Rocket Propulsion

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Lecture No. # 24

Design Aspects of Solid Propellant Rockets

Good morning, we will continue with our subject on solid propellant rockets. What is it we have done so far? Let us take a quick review; we know that either whether the propellant is composite, whether it is double based or it could be night journey or it could be composite modified double based.

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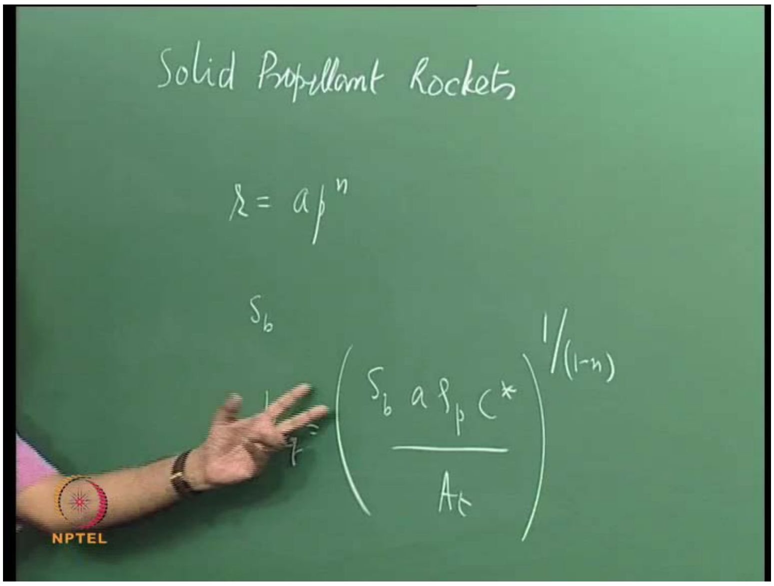

I can write the burn rate as a linear regression rate, r is equal to a p to the power n. We also told ourselves that well if I have a rocket in which the burn surface area is S b, the equilibrium value of pressure, I can derive as may be the burning surface area into this particular exponent a, we had something like rho p here we had C star here, divided by A t over here to the power 1 divided by 1 minus n. How did this come?

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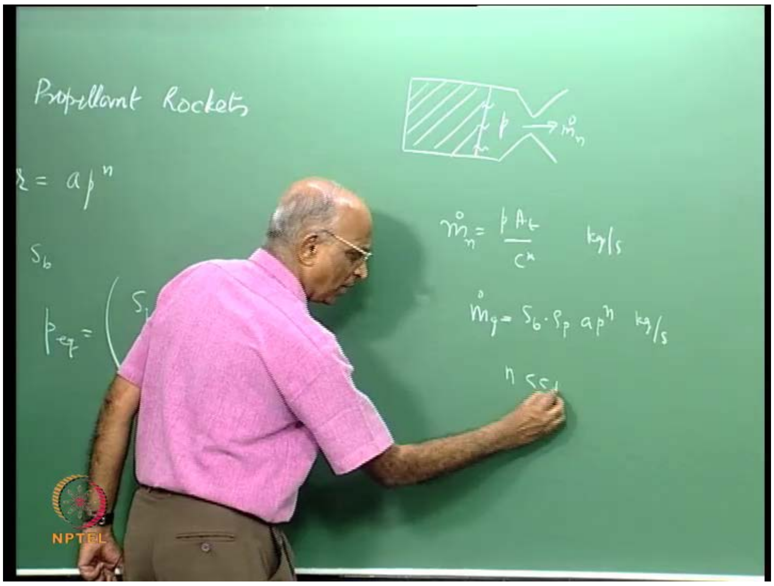

We said if I had a rocket let say we considered a simple scheme where in I had propellant, which was enclosed in a case, I have something like a nozzle here, we said the rate at which the mass leaves through the nozzle m dot n can be written as p A t divided by C star, where p is the pressure here.

And we said it is always choked at the throat therefore, this is a volume I am considering and that is the mass, which is leaving the nozzle. And the rate at which the mass is getting generated from the burning of propellants, we got that as equal to the regression rate is r and that was equal to S b into the propellant density into the burn rate r, which is equal to a p to the power n so much kilograms per second, this is so much kilograms per second. We equated the two, and then we find I take p over here I get 1 minus n, I get S b a rho p into C star divided by A t, and this is the equilibrium pressure.

What does this tell us? Let us take a relook at this equation, we looked at it from the point of view of n told ourselves well, n cannot be anywhere near 1, because then what happens for any small change here, there is a large magnification here. And therefore, n should very much less than 1, because if n is near 1, I get a very large exponent here and a small change can magnify into a large changing pressure. Therefore, we say from stable considerations, well n must be very much less than 1, but we also learnt to look at it graphically.

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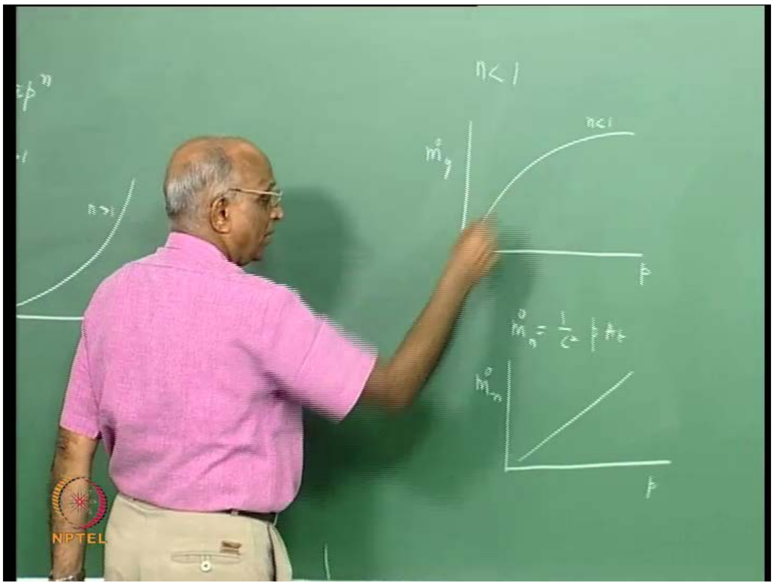

And what did we tell at that point in time, when we have n in the burn rate equation r is equal to a p to the power n, if n is greater than 1, how does the burn rate law change as pressure changes let us make a plot or rather; as burn rate increases, if you have a given burning surface area a given density of propellant we can now plot the rate at which mass flow is generated due to burning. In other words, mass flow is get generated depends on the burning surface area into the density of the propellant into the burn rate law, we want to plot it as a function of pressure.

Then n is equal to greater than 1, well it keeps increasing as n becomes greater and greater rather when the value of n is greater than 1, when for higher large value is of n the mass generation rate rapidly increases. If however, n is less than 1 like for instance in the burn rate law n is less than 1, then the same mass generation rate will have something like a grouping characteristic. This is for n less than 1 with variation of pressure; this is for n greater than 1. How does the mass flow rate, which leaves the nozzle? Change the pressure we always told, we have been writing this on and off m dot is equal to 1 over C star into p into A t rather the mass flow rate through the nozzle, if now I call it as m dot n through the nozzle with respect to pressure will increase as a straight line.

Therefore, let us now plot the mass which leaves a nozzle as a function of mass generation rate for m is greater than 1, m is less than 1 and see whether we can cross some conclusions on the type of exponent n which we require.

