Rocket Propulsion

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Lecture No. # 35

Introduction to Hybrid Rockets and a Simple Illustration of Combustion Instability in Liquid Propellant Rockets

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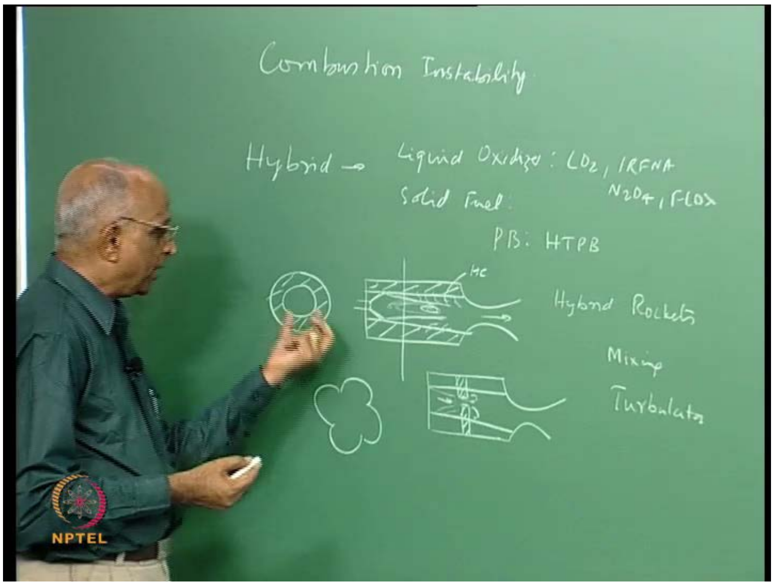

Good morning. I think today we will talk on combustion instability in chemical propellant rockets. Namely, combustion instability in solid, liquid and other types of rockets, but I thought in the last class we had not really finished hybrid therefore, let me before coming to this topic just spend a couple of minutes on the hybrid rockets. The very name hybrid means, it is a combination of two phases, maybe a liquid and a solid, or a gas and a liquid and so on.

But what is normally used in hybrid is maybe a liquid as an oxidizer and solid as a fuel, and we all know solid fuel is like polybutadiene HTPB P ban and all that could be used, but essentially what is used is maybe a polybutadiene, and normally it is hydroxyl-terminated polybutadiene which we used as a binder in solid propellant rockets.

The liquid oxidizers which are used could be anything, could be liquid oxygen, could be let us put it down, could be liquid oxygen, could be nitric acid, inhibited red fuming nitric acid, could be N 2 O 4, and extra maybe we talked in terms of flox, we had liquid oxygen which was in which fluorine was added to make it more powerful and these are the liquid oxidizers, but mainly solid fuel is essentially polybutadiene which is HTPB. How will the construction look like? You have a cylinder, a case, you have a nozzle, you have the solid fuel which is kept over here could be any configuration.

Let us take a star configuration or a circular configuration, I take a section over here this is my solid fuel. And what is it I do? I spray the liquid oxidizer onto the fuel, and in case the liquid fuel like let us say I RFNA is hyperbolic with respect to the fuel then it begins to react at the surface, vapor is evolved and I have vapor which keeps coming out, and here it is oxidizer rich, here it is the fuel vapor which is coming out and therefore, what is going to happen is here you have a stoichiometric mixture or a mixture between the oxidizer and the fuel vapor it burns and then you get the thrust, this is the principle of hybrid.

Essentially, somewhat in between a solid rocket and a liquid rocket, but in this case what happens is provided the surface is hyperbolic, the moment you inject it chemical reaction begin to occur, the heat feedback to the surface generates the fuel vapor say hydrocarbon vapor, and that hydrocarbon vapor mixes with the oxidizer and you get combustion taking place and that is exhausted through a nozzle, and this is the principle of hybrid rockets.

In case, the surface is not hypergolic with respect to the oxidizer, then in that case I have to quote something on the surface which initially will start combustion by having a paste, which is hypergolic with respect to the liquid oxidizer heat gets generated, and once heat is getting generated thereafter it is the heat transfer, namely the convective heat transfer which evaporates the fuel generates the fuel vapor mixes and goes over here.

Therefore, the controlling thing in these hybrid rockets is essentially mixing. What is mixing? At the surface you have hydrocarbon vapors mixes with the free stream of oxygen vapor, because when I inject it this also due to the heat vaporizes and therefore, mixing is sort of a problematic. And what happened is? There was one person by name doctor Dadi Yu in Germany, and what he suggested was whenever we make hybrid rockets, since mixing is a problem why not we put something like a turbulator. What is that turbulator? Let us again do its very common in all engineering situations. Namely, when I look at this particular thing fuel is getting evaporated here oxidizer is coming over here.

