Rocket Propulsion

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Lecture No. # 23

Burn Rate of Solid Propellants and Equilibrium Pressure in Solid Propellant Rockets

Good afternoon. I think we will quickly recap where what we were doing earlier, and then go to the burn rate of composite propellants, and also address ourselves. Are they some particular values of n, the exponent n burn rate law r is equal to a p n, which is necessary, when we design the solid propellant rockets.

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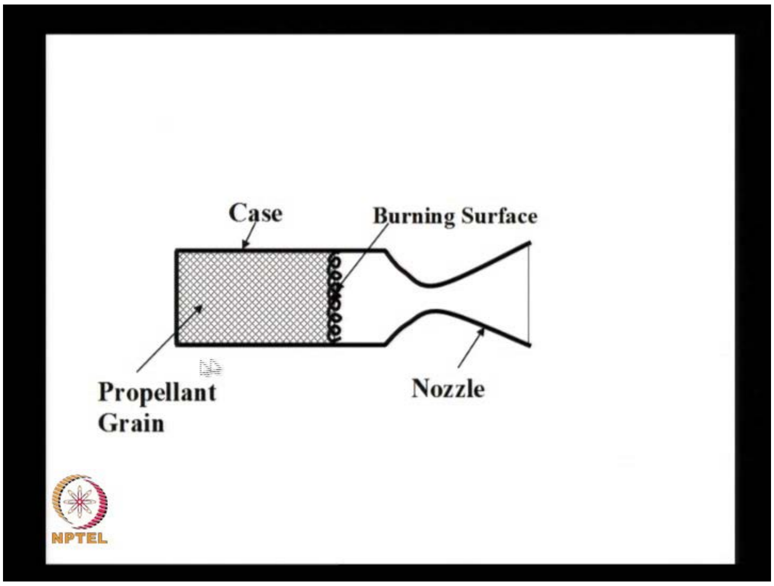

What we did in the last class was we told ourselves, this is the propellant grain which is their inside the particular case. This is the nozzle we ignite the surface of the propellant, the surface regresses at a particular rate which we call as the burn rate.

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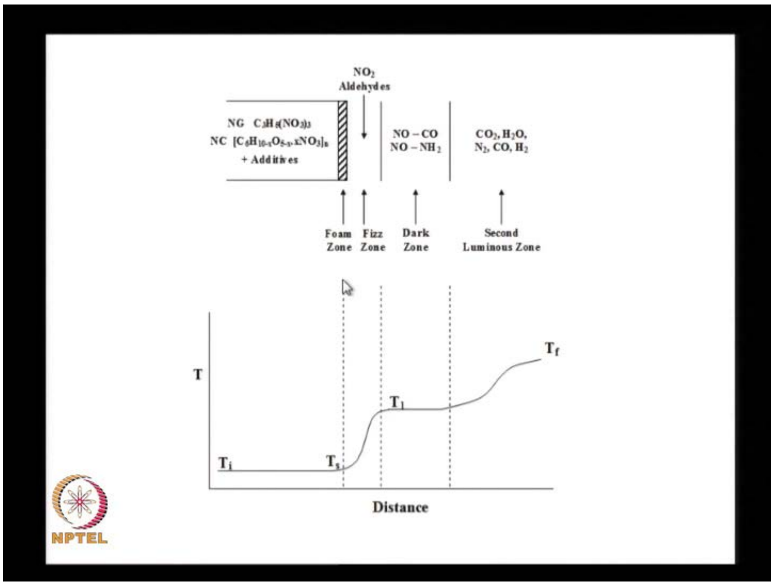

And what did we do for the case of double based propellants? We told ourselves we have nitrocellulose and nitroglycerin, we had N O 2 aldehydes and also some other constituents here, which react. Take the temperature from the surface temperature to the temperature at the edge of the fizz zone. This was the foam zone, which is preheated solid surface of the propellant. And then you had the fizz zone where the temperature went to T 1 there after it remain constant at this value in this dark zone where in essentially the reaction between N O and N H 2, N O and CO do not generate much heat. But thereafter the reactions generate heat and you get C O 2, C O, H 2 O and all and in this case the temperature went to the final value. This zone, this particular zone of dark zone is there only for pressures less than around 10 mpa or 100 atmospheres. And at higher values, what we got was the temperature directly goes to the value of T f.

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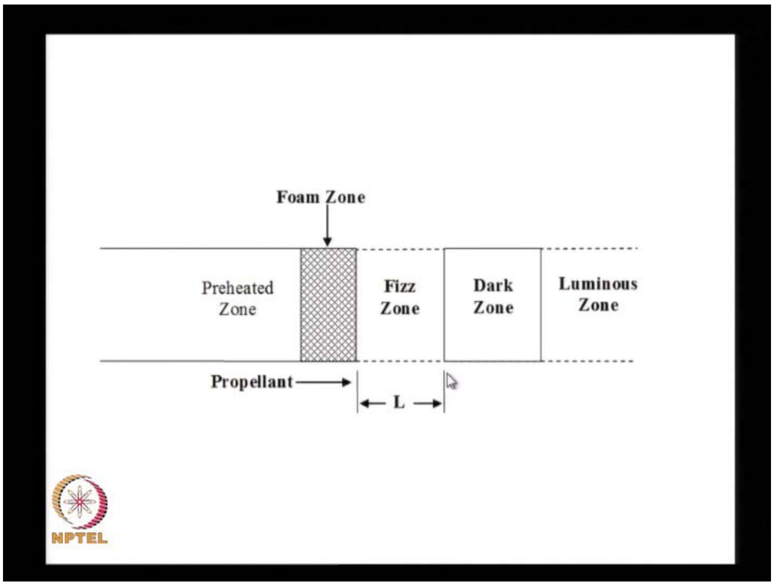

Foam Zone

Preheated Zone

Fizz Zone

Dark Zone

Luminous Zone

Propellant

L

Having said that we wanted to translate this into a into the burn rate and therefore, we again looked at the preheated zone, a foam zone, a fizz zone, a dark zone and a luminous zone.

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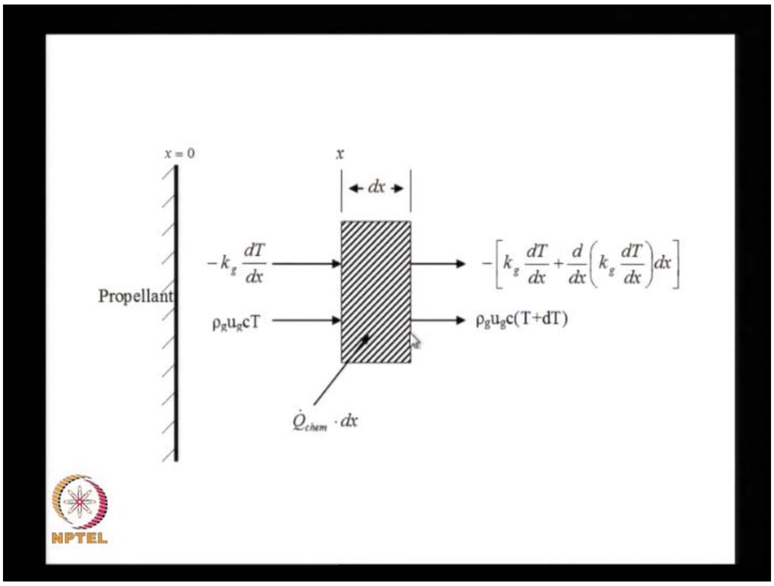

$x = 0$

x

dx

Propellant

$-k_g \frac{dT}{dx}$

$\rho_g u_g cT$

$-\left[k_g \frac{dT}{dx} + \frac{d}{dx}\left(k_g \frac{dT}{dx}\right)dx\right]$

$\rho_g u_g c(T+dT)$

$\dot{Q}_{chem} \cdot dx$

We wrote the equation in this particular zone and what did we write it as, we took a small element in the fizz zone. This was at x is equal to 0, fizz zone extended up to x is equal to L mind you, the length of these zones are fraction of an mm thick. We just extended it to be able to do all this. We had a small element here with unit surface area and we said (( )) heat enters, heat leaves. The heat leaving is in excess by this amount and the enthalpy that is the rate mass generation rate is equal to rho g into u g into specific heat into temperature. It leaves at a temperature T plus d T and therefore, this is equal to the excess heat transfer and the heat generated by the chemical propellants and we were able to write the equation.

