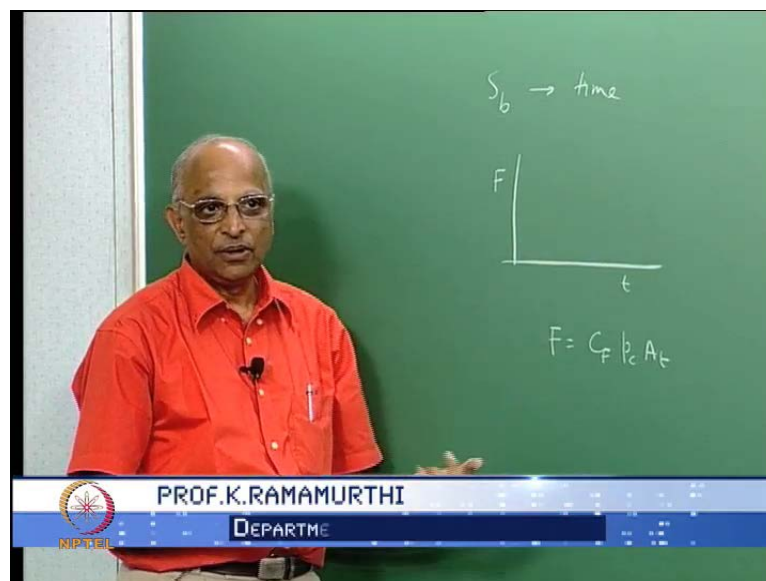


**Rocket Propulsion**  
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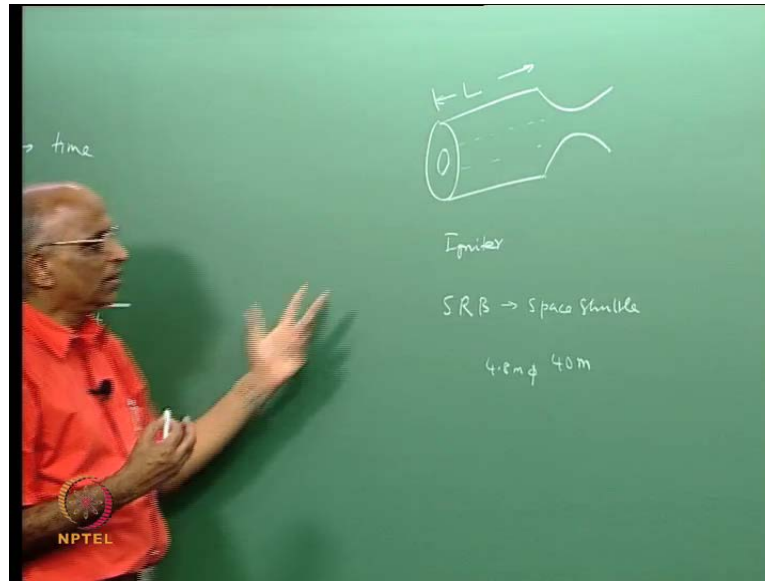
**Lecture No. # 26**  
**Ignition of Solid Propellant Rockets**

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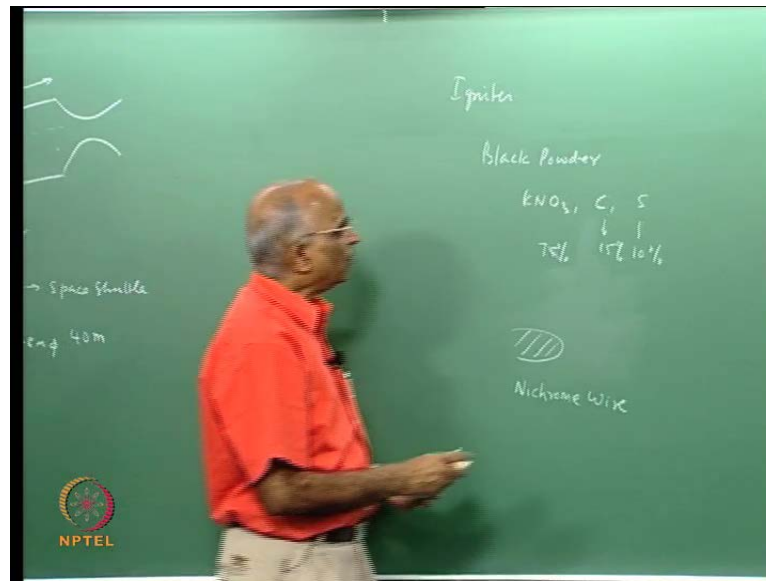
Good afternoon, then we will continue with solid propellant rockets. We have seen how the burning surface area can be calculated to evolve with time and therefore, I can now say or I can now determine how the thrust of a solid propellant rocket will change with time, because once you know the burning surface area, you can calculate the value of pressure in the motor, then the equilibrium pressure and equilibrium pressure into thrust coefficient into the throat area namely  $F$  is equal to  $C_F$  into chamber pressure into  $A_t$  is what gives me the thrust. And now we have we how did we calculate the equilibrium area, based on the burning surface area which we have considered in the last, but one class. Having said that let us now go back and ask ourselves some more questions, see we have considered propellant grains of different shapes.

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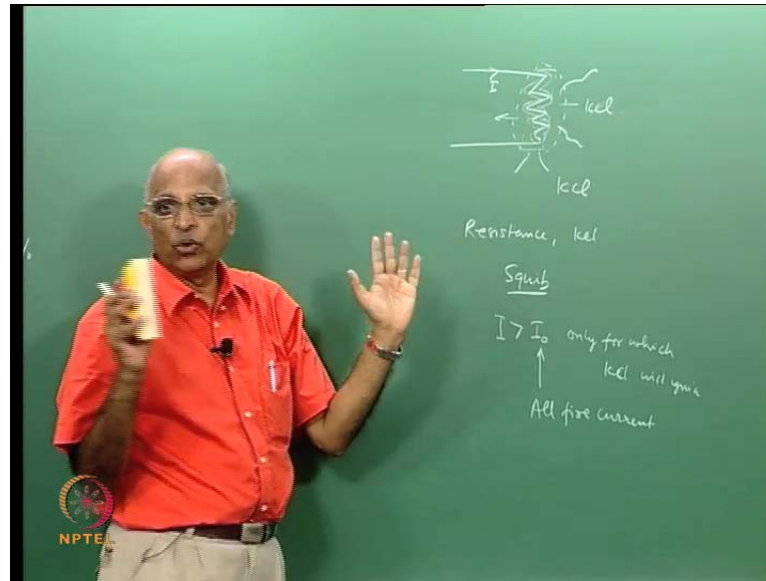
And we said well I should have a nozzle. We said the grain could be internal burning or the end burning and all that. Now the question is how do I ignite the propellant grain, how do I start the burning in the grain, how do I ignite it. Namely, I must use something like an igniter, which will start the burning process, but then we also know that the grain surface is quite large may be it could be something like several meters, like for instance if I take an example, of the world's largest solid propellant rocket, we call it as solid rocket booster for the space shuttle. It is something like 40 meters long the grain therefore, the question is how do I make sure that the grain surface ignites, and how do I ignite a motor that is what I will be talking to you in the first half of the class.

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Namely how do you make an igniter, what should be the igniter of? Well, we can immediately say igniter is something which catches fire soon. And therefore, maybe I will use something like a black powder, which is used for fire crackers, and this consist of may be potassium nitrate, may be some amount of carbon some amount of sulphur, typically around let us say 15 percent carbon may be 10 percent sulphur and the balance what I have 25 may be 75 percent of K N O 3 here. And this is something where you can easily ignitable composition, you know when we make these fire crackers with just a match box it begins to flare up you have something like a like let us say I put the black powder over here, I put a small brick I light it over here, and I get sparkles and things coming over here. Therefore, maybe this could be one of the contenders and therefore, if I were to use it how will I use it in case of solid propellant rocket is the question. Therefore maybe I could have a bag, and in the bag I put this particular composition, but I want to ignite it, how do I ignite it may be I take a resistance wire. May be an electrical resistance wire may be a nichrome wire. And why nichrome wire nichrome wire has high resistance, if I pass a current through it is get hot heat heated soon.

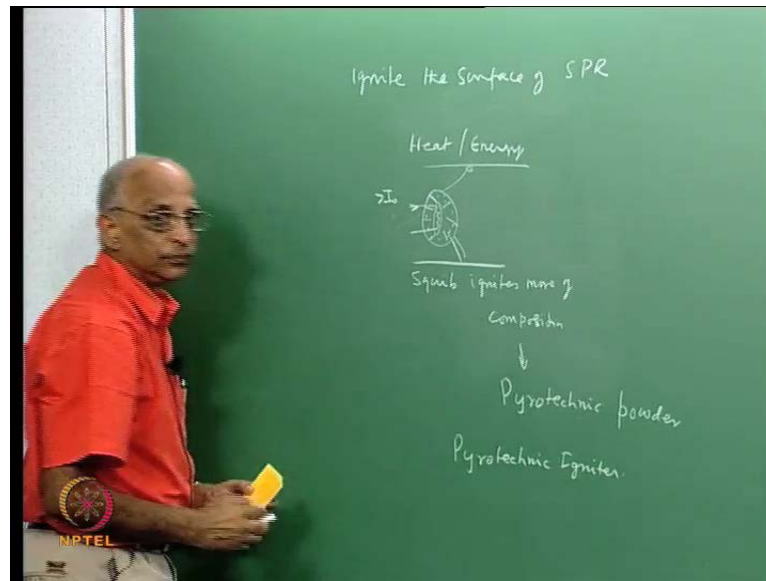
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And therefore, I take this particular resistance wire over here, I coat it or I put on this surface of it some easily ignitable composition may be black powder or something like  $KCl$  which immediately catches fire that means, I have something like  $KCl$  which is in the vicinity of the nichrome wire I coat it with this. And then may be surrounding it I put more of this composition,  $KCl$  or the black powder which I am just talking of and then I pass a current through it.