I maybe make this constriction over here in the form of let us say petal, like petal is something like this. I put this configuration over here in this pole over here and therefore, this is let us say a turbulator, and what happens when the gas is flowing across it? It creates eddies and it helps to mix and therefore, we need something like a turbulator normally for mixing the gases and so that combustion gets completed. Therefore, you know in the early stages whenever people were working with hybrid, they never really used turbulator they got very low performance, after using turbulator somewhat mixing is better, but still the performance is not as good as in liquid propellant rockets but it is better than solid fuel rockets, I think this is all about. But there is one distinct advantage in using hybrid rockets, what are the advantages?

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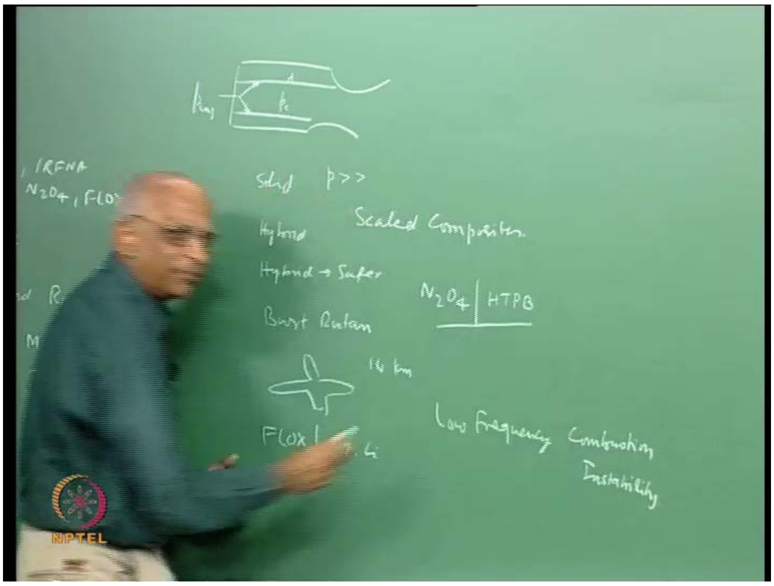

Let us say you have a solid fuel, and the solid fuel has a crack in it or some opening in it therefore, the surface area increases and you are spraying liquid oxidizer onto it, because the surface area is increasing the pressure gets generated, when pressure gets generated automatically the pressure drop across injected is increased is decreased, because pressure here is higher this is constant pressure, pressure gets decreased and therefore, the flow rate of oxidizer decreases and therefore, it is self regulating.

In other words, solid propellant rockets if there is a crack, if surface area increases pressure continues to increase. In the case of hybrid if there is some surface defect, what happens is pressure gets generated, once pressure gets generated automatically the pressure difference between pressure at injection, and pressure in the combustion chamber now it is higher increases and therefore, the flow rate decreases, the flow rate decreases automatically the thrust decreases and therefore, I would say that the hybrid rocket is a little safer and it can take certain surface defects. And it is these things which prompted more recently a person, a millionaire, a British millionaire by name Burt Rutan. He formed a company known as scaled composites, and he used hybrid rockets for ferrying people to space, maybe he says space can be used for enthusiast were interested in looking.

Therefore, he says it is something like a voyage or something like a place where you can go and view the earth from top and therefore, he used that white knight craft which I showed you in the last time, in the belly of it he mounts a space capsule powered by a hybrid rocket, and this hybrid rocket takes him; the white knight aircraft goes up to 15 kilometers height, from there he goes a little bit higher maybe people are able to see the earth from top and then it comes back in a comes back to the earth and therefore, the purpose of the hybrid rocket there is for ferrying people from fourteen kilometers to the ground back, and this is where the hybrid rocket is used.

The company which does it is a private company known as I think scaled composites. Last week officially it was the launch pad was inaugurated in the desert in California, I think these are something which we can keep in mind. Therefore, I would say well hybrid rocket has not been very much used in practice; however, it is something like a convective heat transfer which vaporizes the propellant, mixes and burns. It has lower performance than liquid propellant rocket; however, it is safer than many of the other types of rockets, because it is self regulating and it is finding increased applications, I do foresee much more applications for hybrid rockets in the years to come.

I think this is all about hybrid rockets, we can even think in terms of other propellant may be FLOX if you want very high performance, maybe with metal embedded in the fuel, like maybe you could have some light metals, Let us say, you have maybe magnesium, you have some lithium and other things you could react it and generate more thrust, but what is generally used is what is used by Rutan in his space capsule is N 2 O 4 as oxidizer and you have the HTPB as the fuel. I think this is all about hybrid rockets, but there is one problem with hybrid rockets because of the slow rate of reactions it is very susceptible to combustion instability in the low frequency mode, and I am going to talk about it away, we will talk it is susceptible to low frequency combustion instability.

Maybe, since this is our next topic, maybe we will not spend some time at this point of time. Maybe, after studying what causes combustion instability it is new for all of us.