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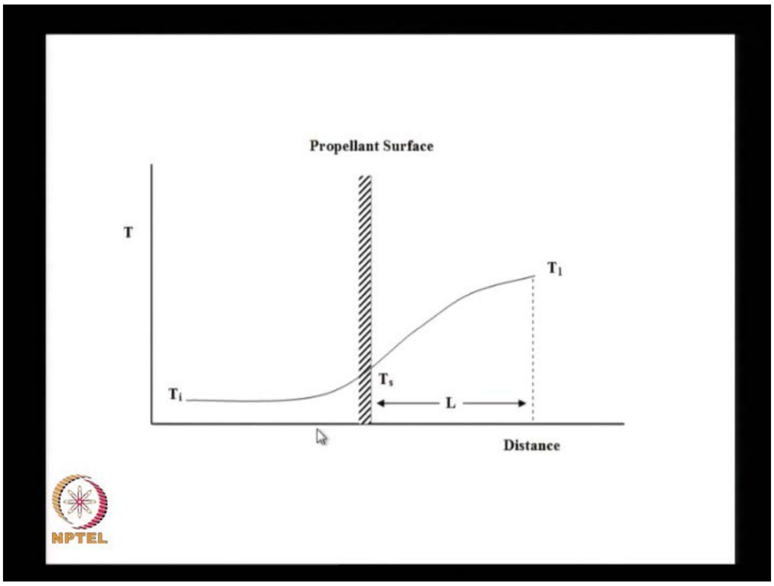

And based on this equation, we said we could get the temperature profiles for given values of u g p g and rho g.

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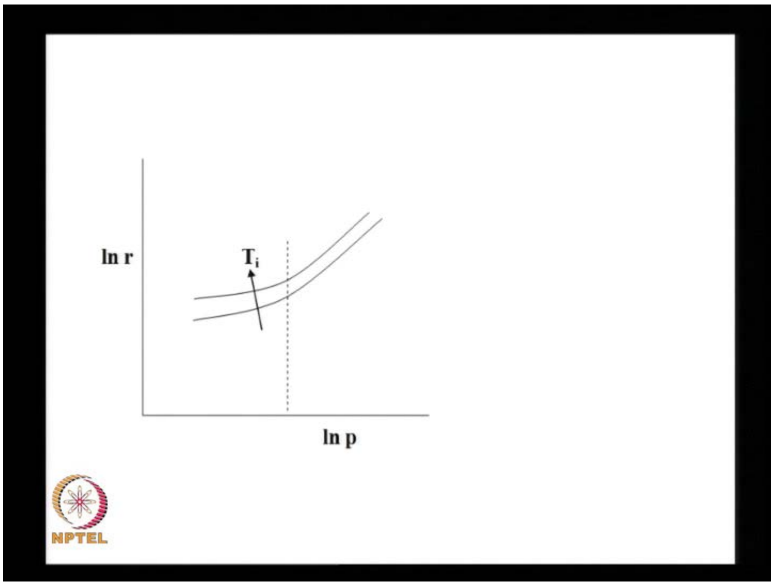

Let us get forward; we also found that in the zone wherein you have the dark zone, the variation of the logarithm of the burn rate versus logarithm of pressure was slightly less. Because the temperature T 1 is supplies heat to the surfaces lower than when I had the final temperature directly supplying heat. And therefore, I have low exponent here and a higher exponent over here.

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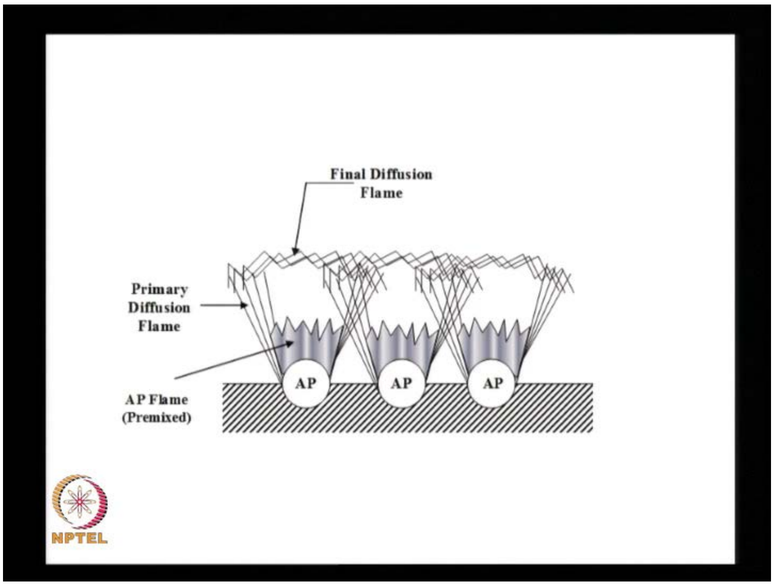

We would like to do the same type of analysis for a composite propellant and a composite propellant is distinctly different from a double based propellant. What is the distinction?

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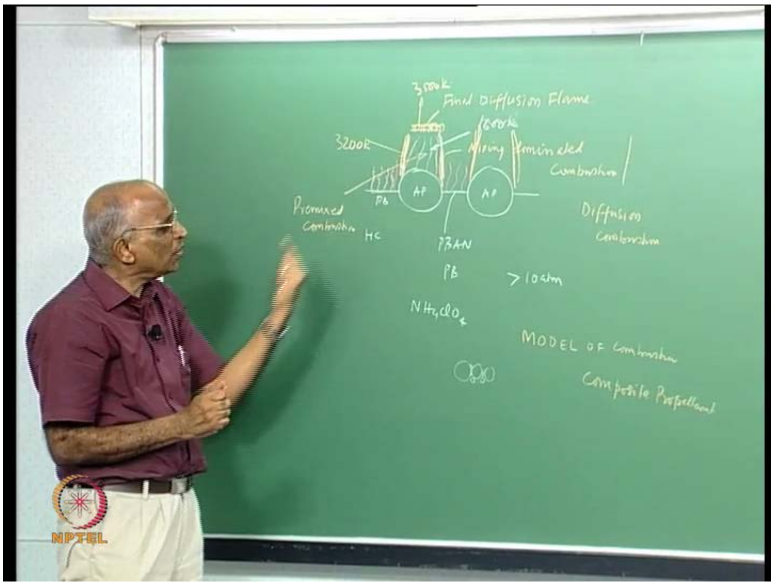

Let us put it on the board, we have we said A P in the form of crystals. And this A P is contained in the polybutadiene, you could also have aluminum just like we saw in this sparkler when there is metal it is much hotter and that is why aluminum is put. Therefore, you have something like A P solid crystals and in between may be it is bounded by the polybutadiene, let say PBAN. And this PBAN could either be H T P B or or let us PBAN it could be some polybutadiene. It could be polybutadiene acrylic acid acrylonitrile or it could be H T P P, it could be C T P P this is the fuel or the binder. And when I supply heat and start making this burn, you know A P is N H 4 C L O 4. It contains oxygen, it contains hydrogen, therefore, I could get a flame something like a monopropellant flame, this itself will burn. But the hydrocarbon which is over here may be the polybutadiene cannot burn because it is just hydrogen and carbon, it does not contain oxygen. Therefore, the volatiles of hydrogen and carbon come over here.

Therefore, let me use a slightly different color over here to show this, may be this is the fuel vapor which is coming out, this is a monopropellant flame and this flame is typically at a temperature of around 1600 Kelvin. Maybe whenever we are talking of this combustion taking place we presume that combustion takes place at a pressure of the order of greater than around 10 atmospheres. Because we are not going to have rockets, which have chamber pressure less than this. Therefore, we are talking of something like 1 m p a pressure. And at this pressure may be I have the flame of A P which is coming over here, I have the vapor coming over here of this and this is the picture I can draw in my mind of what takes place.