When I pass a current through it the wire gets heated  $KCl$  begins to burn it generates heat and it generates flame, and this flame could be used for ignition. A simple way this particular arrangement of a resistance wire or heated by electricity or electrical energy and using some  $KCl$  or black powder is what we call as squib, but you know whenever I use electrical current for heating it is also possible that I could have something like in a when I have some disturbances or electrostatic discharges. I could have a current and even when I do not want to ignite I could have a small current which could heat it. Therefore, it is necessary to make that I should have  $I$  greater than some threshold value only for which the composition like  $KCl$  or black powder will ignite. And this threshold value of current is known as all fire current. And if the current is less than the threshold value even by stray electrostatic discharge it will not catch fire and my motor is safe. Why am I telling all this let us again ask ourselves a few questions?

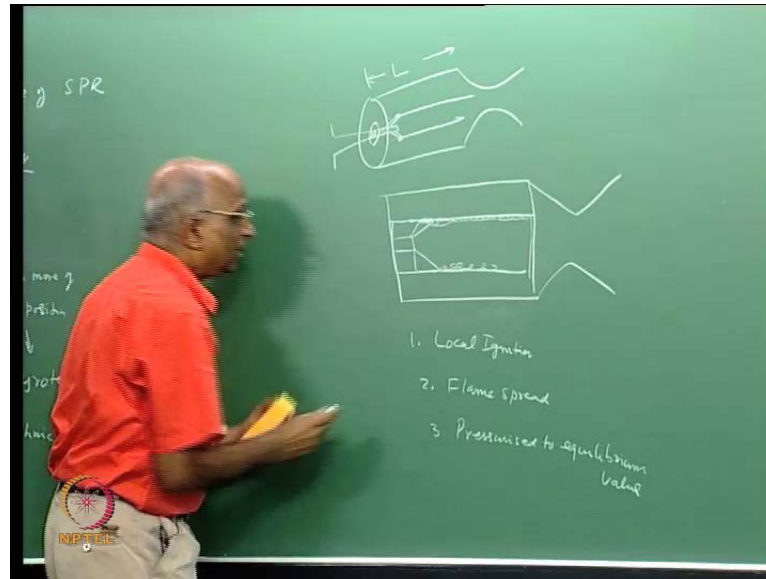
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To respond to this all what we are saying is maybe I want to ignite or burn the surface of a solid propellant rocket. And to be able to ignite this solid propellant rocket I need some heat or some energy. And therefore, all what I do is I need to release the energy I cannot go and put a fire inside it. Therefore I have a composition I put something like a nichrome wire start the ignition process this generates heat over here, but then a squib has only a small quantity of charge like KCl charge or black powder charge. I put maybe some more charge surrounding this therefore, the squib when I pass a current greater than some threshold value of current I need about 100 Ams. It starts a chemical reaction of KCl which is easily ignitable it burns more of the black powder or some composition around that is squib ignites, more of the composition more of a composition which is again easily ignitable and then a fire is formed and this fire impinges on the propellant surface and makes it catch fire. That means, I have a squib surrounded by more of these powder and this powder or composition is what we call as could be black powder, we call it as pyrotechnic composition or pyrotechnic powder.

And therefore, such igniters which make use of a squib with little more black powder around it to generate sufficient energy to ignite the propellant is what we call as pyrotechnic igniter therefore, we tell ourselves well igniter could be simple, but to be able to look at this problem a little more.

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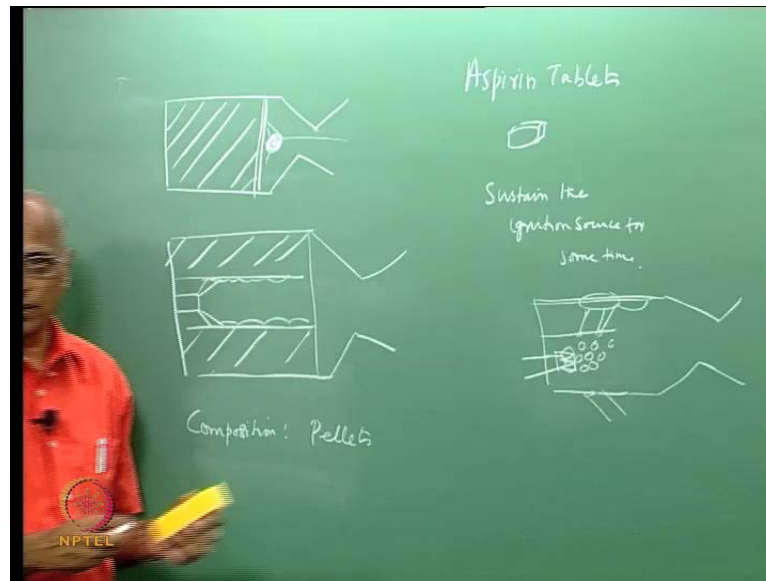


In detail what we are saying is well I have some electrical current, I have a place where in I have some battery or something. I energize it I put I put something like an igniter over here. And then I pass a current through it I ignite the squib the squib ignites the powder which is around it, and it sprays the flame that is a plume which is formed come and ignites here this propellant surface ignites the volume pressurizes and the flames spreads over and therefore, it ignites. See now I told something else what is it I said this is the internal surface of the grain, this is my outer surface and just for the sake of simplicity taking a radial grain, this is my nozzle I put an igniter over here, I pass a current may be some flame is coming like this from the bag, it ignites it impinges over here ignites this surface when this surface ignites then next fellow ignites here, this fellow ignites here again this fellow ignites this.

And therefore, something like the entire surface ignites in other words I have first something like local ignition, where in the sparklers or the plume impinges on it. Then I have a flame which is spreading over the surface, and once the flame spreads the pressure may still not be the equilibrium value, and the last one is the pressure that is the chamber gets pressurized to equilibrium value.

These are the 3 things which could happen, if I have a bag like this, but you know having a bag is not the answer for instance. Supposing I have a small rocket like an end burning charge or an end burning grain let us let us plot the case.

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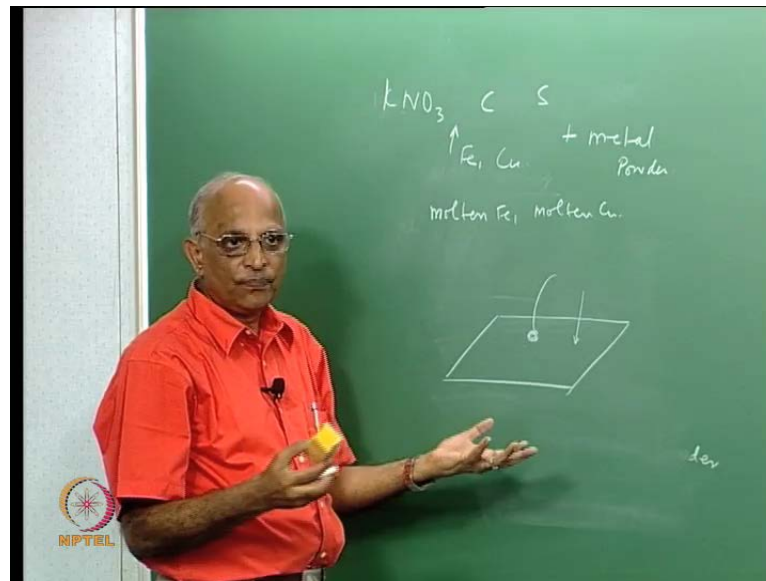


Where in I have a solid propellant like this but it might end burn in charge what I have I have the nozzle over here. I want to ignite it what I do is from the nozzle side I put a bag of pyrotechnic charge, inside the bag I put a squib I ignite the pyrotechnic charge over here, a flame is formed it impinges over here it makes this fellow catch fire.

Whereas if I something like a radial burning grain or a grain like a star grain which burns from inside to the outside. I put the igniter over here; maybe I would like a part of the thing to catch fire. And then this spreads over here the entire propellant catches fire and that is how I ignite a propellant surface. Now if I have a bag I cannot have controlled burning and therefore, very often this composition or pyrotechnic composition. What we have is compressed and made in the form of pellets. What do you mean by pellets? You know we take this Anacin and all that it is in the form of some pellets which we say is aspirin in that as Aspirin tablets. Instead of having a powder you form pellets like this small pellets like this. And what is the advantage of having solid pellets like this. The burning surface area can be controlled it does not like a powder immediately burn it take some time to burn and therefore, it can give better ignition. That means, I can sustain my, sustain the ignition source for some time the instead of having a pellet what we can do is, I could have something like a firm bag or a let us say a cylindrical thing something over here.