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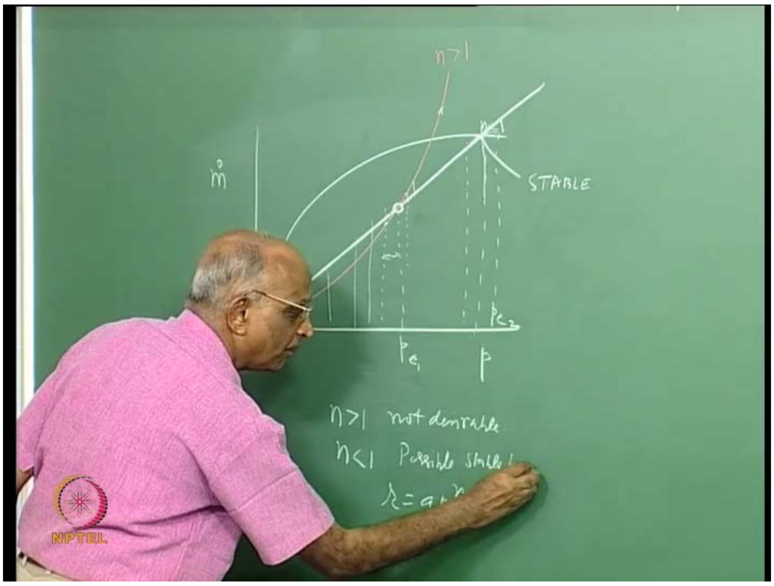

Let me now plot all the three together on a single graph let me have my axis here, x axis which is mass generation or let us say mass which is leaving the nozzle as a function of pressure. Let me put the value first for n less than 1. I get a curve like this if I have n greater than 1, let me write it is n less than 1 if I have n greater than 1, well curve like this. And what is the rate at which mass leaving the nozzle? I show it by a white line again may be the curve is like this. Now we find that for n, which is let which is greater than 1, and the mass which is leaving the nozzle it is same at this point. Therefore this will correspond to let us say equilibrium pressure for the case when n is less than 1; n is greater than 1 I am sorry red line is for n greater than 1 therefore, p equilibrium corresponding to let us say case 1.

Let me also say tell that may be for n less than 1 may be this the p equilibrium value; let us say p equilibrium too. I would like to examine whether are these two stages are possible? Well theoretically, this is the rate at which mass is leaving the nozzle, the rate at which mass is getting produced in the chamber and therefore, this equilibrium pressure. Similarly, for n less than 1 this is the pressure, let us try to get some idea whether these stages are possible, and if so are there some problems in these equilibrium pressures?

When n is greater than 1 let us say I have a small perturbation impression a small perturbation can always come and let say that the pressure reaches this value, that means the pressure is slightly higher than the equilibrium value. Since the pressure is slightly higher, what we find at this point the mass generation rate is higher than the mass which is leaving the nozzle. Therefore, the pressure will increase; when pressure further increases, the mass generation made is further higher increase. Therefore, this point cannot be a stable equilibrium point, but any small perturbation will make the pressure increase further and further till the motor are the (( )).

If I now considered points to the left of the equilibrium point, let me say that by some chance there is a small pressure perturbation and the pressure in the motor falls to a value less than the equilibrium pressure p e 1, then what happens the mass generation rate is lower than mass, which is flowing through the nozzle. In other words mass flowing out through the nozzle is more than what is generated. Therefore, the pressure still falls, the pressure still falls, pressure still falls and it comes to extinction.

Therefore, we tell ourselves well in case n is greater than 1, I cannot really get an equilibrium pressure and even if I get small changes in it will allow it either to explode or to quench. Therefore, we tell ourselves n greater than 1 is not desirable or not possible. Let us examine the case, when n in the old law, what was the burn rate law? R is equal to a p to the power n, what is the value for, what is the value when it is less than 1, what is the state of equilibrium pressure? Let us put the same arguments again, If the pressure when to fall slightly less than the equilibrium value; well it has fallen, but now I find that the mass generation rate is higher than the mass which is leaving the nozzle and therefore, it will go back to the particular state.

If by chance it exceeds the state over here and comes over here, again I find well, the mass leaving the nozzle is higher than the mass generation rate and pushes it back; therefore, it pushes it back pushes it back to this point; therefore, this point becomes let us say a state of equilibrium value. Therefore, the value of n less than 1 is possible and it gives to rise to what we say is a stable burning. Therefore, in the burn rate law r is equal to a p to the power n n must be less than 1. This is all what we have done.

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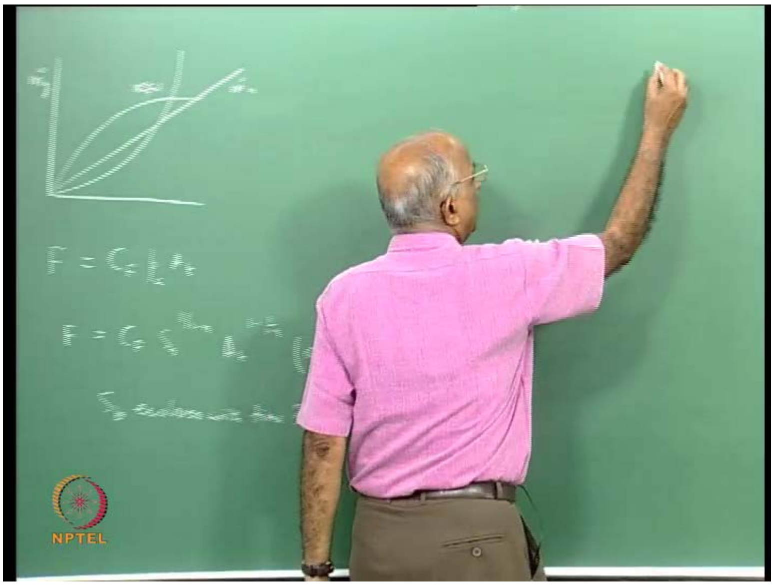

Now our time has come, when have to put everything together; and say how do I design a motor? What is what I mean when I want to design a rocket? I must be able to generate a given thrust from the rocket, and what is the thrust? We said is equal to C F into p into A t. I drop the subscript p c first we said I can put it in terms of the nozzle effectiveness into chamber pressure and A t this the thrust which is developed therefore, if I want a rocket to develop a particular thrust, I know that p equilibrium goes as S p all this is constant o p is density C star is the particular case of operate I can evaluate it I know the throat area. All what I need to do is, configure the surface area such that I need a particular thrust; and this is all a solid propellant rocket thrust. But it is not that easy as we shall see in the in the subsequent class, when I go through it. We find here there is something much more to it you know I cannot just say therefore, I can write this equation as C F into I take the value of p c from here, I can now write it as S b to the power 1 over 1 minus n; what did I do?

I take p equilibrium going as a S b to the power 1 minus n therefore, I write it here. Then I write the other terms together namely, I get A t. Now At is in the numerator here A t is in the denominator here therefore, 1 minus 1 by 1 minus n. I have just substituted the value of A t over here minus 1 over 1 minus n there I had it in the numerator. And then I solve the other parameters namely a value of rho p into C star to the power 1 over 1 minus n. Therefore, you know this a constant throat area is given, if I know the evolution of burning surface area as the surface regresses, I can find out the value of thrust, and that is how a solid mountain is designed. It is a simple geometric problem, we must be able to calculate, what will be the value of S b.

And how s b the burning the surface area in meter square evolves with time and this is what will be considering in today's class.

