Rocket Propulsion

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Lecture No. # 25

Burning Surface Area of Solid Propellant Grains

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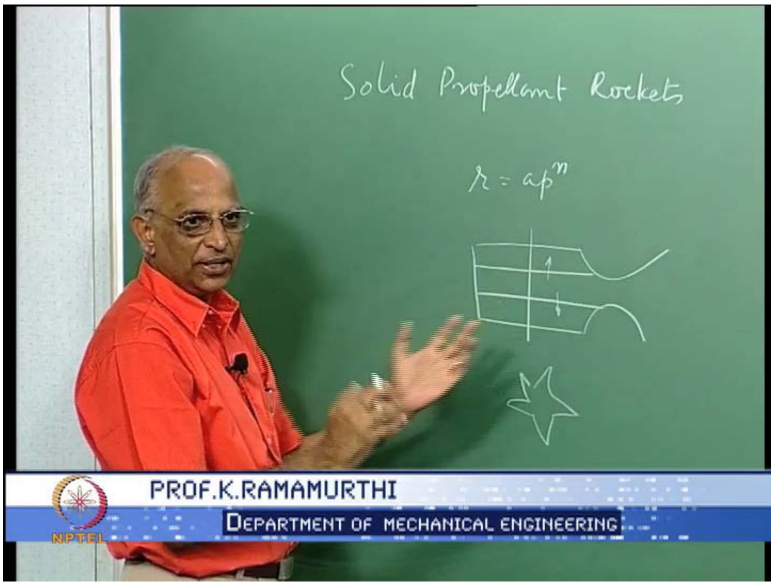

Good morning. In today's class, we continue with our discussions on solid propellant rockets. By now, we know what the burning rate is linear regression law r is equal to a p to the power n; for both composites and double base and also the other forms of may be composite modified double base and also nitramine propellants. What we were discussing is, how do we assemble the propellant grain and what should be the configuration of the grain. We saw in the last class may be even if I have something like which is burning internally and radially, I would like to wrinkle this surface into something like a star or some other configuration, such that I get increased burning surface area and therefore, increased pressure and therefore, increased thrust; the aim is to get a large thrust.

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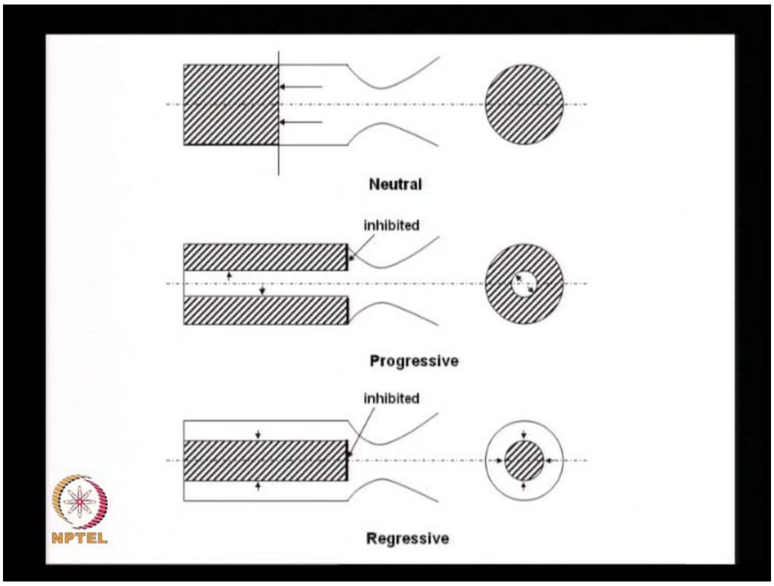

Therefore, let us quickly recap where we were. We talked in terms of let say neutral burning grain; that means, the burning surface area is constant as it keeps burning. We called it as neutral burning, because both the pressure and the thrust were constant for all times. We also discussed this question of in radially burning grain, burning from inward to outward and what did we find? We found that the pressure progressively increased and the thrust progressively increased. In other words, burning started at the inner surface and progressed to the outer surface; we called this as progressive burning. And, we also told ourselves the same thing may be the final pressure it could be on the outer side and if I could somehow get the burning to start from the outer and progress inward, the pressure will keep falling with time, we called it as regressive. Therefore, we talked in terms of neutral burning, progressive burning and regressive burning.

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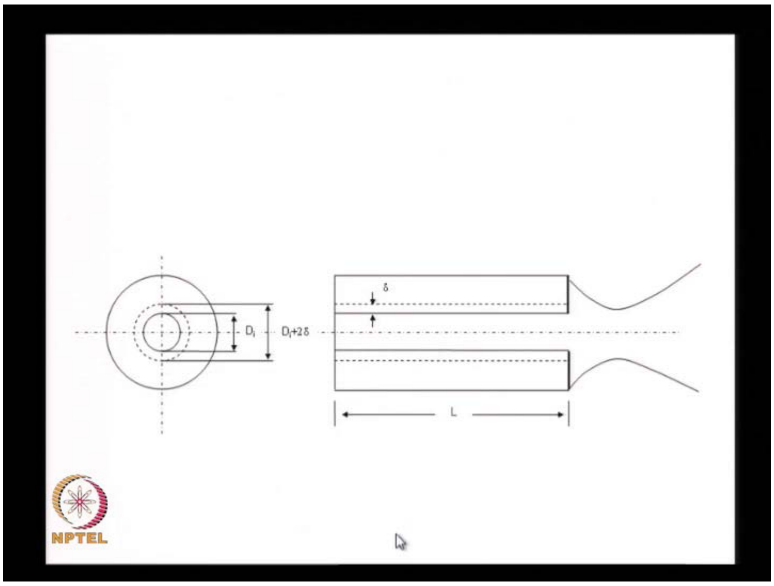

And, thereafter, we also evaluated how we could determine the pressure at different instance of time for the radial inward burning grain and also the time taken to consume the propellant from the inner diameter D i to the outer diameter D o.

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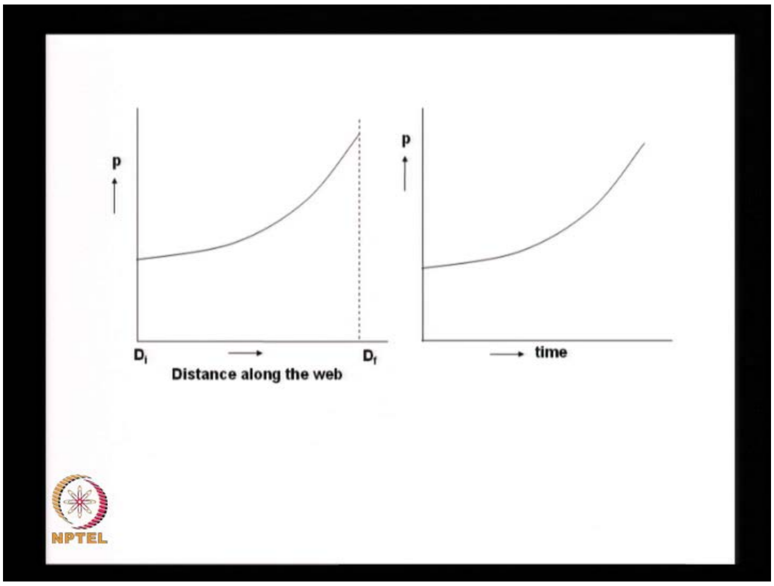

Having said that it is not possible for us to determine the pressure variation along the grain, let us go back. You know the distance between the inner surface and the outer surface is what we call as the thickness of the grain and the minimum thickness is what is known as a web thickness; I will take a look at it again today. And, if I say this is the inner diameter this is the outer diameter, the pressure keeps increasing and therefore, as distance along the grain increases the time increases. Therefore, I was able to plot pressure as a function of time and we find that the thrust of this grain, thrust going as CF into p into A T keeps increasing, with time over here as is shown here therefore, this is the progressive burning grain. This is what we did in the last class.