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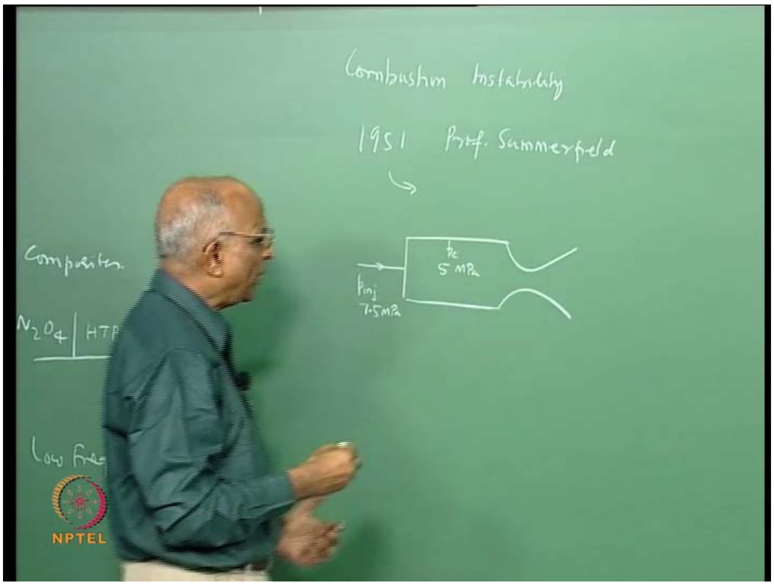

Let us study this and then come back and see what is the real problem over here. Maybe, with this I go to this new topic, a very fascinating topic of combustion instability I wrote it earlier, maybe I write it again because we must be glued on to this topic. Maybe, I will start with the very small example and this example is not mine, this example was given in 1951 by one professor Summerfield; he was at Princeton and he published a paper in 1951 I think at that time (( )) journal was not there, journal of I think rockets and aircraft or something like that, as a it was a it was a journal which came much before AA journal started coming, a journal of AA started coming. In this paper he gives an example, I modify the numbers, because these numbers were in f p s system and not that that easy to work on the board, maybe I take a typical liquid propellant rockets, but as I say I follow what process some of you did in his particular paper. He says let us have a liquid propellant rocket, let the min chamber pressure be 5 MP a; 5 mega Pascal. 5 mega Pascal is something like chamber pressure is 50 bar.

Let the fuel and oxidizer both of them be injected into the chamber at let us say p injected, and this value let it be 7.5 MP a, I just choose these numbers to illustrate some phenomenon; that means, propellant is injected at a pressure of 75 atmospheres, and the chamber pressure pc is equal to 5 MP a or 50 bar.

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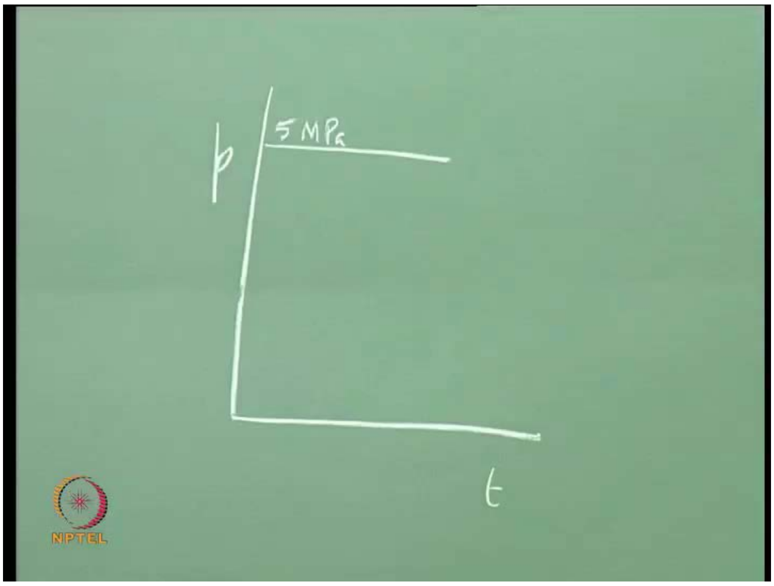

Now, what is it we are talking of let us put it down on the board. Well, we expect maybe pressure with respect to time it's burning steadily, I expect the chamber pressure to be always 5 MP a this is what I get, I start burning it keeps on injecting propellant and something is leaving.

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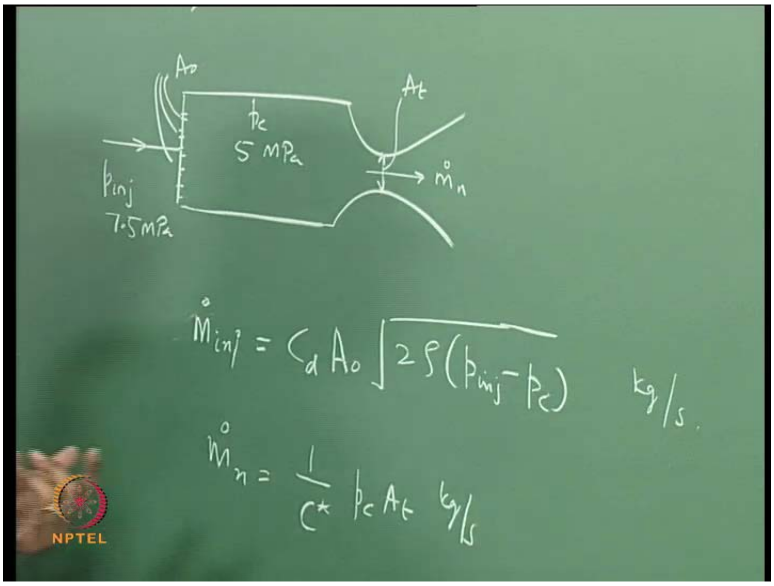