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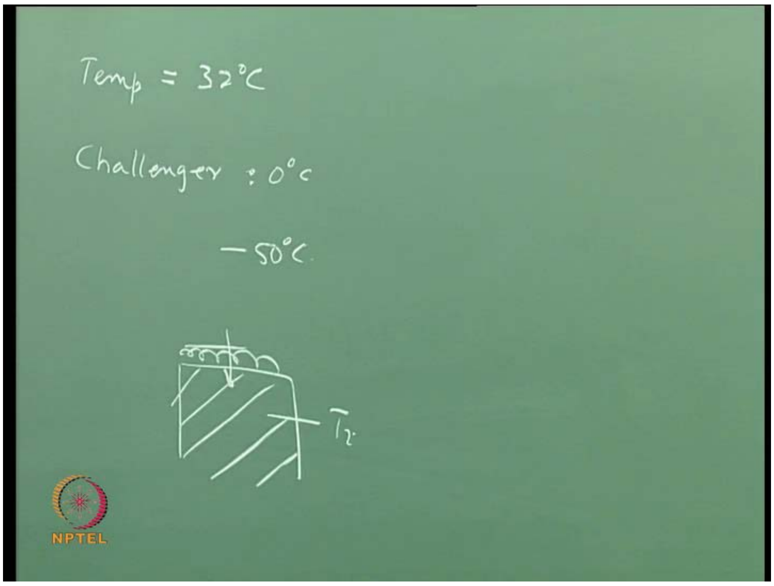

But before I do that so far all what we have done is we took the burning rate law r is equal to A p to the power n. We considered explicitly the effect of pressure alone. But we said A includes the effect of temperature initial temperature of the propellant. Let us take an example; see we can consider a temperature of the propellant to be ambient that is a rocket motor is tested today. The temperature quite hot today may be 32 degree centigrade. Well, you all would learnt about a solid propellant rocket, which misbehaved in space shuttle it was a challenger rocket, which was launched on a very cold day, when the temperature goes around 0 degree centigrade. And we will look at the failure after completing this portion on solid propellant rockets that day was quite cold around 0 degree centigrade. Well, we could have a missile, which is operated from mountains in and near him always, where the temperature could be as low as minus 50 degree centigrade.

What is the effect of burn rate on temperature that is the initial temperature of the propellant itself? I am not looking at the flame temperature all what I say is, well I have a propellant block, the initial temperature of this block before it burns or just begins to burn is what I call as initial temperature; and here I have the flame and I am looking at the effect of initial temperature on the regression rate of this particular propellant; may be let us take a look at it how I do this problem or how we can do it?

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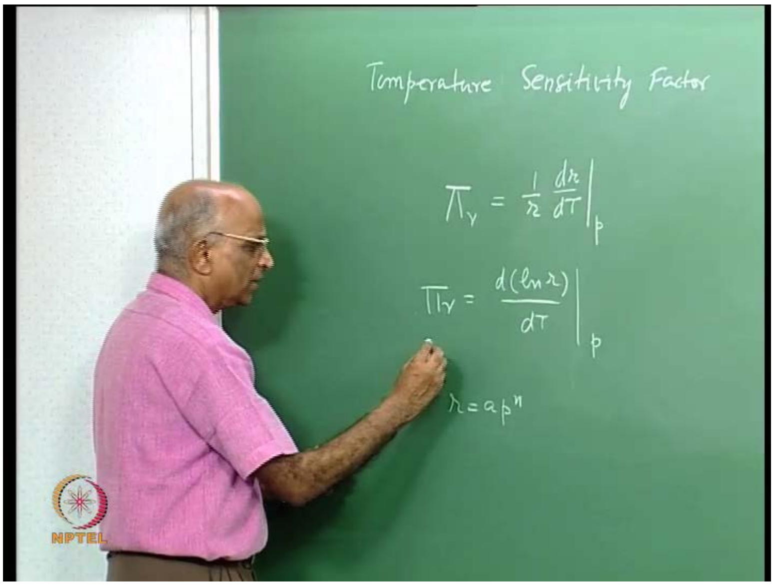

We define a term known as temperature sensitivity factor. Let us see, what it is. We would like to know how sensitive the burn rate is to temperature changes. Therefore, we were interested in finding out how the burn rate d r changes with temperature d T. And since I am considering the effect of temperature alone; I am a considering at constant ambient pressure. But then instead of just saying burn rate variations the temperature or the variations in burn rate due to unit temperature. I now say fraction variation in burn rate, and this is known as a pi r or temperature sensitivity factor for burn factor burning rate. In other words, we define a term like pi r is equal to this d r by r is d ln r divided by dT at constant pressure is defined as the temperature sensitivity factor for a solid propellant.

And just like how did we did measure r is equal to the power n we did two experiments at pressures p 1 and p 2 measure the burn rate r 1 and r 2 and n was equal to the value ln r 1 minus ln r 2 divided by ln p 1 by ln p 2 and the same way, this is also measured a different temperatures and typically the value of the factor pi r, which defines the sensitivity to temperature is around 3 into the 10 to the power minus 3, what should be units?

Well ln r has no units; d r units cancelled, it is only d T, therefore degree centigrade inwards for 3 into 10 to the power 3. Typically for most composite problems this equal to something like 5 into 10 to the power minus 3 degree centigrade for double based propellants. And for H m x based problems it is even lower, it is 2 into 10 to the power minus 3, and that is why for missiles when we go for H m x propellants, because it is not that sensitive to variations in temperature for degree centigrade inwards degree centigrade inwards for H m x based on (( )).

Well, I can go back and integrate this equation and find out explicitly, how the burn rate changes with temperature, let us do that I take the expression over here. I write d ln r burn rate is equal to pi r into d T at constant pressure all I have taken is the change in the logarithmic burn rate is equal to pi r I just write this equation and let us solve this equation, let us say I do 2 sets of experiments I have a temperature T 1 the burn rate is r 1 at temperature T 2 the burn rate is r 2 and I am interested in finding out the burn rate at a temperature T 2; and therefore, I just integrate out this I get ln r 2 minus ln r 1 is equal to pi r into T 2 minus T1 ln r 2.

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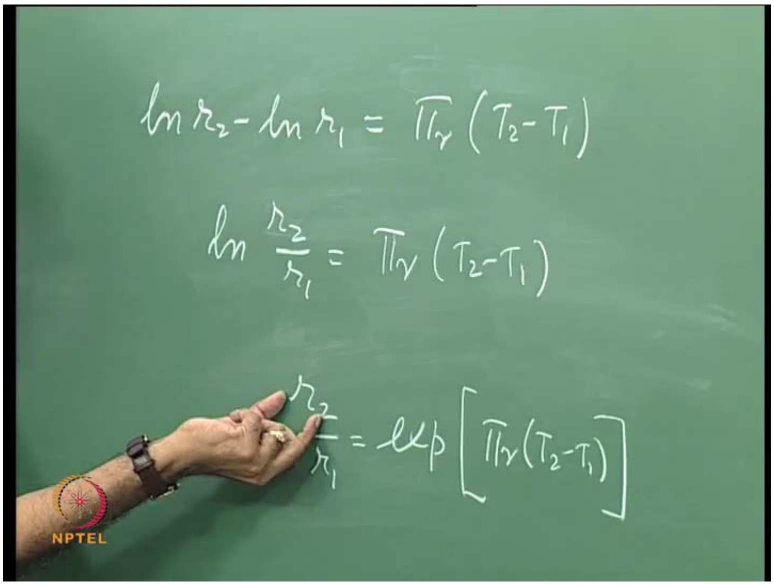

ln r 1 is ln of r 2 by r 1 is equal to pi r into T 2 minus T 1 or rather I get r 2 by r 1 is equal to exponential of or e to the power of pi r into T 2 minus T 1. Therefore, if I know the burn rate at temperature T 1 using the value of the temperature sensitivity factor I can find out the burn rate at a temperature T 2, and this is how the effect of the variations of temperature is taken into account. I think we are now in a position to go back to designing solid rockets that means essentially, find out how much areas is required, but before I do that.