Mind you, in practice when I look at the propellant, I will have A P then A P here, I have A P here, I have small particles of A P here, I could have aluminum in between, I have small things, I am just magnifying this zone and separating it corresponding to this distance and putting this scheme to be able to formulate a model. I am interested in having a model which I can solve for writing an equation that is what I have fuel vapor coming over here. I have oxidizer vapor which is terrible oxidizer rich and when the fuel vapor meets over here, I could have something like a zone. Wherein I could have something like a zone over here, wherein again burning will take place between the oxidizer rich flame which is coming from A P. And this will be something like mixing taking place or a mixing dominated combustion (No audio from 06:46 to 06:54) whereas, over here it is just premixed.

Whatever A P is giving out as vapor, it comes out this therefore, over here I have what I will call as premixed, there is no mixing involved. At the edges, I transport the fuel here, I transport the oxidizer rich vapor here when they meet they mix and burn therefore, we call this mixing dominated combustion as diffusion essentially, controlled by diffusion combustion. Therefore, above the vapor I have premixed combustion at the edges wherein the vapor comes and meets these hot gases I have mixing dominated combustion. And the temperatures here are typically at these pressures greater than 10 atmospheres, around something like 3200 Kelvin. Well, gases are still reacting, this is still oxidizer is some of these a again products of combustion, again meet over here. Products from this come over here again meet it therefore, I have another mixing combustion here and this is the final diffusion flame. And the temperature here is typically around 3500 Kelvin.

Therefore what is the picture I am trying to draw? I am trying to draw a picture wherein or a model in my mind, the mind model wherein I have A P getting the oxidizer rich products which are essentially decomposition products, which are premixed, they form a diffusion flame over here. The first diffusion flame a diffusion flame over here, if I look over here, I will again get a diffusion flame over here, I will again get a diffusion flame over here well, I will get a final flame over here. And therefore, this flame and this flame are at 3200 this is around 1000 a dot Kelvin and the highest temperature is reached over here. This could be my mental picture or model of combustion or burning taking place in a composite propellant. Well, this is distinctly different from what we had for the double based propellant. Wherein we had may be aldehydes, may be N H 2 N O coming over here, you had a fizz zone, you had a dark zone, you had the second luminous zone whereas, here I have a premix zone a first diffusion and a second diffusion flame. Now, how do I solve this, it becomes a little bit erratic.

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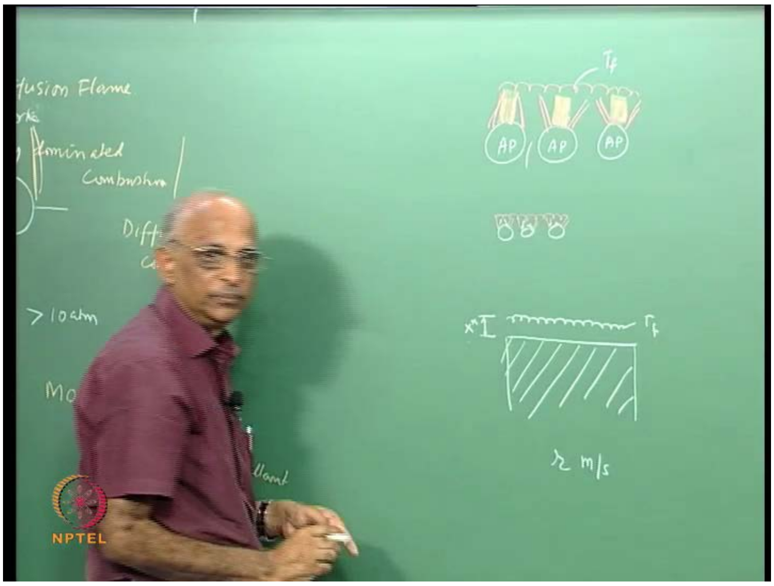

It is becoming a little complicated because now, I am talking in terms of a series of oxidizer particles over here. I have something like remixed combustion taking place just above It is low temperature, I have something like a diffusion flame over here at the edges over here, may be something like this. I could have been either way and then I have a final diffusion flame over here.

Now, what is it I am talking of? I am telling well A P here may be the polybutadiene A P over here and this is what I expect. If I (( )) have A P of smaller particle sizes, maybe I have smaller A P over here. Well, the zones will be something like this, something like this and my final heat release zone will be nearer. In other wards it tells me that the final diffusion flame because the length of this will be proportional to the size of A P. If A P size is less, then the final diffusion term will be nearer. I will still have these diffusion flames over here, the premix over here and I could still have this. Therefore, A P size will decide the distance at which the final temperature will take place.

Now, let us go through the same assumptions again. Let me assume that the final temperature of the combustion products is T f. And now I want make an assumption how do I solve this? I cannot have all these particles and keep solving it therefore, why not make an assumption? I say this my propellant surface, it is composite I know the size of this is around 300 microns, the fine is around 30 microns, I cannot see that closely. Therefore, I have a something like a distance of a centimeter or 2 centimeters or a meter. Now, I tell myself well the final diffusion flame formed is at a temperature T f and it is formed at some distance away something like let say X star, a standoff. It forms after some distance, why does it form after some distance, why does it not form at the surface?

Well, I have the A P flame first which forms then the mixing takes place then the secondary mixing takes place and then the final diffusion flames forms at some distance away. But this distance again is a fraction of an mm in practice, just like in the sparkler. We found, we thought that it is at the surface, but if I take a magnifying glass and see well there is a distance over here. Therefore, now I think I am in a position to write an equation for this, the final temperature is T f well, this is the propellant surface over here, I am interested in writing an equation for the burn rate so many meters per second.

Now, these we can readily do let us let us look at this slide again. This where I show I have three A P over here, I have the shaded portion which is the monopropellant flame that is the premixed flame. And then I have the vapor coming from this hydrocarbon, this is pure vapor when it mixes with the oxidizer rich gases, I have a zone of diffusion flame over here. And the products from this diffusion flame and this again be and give me a final diffusion flame here. And the distance between this final diffusion flame and the surface is what we call as the standoff or the height X star.

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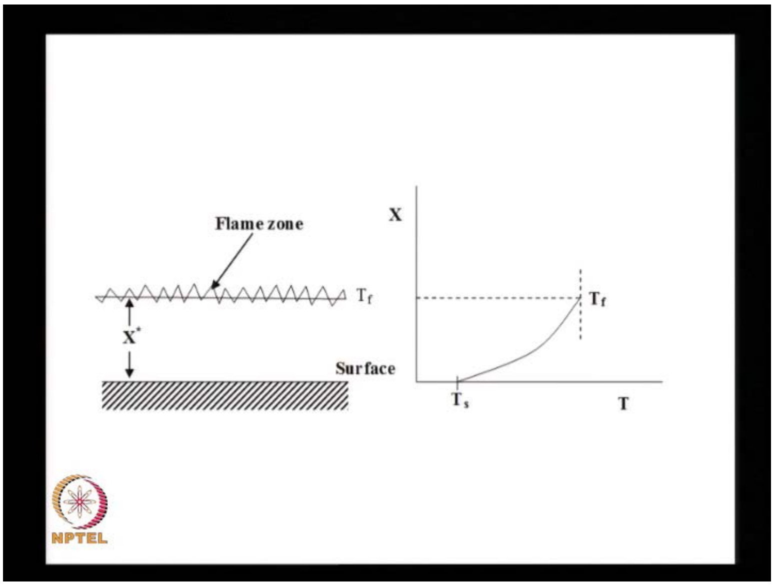

If this part is clear I go to the next one. Well, it becomes quite simple now for me, I say this my final zone, I have standoff distance as X star and this is my surface and the surface temperature which is T s. Mind you the propellant get heated here from initial temperature to the surface temperature and in this gas zone, I say that the temperature varies between T s to the final value. This is a mental model I have that is the model for the combustion or burning of a propellant. At this point I thought I should illustrate what happens, many of you are working in combustion. These are the experiments which we were doing in the lab; I have something like butane gas coming over here.

And when I ignite butane gas in an orifice at the orifice itself, I have a flame like this. I increase the velocity well at the surface I do not have combustion, but it takes off after a certain distance. I have mixing taking place in this zone and thereafter I have combustion and still at a higher velocity. Well, I have this zone, is the zone wherein mixing taking place is at high velocity, the heat is insufficient to propagate the flame into this; I have something like a standoff distance here. Well, the standoff distance is something on these lines, but not precisely because of velocity, but it takes a certain distance for the final diffusion flame to form.