I put lot of pellets in it pellets of the pyrotechnic I put the squib over here the squib is coated with K Cl I ignite it may be I make some holes here, through which the flame plume goes out, ignites on this surface have local ignition followed by flame spread and that is what is the function of the igniter, but what is the requirement of an igniter. You know if I have something like I told you may be black powder can be used

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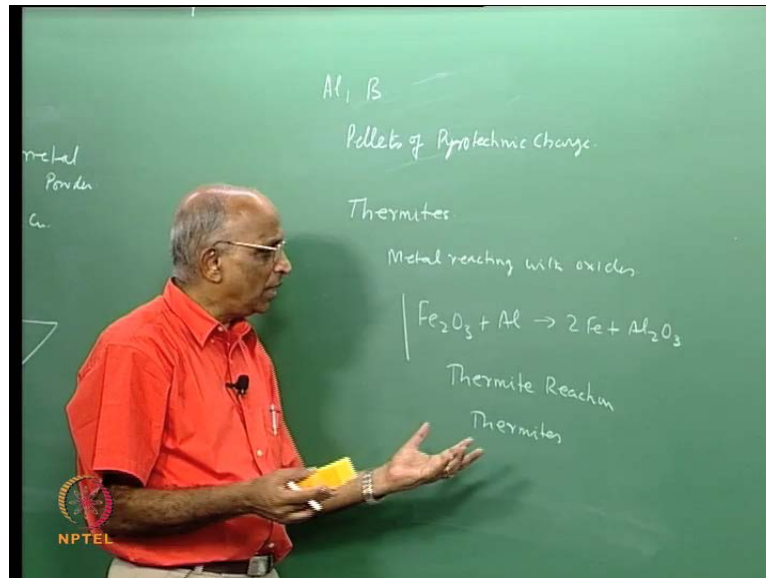
And it consists of K N O 3 carbon and may be sulphur. You know the products are essentially gaseous except for little bit of carbon over here, but all of us know that if I can heat a metal. If I can put some metal into it like let us say I put iron or I put something like copper into it. You know when I heat iron or copper the temperature or if it is high I get molten iron may be molten copper. And what is the advantage of using this metal powder is a gas cools down when it expands, whereas a solid retains heat for some time. And also if I have a surface and on the surface may be a molten iron falls it will translate the heat of the molten iron into the surface more effectively then a gas will transform it.

And why is it a molten iron or a molten hot substance is in better contact with the surface, and it is able to able to sort of conduct the heat to the surface much more effectively than a gas. Therefore for most of the compositions which we talk of we also add metal powder. And why do we add metal powder the reason being a metal sort of conducts heat much better onto a surface than a gas. You know the simple experiment



which we saw when we had a sparkler, when we did not have metal it was not that violent, but when I put metal filings on to this sparkler. You know it sort of burnt my hands if the **if the** sparkles fell on my hand and that is the reason metal is able to conduct heat much better. Therefore, the pyrotechnic composition will consist of may be some metal powders and metal powders which are used include aluminum and boron. These are also used in the along with may be some stuff like this, plus may be pellets of the pyrotechnic powder.

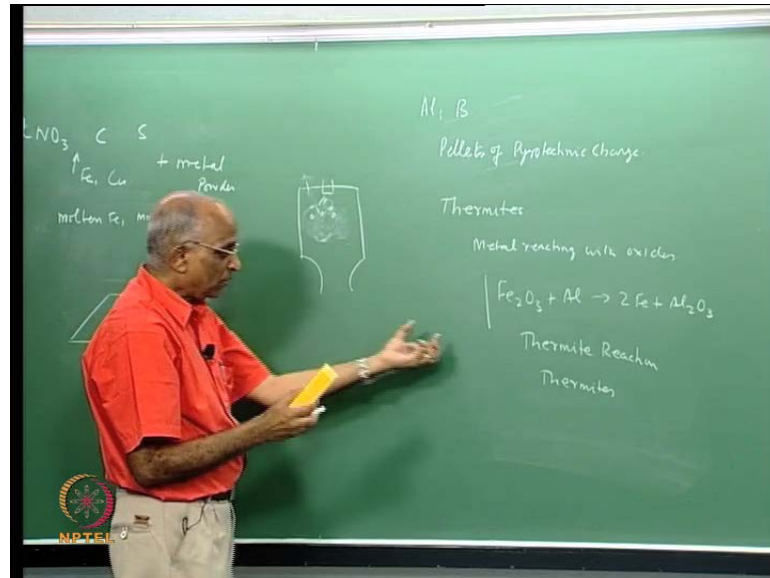
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We call this as charge and this will also include some metal powder in it, this is what an igniter should consist of, but there are some basic issues and let us let us try to result some of these issues. You know instead of having metal as such you know there are certain substances known as thermites, you know this is something which is particularly interesting. Thermites are substances in which I have metals reacting, metals reacting with oxides. What is it I mean, if I take something like  $\text{Fe}_2\text{O}_3$  and react it with aluminum what I form is  $2\text{Fe}$  plus  $\text{Al}_2\text{O}_3$ . This reaction is very exothermic and what I form is molten iron and aluminum oxide, and if I can use this as an igniter well it is a it has a metal constituent in it will touch the surface it will ignite much easier and such reactions are known as thermite reaction and this are known as thermites actually.

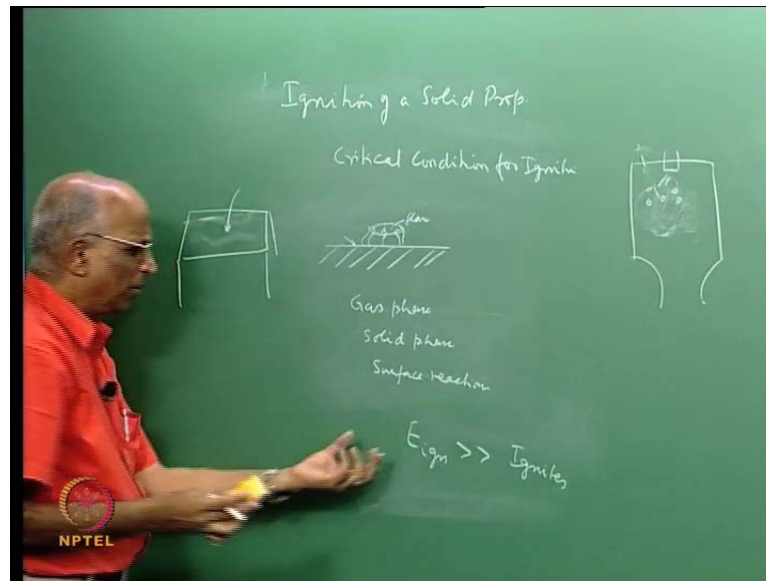
You know this is the time to look at it, because we find work going on or research work going on in the area of nano thermites and stuff like that which are more effective in producing heat. In fact, if I want to ignite now instead of a solid propellant rocket.

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If I want to ignite let us say a hydrogen oxygen rocket. I have hydrogen oxygen which must be ignited. And one of the contenders is this you just put the thermite in it may be I have an igniter over here. I spray Fe<sub>2</sub>O<sub>3</sub> plus Al and I ignite it, I get molten substances and molten substances retain heat for a long time it will ensure that the gas gets ignited. Therefore, this we will call as a thermite igniter. Therefore what is it we have considered so far? We said an igniter could consist of a composition which generates something like a plume or a hot gas jet or if I were to put metal in it will also create some metal or some hot metal which will transfer heat to the surface better and I can sort of give rise to ignition of a solid propellant.

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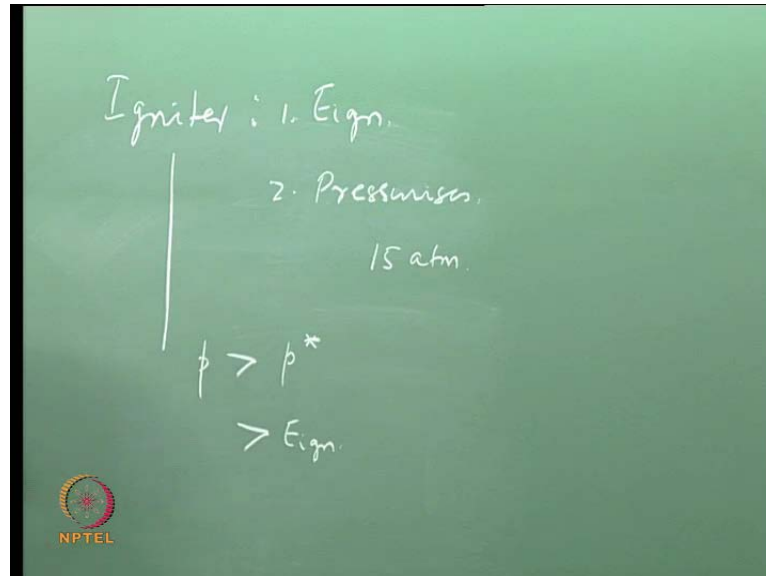


When we say ignition of a solid propellant, what is the mechanism by which a solid propellant ignites we will not go into the mechanism, but all we will say is. Well when heat is transferred to a solid propellant some vapors of the hydrocarbon may come a p could dissociate into mono propellant flame, these things could mix together form a flame. And therefore, the process of ignition could happen in the gas phase, where in vapors ignite it could happen at this at the **at the** solid surfaces. Where in may be some dissociation could take place, it could happen in solid phase, or it could be something like a gas contains some substances which could readily react at the surface giving rise to surface reactions. All these three are possible, but it is difficult to say which one dominates where. And we will assume that all 3 reactions take place which lead to ignition of a solid propellant rocket.