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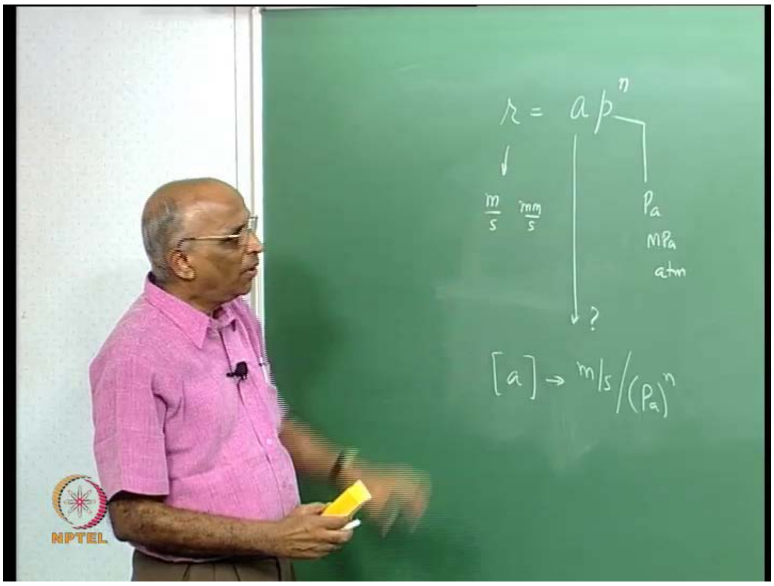

Let us go back and see are there any concerns we have about the burn rate equation, because you know we are going to left and right use the value of burn rate in let say millimeters per second or meters per second given by a p to the power n. What is the unit for r? We say yes meters per second if the burn rate is more millimeter per second, if the burning rate is fast may be centimeters per second, this is burn rate - linear burn rate, we will be a illustrated in by having something like a propellant, and we found that it keeps regressing and the rate of regretion is the burn rate. What is the unit of pressure here? Could be Pascal. Let us say it will reflect Pascal could be mega Pascal, could be atmosphere, then what is the unit for a? The constant which depends on temperature, which depends on composition.

The unit is given becomes a very clumsy, it becomes something like, meter per second unit of a meter per second divided by let us say Pascal fundamental unit to the power n, and this is not a correct way of doing something. You have a unit which is a constant, which is a function of the parameters or the or the units what I use here, and it is a I cannot say that my constant is a so much meter per second to the power 3.32, and all that it becomes little difficult units. Therefore, how do I get over this problem, because the in the center of form and we derive this form of equation is found to be true. How do I get over this problem? Well if I can find the burn rate let us say I use a temperature or a pressure I call as reference pressure, and I evaluate the burn rate r reference at this particular value a into p reference to the power n.

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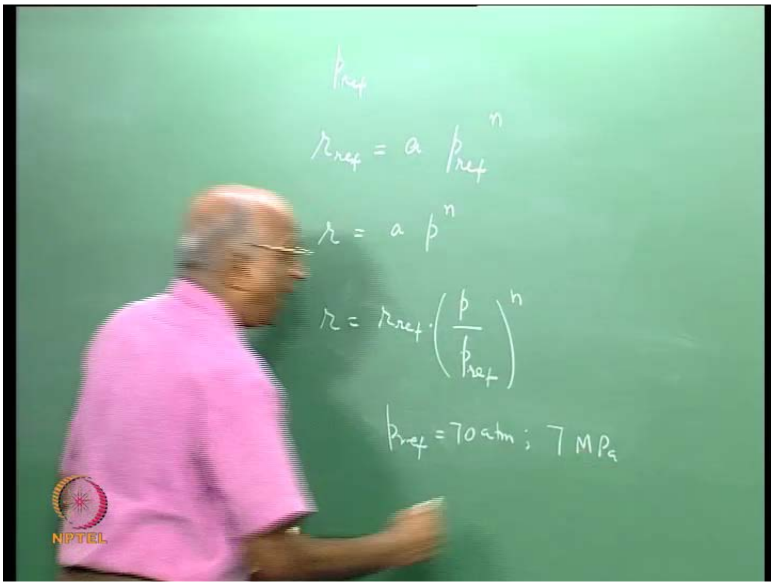

And then I say I am interested in burn rate r, at a pressure to the power n, then I can write the value of r, now as equal to a instead of a I put r reference into p by p reference to the power n and this is one way I get over it. In other words, the exponent what I have could be the burn rate at a given reference pressure provided the reference pressure is used for non dimensionalising the value of pressure, and this is how the burn rate is expressed through a one-dimensional pressure, which is based on the reference pressure and this reference pressure is normally taken as 70 atmospheres.

You remember when we did nozzle we said under sea level conditions when we evaluate this specific impulse or under vacuum, we took the chamber pressure as 70 atmosphere is, because that is the normal pressure at which a solid rocket or a good performing solid rocket works, now this is equal to 70 m p a. And therefore, the burn rate law can now be written as r is equal to r at 70 bar or 70 atmospheres into pressure divided by 70 provided p is in atmosphere to a power n or if I am expressing in terms of mega Pascal r at 7 mega Pascal p is in mega Pascal 7 mega Pascal to the power n. This is at 7 mega Pascal same as 70, and therefore many books write the value of r as r is equal to a 70; they say this is your reference at 70 into p divided by 70 to the power n you must remember immediately this we are talking of atmospheric pressure.

Therefore, the question is what is a 70? A 70 is the burn rate at the reference pressure of 70 value, this is all about burn rates effect of temperature, but there are many more problems which may be I think I postpone it slightly like for instance in a rocket chamber; there could be velocity, there could be illusions in there could be external heat flux. This seems I think I come back a little later, but let us go back and finish this portion on how do I know about designing a solid propellant rocket, and that is the thrust of what I want to do today, but we have already told the answer to get a particular thrust all what I want to find out is the evolution of the burning surface area. Let us do a simple problem, and then go to something which by which I can evolve for a given thrust, let us consider a propellant block which is continue in the motor case, and this is what we said constant for solid propellant rocket, and here I put insulation I put a nozzle here. Well I have a solid propellant rocket as it is...

Now, this particular propellant block is ignited over here, and it burns from the end that means the end of the propellant is ignited we call it as end burning, because it burns from the end it does not burn from here, because this is prevented from burning it cannot burn from this side, it is prevented the flame can go normally in this particular direction. Now supposing the throat area of the nozzle is A t. What is the value of pressure over here? We have already done it equilibrium pressure, and that is equal to let us write it down rho p a c star s b divided by A t to the power one over 1 minus n once I know the equilibrium pressure for a given value of S b, I know the value of burn rate r is equal to a p to the power n, I know the burn rate. And therefore, if my grain has or the solid propellant n burning grain has a length l the tank over, which the motor burns is equal to p is a constant, because the S b is a constant, p is a constant.

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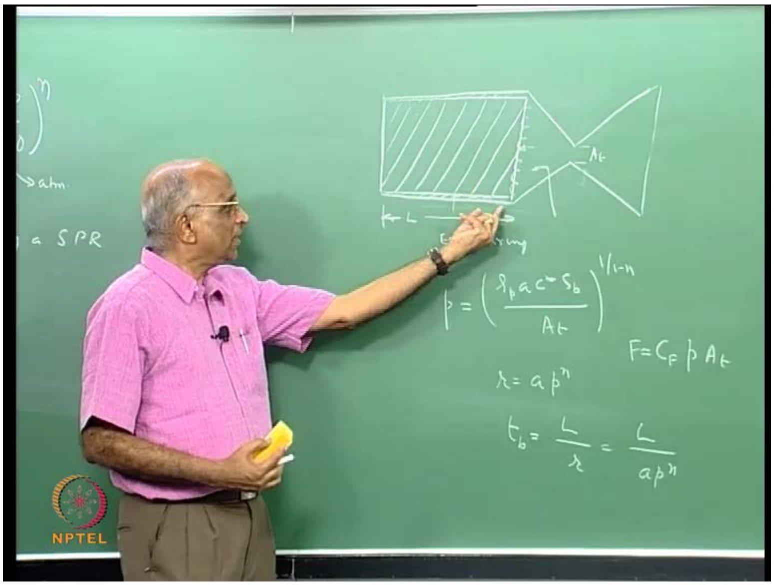

And therefore, t b is equal to the value of L divided by the length of the grain divided by the burn rate r, which is equal to L divided by I have a p to the power n; that means, the thrust developed by this particular rocket I know the value of pressure, and the thrust develop is equal to C F p into A t what do I find? If I want the thrust we said solid rockets are generally used when we want large thrust as booster stage; suppose, I want thrust of several 1000 tones, then in that case we must be huge several kilometers lighting of high solid propellant rocket whose diameter is going to be extremely extremely large. What is the diameter? Tell me we find that the thrust of an embedded charge as it were is going as burning surface area to the power 1 minus 1 by n, if my thrust is going to be large, S b is going to be large well I need something like a large diameter.