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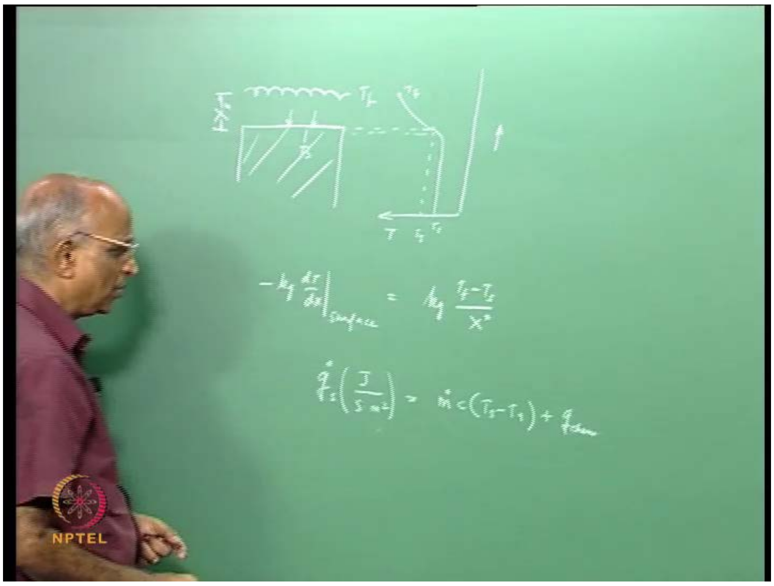

$$-k_g \frac{dT}{dx}\Big|_{surface} = k_g \frac{T_f - T_s}{X^*}$$

$$\dot{q}_s\left(\frac{J}{s\,m^2}\right) = \dot{m} c (T_s - T_0) + q_{chem}$$

Therefore, I would like to write an equation for this. Let us write the equation, the my equation is the we have the surface here, we have the final diffusion flame over here, we have the propellant over here T f, we have the surface temperature T s. And we also found that if were to plot the temperature distribution along this. That means, this is my propellant here, I show this as temperature, this as the distance over here, in the depth of the propellant the temperature is the old value that is the initial temperature. And then what happens near the surface, the temperature increases to T s at the surface and then in the flame zone, it further increases to something like T f over here. Mind you this is my increasing direction of T and this is the distance from the in depth propellant wherein the depth of the propellant is still at the initial value at the surface, I have an increasing temperature T s and it goes to the gas value.

Therefore, now, I want to know what is the heat which is coming on the surface, I am interested in finding the value of k g d T by d X at the surface of the propellant. And this I can approximate it by saying that the temperature increases to T f from the surface value over a distance. And if I say this distance is like something like X star, I just use this word X star I will give you the reason for using, it is equal to k g into something like T f minus T at the surface divided by X star. This is a very simple method. You know there are various flame models which are used for describing the combustion of composite propellants. But this particular one was formulated by one professor Hermann, he was at Waterloo University and it was very simple and is very illustrative therefore, I just use this.

There are various models we say the granular diffusion or some flanges model, but I do not see anything more coming out of those models compared to this simple model. Well, this is the heat which is coming to the surface I say this is equal to q dot s I consider unit surface area therefore, it is so much Joules per second per meter square, this is the unit of this over here. Mind you a is unit surface area and where does the heat go? The heat goes to increase the surface temperature from the initial value T i to the T s value. Therefore, what is it? Let us say that the rate at which the propellant regresses or burn is equal to m dot then what happens, the temperature I have specific heat into it increases to the surface value from the initial value plus some heat is also supplied.

Suppose, at the surface I have some endothermic reactions taking place because I have the binder, I have something like binder over here which I call as polybutadiene. It has to get heated, it has to supply the it has to vaporize and all that I need to supply this therefore, I say I need to supply some q chemical to it. So, some endothermic reaction q chemical and therefore, this heat goes to supply this.

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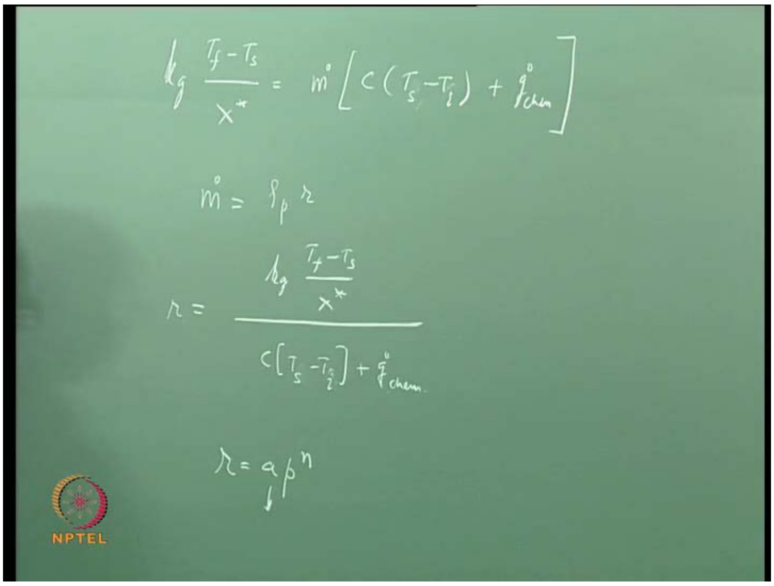

$$k_g \frac{T_f - T_s}{x^*} = \dot{m}\left[c(T_s - T_i) + \dot{q}_{chem}\right]$$

$$\dot{m} = \rho_p r$$

$$r = \frac{k_g \frac{T_f - T_s}{x^*}}{c[T_s - T_i] + \dot{q}_{chem}}$$

$$r = a\underset{\downarrow}{p}^n$$

And therefore, I can write my equation as thermal conductivity of the gas into the flame temperature minus the surface temperature of the propellant divided by the plane standoff is equal to the mass which is getting released at the surface into the density at the surface no, into the specific heat of the surface mass into specific heat into the surface temperature changes to T s from the initial value T i plus I have heat which gets released at the surface might be an endothermic reaction at the surface q dot chem. And therefore, this gives me my energy balance equation. Namely, this is the heat which gets translated or transmitted from the flame to the surface. And this helps to increase the surface temperature to T s from the initial value and also supplies the energy required to vaporize this surface or to convert the surface from solid to vapor through a set of endothermic reactions.

Therefore, now, using this particular reaction if I have write the value m dot, m dot is equal to the thermal conductivity of the gas region then I have the flame temperature minus the surface temperature divided by the flame standoff distance divided by the specific heat into the surface temperature minus the initial temperature of the propellant plus the value of heat release due to chemical reactions at the surface. Actually, in this I need not even put dot here because it is just the magnitude of heat release and therefore, maybe I should do away with the do here. And just write k g into T f minus T s by X star divided by C that is the heat require plus the endothermic reactions at the surface which causes it.

Now, I want to discuss this particular value of mass release and the mass release at the surface can again be put since I considered m dot, maybe I consider units unit area over here, maybe I say m dot is equal to rho p r. Or rather the burn rate can now be written as k g into T f minus T s divided by X star divided by m dot is equal to rho p into r because the you have so much meters per second, so much kilograms per meter cube. Therefore, I have density of the propellant coming over here into C into T s minus T s plus the value of q chemical.

Now, let us examine this equation under different conditions. I find that there is a flame which is standing off at a distance x star from the propellant surface. And this the change of x star let us again sketch it, we have seen this sketch several times during this class. We have the flame at a temperature T f standing at a distance X star now, if the size of the ammonium perchlorate particles size is small then the mixing will take place immediately near to the surface and the flame will be near. Point one, if the chamber pressure, on the other hand or if the pressure at which the burning rate is evaluated is high, if the burning if the pressure is high, you have more number of molecules which can react and therefore, the X star can come down.

Therefore, what is the inference which can we can draw? Well, if I have fine A P particles, fine ammonium perchlorate particles in the composite propellant, then it is quite possible that X star will be smaller and the burn rate will be higher point one. If the value of pressure at which the burning takes place is somewhat higher or large let us say, then chemical reactions get finished in a very short time and therefore, x star will be small. And if x star is small well, the burn rate will be higher. Therefore, a higher value of chamber pressure also leads to a higher value of the burn rate r.

