is the value which is a constant. Mind you the mass of the earth is a constant, g is a constant, radius of the earth is a constant and this is what we say f is equal to minus m g. Therefore, we are really not telling acceleration due to gravity we just tell ourselves as per the universal law of gravitation whenever there is a heavy mass it attracts a lighter mass and it is in a field. That means, we are talking of something known as gravitational field as it were and this is the same thing which happened when an asteroid is going, I create a heavy mass here. It deflects the asteroid and I make it go l square maybe we should do some problems relating to the, but, all what I want to tell you is force is equal to m into g.

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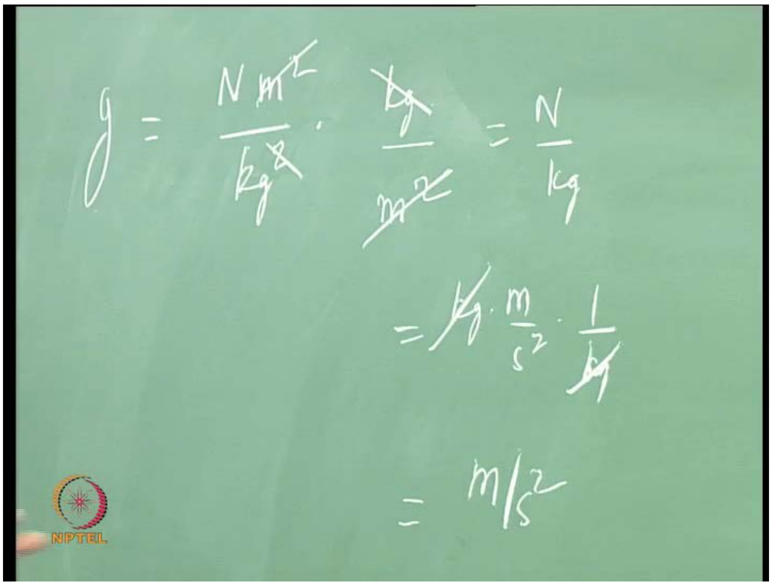

G has units. Let see what is the units of g? We put the expression down. The value of g the constant g was equal to Newton meter square by kilogram square into the mass of the earth in kilogram, radius of the earth by meter square. Meter square, meter square gets cancelled. Kilogram comes here, you have Newton per kilogram. Newton is equal to kilogram meter per square divided by one over kilogram this is equal to meter per second square. Therefore, the unit of the gravitational field its so works out, comes out to have units of acceleration and therefore, many people refer it as acceleration due to gravity whereas, it is just a felid due to a particular mass. See so far you know we have not said anything extra. We just told may be the astronomer Johannes Kepler had the three laws for planets going around; the sun as it were sun at apogee and the planets going round it. May be Newton saw that he was able to relate that the planet was freely falling towards the sun just the same when apple is falling and therefore, he formulated this particular law.

He also told ourselves based on observation and since is based on observation the law is not something which is really universal, but, it is based on phenomena or phenomenal logical law. We also talked of (( )) which which are some bodies in space which travel at a speed near to velocity of light when those things are traveling at such high speeds the law breaks down and the universal gravitational law is no longer valid. Therefore, you know whenever we base anything on observations; the observations the conditions of observations must be related to the particular law and we tell ourselves well, I can use these laws only under conditions in which we are talking of masses that separated by distance. We are talking of low velocities and only for those conditions is the law valid.

Let us keep it in mind, but, still we have not really told ourselves anything about how this law, how will you justify that a heavy mass can attract a lighter mass. How can you justify? Can we do an experiment to show that? And who is the person who did some experiments and showed the validity of the gravitational law? Still it is not proven.

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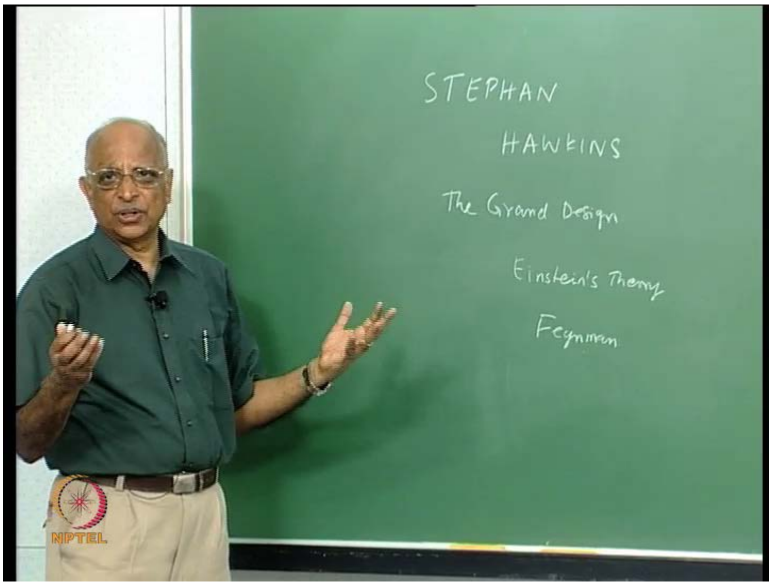