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Having said that we also told; now I show a picture of this grain. See, instead of having the grain which was circular on the inside, I sort of wrinkle the surface such that I have something like a star shape and this star shape is happening all along the grain surface. And, now for this what is going to happen? The inner burning surface area s b is going to be much larger, in other words s b is going to be the perimeter of this particular star multiplied by the length along the grain that is going to be the initial burning surface area. And, therefore, the pressure will be much higher, the thrust will be much higher compared to what it would have been had it been pi d not over here; therefore, these are wrinkle surface. And, in today's class we will try to calculate how the burning surface will evolve for this particular star grain, but it need not be a star alone.

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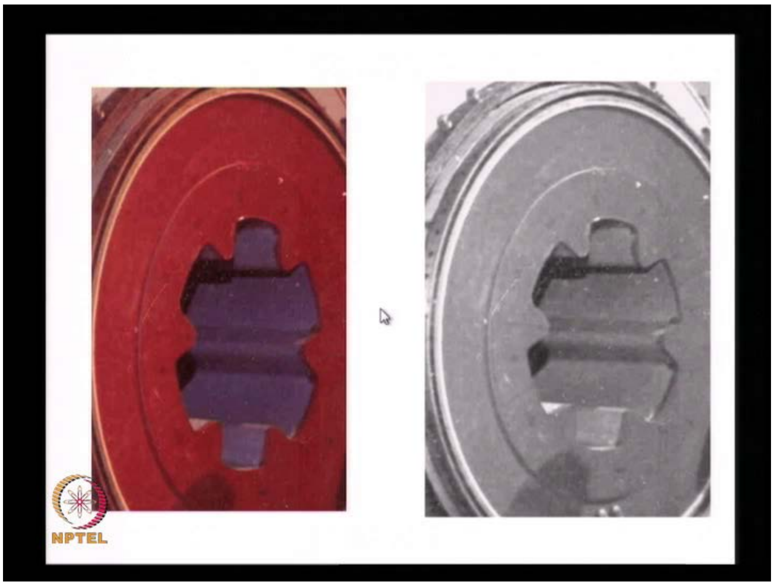

I could have grains, it could be of different shapes; maybe I could have lobe like this instead of having a star over here, I could have a lobe like this and what for what will happen in this case? The burning surface area, this is the inner surface of the grain, this is the length of the grain in this direction, the evolution of surface area would keep on evolving like this, keep on evolving like this until it touches the outer surface of the grain.

The minimum thickness from this to the case is what is known as the web thickness or the thickness of the grain. Mind you, there are several thicknesses it could have from the vertex (( )) to this or it could be from this to this, the minimum thickness is what we call as the web thickness. The same grain I show over here just to make sure we understand, you find that the inner surface has something like a projection here, a valley here, again a projection here; this projection is all along the surface and the grain burns radially. That means, it burns into the grain in this particular fashion and all along the surface, the surface keeps detracting like this. And, I will consider a few examples such that we are very clear how to calculate these things.

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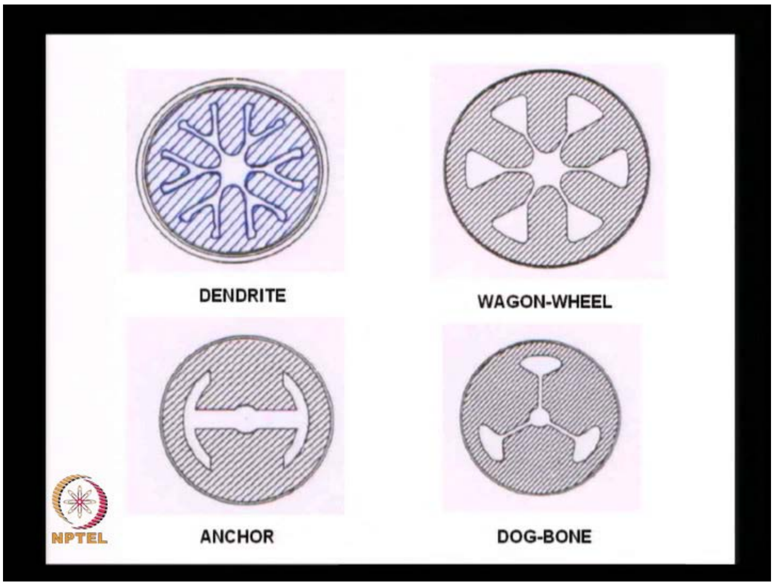

Well it means that grain surface need not always be a star or a shape, like what I just now showed which is a lobe like this. But it could be any shape it could be a dendrite, dendrite is a crystal shape which is like this. And, burning surface starts or the inner burning surface is something like this to begin with; this is the perimeter into the length of the grain over here. And burning proceeds in this direction; this is known as a dendrite grain. You have a wagon-wheel in the shape of the wheel of a wagon, see you have the inner surface which is like this and the length of this is along this particular direction. Therefore, the perimeter along this is not quite large compared to what would have been for a circular diameter over here, multiplied by the length is what gives me the burning surface area and this is known as a wagon-wheel.

I could have different shapes, any possible shapes; I have the shape of an anchor, instead of having a circle like this, I sort of extended make it shape like an anchor and what is an anchor? You drop an anchor when a ship is sailing and this is the shape of an anchor and if this burns, well the burning shape will keep on evolving like this and I can find out how the burning shape evolves with time and therefore, calculate how the, what will be the thrust as it keeps burning. I could have dendrite, I could have wagon-wheel shape, I could have anchor shape, I could also have the shape of a bone, the dog-bone where this is the shape of the bone which is shown, this is the inner diameter. Now, how is this grain going to evolve? It is going to come like this, go like this and go like this at the next instant of time, I should be able to calculate it and thereafter calculate it till this fellow touches here and then the burning surface area decreases.

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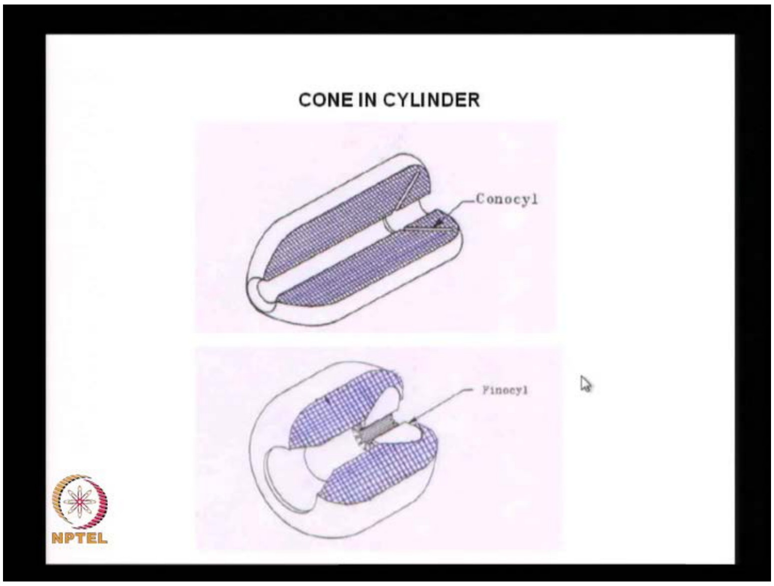

Therefore, we could have different shapes. Well, we could also have something like a cylindrical shape and at the end of which I give something like a cone; that means, I have a cone within a cylinder and this is known as Conocyl; that means, I have a cone within a cylinder. How is this going to evolve? The surface air is going to keep decreasing like this, this is going to go like this, this is going to go like this, this is also going to go like this. Therefore, the evolution of burning surface area is what we are basically interested. In addition to having sort of a cone in a cylinder, I could also have a cone like this and this cone has this cylindrical portion; in the cylindrical portion I make ribs like what I show here.