And why do you get a chamber pressure? After all, you add some mass let us say mass of propellant which is injected, and how do you calculate the mass of propellant which is injected? You inject something; that means, you have injector orifices having total area a 0, let the C d value of the oxidizer and fuel orifice be the same C d into a naught into under root of 2 rho into you have p injected minus p chamber, this is the rate at which mass is injected so much kilograms per second. Under steady conditions, and what we said is C d A 0 2 into delta p by rho is the volume and multiplied by density C d A 0, and this we must know all the time. And what is the rate at which gases are leaving m dot through the nozzle? It is leaving at the rate, we have (( )) known terribly p c into A t, this is the rate at which gases are leaving so much kilograms per second.

Therefore, we now say this is the value of 80, and the number of holes for the fuel and oxidizer, total number of area flow of injection is A 0 all put together. Now, I ask myself one question. See, so far what did we say to be get the equilibrium pressure mass injected is equal to the mass which is leaving. But, I have a strange problem, the fuel or propellant which is injected take some time to burn. Suppose, I inject a parcel of propellants at this rate here, it is going to start burning after sometime because there has to be a delay, there has to be something like a combustion delay, let me go back on that side again, because this point is important and this will be central to all our discussions.

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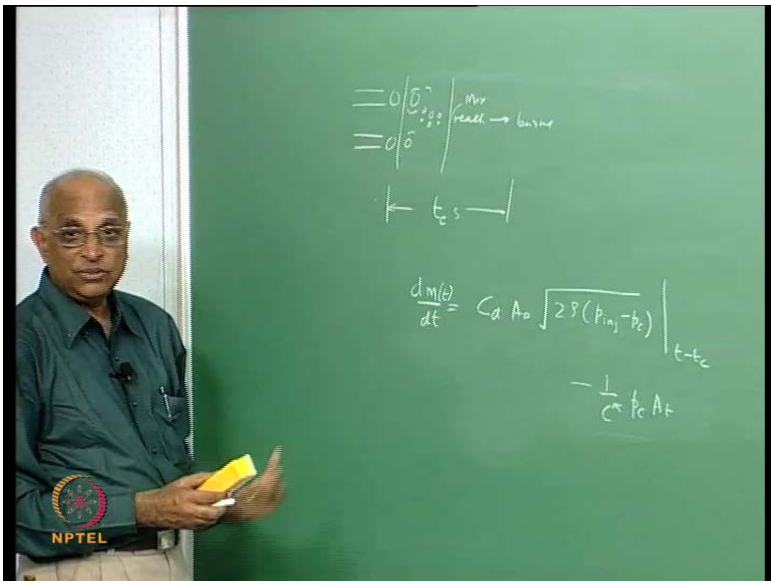

All what we are saying is there is a certain delay, because whenever I inject something maybe I inject a liquid, it breaks into droplets maybe fuel breaks into droplets, then it evaporates, the fuel and vapor mix together, then they react and then outcomes the burning gases. That means, this we said occupies a quite a tremendous amount of time, the vaporization takes time, atomization is fast, reaction maybe fast, mixing before that also takes sometime therefore, if I say the entire process of some injection to burning takes a time of let us say t delay, t due to combustion delay is t c seconds.

Then I need to make some changes in this equation, what is that I am telling? Something is entering, it has to form gases before it leaves the nozzle and therefore, if I were to write an equation which takes care of the rate of mass variations, what is it would be writing? I would be writing under normal circumstances dm by dt the rate at which mass is getting accumulated in the chamber is equal to C d A 0 under root 2 into density into p injected minus p c, this the thing which is coming in, and what is going out? Minus 1 over C star into p c into A t over here.

But now I say this equation may not be really correct, why? Because, what comes out can happen only t c later, or rather I must say the quantity which comes here is equal to t minus t c is what goes out at time t, this is the equation let us say dm by dt, mass at time t, this leaves at time t, but what should really burn is what is injected t c time earlier therefore, this is my dynamical equation or this is my actual equation, and to solve this equation is difficult. But, we must have a procedure and that is what we will be doing in this hour. But therefore, we find that there is a time delay before something happens and therefore, let us get back to this example, with this background lets come back to this example again. What is it we are saying? Mass is injected. Therefore, initially let us say chamber pressure is pi bar