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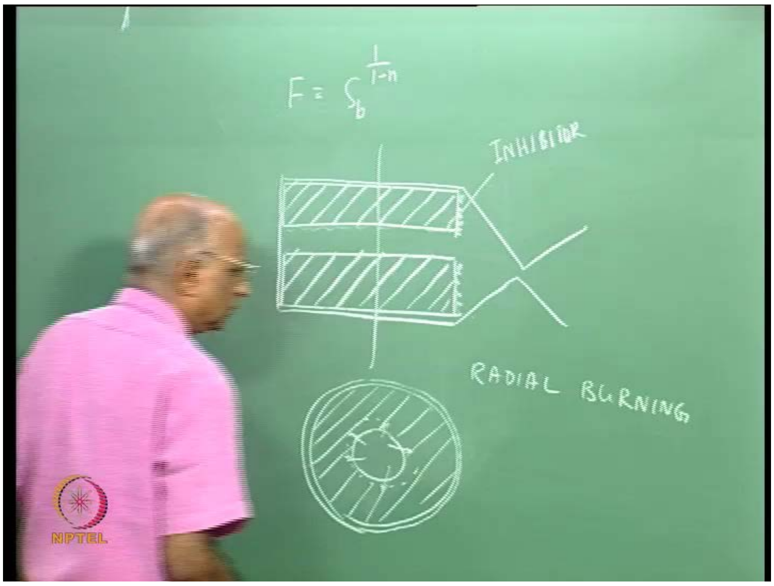

Rocket which I cannot really carry for it; therefore to be able to get some meaningful values well, why not? I try some innovations in this, I have the propellant block the same propellant block I take and may be what do I is, instead of burning it from the end I burn it radially I make a hole in the propellant law I again put it in a motor case it is my nozzle and now this is my propellant block here, I coat the propellant block here with some material which is an insulator which will prevent it from burning on this side, we call it as something which prevents inhibit burning inhibitor. I coat it with a substance which will prevent the propellant from burning here, then what I do I ignite this surface area the inside surface area; both these areas and then we told ourselves.

Then If I take a cross section out, then what is it I get I get may be this outer surface, I have the case over here, and then I have the inner diameter over here, and this is my propellant. I take out cross section here. I ignite this internal surface of the propellant, and the propellant burns we said normal to the surface it burns radially out, and this type of burning is known as radial burning. Let me illustrate this you know I thought you know I want to get into little more complicated areas or what we said is here I have something like a… Let us say propellant block like this, this burns from this side; therefore, it gets consumed as it goes on this we say is n power the same thing, the same area I take I also in a slightly larger diameter may be I take this, I put it inside over here. And now I have the annular area between the outer, and inner which is propellant, and now I ignite the surface area of this particular one. And then what happens is the burning will progress, let us say normal that that I put it over here, I have the annular surface area, and the burning will progress lng as surface area going like this.

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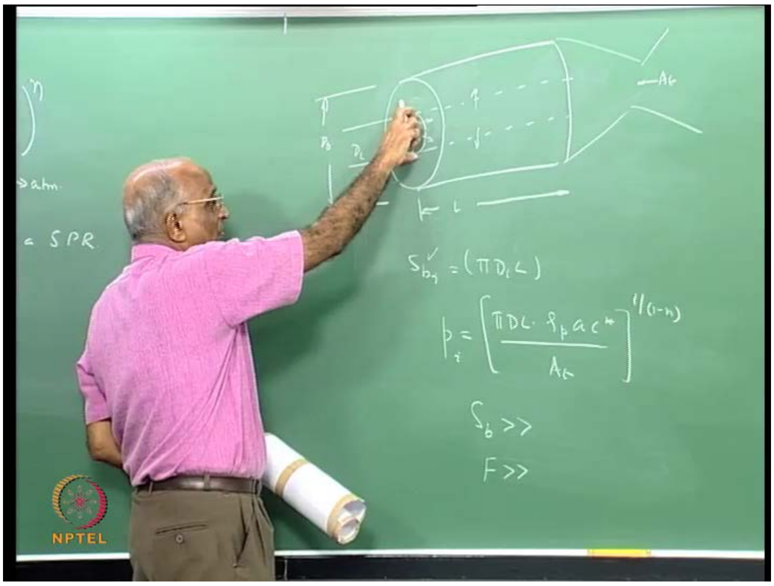

And therefore, now I have the initial burning surface area, if I were to write of this expression. Let us write it down I use this part of the board what did I have this my area, I have the inner area here the propellant block like this, this is my length; all what I am saying is I have this 1, all what I am telling is the annulus between the inner and the outer is what is getting burned this I say as diameter D i, outer diameter is D o, and the length of the propellant grain is r. So, how does the burning takes place it takes place like this, normal in other words burning proceeds, and then I have thing burning over here therefore, what is my pressure over here.

The initial burning surface area, I have to calculate thrust initial burning surface area is equal to… You could now tell me is equal to pi D I which is a perimeter into L, and what is the final burning surface area is equal to p i D o into the perimeter. And therefore, the pressure equilibrium to begin with p i is equal to pi D L into again I have carry over S b a c star divided by A t to the power 1 by 1 minus n.

That means I am housing this in a nozzle, the throat area is equal to A t over here. What is the change we are made, now I have the entire length the pi D of it is burning, and therefore, I have a much greater burning surface area. And therefore, radial burning grain which burns in this particular direction, I can get s b to be much larger than in an n burning grain. And therefore, I can get a large value of thrust this is 1 modification I can do, but you know there is some limit to it even I have area like this, I think I still have I still need to have a particular diameter in the length is it possible to increase this area even further. In other words all what I am asking is this represents my initial burning surface area for a radial burning grain is it possible to increase, this surface area by some means how can I do? If I can wrinkle these surfaces I can wrinkle it in some form, and how do I wrinkle it I try to make a small model may be this was my original perimeter over here, I wrinkle this surface I make stars in the inner surface. In other words instead of having something like a cylinder over here, I make the inner surface in the form of a star. Now I find well the star this is something like a 5 vertices are there; 5 vertex star. And therefore, now I find my surface area has increased enormously, and therefore now I must be able to evaluate how the burning surface will evolve as it continues to burn.

In other words this my outer diameter; the outer diameter will come over here, and the burning surface will evolve along these surfaces, and this is the problem which we must do, but however before doing this problem. Let us do the simple problem of may be a radial burning grain burning from inside to outside, what is the value of pressure and what is the time required for burning this, because in the case of n burning what did we get we got t o is equal to length divided by a p n, we knew how to calculate the pressure I knew the plan technical border can I do the same thing for a radial burning cylindrical grain.