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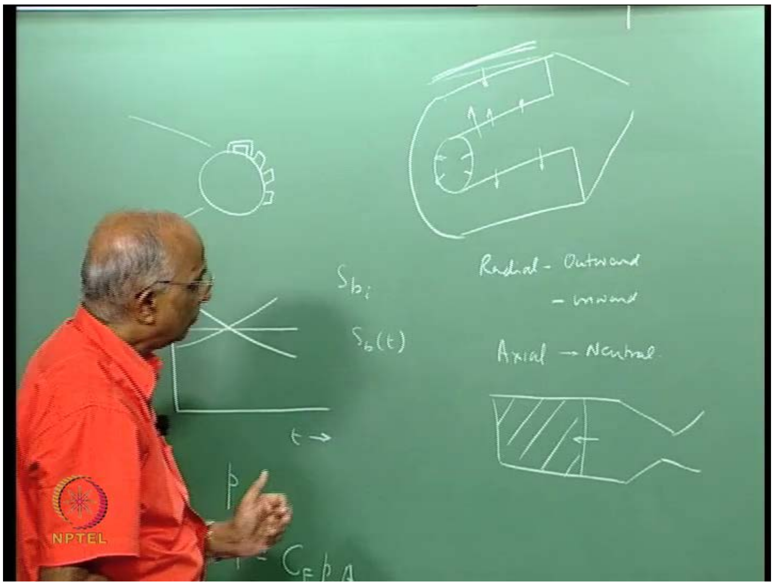

In the section shown, I make ribs like this; something like fins. Therefore, in addition to having a cone on which this is situated and then, I have something like a cone over here, coming to a cylindrical portion on which I have ribs; therefore, here also burning could evolve lying this, I am trying to maximize the perimeter and the burning surface area. And, this when I have a fin in a cone under cylinder arrangement, this is known as Finocyl, that is I have fin in the cylindrical portion. Therefore, there are various grain shapes, but what is maximally used amongst this is something like a star grain is used quite intensively, we will see the reason in this class. We also find such shapes are used quite extensively and these are little rare, but though the wagon-wheel tends to be used in some specialize cases.

Therefore, we could have anything and why do we have these shapes? We want to increase the burning surface area to the maximum possible extent, initial burning surface area and the progression of the burning surface with respect to time. In other words I am interested as time or as burning proceeds, my burning surface area should initially be large such that I get a thrust; you could either evolve like this, either it could evolve like this or it could be a constant. In other words this is progressive, this is regressive, this is neutral; because burning surface area directly translates into pressure and pressure directly translates into thrust as F is equal to CF into chamber pressure into AT over here. And, what was p? We got an expression in terms of S b to the power 1 over 1 minus and therefore, it was directly connected.

Having said that let us now go to the star grain which is our primary interest. Well, I forgot one grain which is known as a slot grain. See, so far when we considered these different grains, we essentially considered let say a cylindrical grain; wherein I have something like a cylinder, this is the length, may be this is my outer case over here and here I put my nozzle over here, we told ourselves this end is insulated it does not burn over here and what is going to happen burning takes place along this surface. And, therefore, my burning surface area the burning is radial and it is inward; oh no, I should say outward from the centre, burning takes place from the inside surface to the outside. If I ignite it on the outer surface may be I have let say a gap and case is insulated, I allow it to burn inward; therefore, I say radial inward, this was progressive this was regressive.

We also had axial burning, by axial burning what did we have? We had something like a case in which I put the grain over here; I allow the burning to take place normal to this surface axially; here also the burning is normal to this particular surface over here. But it is in the radial direction, this is in the axial direction; therefore, burning could also be axial. And, for the neutral burning, that is end burning over here burning from end to end, it is sort of end burning which is neutral. It is also possible to have some configuration like this, let slightly modify this configuration. Let us try to see, how I can slightly change this; supposing I have a grain like this let us now shade it such that we are absolutely clear what to do. This is the inner surface of the grain and the grain is something like this, this is the grain over here, let say it is this outer part; the outer is the case over here, this is the grain over here.

And now if I make some slots here see I make a slot here, I make a key hole here something like this; I make a slot here. Then, how is burning going to proceed? Burning goes radially here, burning grows actually here; that means, at the next instant of time the surface is going to be something like this, coming over here, surface comes here something like this. It goes both axially and radially and these are known as radial cum axial burning or three dimensional burning surfaces. In other words, it is both axial axisymmetric as well as axial over here and therefore, these are known as three dimensional burning surfaces. And I show a three dimensional surface over here which is a slot and it continuous to burn in this direction, in this burn direction, but the problem is a simple geometric problem.

I want to find out how this initial burning surface area, this is the perimeter into the length is the burning surface area; how this keeps increasing as the regression of these surface continues? This is the slot if I just put number of these things together I allow burning over here, I allow burning on the outer surface, I allow burning here through a number of these things tagged together in a case; I am not having a particular burning, I am having unrestricted, because it is burning from radially inward, radially inward over here, radially outward over here, radially inward here; it is sort of unrestricted. And, therefore, I get a large surface area, but then you know the problem is, how do I control the burning taking place; such type of things have not seen being much used in practice. But when we see may be rocket assisted takeoff for planes and all that we will take, we will try to see whether some of these things can be used.

Therefore, to summarize we talk in terms of neutral burning, progressive burning, regressive burning and everything decides on how the surface area keeps changing. Having said that let us come to this particular problem of a star grain. See, here I show the end view; let us keep our discussions clear.

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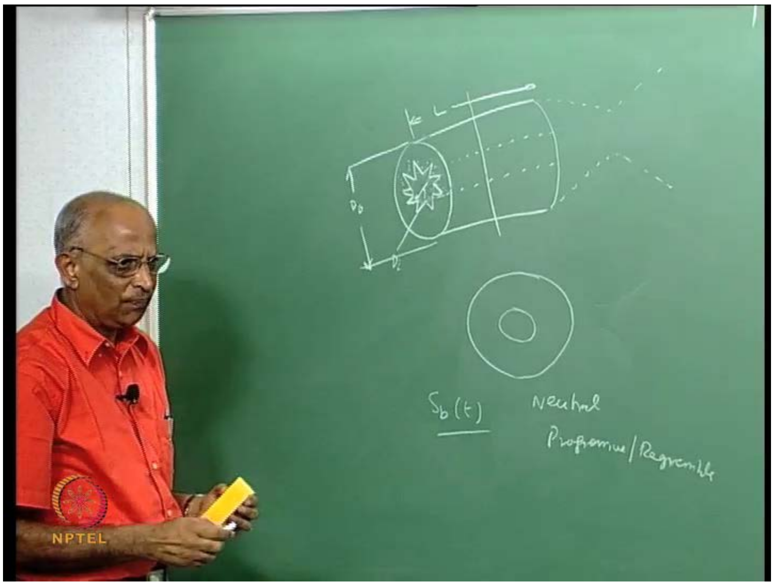

All what we are doing is, I have a grain like this; this is the length of my grain, propellant grain over here and here this is where I put my nozzle to give me the thrust. We were able to calculate, when I have may be a section I take over here, the section is a circle corresponding to the outer diameter and I have a hole at the centre corresponding to this.