We have scientists, may be some very famous scientist like Stephan Hawkins. He recently published a wonderful book known as the Grand Design. He talks in terms of a unified model for explaining the laws of nature. In fact, in this particular book may be the Grand Design, he talks about the phenomenological theories of Newton. Then he goes goes ahead to Einstein's theory is about and also about the pioneering work of Feynman. May be it is it will be nice to read through. But, all what I want it to say was well Stephan Hawkins has looked at may be the forces in nature and he has also contributed to evolving the reason why such forces exist in nature.

And these people are trying to prove how this law comes through. But, can we express it? You you had Einstein's who who gave an explanation for it and the explanation given is looks to be easily understandable because we should be able to get a force from gravity.

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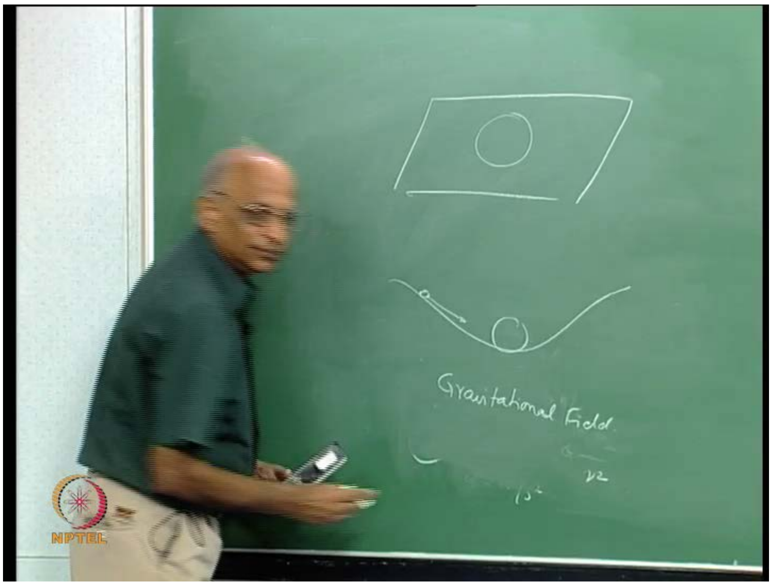

Let let us try to see this. Supposing I hold something like a towel or rubber sheet something like this. I hold a rubber sheet like this. In the center of the rubber sheet I put a iron ball what is going to happen? The sheet is going to come here. The heavy ball is going to come over here. Now, I put a small ball here it will roll towards it; that means, this heavy ball creates a feel which helps in the motion of this small ball and this is an analog to how a gravitational field can exist. But, one has to do one has to really go through some theories. But, the the reason for gravitational field is still not understood. We can only understand it through an example like this. May be when I have a heavy mass I have something like a gradient and that gradient attracts a smaller mass. Therefore, now we ask ourselves one last question yes in today's class we have been looking at may be some parameters. We looked at the gravity gravitational constant and can we do some simple problems of forces and may be think in terms of rocket using these forces?