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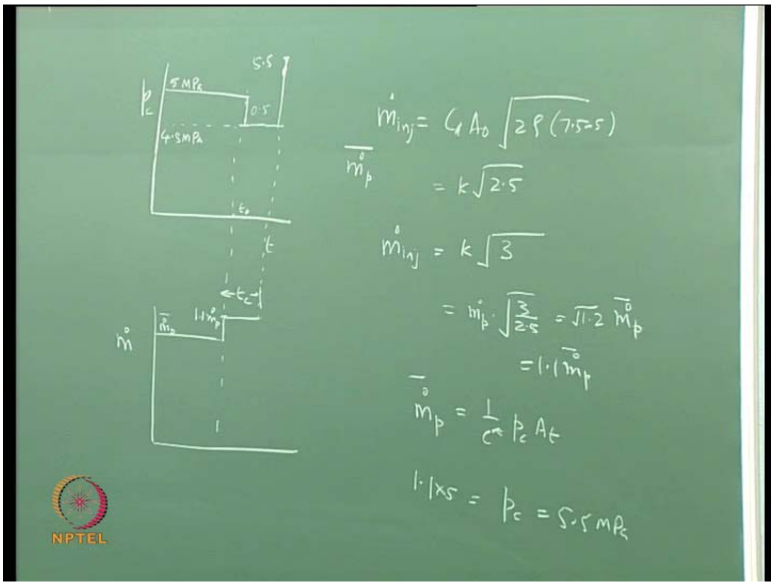

and therefore, I can now write the mass which is injected is equal to again I write C d A 0 under root 2 rho into, now I just put the units because after all it'll constant, I have some chamber pressure I say is 5 injection pressure; that means, 7.5 injection pressure; that means, 7.5 minus 5 is the value or rather I say it is equal to k I take all the constants outside root of 2.5 is the mass of injector. I take all these constants k, maybe I have to multiply by 10 to the power 6 over here, all those things I take constant put it into the k value.

Or therefore, beneath it I therefore, now plot the rate at which I am injecting the liquid in this, and what is the rate I am injecting liquid? I say this is my nominal value, I call it as m dot of propellant which is getting injected nominal value is what I am injecting, and I come I keep on injecting and this is the value I am injecting m dot p. Let at this particular instant of time t 0, let the chamber pressure drop by let us say 0.5; that means, this magnitude is 0.5, something happens in the chamber, maybe there is some problem with the injector or something happens within this chamber and therefore, what is happening is I am now allowing the pressure to fall from 5 to 4.5, see instantaneously it falls, let us just assume something like this happens. Therefore, now my chamber pressure which is p c now becomes 4.5 MP a.

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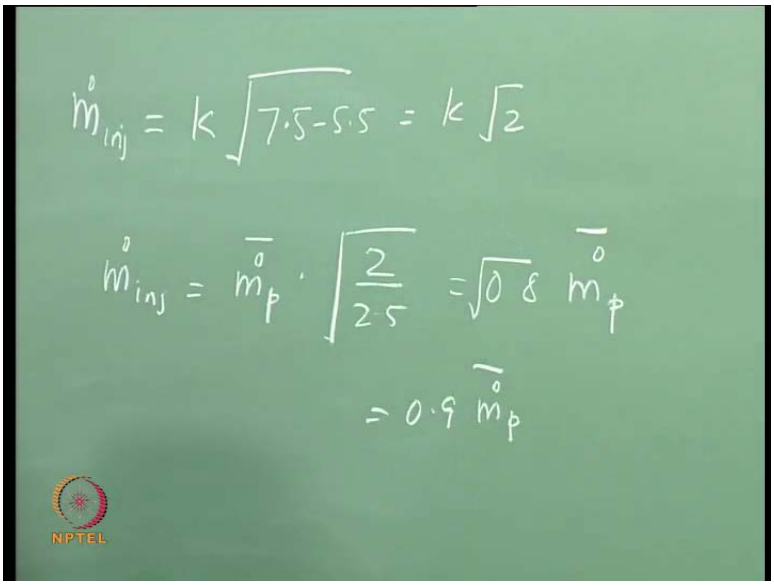

If it is 4.5 MP a, my mass which is injected is going to be k times, now 7.5 minus 4.5 which is 3 under root 3. Therefore, if I were to put it in terms of the steady value of m p dot which was the steady value, it is going to be m p dot into under root 3 divided by 2.5, which is equal to something like 0.1 times under root m p dot, which is equal to 1.1 times the value of m p dot which is the nominal value at a steady.

That means, all of a sudden I am injecting little more which is now 1.1 times m dot p over here. See, the pressure has fallen, because of that the quantity which is injected has now gone up by 1.1 times, it takes a certain delay let me enlarge it, see it takes t c time to evaporate mix and burn together, and when it mixes and burns what is the chamber pressure I get? Originally, I had when the nominal value was m dot p, I have the chamber pressure which is given by 1 by C star into p c into A t, and now I get a value which is 1.1 times and therefore, my chamber pressure will be equal to 1.1 times the value which was 5 earlier, which means after this delay time it continues at this value, after this delay time it increases to 1.1 into 5 which is 5.5 MP a.