And, therefore, now my section is something over here; this is the length of my grain, this was my D I, inner diameter and this was my outer diameter right.

Now, if I sort of wrinkle this surface and I showed you how a wrinkling is done, instead of having this surface may be now I put a multi star configuration over here. That means, I have 1 2 3 4 5 6 7 8 or 9, 9 star, 9 vertices of the star and I put this; that means, I remove this, inner diameter now I put this. And, now I find that I have a much larger burning surface area and how is it going to evolve, at the next instant of time it is going to come here, it is going to come here, it is going to come here, it is going to come here. And, therefore, I find it regress this in a slightly different shape and therefore, I am interested in finding, how the burning surface area evolves with time. Question is, is it going to give me a progressive burning, is it going to give me a regressive burning; I must be able to do that and it is possible to configure the star grain to give both neutral as well as progressive or regressive, we will take a look at that.

And, how do I predict the thrust developed by a star grain; that is what I want to do. Therefore, you know if I take a look at this picture again, what is it I see over here? This is the star grain, see in practice to get a point is difficult and therefore, they have slightly curved the point over here. And, therefore, the next line over here shows, after sometime when the burning has progressed it goes like this. And, therefore, I need to calculate this perimeter and if I calculate this perimeter and multiply by the particular length, I get the burning surface area S b at time t. At the next instant of time, well the surface is again evolving, it goes over here, comes over here; I can calculate this and this is what I am I wish to calculate. Similarly, may be towards the end it comes over here, this is the shape; see, initially I have these surfaces, but as it progresses it becomes something like a circle. And, why does it become a circle, because of point when it burns it goes into a circular fashion. And, therefore, to be able to understand this type of burning, I first deal with a very simple case.

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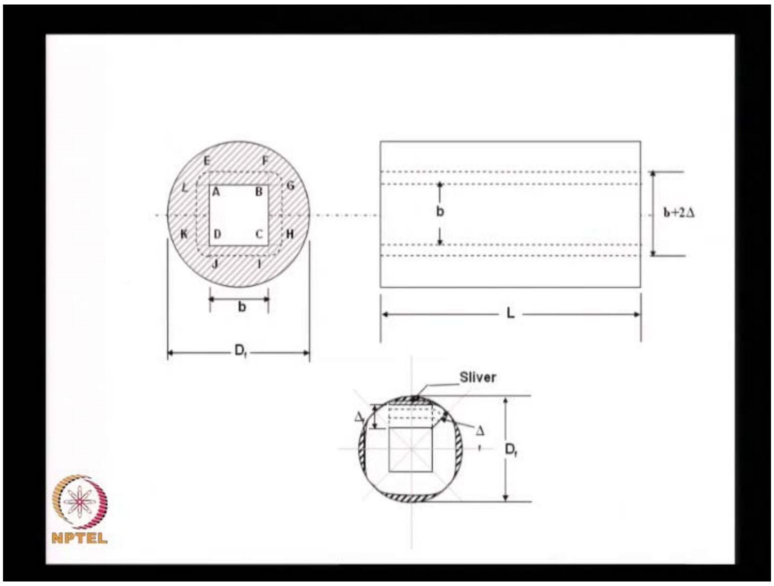

I deal with a case wherein I have a vertex; in other words I put a rectangle opening. That means, this is my opening over here, I will try to calculate how the surface evolves with time and based on the experience I gain in this particular case, I will go back and take a look at how a star burning takes place; let us do that. In other words, let us keep (( )) the mission for this class very clear.

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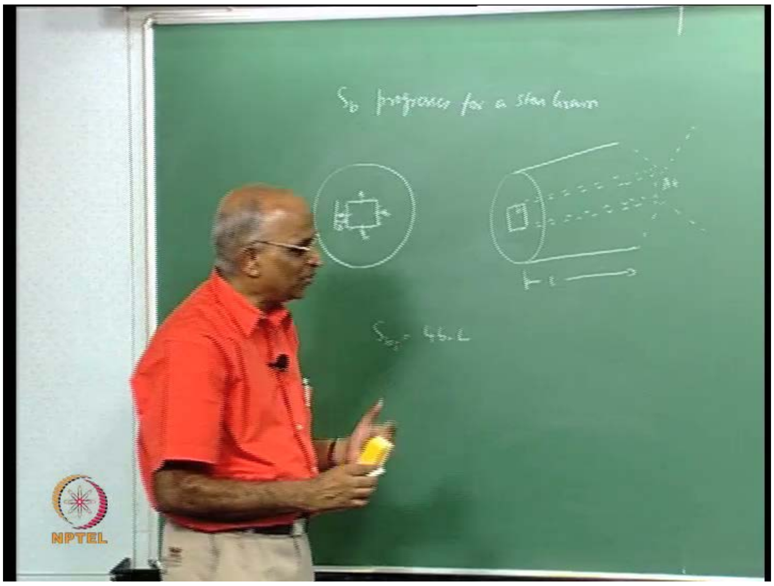

We would like to find out, how the burning surface area progresses for a star grain; what is a star gain? It is one in which the internal surface is sort of wrinkled such that I have larger number of surfaces. If I know how to do a star and if I know how to do further case of 1 or 2 cases which I will do on the board, maybe we can do for any surface and be able to find out how the thrust of a solid propellant rocket can be varied.

Let us do this simple configuration; let me take a cut view that is an elevation of a grain like this and instead of having a circle here which we have already done, radial burning and radial burning may be we considered the case of a circle and it burns normal to the circle; we have considered it. Let us now consider the case wherein at the centre symmetrically, I put a square hole; instead of having a circular hole I have a square hole of dimension b b; that is square hole. In other words, if I have to take the view as it is for grain, I have the grain of length L, anyway I have the nozzle over here, area A t and what I have this grain shape is something like a square. And, therefore, may be this is the shape of the thing and I would like to know, how this surface evolves? How these 4 surfaces which are straight lines evolve?

In other words, I want to know how this surface, what is the area of this surface? We said that the length is b therefore, the surface area is L into b; therefore, the total surface area at the star of burning will be, 4 b is the perimeter into the length, I think this we must visualize; if you are clear about it we can do any problem. See, what is the surface area with which we are igniting the grain, the inner surface area corresponds to b into L, b into L, b into L, b into L or rather perimeter is 4 b into length; that is the initial surface area which begins to burn, I just took a section over here. And, how does burning takes place? Burning takes place linearly at the surface. I want to find out, what will be the burning surface area after a certain time; let say when the brain, when the surface moves through a distance let us say delta, what would be my burning surface area? Initial value is 4 b into L. Is this clear, are there any questions?