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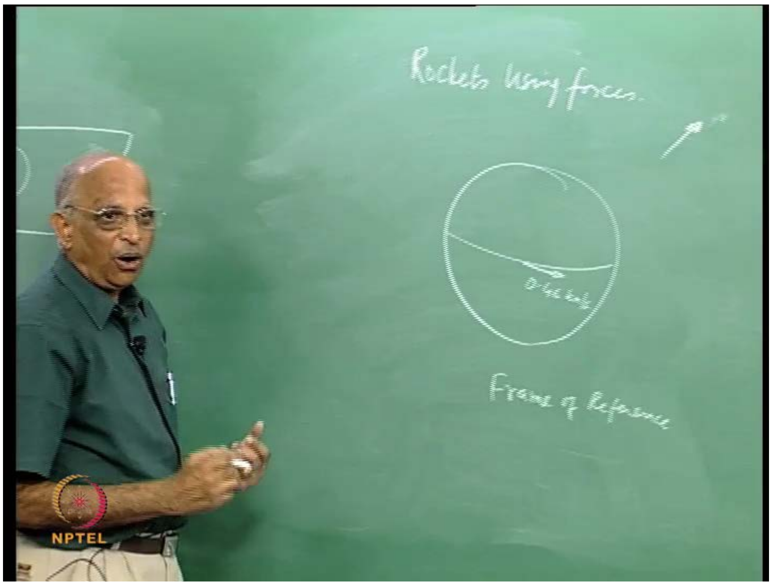

You know there is one problem which I did not bring out to you earlier. The problem is when I have the earth as it where and I see an object traveling at a particular velocity v or I, some object which is travelling. I cannot define the velocity now. I am on the earth, earth is rotating and when earth is rotating I am also rotating along with the earth. That means, I am also moving. My velocity of movement is something like 0.46 kilometers per second. This is the speed with which I am moving because you have the earth, the we said that the radius of the earth it rotates once in 24 hours. This is the speed which I am rotating.

Now, if I am rotating at 0.46 kilometer per second and as I am rotating this body is moving. How do I say? How I am able to relate the velocity of this body with respect to me? It is going to be difficult to even determine the velocity. Therefore, we have this problem. And therefore, I need to have something like a frame of reference in mind. How do I define a velocity? How under what condition will the velocity be correct?

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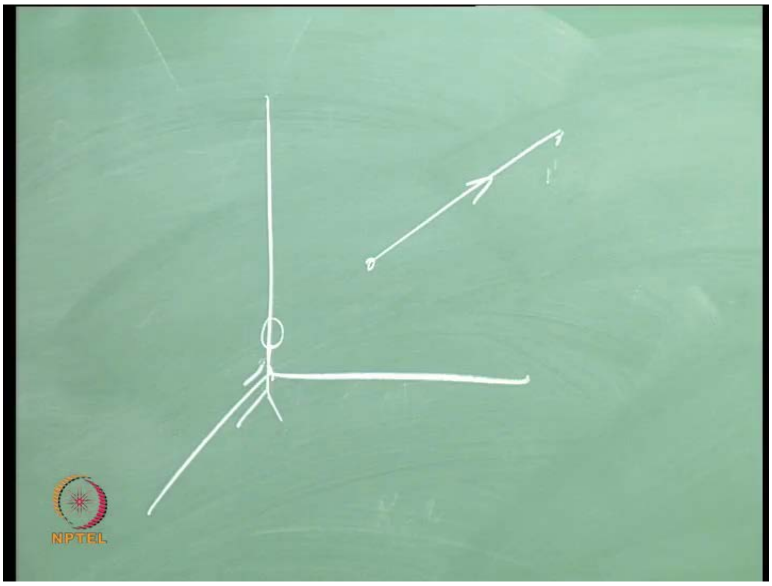

You will immediately tell me well, if I have if I am absolutely stationary like for instance I am I am standing here. This is my coordinate system. May be I stand over here I am absolutely stationary and a body is moving; then I can say that distance travel divided by times gives me the velocity.

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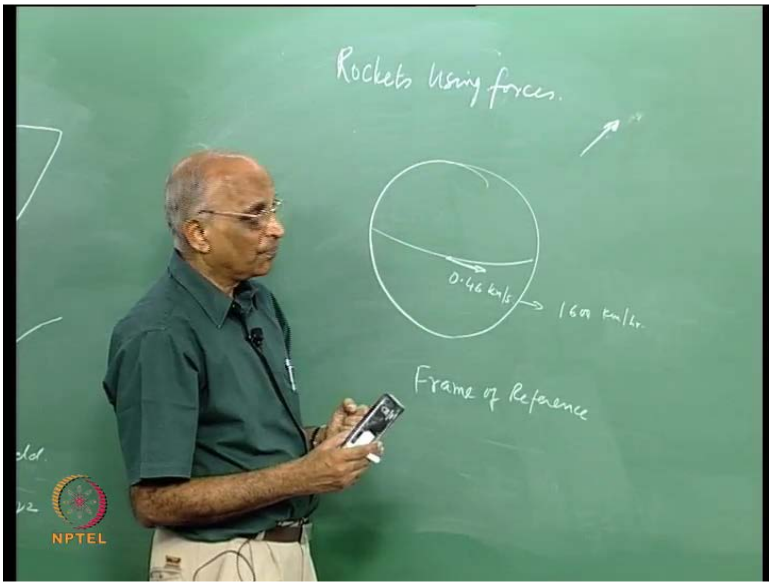

But if I am on a plain like this and what is my speed? Mind you 0.46 is kilometer per second will translate into something like 1600 kilometers per hour which is going to be faster than the fastest car which can travel that is the speed with which I am moving.