In other words, at after this value of t c let us now put it together; the pressure will now increase to 5.5. What is now the implication? The implication is maybe at this point in time when I said the chamber pressure is 5.5, the mass which is injected is now going to be the value of k into under root, the supply pressure is still 7.5 minus 5.5 that is equal to k root 2 and therefore, if the mass injected has is now k root 2, the value of mass injected in terms of the nominal value, which was m p dot which was based on the difference of 2.5 is now equal to m p dot into under root 2 by 2.5 which is equal to 0.8 under root of m p dot, or rather this is equal to 0.9 9 into 9 equals to 81 of the old value m p dot which is the nominal value.

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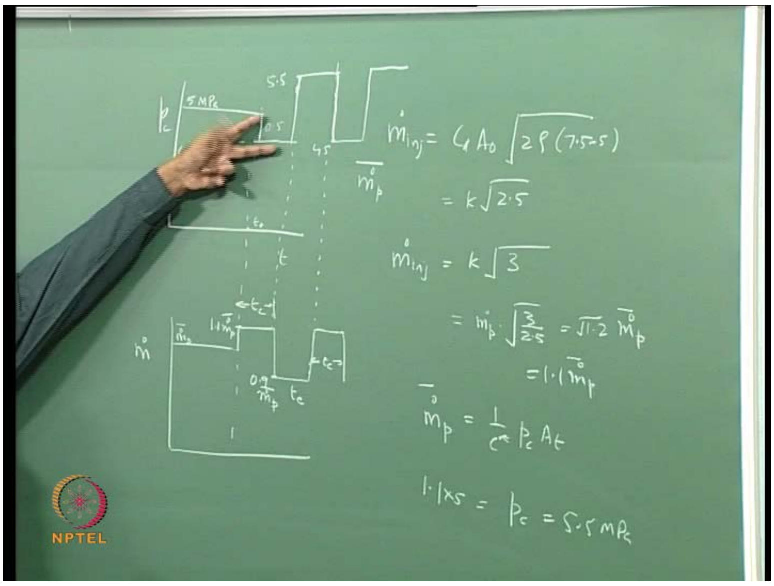

And therefore, what is going to happen when the pressure increases? The mass flow rate now drops to a value 0.9 of the old value corresponding to m p dot over here. Now, what happens? Now it has come back to 0.9 and therefore, this continues this liquid evaporates burns over a time t c, and till it burns nothing really happens, and when this quantity burns over here the chamber pressure is now I get mass which is injected is 0. 9, the vapor which is formed or the gases which is formed is 0.9 and therefore, 0 .9 into the value of phi will be 4.5, the chamber pressure now drops to 4.5 again. And this sequence now again it is 4.5, again after a delay time it increases goes over here, goes over here and so on, this fellow also goes to 1.1 again, from 1.1 it falls to 1 and this delay is t c. And therefore, you find that a momentary drop of 0.5 makes the chamber pressure oscillate between a value from 4.5 to 5.5 to 4.5 to 5.5.

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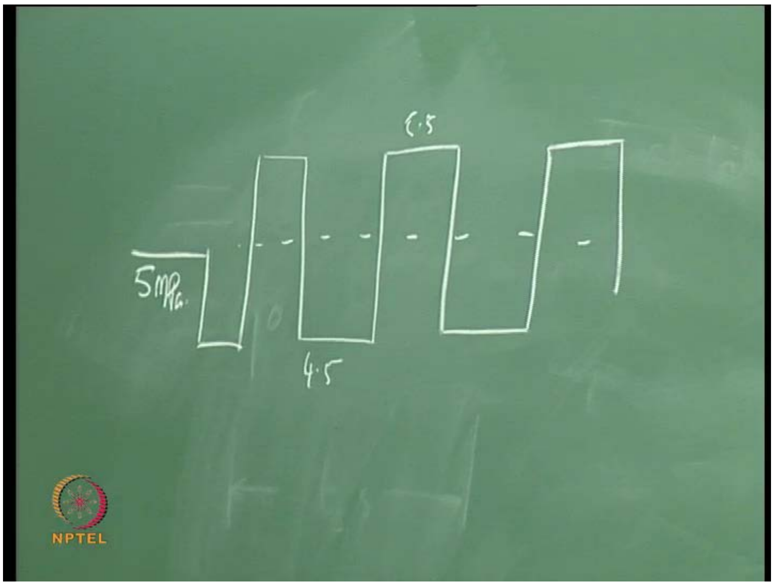

And out of a steady situation, when you have this delay term, what did you do? You started getting oscillations. Well, the oscillations are neutral in that, maybe it keeps fluctuating between a value of let us say 4.5 to 5.5 whereas, the nominal value before the test, before when it falls down was 5 MP a, is the process clear? I think if this is clear I think we would have understood some part of combustion instability. Namely, a dropping chamber pressure whenever there is a delay causes this problem, I will come back to it, let me give a physical example before I proceed with this, because we are talking of something like combustion and all that.