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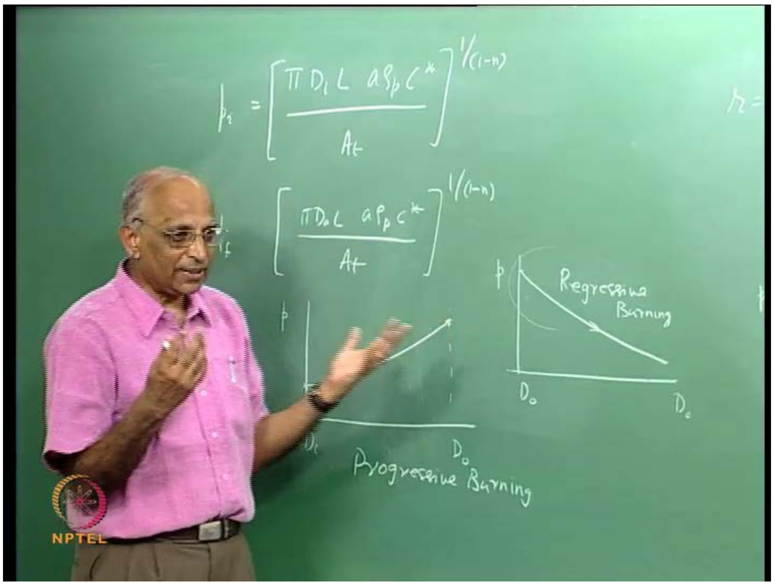

Let us do that and let us also find out how the pressure will change. Well I have the initial value of pressure is equal to whatever I have written there, let us rewrite it pi the D I which is the initial perimeter, L is the initial burning surface area a rho p into c star divided by A t to the power 1 by 1 minus n. Now, what is the final value of pressure when the burning has progressed? In other words the burning progress from the inner cylinder and reaches the outer cylinder what is the value the final value of burning surface area is going to be pi D o is the perimeter into L is the final surface area a rho p c star divided by A t to the power of 1 divided 1 minus n.

Now, is the pressure is constant like in n burning model or is a variable how would you locate it? That means you know D I has increased to D o. In other words, if I were to now plot the pressure and what is the pressure initially the diameter is D I the motor burns out till the diameter D o is reached. Let us let us keep our terminology very clear, this is the inner surface it is over here in between the 2, I have a narrow list it starts burning from the inner diameter which is the outer diameter, when the diameter is D I the pressure p i when the diameter is D o the pressure is p o.

And therefore, I know that D I is very much less than D o or less than D o; therefore, initially the pressure is this, let us say when it reaches the final value the pressure could be higher, therefore the pressure increases. Whereas, in then n burning grain what did we have we found at all the angles we had the same pressure versus; let us say note the time or the distanc,e let us say time I had a constant pressure is this. If this is now I will ask myself one last question if instead of having the grain start burning at the inner diameter, and progressly burn to the outer diameter I somehow put the case over here I ignite the outer surface, and then the flame propagates inward.

Then what is going to happen to me? I am going to get just the opposite I am going to get the pressure as a function, now D o gets ignited then it gradually the flame comes, and comes to the inner part; that means I start with a value of p final that means the initial value is now higher, the value is higher the pressure drops. In other words while the in burning grain gave me a constant pressure; and therefore, a constant thrust a radial grain burning from inside to outside gave a progressive increase in pressure or a progressive increase in thrust, while a inner burning.

That means burning from outside to inside gave me a falling pressure; and therefore, a falling thrust therefore, I could have 3 types of thrust evolution in such rockets. And therefore, this is known as neutral burning. What should be the name for this progressively increases, that is progressive burning, and this it will keep decreasing that is regressing; therefore, all what we have telling in last few couple of minutes is, well if I have a propellant which burns from toward end to end; that means actual burning well the type of burning is neutral constant pressure constant thrust, if it is burning from radially from inside to the outside the pressure keeps increasing.

While if it burns from outside to inside well it could be regressive. Therefore, we could think in terms of 3 types of rockets; neutral burning rockets side propellant rockets progressive, burning rockets and regressive - burning rockets is it clear? If this is clear question is you know what type of burning? We required see I cannot say I making a rocket which is regressive it is going to fly mission will demand something, and what will mission demand, when the rocket takes off the thrust must be high and as it goes up the thrust can come down.

Therefore, may be something like this might be better than this, because when the rocket goes up it is mass is higher I cannot expect to go with a higher acceleration in the beginning itself. Therefore, we have to somehow get balanced what type of burning is required for a given system, and that is were when we go into some shapes when we give these shapes to the internal that is what did I do here? We just took the inner surface we just wrinkle I could have wrinkled it in any shape why should I have to give a star, star is one of the shape I could have wrinkled into some other shape instead of giving a shape of star over here. I could have given something like this I could have given whatever shape, I want and all what I have to calculate, how does the burning rate evolve around this surface can we find out how S b changes with time. And once I know how the S b changes I know how the pressure changes with time, and I will find out how the force or the thrust which the rocket will develop with time, and that is all what do I by designing solid propellant rockets. Therefore to be able to do this let us do a simple example today.

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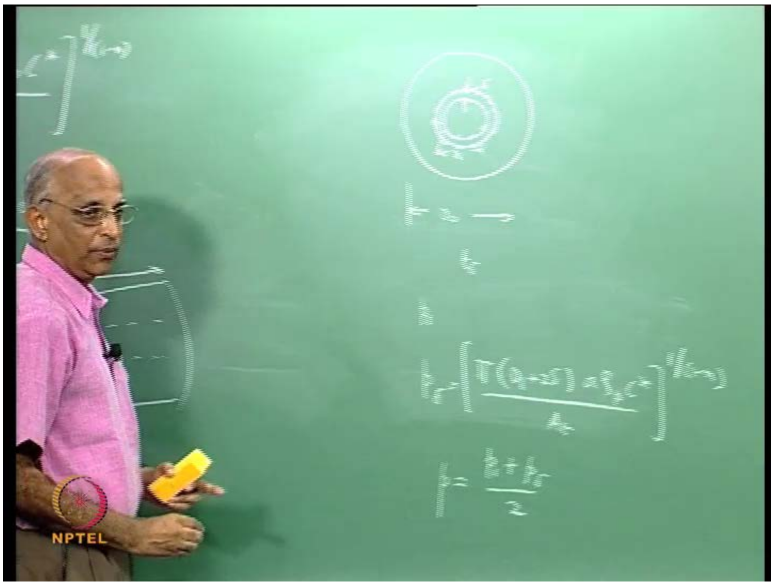

Let us find out the time taken for a cylindrical grain what do I say by a cylindrical grain; the first example, I have a end like this I make a hole here I have internal burning it burns through like this, the diameter is let us say D I as what I have written here the outer diameter is D o, the length is L may be I am repeating this figure question is can I predict how the pressure will change with time. And how do I do it, and well it is a simple problem well, let us lets do it in a minute, let us say I have the outer diameter here D o here is the initial diameter with which we get started with D i.

Let us consider a small part of the propellant gets burnt, and let the small part p of thickness delta over a time; let us say t delta, and then at the end t delta I will calculate the pressure again at the end again 2 delta. I will calculate the pressure again like that I can progressively calculate, and how do I do it. Well I know the pressure to begin with p I is this value, what is the pressure at the end instead of p i, I say what is the value when the grain has used this it has started burning from inside it has come to this level. What is the value of p delta I want to use this equation and write it can you tell me well I say pi what is the value of D, when the thickness burnt is delta yes D 1 plus 2 delta into I carry these things, all other are all constant a burnt rate constant rho p c star divided by throat area to the power 1 by 1 minus n; that means I know the value of pressure this point, I know the value of pressure at this point. I want to know the time taken for delta of the problem to get consume.

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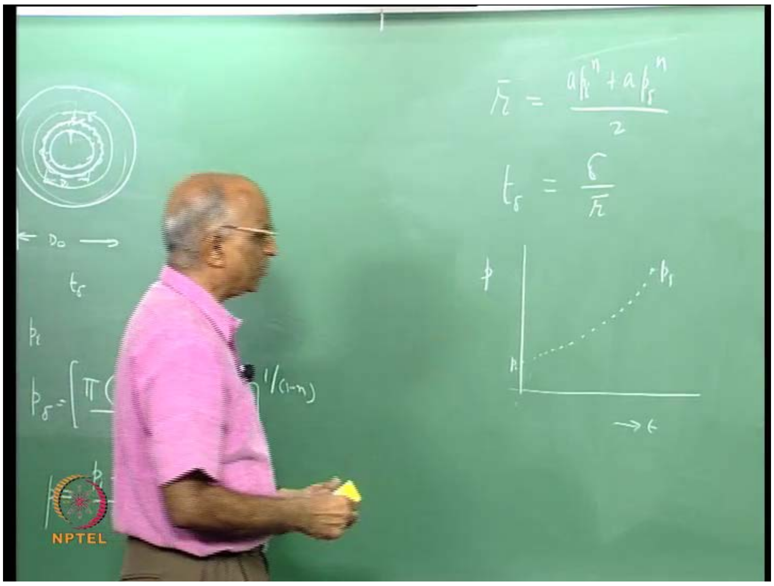

Therefore, the mean pressure between this and this is equal to p is equal to p i plus p at delta divided by 2. And therefore, the burn rate the mean burn rate between the initiation to the that from D i to D i plus 2 delta is equal to r bar mean value, and what is the mean value equal to a p i to the power n plus a p delta to the power n divided by 2 r 1 is the burning rate at the initial condition.