Now how do I find that this particular velocity? See, I am also moving on the surface of the earth as an object is moving in space and my appreciation of the distance travel by m depends on my movement.

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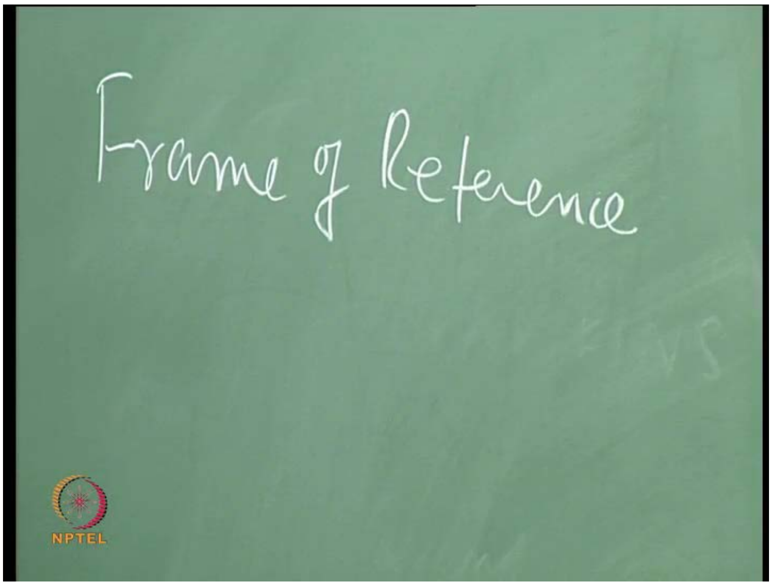

Therefore, we are all relative and therefore, we need a frame of reference to be able to describe the motion of bodies in space.

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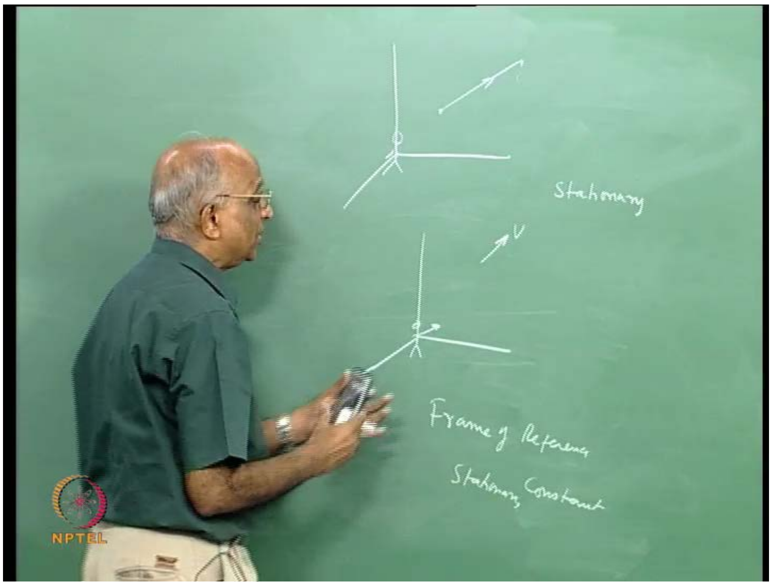

Therefore, in order to we able to do I must either be totally stationary which is difficult or else I must also tell myself if I am not stationary, if I am moving at a particular constant velocity.

Supposing, I were to move at some constant velocity and I am observing a body at a at a different velocity; the change in velocity of this with respect to my constant velocity will always give me the same change. Therefore, my frame of reference, whatever I consider must either move at constant velocity or be stationary. Then only I can say what is the change in the velocity of this body. Otherwise if I am here on this earth and it is rotating and all that my my translation velocity is continually changing, I cannot I cannot really monitor the change in momentum or change of velocity here and a frame of reference which is either stationary. Let us write it down.