We would have seen many toys in the market; I just brought one such toy here. Let me get back and show you this, because it is very illustrative of the phenomenon. You have these toys which are available in the market, and what you do is you just take this toy, you bend it once it'll keep on oscillating up and down, maybe I put it on the table, I push it here it goes, it keeps on rollicking up and down, it is in a state of neutral oscillations. Why does it have to do this?

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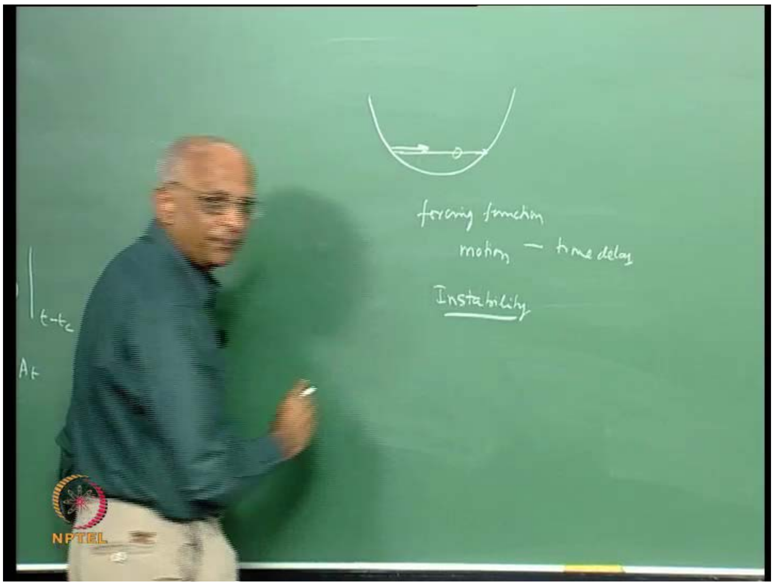

Let us take a look at what is the construction of this particular one, there are very various ones. Maybe, there is a small marble here that is what is making this particular noise, let us say I shift it, when I shift it there is a sling and a marble over here, when I shift it here the marble comes and hits over here, and I hitting is much before it has bent over here and therefore, this fellow comes and hits here and gives a force to this, but when it hits it rebounds back, and even before it will come here, it comes here rebounds here; that means, there is a delay between the forcing function and the motion, there is a time delay, and it is this time delay which keeps this oscillating once in once you oscillate, it keeps on going.

This particular toy is different from a toy in which at the bottom you have a mass, sometime because of center of gravity you do, but in this case there is a marble inside and it is this which keeps it going up and down, and it the friction essentially stops it, and it is exactly the same thing what is happening here, you inject propellants there is a time delay, the time delay precedes whatever be the change which happens, and this is what happens in the case of an one case of instability. Can I say this is understood? If this is understood let me go back to the next example. I just do once more and then I will say we can generalize it.

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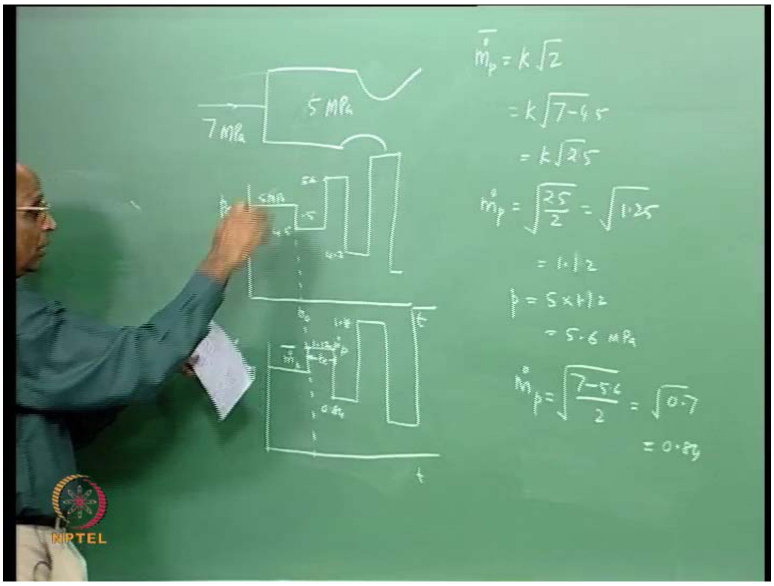

Let us now assume that now I have another chamber; another rocket chamber in which my injection pressure… chamber pressure is still the same thing, I keep it at let us say 5 bar, 5 mega Pascal. Let me reduce the injection pressure to 7 M P a, and ask myself what is going to happen in this case? Now, I do not need to repeat much, all what I want to do is maybe make these 2 plots, maybe the chamber pressure as a function of time, maybe the mass which is getting injected as a function of time, the initial pressure is let us say 5; 5 MP a, and maybe at time t 0 we reduce the chamber pressure like in the previous example, I reduce the value to 4.5; that means, I decrease it by 0.5 to 4.5.