That means, a solid propellant if I have a slab of solid propellant this is the surface I consider may be I transfer some energy to it, and the solid part of it some reactions take place in the gas some reactions takes place, some surface reactions takes place. I could model it using any of these three theories or combination of these two or three theories and I could find out what is the critical condition for ignition. I will not get into details other than say well I should supply some ignition energy greater than some limit, such that a propellant ignites. This is a subject by itself, but we know that if my energy is sufficient well what is going to happen the plume ignites on a part of the surface and wherever the plume ignites my ignition gets over, but one thing which we all can readily

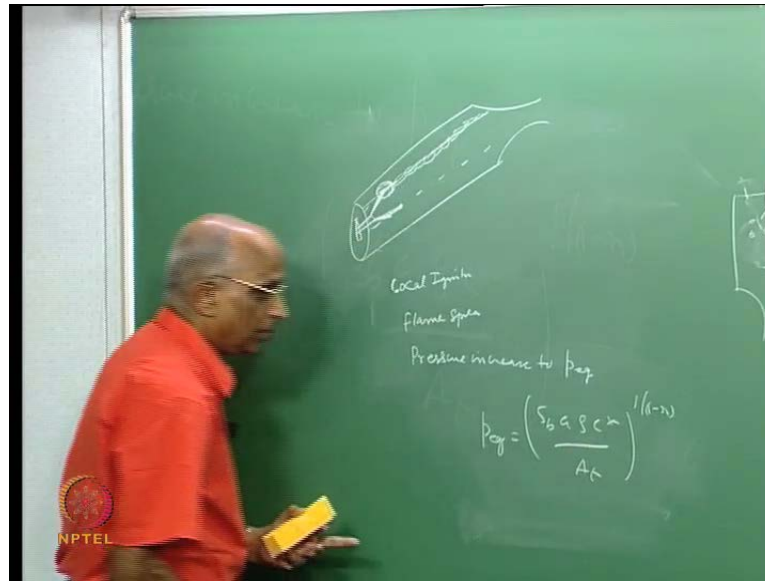
say is. If the pressure in this cavity is low my ignition energy must be large, if pressure is higher the flame surface will be nearer the surface therefore, I would like to have ensure that I have something like a minimum pressure.

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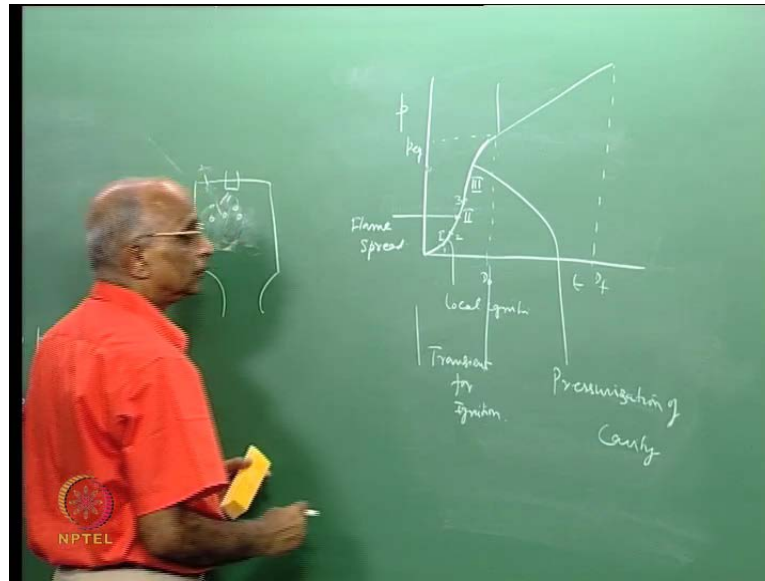
That means, the igniter role of the igniter is to make sure that I supply some ignition energy to this surface one, may be it pressurizes the chamber to some threshold value let us say 15 atmospheres or so such that what happens is when the pressure is low we say that standoff is little higher, when pressure is higher the standoff is lower. I make sure that the standoff distance is small such that the propellant surface ignites much better and this we will see when we study the question of instability. That if the pressure is greater than some threshold value the solid propellant combustion is more stable and therefore, the role of an igniter is to ensure that it pressurizes the chamber to a value greater than some threshold limit and also supply some energy greater than some threshold value this is all what is required. And what does the igniter therefore, do all what it **what it** does is suppose now I take a propellant grain I put an igniter here, it generates a plume may be I put some pellets here it impinges over here.

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If the energy of this surface or transfer to this surface is greater than ignition energy. And the pressure in this cavity is greater than some limit well this fellow ignites. Therefore, I have something like local ignition of a small part of the surface. And when this part ignites heat which is generated by this, plus the heat which is generated by the igniter helps to increase the adjacent amounts and therefore, the flame keeps on spreading. And I have a zone which I wrote as flame spread. That means flame spreads from the local ignition area over the entire surface of the propellant. And thereafter the pressure is still not equal to the equilibrium value and therefore, then thereafter the pressure increases to the equilibrium value. And we calculated the equilibrium value as equal to we said  $S_b \rho c^*$  by  $A_t$  to the power 1 over 1 minus  $n$  rho p.

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Therefore let us put this down on the on a on a figure such that we clearly understand it. What is it we are telling in the earlier lectures, we said that the pressure with respect to time may be the equilibrium value may be I am considering a radial burning grain? This is my pressure this corresponds to  $d_{naught}$ , when the surface is ignited this correspond to the burn out value corresponding to  $d_F$ . Now when I ignite the motor I start with ambient value, and then what is it I do I ignite the motor may be I bring it to this value.

Initially I have the igniter composition increases the pressure to some threshold value. Let us say one when it increases the value and heat transfer takes place I have let us say one to two, which is local ignition of a small part of the surface. That means, only this surface gets ignited initially, because the plume is impinging on it the metals impinge on it hot metals impinge on it ignites this and, because energy is released further from here further flow takes place ignites the balance and therefore, I have something like a flame spread two to three flame spreads over the surface entire surface over the propellant, I call this zone as flame spread and when once flame spreads over the surface I have reached this pressure which is still less than the equilibrium pressure. And then the pressure in the chamber increases or there is something like pressurization in the cavity.

These are the 3 processes namely local ignition followed by flame spread followed by cavity pressurization to the equilibrium value. And this is the equilibrium value to get started with and there after progressive burning takes place. We would like to write

equations for these 3 how I do it. How do I determine this pressure revolution and this is the portion wherein ignition takes place, and this time is known as the transient for ignition? Well let us try to write an equation for some of these processes, may be the thing is quite simple if we really get into the details it is nothing complicated we have done much more difficult things than trying to determine how the pressure should evolve

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$$\frac{dm}{dt} = \dot{m}_{ig} + \dot{m}_p - \dot{m}_n$$

$$\frac{dm}{dt} = \dot{m}_{ig} - \dot{m}_n$$

$$\dot{m}_{ig} + \dot{m}_p(t) - \dot{m}_n$$

$$\frac{dm}{dt} = \dot{m}_p - \dot{m}_n$$

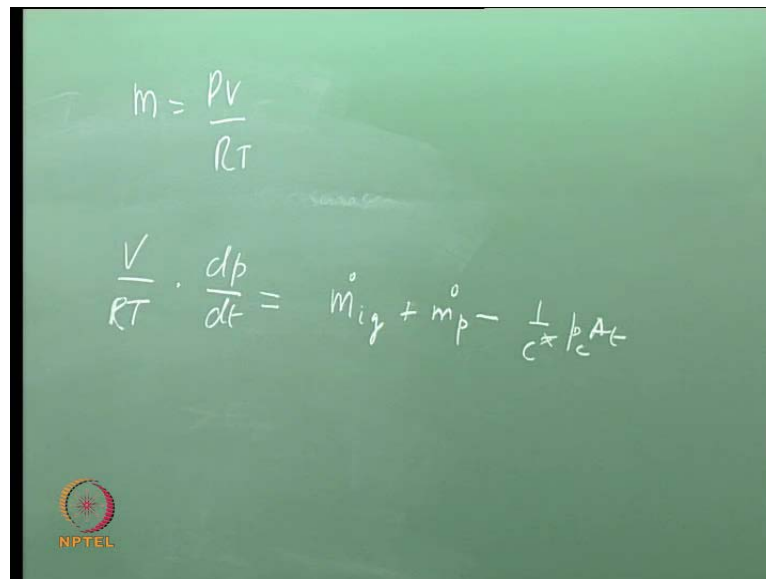
We would like to solve this. Well we say the rate at which mass is added by the igniter. Mind you during the process of flame spread, mass is not only ignited added by the igniter, but the initial burning phase is also adding igniting and as the flame spreads we are adding more and more of the propellant getting burnt.