Now, let us do this problem. And, if you are clear about this, I think you are doing star or any other configuration is quite simple and we will see, why this is not used in practice though it is so illustrative. I think we should determine this.

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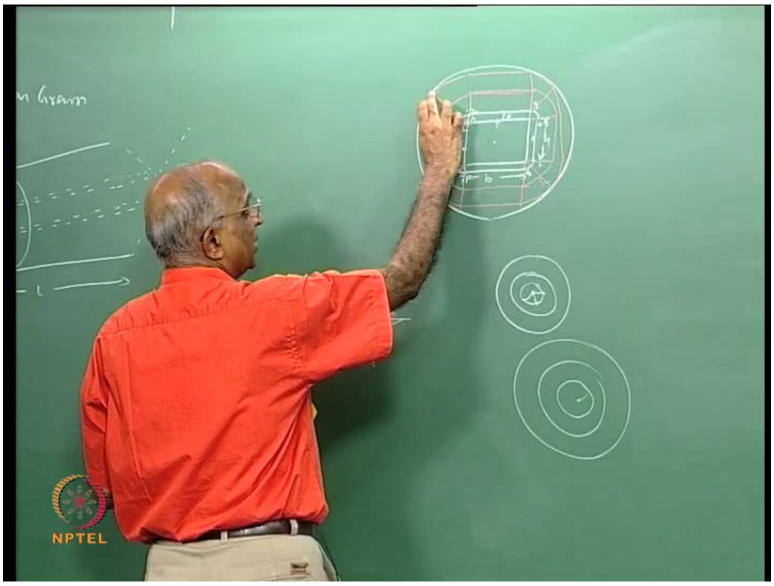

Let me draw a slightly bigger figure. Now, this is the centre, all sides are b. Let us assume that the grain moves through a distance or burns through a distance delta. Therefore, this straight line comes over here, this straight line comes over here, this straight line comes over here; again mind you, this is delta, this is delta, normal burning therefore, this is delta over here. How does this points evolve, how does the vertex of the 2 lines evolve? We said burning is always normal, therefore, how should it evolve? Let say, I have this vertex here, burning surface is normal here, burning surface is normal here; this is the surface which burns. How will this point evolve? Normal? Point will evolve like what? How would you look at this problem? May be you all should give me an answer.

Two things; this surface goes straight, this surface, how will a point burn as it proceeds? See, the point you know, when I have a point then it should be normal to the point; that means, burning should take place here, what happens over here? That means, a point will go as a curve of particular radius; that means, when this moves through a particular delta may be between this to this is there, it will form a quadrant here when because for a point we say burning is always normal to the surface; for a point it could be either way. That means, if a point is going to burn it has to burn like this right; normal to the point could be in any direction, therefore, normal to the point will go as a circle, but if I look at this, this surface is already evolved like this, this surface is evolved like this, this point will evolve like this and therefore, it will be a circle.

Similarly this point will evolve as a circle over here; that means, only a quadrant because when it meets here, it is already reached this point; when I talk of this particular point, well it is a quadrant over here and if I move over here it is something like this is the initial surface, this is the outer surface well it is quadrant over here. What is that radius of this quadrant of this particular circle? It assume all through a distance delta, delta therefore, radius that is delta again; that means, if I have a point the burning rate is r, in the first second it comes to r, second second it comes to 2 r, third second it comes to 3 r and so on. Therefore, this has now regressed by delta in all the directions delta, delta over here. Therefore, when the surface has sort of gone from the initial point to the final point which is delta away, what is the value of the burning surface area?

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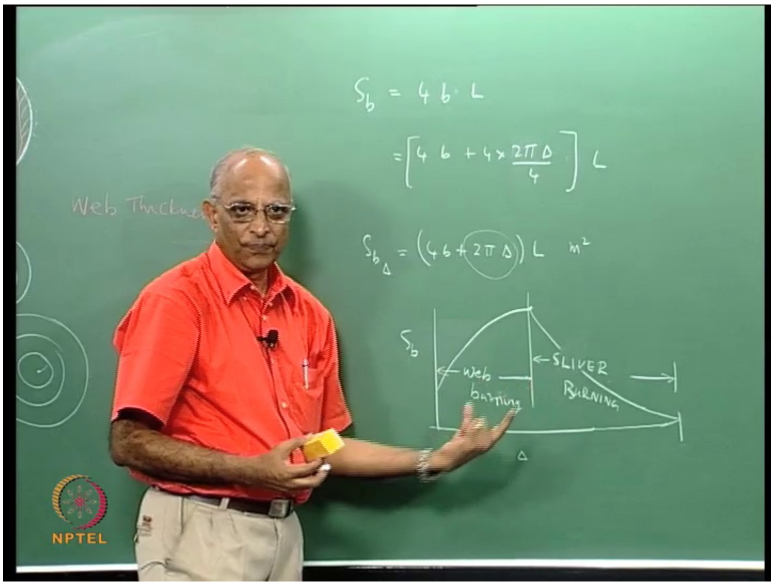

We told ourselves at the beginning it was just 4 L into 4 b into the length of the grain, how does each b evolve? Each b now becomes 4, b is as it is; this remains b, this remains b, this remains b, this remains b, what is the additional value I get now? I get four quadrants, four into each quadrant has a radius delta; that means, pi D by pi delta or radius is delta; therefore, 2 pi r, 2 pi delta divided by 4 is the perimeter of my quadrant into the length or rather I get the burning surface area, when the grain has regressed by delta is equal to 4 b plus I have 2 pi into delta into the length L, so many meter square. I think we should try to understand this; you know all what we are saying is, if I have something like a square with a vertex over here, the vertex burns in being a point it evolves as circles. But this surface evolves as a plane because burning is normal here.

Therefore, the vertex as it burns it meets here, it meets here it meets these two surfaces here. Therefore, I have a quadrant, the perimeter of this quadrant is equal to 2 pi into radius 2 pi delta divided by 4 is the length of this quadrant, let say from 1 to 2. Similarly, from 3 to 4, the length is 2 pi delta by 4, here I have 5 to 6, the same thing and I have may be 7 to 8; whereas, surfaces 2 3 is equal to the initial perimeter over here, this is equal to the initial perimeter, this is equal to the initial value, this equal to the initial value; therefore, I still have 4 b plus 2 pi delta into L meter square.

Therefore, I find that the burning surface area now keeps increasing for each delta like this and how long will it keep increasing? Let us now say let it burns for some other distance, let us just do that; such that we are very clear about it. If it, if the burning process let say by some amount delta over here, now my new delta is delta 1, I know a delta is delta 1, delta 1 over here, delta 1 over here, delta 1, delta 1, delta 1, delta 1. That means, I have another arc of a circle, another arc of a circle, straight line, arc of a circle straight line, straight line over here, arc of a circle. And, therefore, my burning surface area in this case is 1 2 3 4 which is same as b plus, now I have this quadrant, this arc of a quadrant, this arc plus, this arc and corresponding to my new value of delta I get this value.

And, I find delta keeps increasing; therefore, S b now keeps increasing with time and rather instead of having the grain wherein diameter, if I had something like a circular hole, you know the diameter directly increases. In this case, I get little effect over here. The burning surface area would keep on increasing still further and I get S b as the value of delta keeps increasing, I will get something like burning surface area keeps increasing like this it reaches this limit. When does it reach the limit? When this particular tip or vertex comes and hits over here, then I have a circle of diameter over here and the plane surface comes over here; then when this comes over here at that time, then this is again a circle over here, comes over here, hits over here, hits over here.