Therefore, you know we are finding certain things may be if the pressure is higher, the thermal conductivity of the gas will be higher and therefore, again burn rate will be higher. Again rho p is the density of propellant does not change, may be the surface temperature effects will come in, may be the endothermic reaction activation energy will come in, but the something we are finding that pressure is a major factor because pressure decides the value of X star. And therefore, you remember when we did double base propellants, we could say that the burn rate law could be expressed as in terms of Saint Robert’s law as r is equal to A P to the power n. We find here also as pressure increases the flame comes nearer and therefore, a similar law that is the same law r is equal to A P n can also be used to determine, the burn rate of composite propellants.

Now, depending on the value of n the effects of pressure are modeled, but what is the value of a? a will depend on the size of ammonium perchlorate particle size. May be the activation energy of the endothermic reactions and may be some of the compositional aspects like initial temperature of the propellant. As the initial temperature increases, I find that the denominator decreases and r increases. Therefore, the effect of the initial temperature may be the activation energy. May be the ammonium perchlorate particle size, A P size and all are embedded in a therefore, we can conclude therefore, that the same law r is equal to a p n which describe the burn rate of double base propellants is also used can be used for composite propellants.

And therefore, the burn rate can be expressed in this particular form. Very simple, but very illustrative and we must be able to write such expressions for any system and do a problem, may be woods mold ring, may be carbon or may be some charcoal burning such type of models are possible. Therefore, let us let us summarize it again in terms of this slide, we said we had a flame zone at a distance X star away then we had two zones like premixed zone and this. And we got the expression for r and let us now go back and examine the value of r, how should it change with pressure? Could I rub this off, is it is it clear to each one of us? (No audio from 25:22 to25:33)

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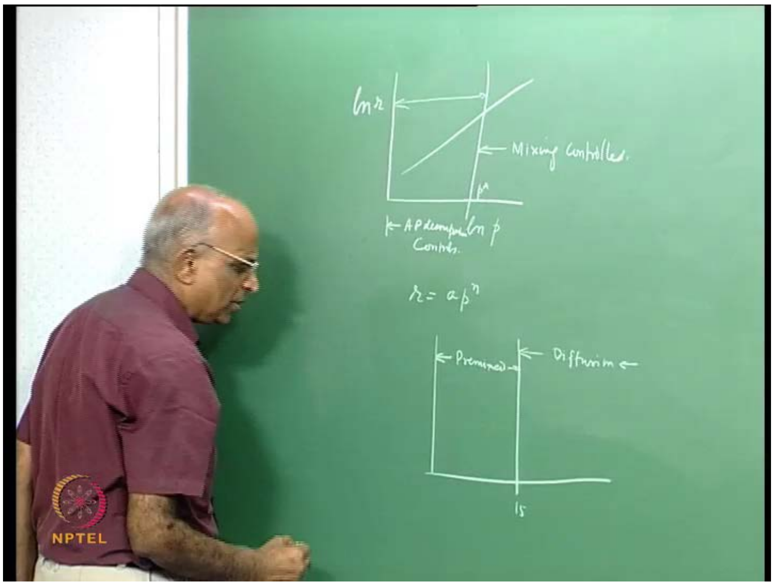

Now, I would like to use that equation and plot the value of the burn rate, logarithm of burn rate versus logarithm of pressure. Because we said well burn rate for composite can also be described by the Filey equation or the Saint Robert’s equation which is r is equal to a p n. We said well r with this, as per this equation is like this, but if I operate at low pressure region, pressure regions in this particular range. Let us say less than some threshold value, let us say p star less than some value I have to still determine this value. What happens at low values? May be A P will decompose, but then the hydrocarbon vapors are coming.

And you know that what happens is the rate of decomposition at low pressures is somewhat limited by the kinetics, why kinetics? We always write q dot chemical is equal to may be A p to the power n exponential of minus E by R p. And this is what we said is small q dot chemical per unit mass over here and therefore, at lower pressure, I have lesser amount of energy getting liberated. Or this becomes the controlling parameter because this fellow still generates vapor, but this fellow is missing. Therefore, this is something which is controlling therefore, at low pressures I can say well, in this case chemical kinetics or A P decomposition controls.

When I talk of higher pressures, the diffusion is independent of pressure at higher pressures I am getting copious amount of this coming out because the reactivity has increased. But the mixing of gases is independent of pressure and therefore, mixing gets controlled. And therefore, in this zone, I have something like mixing controlling the final diffusion flame. All what I am trying to say is, over a normally I would get the same n over the region this is what I was expecting, but the phenomenon is changing. At low pressures, I get premixing or premixed flame dominating or this is the limiting factor which controls the final temperature. Whereas, at very high pressures diffusion is less and therefore, since it is less, it becomes the controlling factor because this is available to you. But this is not allowing the availability to get back into the final temperature and therefore, I say it is mixing control.

In other words, if I were to again re-plot this I have a region in which I have premixed combustion controlling. And at higher pressures, I have something like diffusion mechanism controlling. And this pressure is of the order of something like little bit more, may be something like 15 atmospheres also. And what happens is whenever it is premixed control, I have a higher value of n because it goes as p to the power n, what is controlling? And if it is diffusion controlled I have something like n which is lower.

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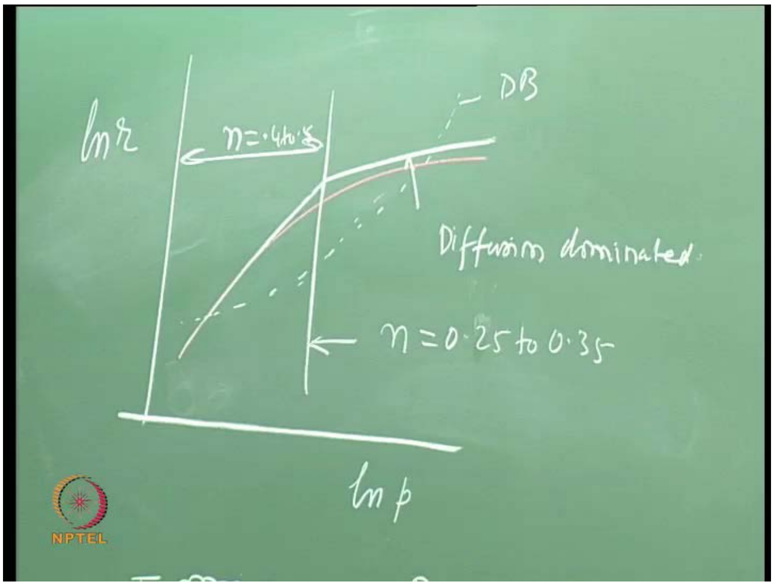

Therefore, for composite propellants, what happens? It is different from double base propellants in that if I were to plot the value of logarithm of burn rate versus logarithm of pressure, I have something like high value here, high value smaller value of n over here. Or rather the burn rate goes something like this whereas, what did we have in the case of double base propellant? It was just the opposite, we had at higher values we had a steeper values something like this and at lower value we had this in fact, we had something like this in the case of double base. Whereas, in the case composite it is the reverse and we have small n at the lower pressures and higher value of n, at the lower pressures and smaller value of n, at higher pressures higher value of n, at lower pressures.

I think this distinction must be clear. But normally most of the rockets we operate at pressures in access of this. And therefore, we will say that n typically is around 0.3 to 0.25 to 0.35. Whereas, in this region, wherein n is nearer to something like 0.4 to 0.5 we have premixed combustion and between this is what we operate and it is diffusion dominated. (No audio from 30:18 to 30:28) I think I show this in this particular slide wherein I said at a pressure less than a threshold value, I have premixed a higher value of n followed by diffusion wherein I have a lower value of n, I think this is about it. And now if this part is clear I can go to the choice of n which is necessary for me to give the particular criterion of I choose a propellant.