Therefore, the rate at which mass is added to the propellant  $\frac{dm}{dt}$ , must be equal to the rate at which the igniter adds mass, plus the rate at which the propellant adds mass, let us call it as  $\dot{m}_p$ , minus of course, the mass at which nozzle leaves this. When we talk of local ignition the only the  $\dot{m}_{ig}$  is adding mass, because we are still talking of this. and therefore, in the first phase it is only  $\frac{dm}{dt}$  the mass added by the igniter is equal to  $\dot{m}_{ig}$  and when I talk of the second phase I have  $\dot{m}_{ig}$  plus  $\dot{m}_p$  which is changing with time as it is going of course, here also I should have had  $\dot{m}_n$  which is leaving, minus  $\dot{m}_n$  which is leaving. And during the pressurization time I have may be by then the igniter function is over I just have  $\dot{m}_p$  which is the whole

surface of the thing is burning, minus  $\dot{m}_n$ , which is leaving which is equal to  $\frac{dm}{dt}$ .

This is during the local ignition phase; this is during the period when I have flame spread also taking place. That means igniter is still supplying energy, part of the propellant surface which is burning is also supplying energy or supplying mass to the hot gases. And some hot gases are leaving and I have may be in the final phase, when the entire surface catches fire, and the pressurization of the cavity takes place. This is the equation to it I could solve these equations and determine the pressure how do I determine it I write  $m$  is equal to from the ideal gas equation.

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$$m = \frac{PV}{RT}$$

$$\frac{V}{RT} \cdot \frac{dp}{dt} = \dot{m}_{ig} + \dot{m}_p - \frac{1}{c^*} p A_t$$

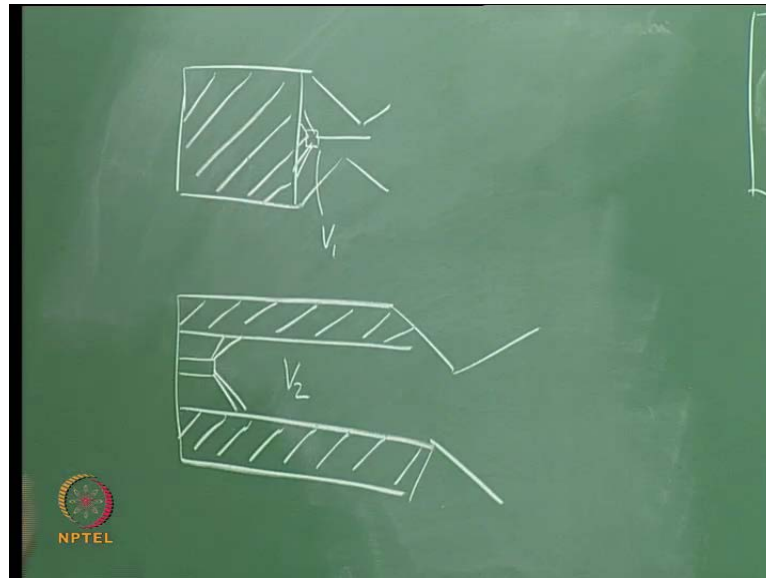
We get  $m$  is equal to  $Pv$  by  $RT$ , where volume of the cavity is the volume over here, that is the cavity volume and therefore, I can write  $\frac{dm}{dt}$  is equal to volume during ignition there is hardly any burning I can take  $v$  volume as a constant temperature of the products I take as a constant. And I write this is equal to  $\frac{dp}{dt}$  is equal to let me write it for the third 1 it takes care of the others, is equal to I take may be I write now  $\dot{m}_{ig}$ , plus  $\dot{m}_p$  minus what leaves the nozzle  $\frac{1}{c^*} p A_t$  where this is the chamber pressure.

And now I can find out what is the rate at which my igniter is supplying mass to it. I know  $\dot{m}_p$  is equal to burning surface area local means, the local area if the area is larger I know this  $A_p$  to the power  $n$  I can solve for  $\frac{dp}{dt}$  and I can determine the



thrust unit. Now, let us take a look at this figure once again, and try to see are there some changes are there some problems we could anticipate, is there I can solve it in some way in a better rational way.

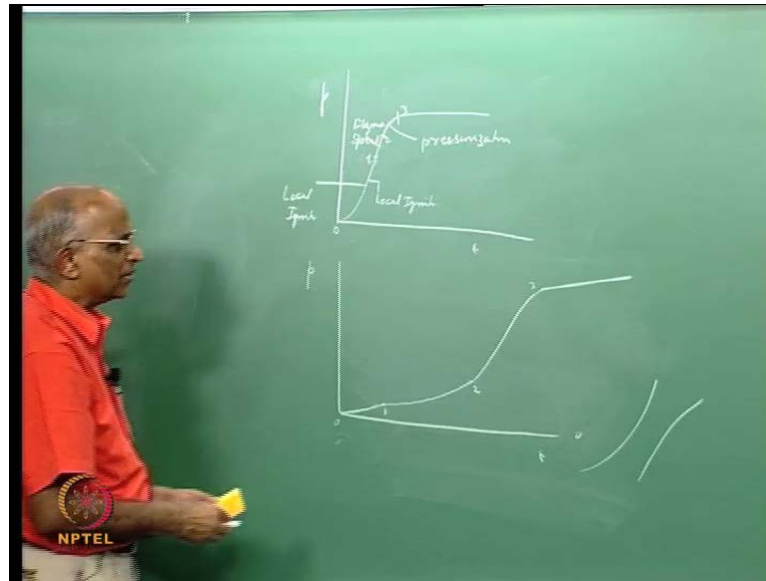
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Supposing I consider a small rocket solid propellant rocket; and let us say I consider for ease may be an end burning grain. This is my propellant the volume is quite small, or the second case I consider is a huge solid propellant rocket much larger. And I say the initial volume is large that means, I have something like this is my propellant over here. I want a large burning surface area, if I give a larger area my volume here, is going to be let us say this is volume 1 this is volume 2 for a large 1. I know the volume here is very much larger than the volume 1.

I put a igniter here may be a controlled igniter. I put the pellets here, and I make sure that the pellets squirts fire on this. I put same pellets over here; I make sure it squirts on the surface over here. Now what is going to be the change in the transient for pressure for the small solid propellant rocket compared to a large solid propellant rocket? Apparently here it will locally ignite whereas, here the moment I put fire almost the entire surface ignites therefore, for a small solid propellant rocket I would expect.

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The sequence of events to be may be pressure with time; I start it immediately goes up may be it comes to neutral burning and most of the sequences from 0 to ignition will be local ignition. In other words, the flames from the ignited directly ignited and there is very small amount of flame spread and it ignites and then it equalizes the pressure therefore, I will say 0 to 1 will be now I say is the local ignition, 1 to 2 is the flame spread very small region is getting ignited, then I have the equilibration taking place, because there is not much to do here therefore, this becomes my pressurization of the cavity and this 1 to 2 becomes my flame spread.

In other words for a small solid propellant rocket it is governed by local ignition. If I have a large volume here, well local ignition locally it ignites. Let me plot it on a plotter at the bottom pressure versus time. Well, I have a large volume therefore, it takes time to pressurize it therefore, it starts very slowly, because I need a minimum pressure and minimum ignition energy to burn it. Therefore, from 0 to 1 which now I call as local ignition it takes place here. Then what happens the flame spreads over the surface the surface is large therefore, it continues at low pressure the flame spreads over here, it comes to 2. And then what is it I do I have to take the pressure from here, to the equilibrium value and I have something like pressurization of the cavity it takes to 3, and then I have the grain which goes in this particular frame.

Therefore the sequence is consisting of local ignition. I have a significant portion of flame spread compared to a small rocket. And this portion also tends to be a large fraction I would like to predict what must be the shape of this curve well this shape should increase here also the pressure should increase. Let us take a look at this particular curve is it does it go like this, does it go like this, how should the pressure change then let us write an equation for it and its quite simple let us do it in the same way.

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$$\frac{dm}{dt} = \rho_p S_b a p^n - \dot{m}_n$$

$$\frac{dm}{dt} = S_b a p^n \rho_p - \frac{1}{c^*} p A_t$$

$$\frac{dp}{dt} = \frac{RT}{V} \left[ S_b a p^n \rho_p - \frac{1}{c^*} p A_t \right]$$

What happens during the chamber pressurization? Well igniter has finished the job it has ignited the surface and flame spread has happened. Therefore, I can write  $dm$  by  $dt$  as equal to the entire surface catches fire therefore, the burning surface area into a  $p$  to the power  $n$ . Into the value of  $\rho_p$  is the mass which is released minus  $1$  over  $c^*$  into  $p$  into  $A_t$  where  $p$  is the chamber pressure it will be we can call it as  $p_c$  or  $p$  whatever you can denote. Now  $dm$  by  $dt$  we have already written as equal to  $dp$  by  $dt$  into  $V$  by  $R T$  therefore, I have  $R T$  by  $v$ ; into  $S_b a p$  to the power  $n$   $\rho_p$  minus  $1$  over  $c^*$   $p$  into  $A_t$  is it all. Let us solve this equation, but I have so many variables can I take the variables out and put it in terms of some non dimensional numbers.