Similarly, this one comes and hits over here, for I have a circle like this; plane surface over here. And, this is the limit till then the burning surface area keeps increasing in the amount 4 b plus 2 pi into the gap which gets burnt. Once, this happens you know the, this surface keeps now decreasing; because it has already reached the case whereas, this fellow continues to burn in this direction. Therefore, once which I now show dotted over here, once the vertex comes and hits the case thereafter the burning surface area should decrease; because at the next instant of time this comes here the area decreases. And, therefore, the burning surface area may be decreases like this and ultimately becomes 0. This is now the way they burning surface area changes.

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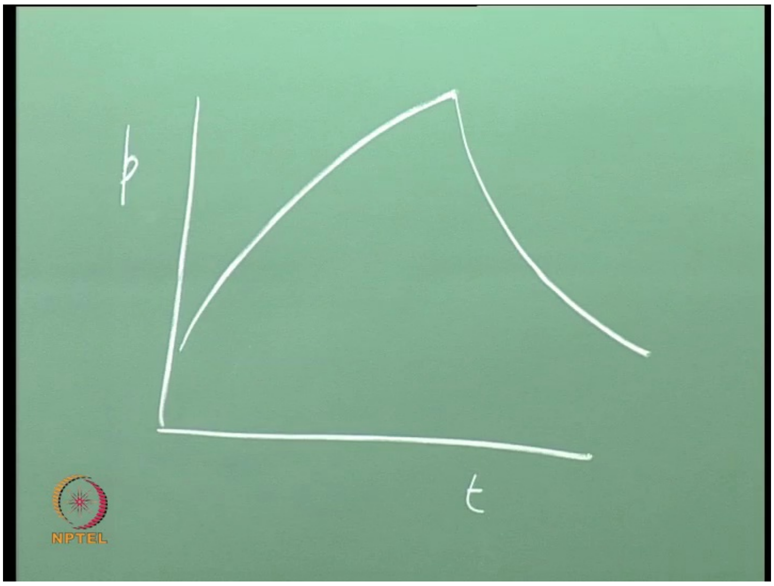

And therefore, the pressure in this particular motor which has a square inside will be, may be pressure with respect to time will evolve something like this and comeback like this. But you know looking back at this let us again define this; the minimum distance between the surface or any point on this surface and the case we defined as web thickness. Let us again take a look at what I mean by this web thickness, I will show it in this figure again. If I have something like a outer diameter over here and the inner diameter over here D i and the outer diameter is D o, the thickness of the grain is equal to D o minus D i divided by 2, right. This is the thickness of the grain; that means, the grain starts burning here, when the thickness is so much it gets totally consumed.

Now, if instead of D i being a circle, supposing I were to put a square over here at the centre; let us now say I put a square here. In other words, now the grain is of this shape. Now, what are the thicknesses I can talk of? This thickness is a little larger, this along the vertex may be from here if I plot a line this is the minimum distance and as the grain evolves, it goes as a circle and first touches here; because here the thickness is larger at that point it is coming only over here. And therefore, this thickness between this point to the casing is the minimum thickness and this minimum thickness is what is known as a web thickness. Why do I call it as web thickness, because still the web thickness is consume, the burning is progressive or the burning surface area keeps increasing and once the surface comes over here; that means, it comes like this and then it comes, comes over here up to this point, comes over here up to this point, comes over here.

Thereafter, what is going to happen this particular perimeter keeps decreasing; whereas, this fellow is still constant. And, therefore, what is going to happen is, the burning surface area will keep decreasing and therefore, the progressive burning is up to the web gets consume; this is known as web burning and this part wherein the web is consumed, but still some propellant is left is what is known as sliver left over sliver burning. Therefore, in such rectangular hold grains, you find that sliver occupies quite some space and therefore, this sliver you know do not want such low thrust, low burning surface area and that is the reason why this rectangular once are not used in practice.

But now if I go to a star with which I am interested, you know what did I do in the case of star? We had something like a circle, I wrinkle the surface such that I get a star shape I have a number of star points over here let say; what is it I am doing? The web thickness will be from this to this, this is the, this is my minimum thickness, this is the web thickness whereas, you know here the thickness is much larger. Therefore, I am interested in making sure that the web burning distance; that means, this particular portion is quite significant and my sliver, sliver is the length left over burning is quite small; this is how a grain design is done. Therefore, what is it? Inadvertently we introduced some words like web burning and left over known as sliver. With this background, I think we are in a position to be able to calculate how the burning should proceed for a let say a star grain, maybe I should do that now.

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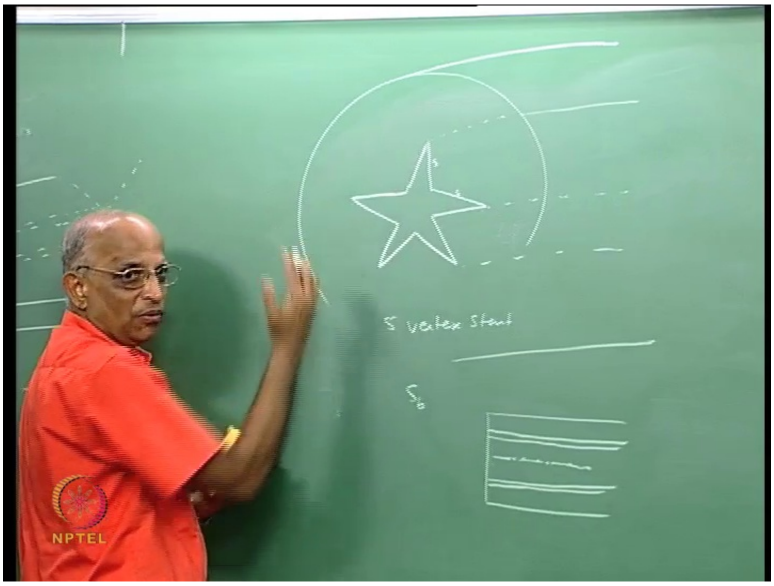

You know all of us by now should be very clear namely by a star what I mean is, I wrinkle the surface such that I get something like a star surface; let say I just have 1 2 3 4 5, 1 2 3 4 5 that is of 5 vertex star. I just choose anything vertex star, it could be 7, it could be 8, many are used you know. How do I characterize; that means, my outer cases over here, I am interested in finding out how the burning surface area keeps evolving with time. Let us say that the sides of the star are s and how does the grain look in the three dimensional plane? Well, this is my outer surface; well, all these are points here. And, therefore, if I were to make a plan view of this, I get a cylinder, I get this my center line, corresponding to one vertex I get a line over here it should be dotted, I get another line over here corresponding to the other vertex, I am interested how the surface keeps evolving? Now, you could tell me, if I were to now calculate the burning surface area initially all what I say is well.