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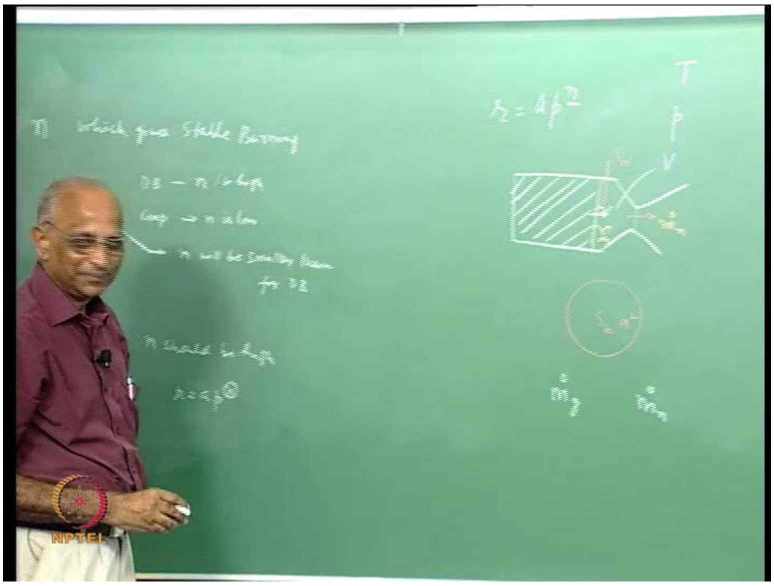

What must be the value of n which gives me, stable burning. (No audio from 31:06 to 31:13) Can I say I have understood the burning rate laws? Now, if I were to question you before getting into this, what should be the burn rate law for a composite modified double base? I know for a double base the burn rate law, I know for a composite the value. We can say composite modified double base should incorporate both these feature and give something like average of this two. In other words, composite modified double base will give a value of n which is slightly lower than for double base, for double base you know it is all premixed.

Therefore, n is high for double base for composite at the regions of interest n is low therefore, for composite modified double base n will be smaller than for (No audio from 32:00 to 32:07) double base. If I were to consider something like nitramine propellants well, in nitramine propellants I had H M X, I had binder which were mixed together both of them are premixed and therefore, n should be high. That means, in the burn rate law, r is equal to ap to the power n, n is higher for nitramine propellants for C M D B it is less than double base, but higher than for composite. I think this is the way one should take a look at it is possible to do the same type of analysis for this and come to the same conclusion.

I think let us keep it in mind. We should work out some numerical problems for burn rate and put things together. But what we have done is for double base yes, we looked at the evolving layers like fizz zone, dark zone and also the second luminal zone that is for composite. We use the simple Hermann’s model wherein we talk of a flame standing away from the surface by a standoff distance. We also know what this standoff mean means. When a gas comes it take some time for things to chemically react and form a flame. I would now like to ask myself as I was telling you, if I have r is equal to a p to the power n, can I formulate a law which says n should be small n should be greater, what should be the value of n which I want, should I have n equal to infinity, should I have n is equal to 0, what is the value which will give me a good burning in a rocket?

Therefore, for that let me go ahead let say I have I now come back to a system of a solid propellant rocket. I tell myself well this is the propellant want I have in a rocket as it is, this is my burning surface area S b. In other words I have if I take a section over here, this is my surface, this is my burning surface area S b meter square. And this is where the burning takes place and gas is getting released, and the gas gets pushed out through the particular nozzle. Let us put some numbers down we will call rate at which mass is getting generated from this surface area as equal to m dot generated m dot g. The rate at which gases are leaving the nozzle, we will call it as m dot n.

Can I write an equation for the dependence on m dot g and m dot n from this? Let us consider the case wherein I have a certain volume let say this particular volume over here is V, let the pressure in this volume be p, let the temperature of the gases be T. How do I write an equation? Mass gets generated over here m dot g it leaves the control volume or this volume at a rate m dot n. What is the equation I have? Mass balance if I were to write, what will I get? Yes. (No audio from 35:36 to 36:43)

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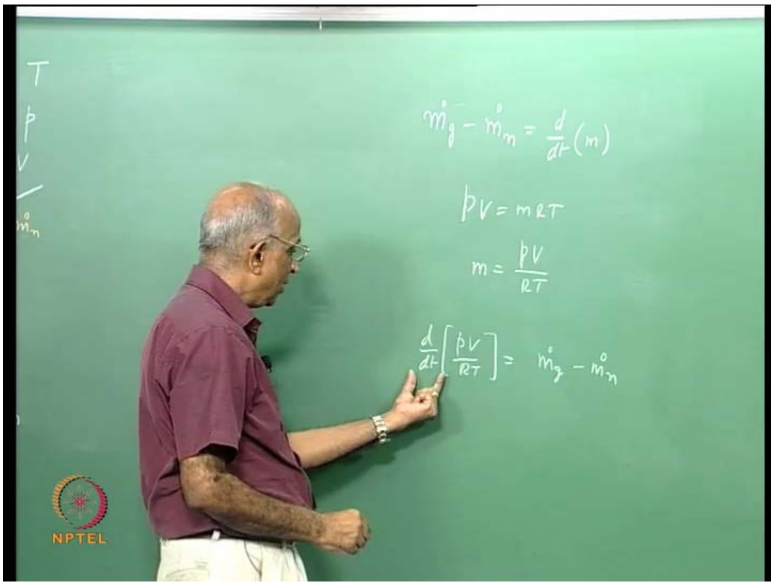

m dot g yes, you are telling it should be m dot g minus m dot n gas leaving should be the rate of accumulation of gas in the particular volume or the chamber, what is the value of m from gas equation? We have p V is equal to m R T. The pressure is p small p the volume is V the temperature is T therefore, m is equal to p V by R T. Therefore, I can now write d by dt of p V by R T is equal to mass of gas which is generated at the propellant surface by burning minus m dot n which is leaving through the nozzle, is it ok.

Let simplify this term before I come into this particular terms, how will I simplify this particular term, (No audio from 36:50 to 36:59) what are the variables? Temperature is a constant flame temperature, pressure I am considering a particular value of pressure it could vary. As the propellant regresses the volume varies therefore, I have two variables pressure and volume well the flame temperature is constant.

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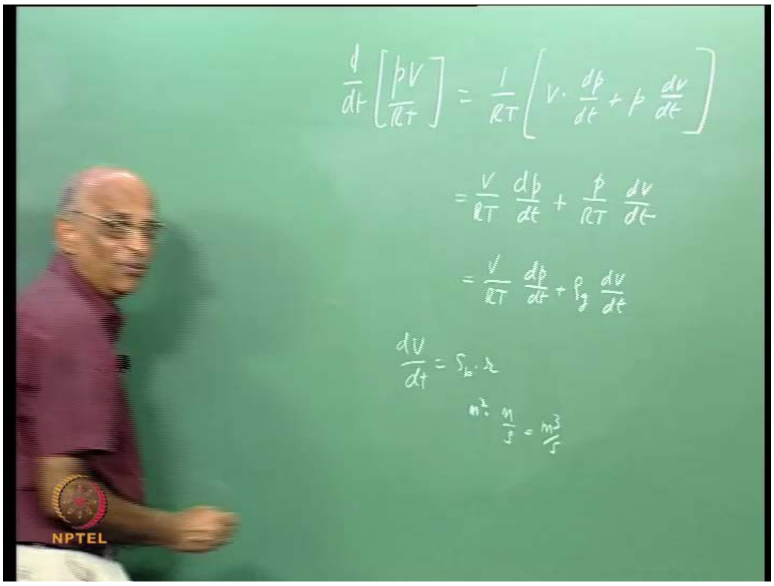

Therefore, for temperature being a constant or anyways a specific gas constant it is constant, I can write this equation d by d t (No audio from 37:26 to 37:32) of p V by R T 1 over R T into V of d p by d t plus p of d V by d t, is it all right. Now, again I simplify this I write it as V by R T of d p by d t plus p by R T of d V by d t, what is p by R T equal to? Let us take a look at the gas equation p V is equal to m t p V by R T is equal to mass by volume which is the gas density, I can write this as equal to V by R T into d p by d t plus density of the gases into d V by d t.