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$$\frac{dp}{dt} = \frac{\Gamma^2 c^* k}{V c^*} \left[ \frac{S_b a c^* \rho_p}{A_t} p^n - p \right]$$

$$L^* = \frac{V}{A_t}$$

$$\frac{dp}{dt} = \frac{\Gamma^2 c^*}{L^*} \left[ p^n \left( \frac{p_0}{p} \right)^{1-n} - p \right]$$

See we already know that  $c^*$  star into  $\rho$  into  $a$  into what else something else was there  $c^*$  star  $\rho$   $a$  divided by into  $S_b$  into divided by the value of  $A_t$  was equal to  $p$  equilibrium. And this was equal to  $1 / (1 - n)$ . And we are talking of the same  $S_b$   $a$  can I can I slightly elaborate on this can I simplify it. We also know that the value of  $c^*$  star what was the value of  $c^*$  star in terms of  $R$  and temperature and capital gamma under root.  $R T$  by capital gamma where capital gamma was equal to under root gamma into 2 over gamma plus 1 and we had a gamma exponent. Therefore, I may be I can get rid of this  $R T$  here; by writing  $R T$  is equal to gamma square into  $c^*$  star square. And if I were to do that my equation now becomes pressure at any point between this pressure at this point, **pressure at this point**, **pressure at this point** I want to reach the equilibrium value I can write the value as  $dp$  by  $dt$  is equal to now instead of  $R T$  I write gamma square into  $c^*$  star square divided by  $V$  volume.

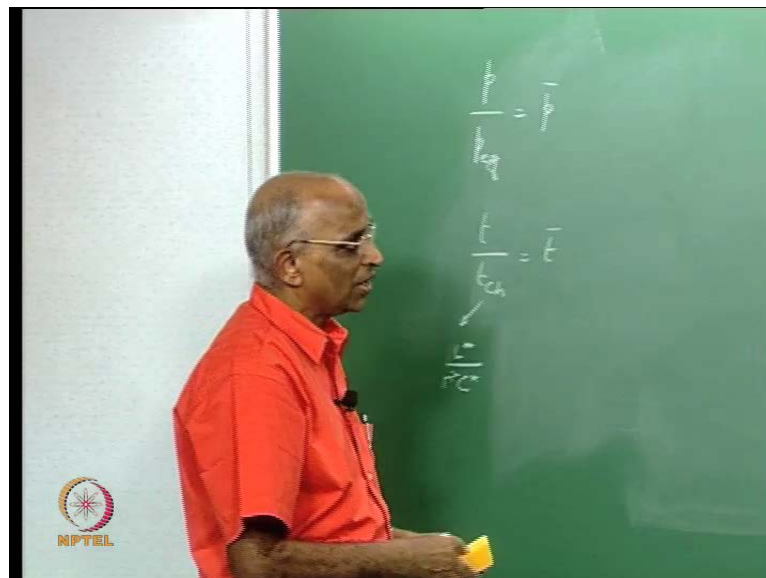
Now let us simplify this. I can take  $c^*$  star outside therefore, I take  $c^*$  star outside please be watchful. And now if I take  $c^*$  star outside I should multiply this by  $c^*$  star;  $S_b a c^*$  into  $\rho p$ . And I also take let us say I take  $A_t$  outside **I take  $A_t$  outside** therefore, this become  $A_t$  and therefore, this becomes only I am left with  $p$  over here please be careful is it all. Now I what I want to do is I want to solve this equation I find  $c^*$  star and  $c^*$  star gets cancelled this two goes off. Now I have volume of the cavity divided by the throat area this gives me something like the length something I say  $L^*$   $L^*$  I define as equal to volume initial volume divided by throat area to begin with I say  $L_1^*$  or  $L$

star is equal to volume by area this gives me unit of length. And therefore, I can write this equation now as equal to  $dp/dt$  is equal to  $\gamma^2 c^*$  divided by  $L^*$  which is something like a length of the equivalent length of the chamber into. I have  $S_b$  a  $c^* a_p$  by  $A t$  is equal to  $e p_{\text{equilibrium}}$  to the power  $1 - n$  because this to the power  $1 - n$  is  $p_{\text{equilibrium}}$  minus the value of  $p$  is there anything I have left.

Here  $A p$  power  $n$ .

Exactly, here we said  $a_p$  to the power  $n$ , because I should have had the burning rate of the propellant therefore; I had  $p$  to the power  $n$  over here. And therefore, it is  $p$  to the power  $n$  over here. Let us be very clear  $S_b$  a here  $p^n$  and therefore, here I have  $S_b$  a  $\rho_p c^*$  I should have had  $p$  to the power  $n$  here. Therefore,  $p$  to the power  $n$  over here and therefore, now can I can I think in terms of reorganizing that equation in some form let us say pressure at any point

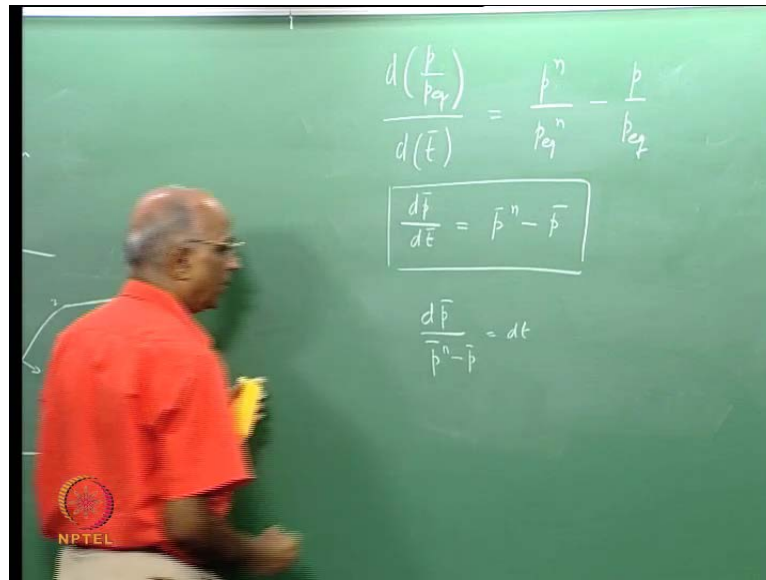
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Or any time is  $p$  I non dimensionalize by the equilibrium pressure I call it as  $p_{\text{bar}}$ . And now I have time  $t$   $dp/dt$ . I non dimensionalize with respect to I have this is the linear  $L^*$   $\gamma$  is anyway a constant. I have characteristic velocity  $L^*$  by  $c^*$  has a time which is again a characteristic time. Therefore, now I say  $t$  by  $t_{\text{key}}$  characteristic is equal to I say  $t_{\text{bar}}$  non dimensional value of time and what is  $t_{\text{key}}$  characteristic I say it is equal to  $L^*$  divided by the value of  $c^*$  and  $\gamma^2$  length divided by velocity

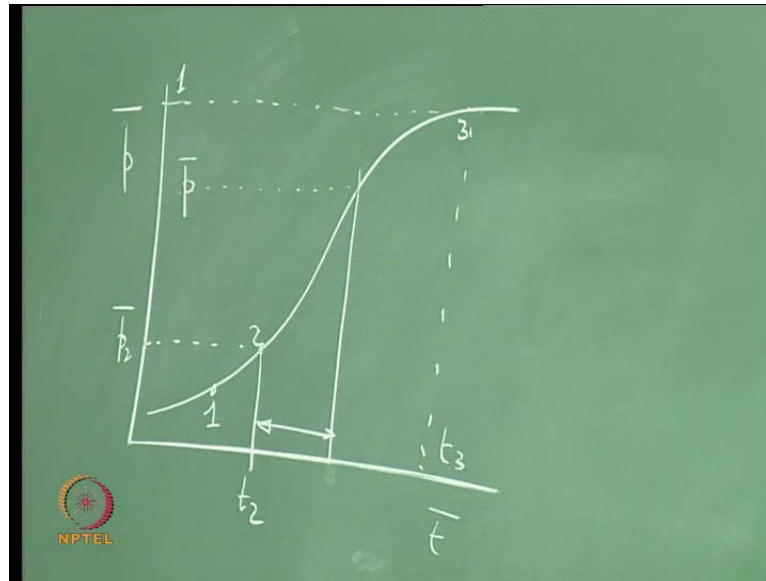
gives unit of time I call it as characteristic time. We will develop on this when I study the equation of combustion instability we will develop on it still further. And now if were to do this two non dimensionalizations. What is the final equation I get let me put it here. I get let us take p outside p over here let us take it out.