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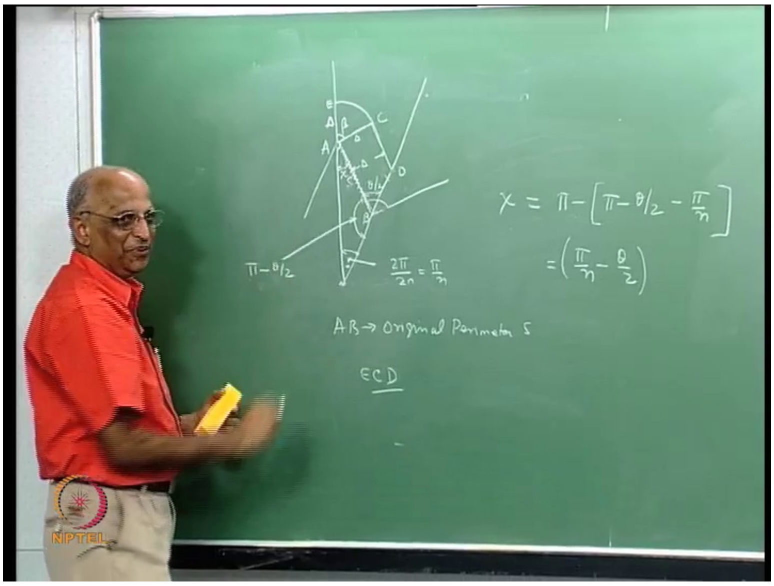

Let me have a star grain with n vertices. And, I tell well, I have something like n of these vertices, this is my second line and each of these lines has a dimension s and therefore, what is my initial burning surface area? It is an n pointed star; therefore, what will my initial burning surface area? The length of the grain is as we said L yeah.

(( ))

Fantastic, yeah 2 into n into s into.

(( ))

L, correct; so much meter square, if S is in meters. I want to know what will be the shape of or what will be the value of this length, as it burns when the grain progresses through a distance let say delta again. Let say the burning surface area now comes over here it moves through a distance delta. I want to calculate what will be the configuration of this particular point? Can I choose some axis of symmetry or something and still do the problem? Let me come back to this figure, may be it will become a little more clear. What is going to happen, we say this is the point it will burn as circle. Here the regression will be normal to the surface; that means, this surface area should come like this. But then this surface area should again go like this, some interference here; I think we have to be very clear. But I can say well, this is going to be my access of symmetry here; because over here and over here it is symmetrical it has to burn like this I cannot have these 2 points.

Therefore, I can have an axis of symmetry here and now with this axis of symmetry, I can now tell myself, well my center is over here, I have an axis of symmetry which is here; this side and this side are symmetrical. Therefore, if I can calculate for this single surface S over here, how it is going to change the value of S; then this value of S, if I can calculate when the grain moves maybe by distance delta, if I can calculate the new value of S 1 let say or the new value of S; then I can say S into L into now I have 2 n of these surfaces 2 n s L will be my burning surface area. How do I calculate this particular new length? Let us do that; the burning surface area should be should proceed normal to the surface, let say it moves by distance delta. Therefore, when it moves by a distance delta this line comes over here; that means, let us now put some names to it, because we are complicating the issue a little bit; this is A B is the original length. (No Audio From: 39:51 to 39:59) S for 1 nth of the total surface, right.

Now, when it has burnt through some distance, this is normal and it has now come from A B to C D, this is the new dimension. What happens to this particular point? Again, I find this is going to be a symmetric line, if I can find out how this area, how this line is evolving; the new length of this line multiplied by 2 n times, multiplied by the length is the new surface area. Now, what is left in this, how does this point evolve? Circle delta therefore, it evolves as a circle, this becomes delta here; let say this becomes delta over here this point is E. Therefore, I find that this particular line on the inner surface now becomes partly the same straight line parallel to this and partly this.

How do I calculate this particular total length, now I need to calculate the length E C to D; how do I calculate? Is the problem clear? All what we are saying is, I have n vertices, I choose a symmetry plane such that I am interested only in examining a single (( )), single line over here. And, whatever happens to this line, will happen to this, will happen to this, will happen to this and therefore, I am looking at this and therefore, I looked at this particular line, it I would like to know; what is the value of E C, when this line has progressed by a distance delta?

Now, you can tell me. As I keep telling, you know the progression of a surface is all what matters it is a geometric problem; there is no combustion there is nothing. You know all what we need to know is what is the value of the new burning surface area, when the burning at the surface has progressed by a distance delta over here? Now, I need to solve this, how do I determine the value of E C? I know that the radius is delta, if I say this angle is let say capital beta or just beta over here, I can now say E C over there is equal to delta into beta or theta. And, I should also calculate the value of C D; these are the two things I must calculate. Now, how do I get the value of E C? You know; well, let us tell ourselves in this particular star let the total angle be theta, in other words I had star doing like this, let this angle be theta; therefore, this half angle which I am talking is theta by 2. In other words this angle, this small angle is equal to theta by 2.

What is the angle at the center which is included by this particular side S, what is the value of this angle? Let us calculate this angle, what is the value? 2 n vertices have an angle which is equal to 360 degrees that is 2 pi; we have n vertices and now each of the n vertices consist of 2 lines; therefore, the angle included is 2 pi by 2 n or rather pi by n is the included angle for this side at the center. What is the value of this angle? This angle we said theta by 2, this is 180 minus theta by 2 or now this angle is equal to pi minus pi is 180 degrees minus theta by 2 over here. If this angle is pi minus theta by 2 and this angle is pi by n, what is the value of this angle; let say this angle is chi; chi is equal to. Calculate this angle, sum of the angles of a triangle is equal to 180 degrees pi minus the value over here is pi minus theta by 2. And, I have a value of minus pi by n or rather this is equal to pi by n minus theta by 2, this is the value of chi. Therefore, what is the value of beta now? See, it is a simple geometric problem, you know and we must be able to do this because any evaluation is done like this.

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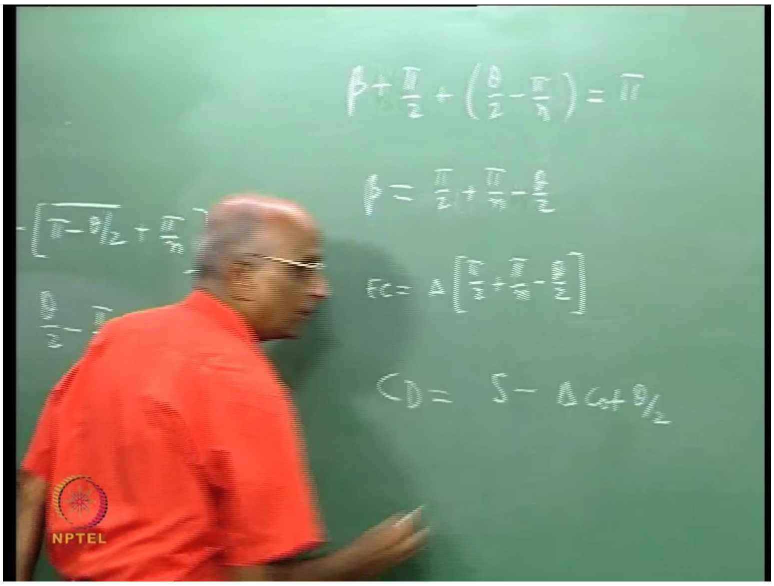

Therefore, now what is the value of beta? This is right angle over here; therefore, this is equal to 2 pi, this is equal to pi 180 degrees pi minus pi by 2; therefore, we have beta let us put it beta plus pi by 2; 90 plus the angle chi, chi was equal to pi by n minus theta by 2 is equal to 180 degrees which is equal to your pi. Therefore, what is a value of beta from this? This is equal to pi by 2 minus pi by n plus theta over 2; beta is equal to pi by 2 minus pi by n plus theta by 2. Therefore, we have found out this angle and therefore, the length of the line E C is equal to delta into pi by 2 minus pi by n plus theta by 2.