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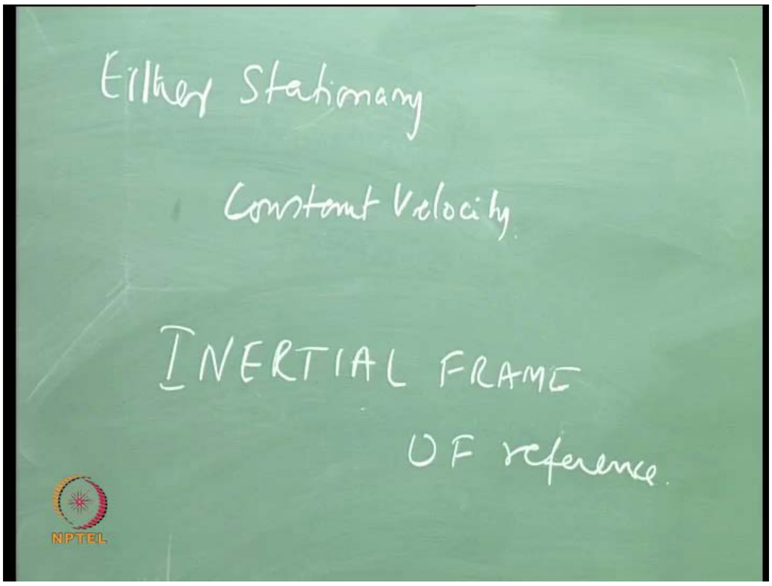

Which is either stationary or moving at constant velocity is known as the inertial frame of reference. It is very essential to have this distinction very clear. I will come back. You all are from a combustion and background I will ask you a question.

What is the difference between flame speed and burning velocity are they the same? What is the difference? Now, frame of reference is extremely important in any engineering problem. That should be a clue. What is the difference between flame speed and burning velocity? Let us do this problem. Even though we are getting away from what what I should be doing here.

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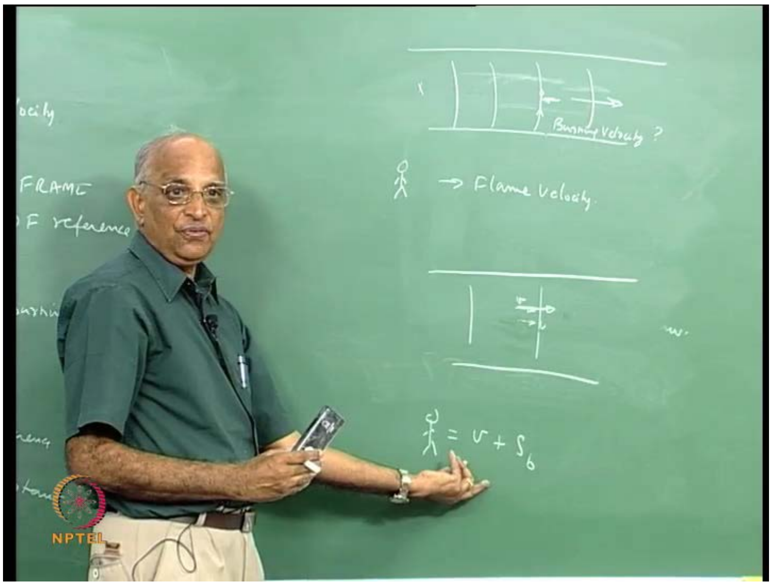

Let us say I have tube filled with a gases and that is what many of you all are doing and I ignite the gases, I allow the flame to propagate. when What gives me burning speed or burning velocity? What gives me the flame velocity?

I slightly complicate the problem. I tell myself well, I am standing here stationary. I, Because this fellow is also rotating at the same speed on the surface of the earth. I am also on the same speed. And now, I observe it from here. I call this velocity as flame velocity. But, if i were to sit over here on the flame and find the velocity with which the gases are coming towards me; I call it as burning velocity. What is the difference? What is I have done? Should it still be the same or should it be different? Now, I pose this question to you; I have the tube, I have the gases which are, this is the flame the flame is pushing the gases ahead of it the gas as some velocity over here. And therefore, by the time the flame comes over here the gases all ready in motion. It has particular velocity and then the velocity of the flame front is with respect to the velocity of the gases over here.