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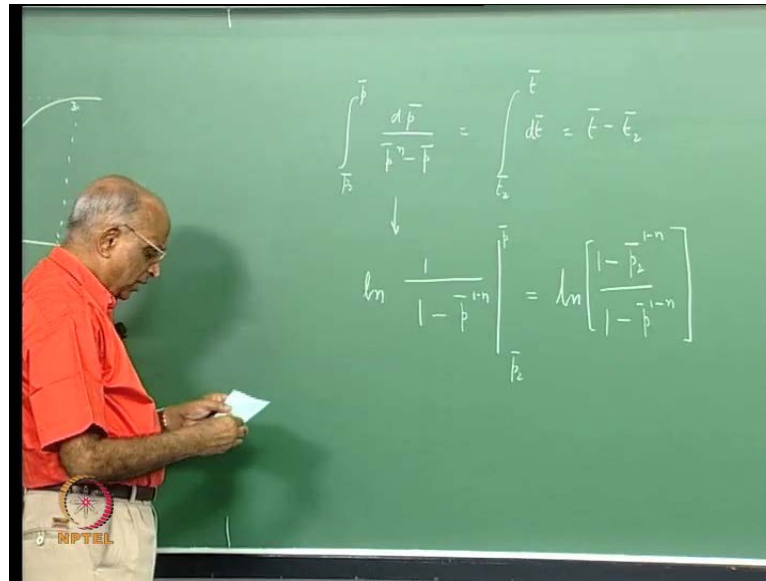
And therefore, I get dp by p equilibrium. And now I have brought 1 p equilibrium here therefore, there it should have been divided by d of the value of t equilibrium t non dimensional that means, I have taken this transport it over here. I took 1 p outside therefore, this is equal to p to the power n divided by p equilibrium to the power n and this becomes again p by p equilibrium. Or rather this equation is now telling me d non dimensional pressure divided by t I have taken d that is an equilibrium time is equal to p bar n minus p bar. This is a final equation I get. And now I can solve this equation by writing as dp bar by p bar n to the power n minus p bar is equal to dt let us integrate this equation from a value.

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Let us put the value down we have already said that I can write the non dimensional pressure. As a function of the non dimensional time starting from a value it goes the equilibrium value. Equilibrium value is  $\bar{p}$  bar is defined as a pressure by equilibrium value which is 1. We have may be the 0.1 at which the local ignition is completed. 2 is the time or  $p_2$  is the pressure non dimensional pressure at which the flame spread is completed. And between 2 to 3 wherein I get the equilibrium condition I am interested in finding out the time taken namely from the value of  $t_2$  to the value of let us say  $t_3$ . Inner you find that you are integrating up to a  $\bar{p}$  of 1. May be first let us find out what is the time required for the pressure to go from the end of  $p_2$ . That is the entire propellant grain has ignited to the condition when there is there is some pressure  $\bar{p}$  bar in the system. Now that means, I am interested in this particular time to be able to do that we integrate the equation.

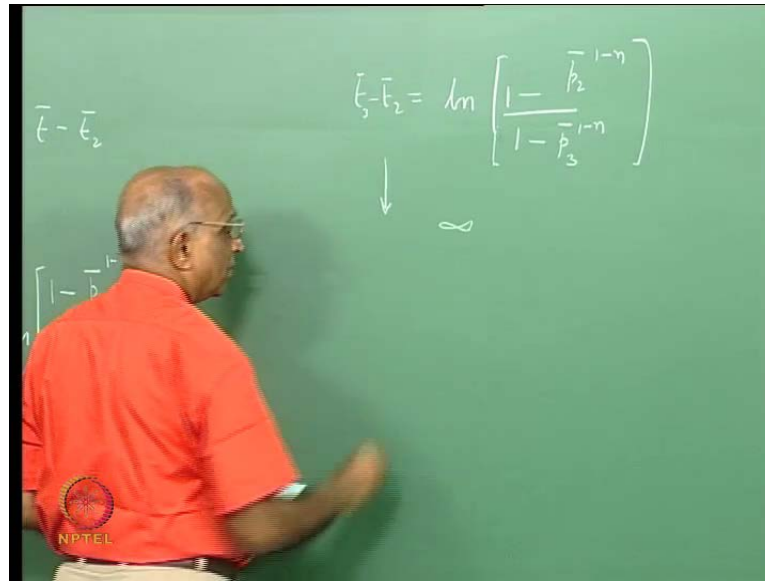
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Again we say I integrate the equation which we had obtained namely  $dp$  by  $p$  to the power  $n$  minus  $p$ , as we are going from this value namely  $p_2$  to any value  $p$ . And for that we are also saying yes the time changes from  $t_2$  over here, to a value  $t$  and if the time is also non dimensional  $t_2$  to  $t$  as the time changes to  $dt$ . Therefore if I if I want to integrate this. We see on the left side  $1$  over  $p^n - p$ . And this is a standard integral which can be integrated by parts. And I can get the value the this particular left hand side value to be  $\ln$  of  $1$  divided by  $1 - p$  to the power  $1 - n$ . And this changes from  $p_2$  to a value of  $p$ . And the net value will therefore, be equal to  $\ln$  of  $1 - p_2$  to the power  $1 - n$ , divided by  $1 - p$  to the power  $1 - n$ . I would like to examine this particular  $1$  and of course, when you when I look at the hand side it are quite simple I get the value between  $t$  minus  $t_2$  over here.

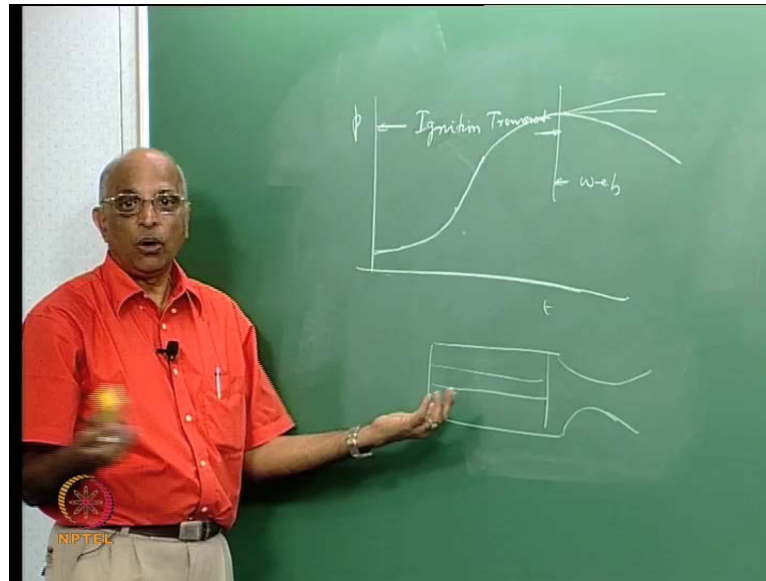


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Therefore, the final expression what we get is. The value of  $t$  minus  $t_2$  non dimensional is equal to  $\ln$  of  $1$  minus  $p_2$  bar to the power  $1$  minus  $n$ . Divided by  $1$  minus  $p_3$  bar to the power  $1$  minus  $n$ . If I am interested in finding out the time to reach equilibrium pressure, well I substitute the value as  $3$  over here; I substitute the value as equilibrium or  $p_3$  here, but the value of equilibrium pressure non-dimensional is with respect to equilibrium pressure itself. Therefore, the denominator becomes  $0$  and therefore, the time taken to reach the equilibrium state is infinite. Therefore the trend of variation of pressure is such that initially there is progressive increase thereafter it droops and it takes infinite time or a long time to reach the equilibrium pressure and therefore, we tell ourselves.

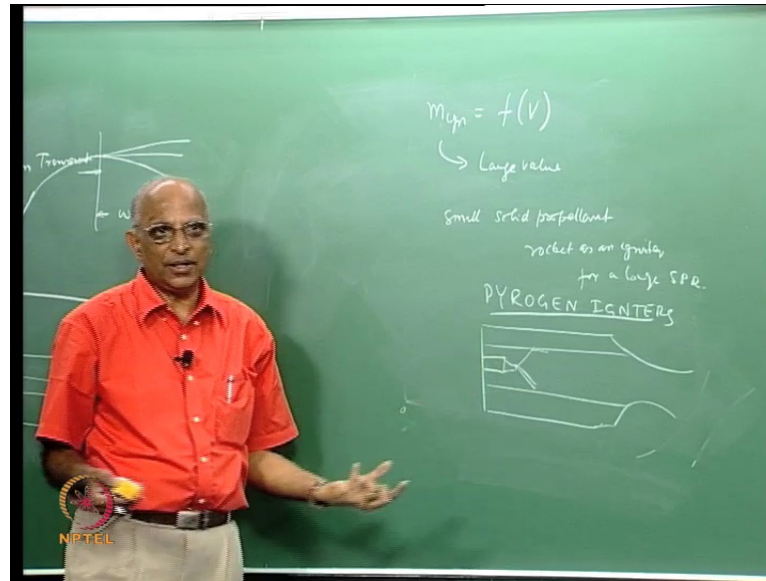
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Well the ignition transient is such that it starts off it droops and comes like this and using this we will try to define some characteristic times for burning for ignition subsequently. Therefore, the ignition transient is seen to be something which has a character which is starts igniting essentially for large motors starts like this, goes off like this. And then again droops back and comes back to the equilibrium value. And thereafter if it is neutral burning it goes like this, if it a progressive it goes like this, if it regressive it goes like this; this is the steady web burn time and this is your on the pressure versus time this becomes my ignition transient. And this is how you predict the ignition transient.