What is d V by d t? Let us what did we tell ourselves the rate at which the volume is increasing and what is the rate at which the volume is increasing? The burning surface area is S b, regression is by r therefore, d V by d t is what? Yes, d V by d t is equal to burning surface area into the propellant burnt rate so much meter square into meter per second is the burn rate law, meter cube by second this is d V by d t over here. Let us follow this, what is this I am telling? In one second, the distance would have been moved by r therefore, that increase in volume d V by d t is r into S b for one second or d v by d t is equal to S b into r. Therefore, now I can rewrite the equations in a controllable manner as R T by V, was it the value yes, V by R T into d p by d t plus I have the value of rho p into d V by d t.

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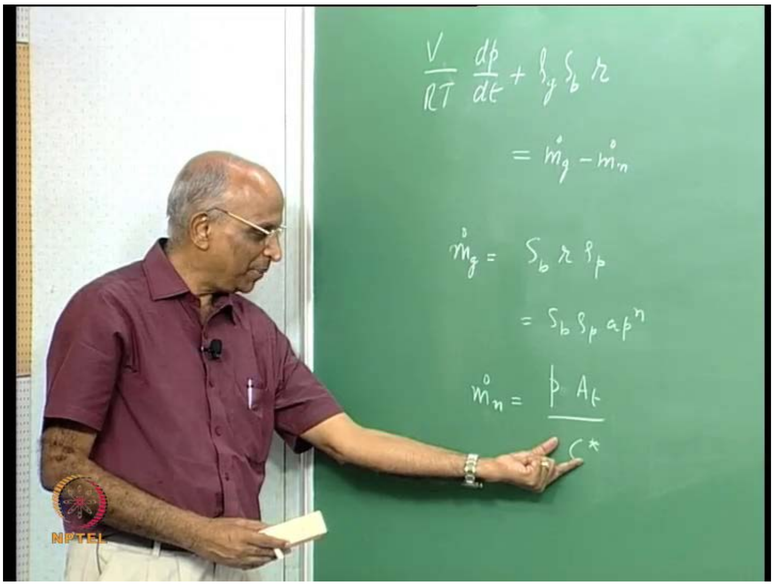

Which is equal to rho p into burning surface area into propellant burnt rate, let us make sure it is correct, is equal to what m dot generated minus m dot nozzle what is leaving, is it ok.

(( ))

V by R T d p by d t.

(( ))

Rho g you are right. We what we had here was rho g over here we multiplied it by S b into r, very good. What is the value of m dot g in terms of pressure or in terms of burn rate?

(( ))

Yes, burning surface area into r into rho p because S b into r tells the volumetric rate at which it is going multiplied by the propellant density is a mass at which is getting generated. Let us be very clear which is equal to S b rho p into a p to the power n. Where, r is equal to a p to the power n I used the law. Now, what is the rate at which mass is leaving the nozzle? We have been doing it all along.

(( ))

C star we used solid propellant property that is m dot n is equal to p c A t now I am telling p as the chamber pressure by C star, this was a definition of C star; that means, we said C star is the transfer function between chamber pressure and mass flow rate per unit throughout area. And therefore, let us just substitute it and get the answers, what we are looking at.

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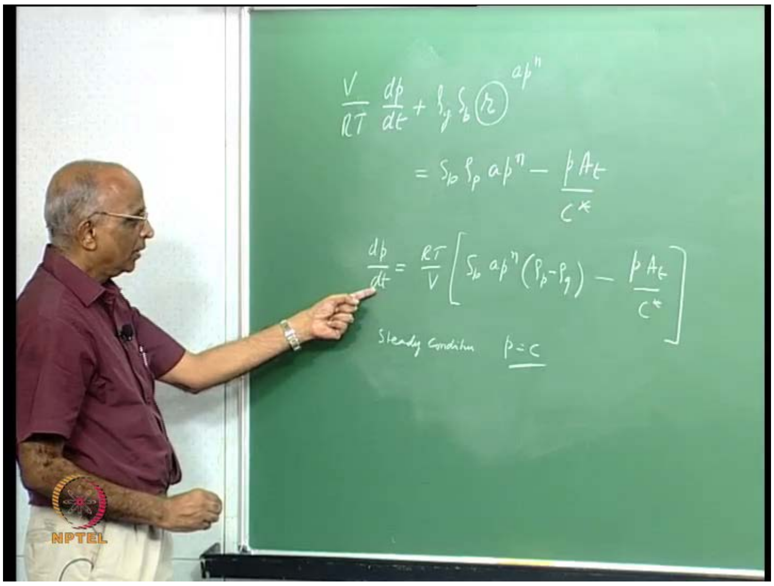

Though I substitute this again I get V by R T is equal to let me make use of this space now is equal to S b rho p into a p to the power n minus p into A t by C star. I can write r here also as a p to the power n burn rate law, and what is it I get? Let us put it together get now I get d p by d t is equal to R T by V into S b into a p to the power n into rho p minus rho g plus no minus p A t by C star. All what I have done is I brought rho g here, because it is the minus sign now, I have S b into a p to the power n and this is my value. That means, I have d p by d t I take R T on this side over here. Is it all right, please check it.

Now, I want to know what happens under steady conditions. By study conditions, I mean the pressure with respective time, it is a constant, pressure does not change therefore, d p by d t is 0 therefore, I get this term must be equal to this term.

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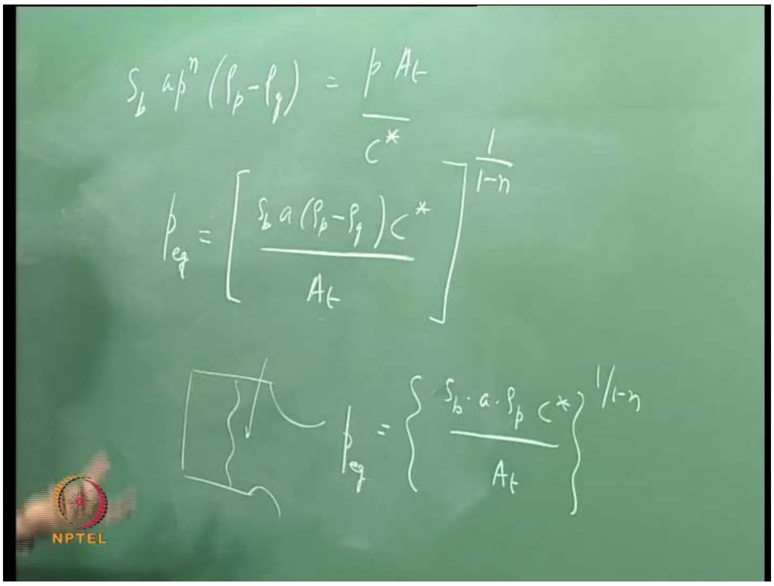

$$S_b \, a p^n (\rho_p - \rho_g) = \frac{p A_t}{C^*}$$

$$p_{eq} = \left[\frac{S_b \, a (\rho_p - \rho_g) C^*}{A_t} \right]^{\frac{1}{1-n}}$$

$$p_{eq} = \left\{ \frac{S_b \cdot a \cdot \rho_p \, C^*}{A_t} \right\}^{1/1-n}$$

Or rather, I get the value of S b into a p to the power n into rho p minus rho g is equal to p A t by C star. If we were to call this steady value of chamber pressure which, is actually the steady equilibrium value as equilibrium pressure the value of equilibrium pressure p equilibrium is equal to I take n on that side I get therefore, S b into a into rho p minus rho g into C star divided by A t and now I take here 1 minus n suppose this becomes 1 over 1 minus n. Please check whether it is all right. I take p on this side it becomes p into 1 minus n and I still retain S a rho p minus rho g and I take C star upstairs over here and A t downstairs over here and this becomes my equilibrium pressure.

What is it I am telling? All what I am saying is when I am burning a propellant whatever be the configuration of the grain what I have if this pressure is constant, the equilibrium pressure is given in terms of the burning surface area, the pre-exponent, the difference between propellant density and this the C star of the propellant divided by A t is gives me the value of the equilibrium pressure. Let us take a look at this normally the gas density will be very much lower than propellant density.

In fact, had I neglected the gas density I would have got this expression to be even simpler I would have got this to be equal to S b into a into rho p into C star divided by A t to the power 1 over 1 minus n. This should have been the value of my pressure equilibrium. This particular case because this will be around one thousandth of the normally the solid has a much higher density than a gas. Therefore, what can I draw some conclusions from this. Now, I tell myself if please be attentive I think you should be able to discuss this point.