It has progressed by a factor delta in which case this becomes more pressure; therefore, the burn rate has gone up divided by 2. And therefore, the time taken to burn a small quantity between D I and D I plus 2 delta is called as D delta is equal to delta divided by r bar. Now I do the same thing, I now take this as initial condition I go to the next step I, find the value the value of t consume I get the value of t consume, and then I can put the whole thing together as a function of time my pressure increases from p i, I know how the intermediate values I know the final pressure is p f which is given over here and this is how the pressure evolves.

In an inner burning rocket we can do this, but this is numerical way of doing this there is no other way of doing and this a way to do a problem, because when I have more complex configuration like what I said, I wrinkle the inner surface I get all these shapes I can find out how the surface should evolve with time. And this how one calculate the variation in pressure in time. Once I know the variation in pressure in time I can readily go ahead and say that the thrust characteristics with respect to time will also change at this particular rate. This is how a solid propellant rocket involve radial grain is made.

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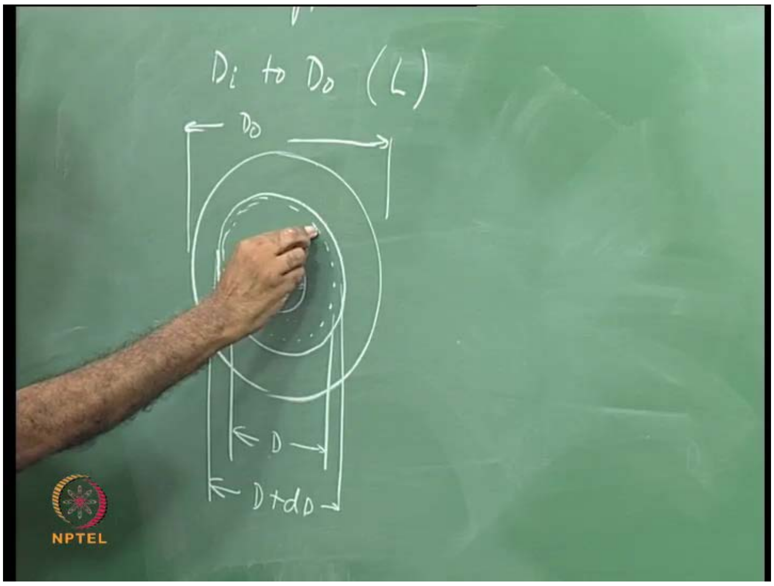

Let us do one small problem I supposing I am asked to find the time taken to burn a propellant to burn a radial. Now I am using slightly radial grain between D I to outer diameter the length of the grain being L, the same thought the same thing what I am considering the initial diameter is D I; the final diameter is D o the length is L of the grain. I want to find the time taken to burn this. I do exactly the same thing you know there is nothing more to do it again.

I consider a section over here, the initial diameter D I the outer diameter is D o why not do a simple thing like what we say is let us consider any diameter D. Let us find out the time taken for the diameter D in between D I and D o to increase from the value D to a value D plus a small change over here, and this I say the diameter has increased to d plus small d D. If I can find the time taken to burn this part I can integrate out between this and this ,and find the time taken and that is what I am going to do; let us do that how do I know it how do I get the time taken to burn a distance D d well I know this distance D d is very small. Therefore, the pressure could be the value at D; and therefore, the pressure is equal to.

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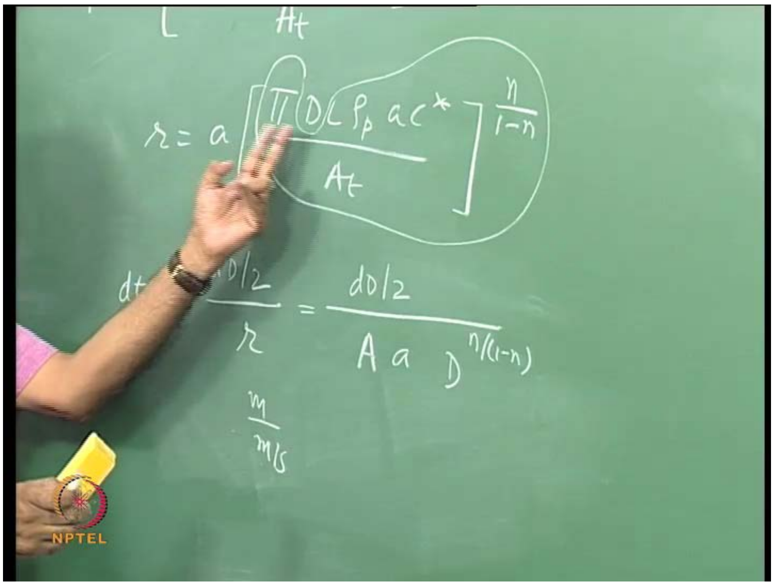

We again write it s b is equal to pi d perimeter into length is the burning surface area into rho p into a c star divided A t to the power 1 by 1 minus n. Therefore, what is the burning rate r is equal to a p to the power n. Now I substitute the value instead of p i substitute this value over here, I get this becomes this 1; let us write it again pi D l rho p a c star by A t to the power n by 1 minus n is the burn rate when it is here what is the time taken to burn a small distance D d. Time taken let us say d t is equal to diameter is increased from D to d plus D d; therefore, this thickness is equal to D d divided by 2 that s i, the special length divided by r which is the time taken, and what is this come out to be. Now I find pi is a constant length is a constant D is variable rho p is a constant.

All these are variables except n; therefore, I can write this as equal to D d divided by 2 divided by, I take all the constants to the power n minus 1 as let say A, and then I have this a over here. And then I write it as D to the power n by 1 minus n. Please let us first check, if it is all what we are telling is this has units of length say meters divided by meter per second this is the time taken. Here you have D d by 2 is the distance propagated through a small distance D d by 2 divided by a into i club all the factors other than this, in terms of a over here, because a is equal to pi l o d a c star by A t to the power n by 1 minus n that is a into D, because a variable into n by 1 minus n I have to integrate out, and what is it I have to integrate, the time taken to burn from here to here it is at 0. Let us say the time taken to burn is t b time taken for burning b, b denoting the burn time.