Whereas, when when I look at the flame from outside I am looking at the gas as it were moving in this. That means, I am looking at the speed of this flame is moving and around around this the flame is moving. Therefore, on stationary flame I have; that means, I look at the velocity of the gasses plus the burning speed over here. That means, the flame speed is a velocity with which is gases are moving plus the plus the plus the burning speed here. Therefore, they have to go into total different. Therefore, we have to be clear you know the frame of reference is extremely important.

If I sit on the flame what I observe is the burning speed whereas, if a sit on the ground and watch something moving I have something which is the flame velocity. So, also the flame of reference in the, is something and we have to consider.

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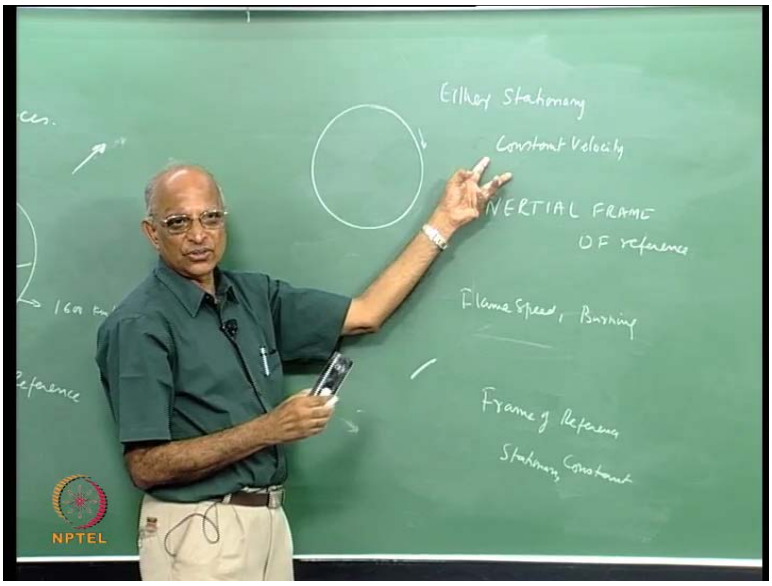

Only the inertial frame of reference and the very fact that we are here in in in a rotating frame of reference, introduces new components and its necessary for us to compensate for this and this is what I will be doing in the next class. To summaries then; what is it we have done so far?

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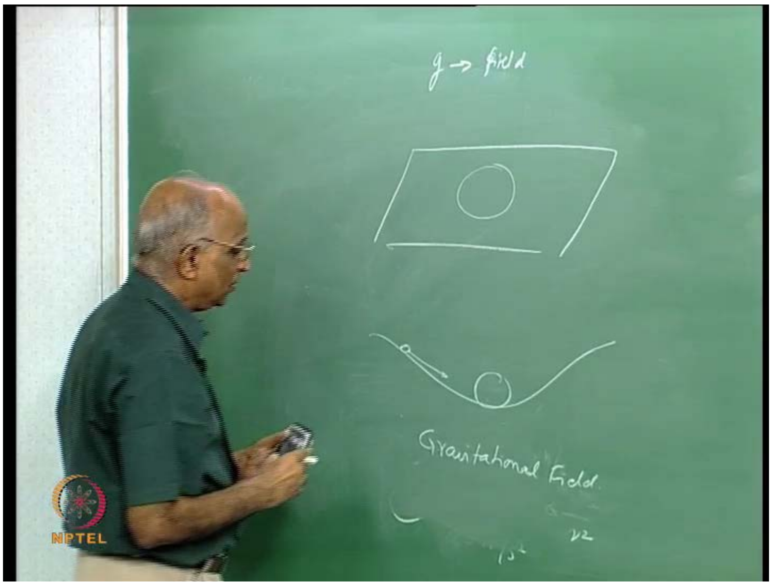

So, far we looked at the parameters describing the motion in space. We looked at the constituents of space. We talked in terms of planets moving round the sun. We told ourselves the planets are in a state of continues fall on to the sun and they are something like a freely falling body. And therefore, Newton saw the commonalty between a freely falling body like an apple on to the ground and the planets and then formulated it the universal law for gravitation. We looked at the value of G which we say is the gravitational field. What is the force in gravitational field is what we told ourselves and towards the end we looked at may be the frames of reference and we said that to be to able to measure momentum or momentum change; its necessary to have an inertial frame of reference.

I continue with this in the next class. We will we will get into a rotating frame of reference, what are the corrections required and that will help us to define orbits and also the requirements of rockets. Well thank you then. I think that (( )).