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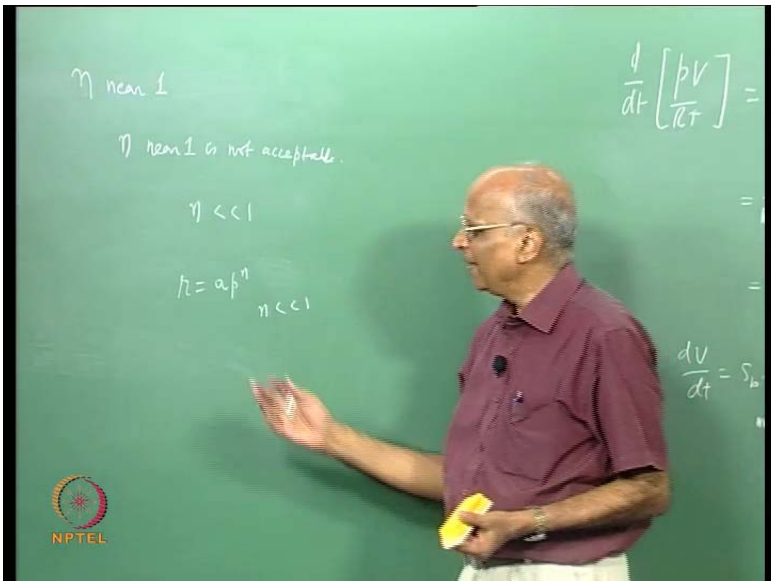

If n has a value near 1, what will happen to this equation? If n has a value of 1 well, I have infinite pressure it is just not possible. If n is around 0.995 very near to 1, then I have a really large exponent and small perturbations or changes in this will give me a high value therefore, a value n near 1 is not acceptable. That means, n must be as small as possible and that is better because I have much smaller value. And therefore, any changes in burning surface area while it is burning, some changes in the gas density while it is burning, some changes in C star or throughout area should not lead to an explosion and getting abnormally high value.

Therefore, we tell ourselves the one of the quality is required in the burning rate of a propellant r is equal to a p to the power n, is n must be a small number a large number is just not acceptable. And we say yes for composites around 0.3 0.25 to 0.4 well, for double base it is around near to 0.5 0.4 to 0.5 therefore, may be composites are little more better than double base propellants. Well, having said that I can also use the same thing in a slightly different format I can look at the value of the value of n which is desirable.

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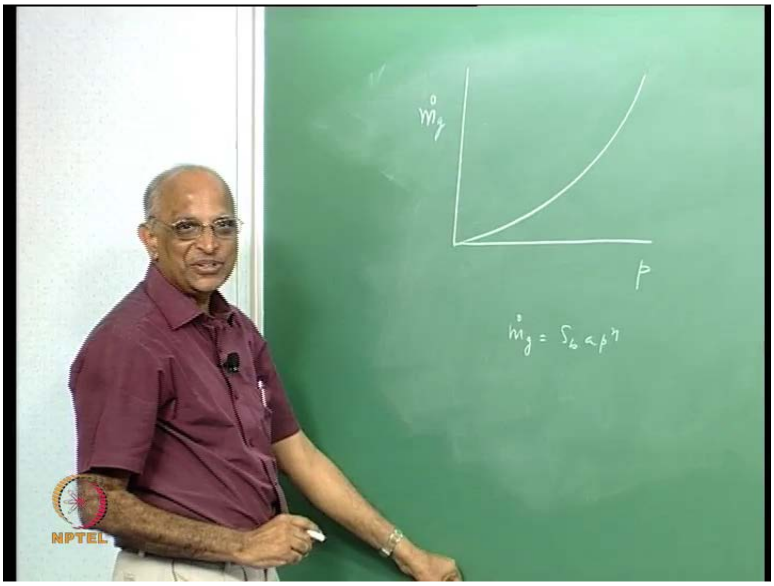

Let us do it graphically, it will take me just a couple of minutes, let us just do it together. Let us say I have something like mass generation rate of propellant as a function of pressure. We said m dot g is equal to a p n into S b and all that; that means, mass generation rate is equal to S b into a p to the power n. Therefore, it will go like this. Here, I must multiply by the density of the propellant rho p, what is the rate at 0 pressure it must be 0 therefore, I should bring the curve down. If I have n which is greater than 1 well this will be my shape. If I have n less than 1, what will be the shape? Yes, we must learn all this.

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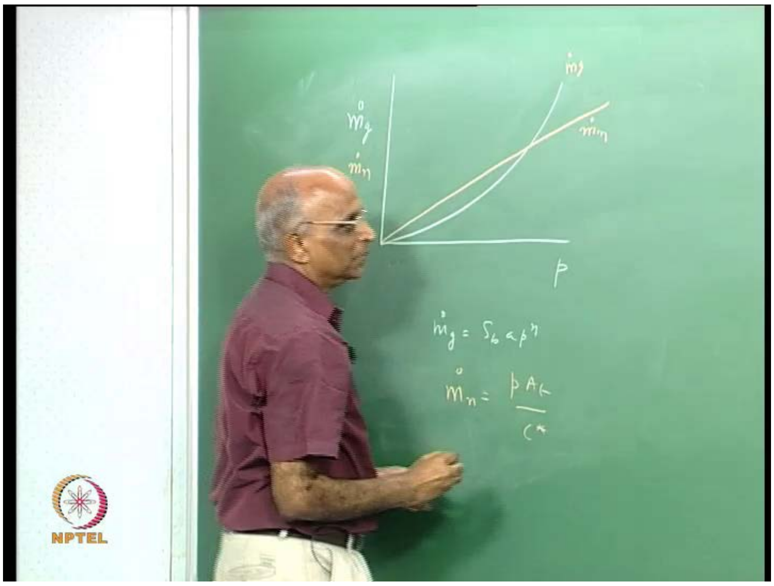

You know we must be able to do these things that is why I think this the basic aim of the subject is to be able to put things together (No audio from 49:23 to 49:30) and less than 1. Now, what is the value of m dot n? m dot n is linear p A t by C star for straight line it is m dot g. Now, I am also putting m dot n on it, same thing I also have a line for m dot n. (No audio from 49:57 to 50:05) I should not have written r here I should have written m dot g and m dot n. Let us take a look at this, what is this point wherein both are equal steady state p equilibrium, what is this point p equilibrium? And we know the expression for the p equilibrium, we have just derived it as saying S b small a into something like C star divided by A t to the power 1 over 1 minus n.

Now, let us examine this particular point. Suppose, in by chance as the motor is functioning there is a small diversion in pressure, pressure slightly increases. If pressure slightly increases the mass generation rate increases, pressure further increases, it goes like this and it explodes. If pressure slightly falls, the mass flow through the nozzle is higher therefore, more mass flow rate compared to mass generation pressure comes down, it comes down, it comes down, it extinguishes. Therefore, when I have n is greater than 1, I cannot get equilibrium, the motor either quenches or it explodes. Therefore, n greater than 1 is not possible, you cannot have a propellant because it is it will not usable I will say.

Let us come back to this and examine this again. If there is a slight increase in pressure, what is going to happen? The pressure has increased the nozzle flow rate has increased therefore, the pressure again comes, it comes back to this and it is stable at this point. If the pressure falls slightly, if the pressure is fallen mass generation rate is higher, it gets back to this point therefore, this becomes my stable operating condition. (No audio from 52:07 to 52:14) Therefore, now, I am very clear n must be very much less than 1 and that is how it is preferred. We learn how to do this using these two plots, we have also learnt how to do this using the equations wherein we had the expression raise to the power 1 minus n. And this is what how we decide the choice of n.

I will continue with this in the next class, but what we did in this particular class was, we derived an expression for the burn rate of composite propellants using the standoff distance X. We found that it is still moldable using the Saint Robert’s law namely r is equal to a p to the power n. We related it to a rocket and we said well n must be very much less than 1. In the next class, we will look at the temperature sensitivity of r, we will also look at some other parameters which are important for r, and then go to designing a solid propellant rocket. And what do we do in a design? We have to have some particular burning surface area; it becomes a simple geometric problem, and this what we will do in the next class. Well, thank you then.