Let us check the angles; is there anything, can we just check it? What did we tell ourselves this angle is pi by n, this angle becomes pi minus theta by 2; therefore, this becomes pi minus theta by 2 plus pi by n, now this is one angle. See, this value is equal to pi minus theta by 2 and this is pi by n and therefore, this is equal to. And, if it is theta by 2 minus pi by n, let us write it out over here, we change this into theta by 2 minus pi by n and therefore, the sign gets changed over here plus and minus. And, therefore, the angle is equal to pi by 2 plus pi by n minus theta by 2 beta plus pi by 2 and this value is theta by 2 minus pi by n is pi and therefore, this becomes positive on this side, this becomes negative and therefore, E C is equal to this value.

What is the value of C D now? How do I get this value? Immediately, I tell myself this is also right angle I can have a right angle over here, this distance is delta. And, now if I can subtract this distance from S; that means, it is S minus and what is it I am subtracting? The base of this, this particular perpendicular over here; that means, base divided by delta must be equal to cot theta by 2 or it is equal to delta cot theta by 2 S minus delta cot theta by 2. Let us make ourselves clear, this is delta, this angle is theta by 2; therefore, the base is equal to base by delta is equal to cot theta by 2 or base is equal to delta cot theta by 2. Therefore, what is the total surface area or total perimeter for this particular single stretch of line?

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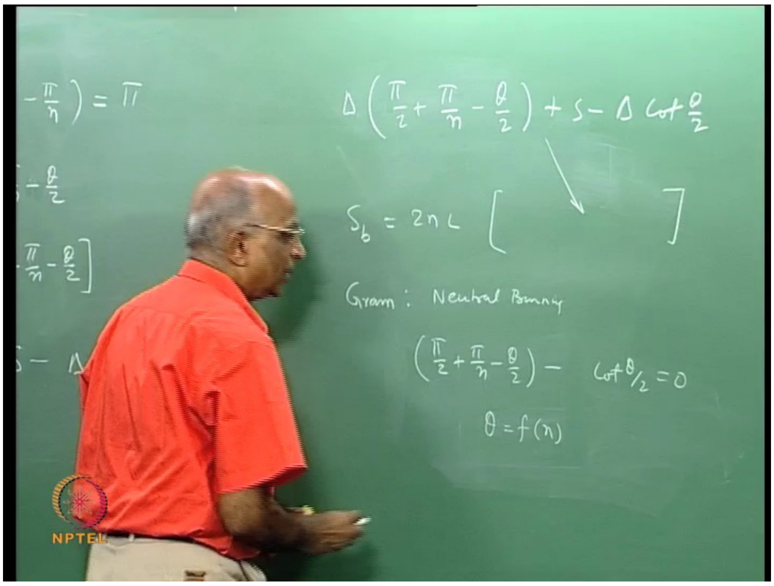

The single stretch of line, now gives me delta into pi by 2 plus pi by n minus theta by 2 plus S minus delta cot theta by 2. Therefore, what is my new surface area, when the grain has receded by delta? S p is equal to I just have n 2 n surface 2 n of the S perimeters into the length into this particular value is what gives me the new burning surface area. If this is there supposing, I were to form; I want to form a grain a particular star grain which is let us say a neutral burning. If it is neutral burning, what must happen? It must always be S; in other words for neutral burning it is necessary for me to have delta into pi by 2 plus pi by n minus theta by 2 minus delta cot theta by 2 is equal to zero. That means, I will have neutral burning. And, what is it I get? I solve this equation, I find delta and delta gets canceled and rather I get cot theta by 2 minus this particular value is equal to 0.

And, now I find to solve this, since it involves a cot term I cannot do it directly, I do it numerically; may be I use the method of steepest descent or something and now I find that theta is going to be a function of the number of vertices.

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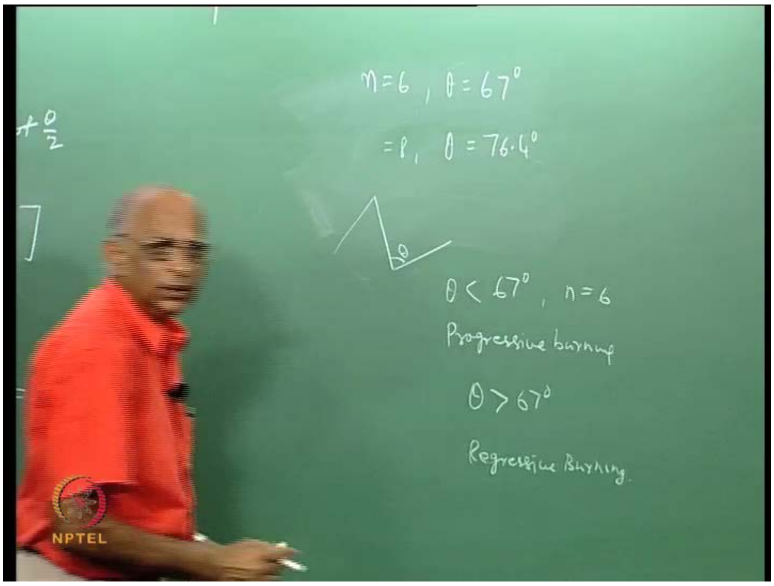

And, if I have a grain which has the value of n; that is number of vertices are 6, the value of theta comes out to be 67 degrees. If the number of vertices is 8, the value of theta comes out to be something like 76.4 degree centigrade degrees over here. In other words depending on the angle theta over here, I could have neutral burning and if the angle is less than theta, what happens? If the angle is less than theta well the surface area keeps increasing because I am subtracting a smaller quantity. Therefore, if theta is less than 67 degrees for something like a when the number of vertices are 6, I get progressive burning.

Whereas, when theta is greater than 67 degrees; that means, when it is equal to 0, I get 67 degrees for n is equal to 6, I get regressive burning. Therefore, what is it I have said. You know star grain, we started with a star grain and now we find the conclusion that depending on the value of theta or rather the thickness of this line, how did we get a star grain; we got like this, I could have had the star grain something like this, I could have a different value of theta. Depending on the value of theta what I have, I could have either neutral burning or progressive burning or regressive burning and that is why star grains are quite powerful; wrinkling has that effect. If I want a grain shape in which initially it should come and something like this, I can always build it into the star gain and that is why it is so versatile.

Therefore, let us just conclude by taking a look at these particular slides. You know this was for the rectangular grain; we said towards the end, when the vertex meets the particular outer diameter then it the balance is what is known as a sliver and you have web burning up to this particular time, differentiated into web burning and sliver burning. And, then we came to the star grain and for the star grain we did the same thing we calculated this angle, we found that we had to we found that this is equal to S minus delta cot theta or this is the total value of the burning surface area. Using the burning surface area, the equilibrium pressure and the thrust of the rocket are calculated.

And, then we got the value as theta is equal to 67 and you should have been, I think I gave the value of 76, it should have been 74.6; please correct it 0.6 degrees for neutral burning. And, this you all can do using a numerical method. That means, we will get regressive burning for larger values of theta and progressive burning for this; therefore, a star has the capacity to be designed for whatever be the type of burning we desire. This is all about evolving burning surface area in different propellant grains. In the next talk, we will look at the other elements of solid propellant rocket. Thank you then.