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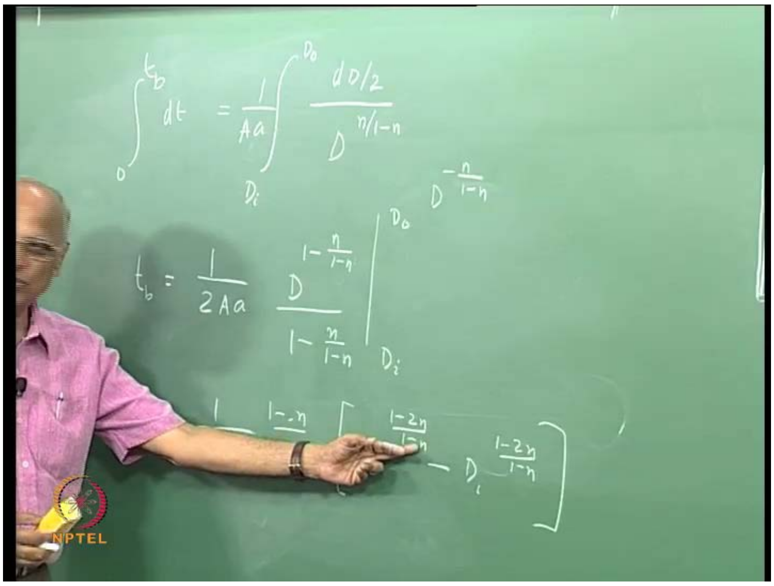

Therefore, I will get d t goes from start to the end, which is the burn time which must be now equal to as it goes form initial diameter to the outer diameter, and the value of ignition is D d divided by 2 divided by D to the power n by n minus 1 and I can take a and a outside is equal to 1 by into b. Well these are constants now what do I get on integration. I get t b minus 0; therefore, t b is equal to 1 over 2 a into the exponent of the burning rate law. Now I integrate it D to the power that is n D to the power I get if I bring it up it is D to the power minus n divided by n minus 1. Therefore, this becomes 1 minus n by n minus 1 is it I have D over here I brought it D n minus 1 I take it up it is d minus n by n minus 1; and therefore, integrated I must also get divided by 1 minus n minus 1 was it n minus 1 or 1 minus n 1 minus n 1 minus n 1 minus n 1 minus n.

What is the final value? Therefore, this is equal to 1 by 2 A a into 1 minus 2 n where 1 minus that 1 minus n divided by 1 minus 2 n in the denominator, I am sorry into D o to the power 1 minus 2 n divided by 1 minus n minus D I into 1 minus 2 n divided by 1 minus n; this is the time taken it is possible to add a post form expression, and this a value please check whether there are any mistakes in this.

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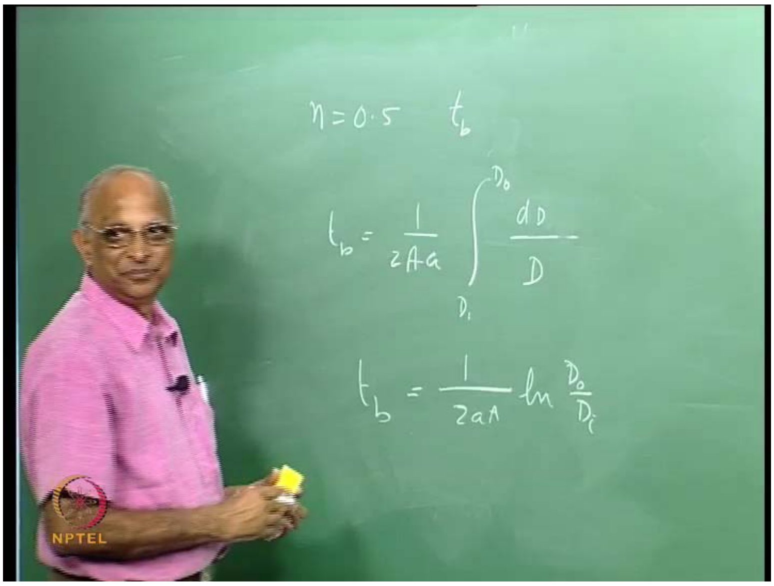

Therefore, we are able to find out the burn time, but now I will ask a question if n is equal to 0.5, what is the time for burning? We said as lng as n is less than 1 it is for us if n is equal to 0.5 what would be the value of burning time? Lets do it 1 minus n is 0, so there is a problem that means this why is it not usable can we ask ourselves, what is wrong there we have done it correctly.

But it is not working at n is equal to 0.5, well can somebody some body come out with an answer well this seems to the problem, and how do I do. I think we should be able to analyze it let us do a our integral is D t or t b is equal to… Let me write that 1 over A a into interior D I to D o of what D d by d 2 divided by D to the power 0.5 divided by 0.5; and therefore, this equal to D d that means I take 2 over here, and this gives me for n is equal to 0.5 I get 2 a ln of D o by D I. That means what happens for 0.5, I need to use a ln form because you know the point is when I substitute this, I will not be able to get for n is equal to 0.5; and similarly, if n is equal to 1 I get all evaluated things and we found n is equal to 1; is not possible.

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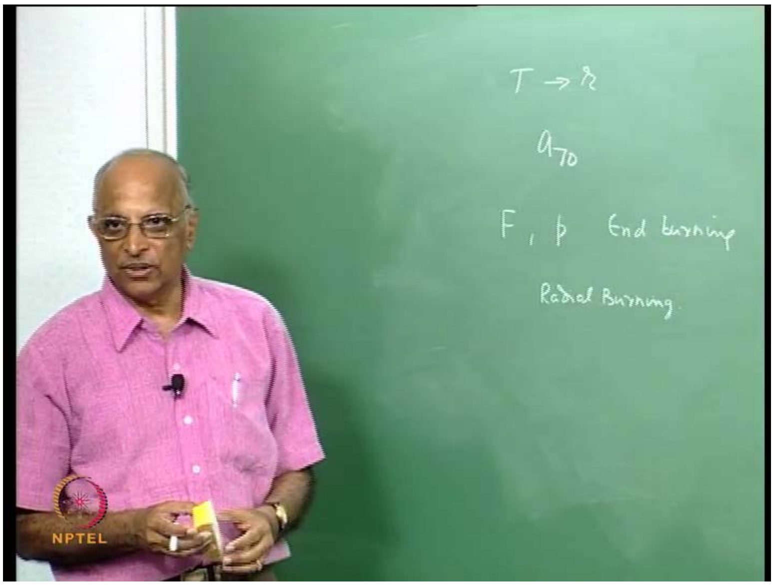

We are able to and this is how a cylindrical grain will be designed; therefore, what is it we have done in today's class we looked at the effect of temperature on burn rate, we also defined the value of a as a 70 at a reference pressure of 70 bar. Then we learnt how to develop an equation for thrust and pressure for n burning grain. We also did for radial burning, but we found for radial burning grain the pressure evolution had to be done by in increments while the burning time.

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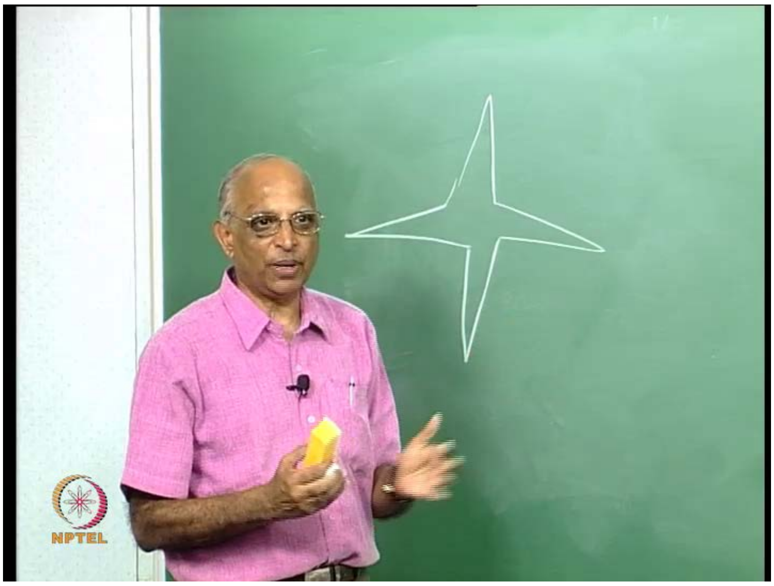

We could solve in this particular equation. I continue with this in the next class and in the next class we will do something a little bit more, we will look at the evolution of burning surface area from something like a star ring may be n n types of what is this?.And also look at the different forms of grain shapes, which are used in practice, and the reasons for it after that we will summarize the whole thing on solid propellant rockets by incorporating; may be the igniter, and other aspects into the solid propellant rockets. Well thank you, then.