Therefore what is it we are telling of? We talked in terms of pyrotechnic charge we talked in terms of metal powders, we talked in terms of thermite igniter essentially to locally ignite it then flame spread takes place, and then it goes like this, but if I have a very large motor and most of our motors, whether it is for missiles, whether is for defense whether it is a launch vehicle, solid propellant rockets the initial volume is quite large, in which case the amount of charge what we want also grows and what is the volume of charge, which I need or what is the mass of charge which I **which I** require?

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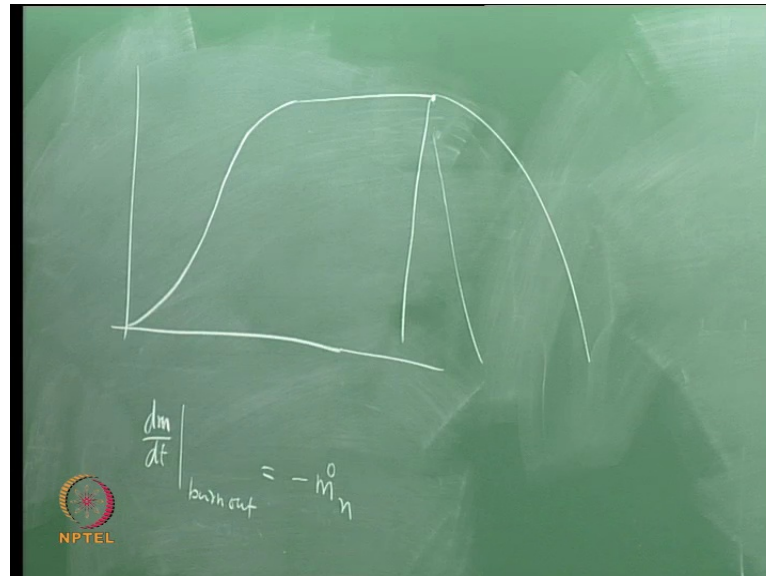
It should be a mass of igniter charge, what I have should be a function of the volume that is an initial cavity volume. If I have larger cavity I must have more charge and the volume keeps on increasing. And you cannot have large amount of propellant of pyrotechnic charge, and large flame which is uncontrolled; and therefore, if I have an igniter which is a large value. Then the question is can I use a small rocket itself, can I use a small solid propellant rocket as an igniter for a large solid propellant rocket.

That means, what is it I am telling you, I am telling you well this is my solid propellant rocket, this is my volume. I find that I need a large value of the igniter composition therefore; I put a small solid propellant rocket over here. I get the plume coming over here, it ignites it and such solid propellant rockets which are used as igniters for large rockets are known as pyrogen igniters. And almost all the solid propellant rockets which we developed will make use of pyrogen igniters, because booster rockets will contain this. And even if we use rockets in upper stages we will ensure that we need to ensure that the propellant what we require when I generate this as a function of volume. I need a few 100 grams or may be a kilogram and therefore, I use a pyrogen igniter.

Pyrogen igniter is a small solid propellant rocket which is used for igniting a large rocket, but this rocket should not contain aluminum, because the nozzle will tend to block it or if the aluminum globules will collect and will eat into the rocket or it will restrict the motor. And therefore, essentially pyrogen igniters use non aluminized

propellants it could be HTPB it could be p bang anything could it could be. Therefore, this is all about igniters, but having said that let us also finish the subject of solid propellant rockets with an example, of what happens.

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When I ignite a motor I know how to go about it how to calculate it for equilibrium I know how to do it. When I stop ignition how does the flame come, does it come like this does it end like this, or does it will end like this we would like to know. And for that we use the same theory again.

We say yes  $\frac{dm}{dt}$  after burn out of the motor, is equal to well igniter is not there. All the propellant has ignited it is also not there. And only thing which is there is  $\dot{m}_n$  which is leaving the nozzle and therefore, my  $\frac{dm}{dt}$  is equal to  $\dot{m}_n$  let us quickly integrate it and what is a result I get.

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$$\frac{dm}{dt} = \frac{dp}{dt} \left[ \frac{V}{RT} \right] = - \frac{p A_t}{c^*}$$

p T

I get  $dm$  by  $dt$  which is equal to the value of  $dp$  by  $dt$  into again. What has happened the entire propellant has burnt out I am at the case it is still at the high pressure over here therefore, I have  $V$  by  $RT$  over here, is equal to  $d$  and this is equal to minus chamber pressure into  $A_t$  by  $c^*$ . I can write this the same thing  $m$  is equal to  $P V$  by  $RT$  volume there is no change in volume, because all the propellant has burnt. I have something like the temperature is till the higher value over here. Therefore, I can write it as  $V$  by  $RT$   $dp$  by  $dt$  is equal to  $m$  dot  $n$  which is equal to  $p a c$  by  $n$  nozzle is leaving the flow I have the negative.

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$$\frac{dp}{dt} = - \frac{RT}{V} \cdot \frac{p A_t}{c^*}$$

$$= - \frac{\gamma^2 c^{*2}}{V} \cdot \frac{A_t}{c^*} p$$

$$c^* = \frac{\sqrt{\gamma RT}}{\gamma}$$

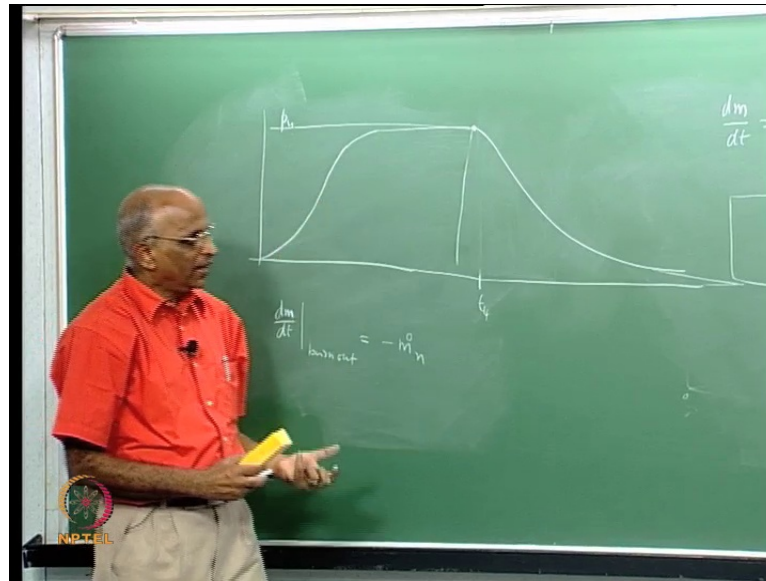
Let's let us solve this equation. The same thing what I have done in this case, I will do here. I have  $dp$  by  $dt$  is equal to  $R T$  by  $V$ ; into I have a minus sign now  $p$  into  $A t$  by  $c$  star.  $r t$  I can write in terms of we said  $c$  star is equal to under root  $R T$  by capital gamma. Therefore, I have gamma square into  $c$  square  $c$  star square is a value of  $R T$  divided by  $V$  into I get  $V T$  over here I have  $c$  star over here I have the negative sign over here and I have the value of  $p$ .

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The image shows a green chalkboard with three equations written in white chalk. The first equation is  $\frac{dp}{dt} = -\frac{\Gamma^2 c}{L^*} p$ . The second equation is  $\frac{dp}{p} = -\frac{\Gamma^2 c^*}{L^*} dt$ . The third equation is  $\ln \frac{p}{p_4} = -\frac{\Gamma^2 c^*}{L^*} (t - t_4)$ . In the bottom left corner of the chalkboard, there is a small circular logo with a star and the text 'NPTEL' below it.

I use the same thing  $V$  by  $A t$  is equal to  $L$  star. I retain the dimensional form. And then I can write it as  $dp$  by  $dt$  is equal to or  $dp$  by  $dt$  is equal to minus gamma square  $c$  by  $L$  star. I have not taken the  $t$  outside by  $p$  or rather I get  $dp$  by  $p$  is equal to minus gamma square  $c$  star by  $L$  star into  $d t$ . And therefore, what is it I get the value of  $\ln p$  divided by the equilibrium value, at this particular point is equal to I say that is equal to  $p_3$  or  $p_4$  now is equal to minus gamma square  $c$  divided by  $L$  star into the time after this particular event  $t_4$ .

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That means, I say this time is  $t_4$  this value is  $p_4$ . And therefore, what is it I see that the pressure keeps coming down logarithmically or rather the pressure should come down with it to reach ambient pressure it is going to take infinite time, or rather the slope would be something like this. This is all about the changing pressure during once the motor has burnt out or once the solid rocket has burnt out this is the transient of ignition. I think we are still left with one or two small things. The characteristic times involved in a rocket and I also said I will go through some examples of some big solid propellant rockets to it may be we will do it in the next class, but this is all about ignition we have covered, and we also covered briefly about once the propellant grain has burnt, how long it takes to reach the ambient pressure. That means, you find that it gradually only comes down it also goes up gradually therefore, we need to find out if I have to say web burning time, I need to get some more times involved and this is what I will consider in the next class.

Well thank you then.