Now, what is going to happen in this case, what is going to be the change? Let us say corresponding to 5 MP a I have a steady value corresponding to m dot p which is steady. Now, I have reduced it the chamber pressure from 0.5 to 4.5 and therefore, I use this s space to work out, now I say m dot which is now injected m dot p is going to be the value of k under steady conditions it is equal to k root 2, and now this is the steady value. When the chamber pressure has got reduced to 4.5, what is going to happen is it is going to become k into 7 minus 4.5, that is equal to k into under root 1.25, no 2.5 or rather the new value of m p dot is going to be under root of 2.5 by 2 which is original, which is equal to under root of 1.25 and therefore, this under root of 1.25, 1.21will be something like 0.12 and therefore, now I find that the at this point t 0 it increases to 1.12 of the value of m p dot, and what is the repercussion?

Well, it is going to burn, it is going to take some t c time to burn ms, and once it gets combusted all the delay is over, then at that point of time the pressure increases, what will be the value of pressure? It is 5 that is the new value of pressure will be the value of 5 into 1.22, which is equal to something like instead of having…. I am sorry it is 1.12, 11 into 11 is equal to 121 and therefore, it is 1.12 over here and therefore, this is going to give me 5.6 M P a therefore, the pressure now goes up to 5.6 M P a, and when it is 5.6 M P a well the mass flow rate now decreases and it becomes equal to with respect to the steady difference of 2 now it is going to be 7 minus 5.6 divided by 2 over here, that is equal to under root of 0.7 1.4 divided by 2.7, the value is 1.12 then it has burnt over here, when it has burnt over here after this maybe the I have a delay time during which it comes, the pressure now comes to 4.2 and then after 4.2 it goes still further, and therefore, it could even diverge it could keep on increasing the value, and here if I were to put it down it is 0.84 this value 0.7 is 0.84, and from 0.84 it picks up and goes to 1.18, and then it comes still further and it keeps increasing and therefore, even the oscillations need not be neutral, it could keep on increasing depending on this pressure difference. I do a third problem in which I choose this value as maybe 9 MP a, this as 5 MP a.

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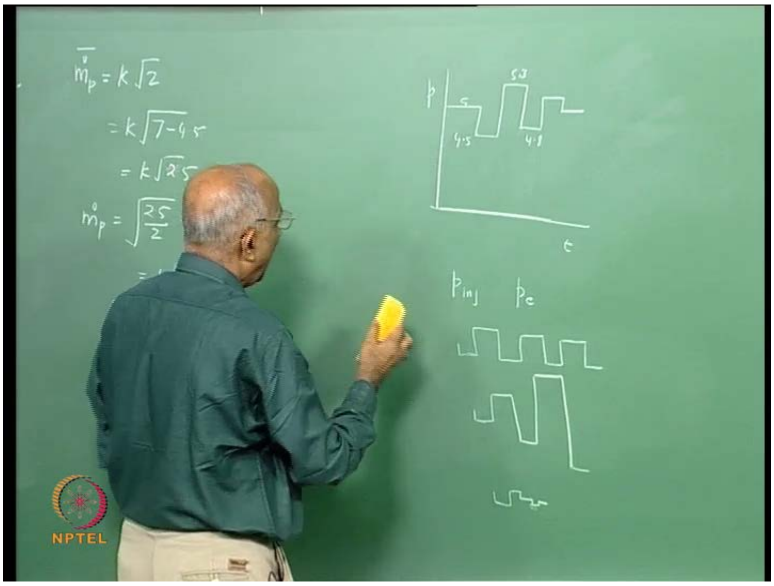

And what I get for this pressure, which I do not need to do at this point in time is maybe I again start with 5, pressure of 5 with respect to time I drop it from 5 to 4.5, then in that case what happens is it goes like this, it goes like this, it goes like this, maybe it does not go all the time, it is it keeps decreasing and it comes back to the neutral. In other words, what did it happen? Because, I had much higher flow rate compared to this, and I do the same sequence I can trace that the pressure with respect to time follows a decreasing trend.

And let me put the numbers through because we should not go too fast also 4.5, the value goes to 5.3, the value comes back to 4.8 and so on, it will be something like 5.1 and so on till it comes back to this. Therefore, what we have just done is? We have taken a simple case wherein, we varied the injection pressure kept the chamber pressure constant, and we found either the oscillations are so formed that it keeps on oscillating in a limit cycle mode, the same amplitude. Or else, we started with this therefore, let us start with this again, it could either go and diverge out like this and ultimately explode, or you I could start like this and it keeps decreasing with time and it reaches this. Either, it could be neutral, it could diverge, it could keep on increasing just like you have equilibrium, because this is neutral, this is diverging, this is something like a decaying oscillations or a stable system, it is possible to imagine all these things in this.

And the example which I say is very similar to what we have seen, keeps on oscillating. Now, let us try to understand this a little bit more, because I think it should be possible for us to write an equation, and what is the equation we had? This was the equation; can we solve this equation and get some inferences? Because, if I were to do it mechanically, if I were to do all these things maybe with 57 57.5, maybe 59 and all that. I will find a system or I will find an equation like maybe

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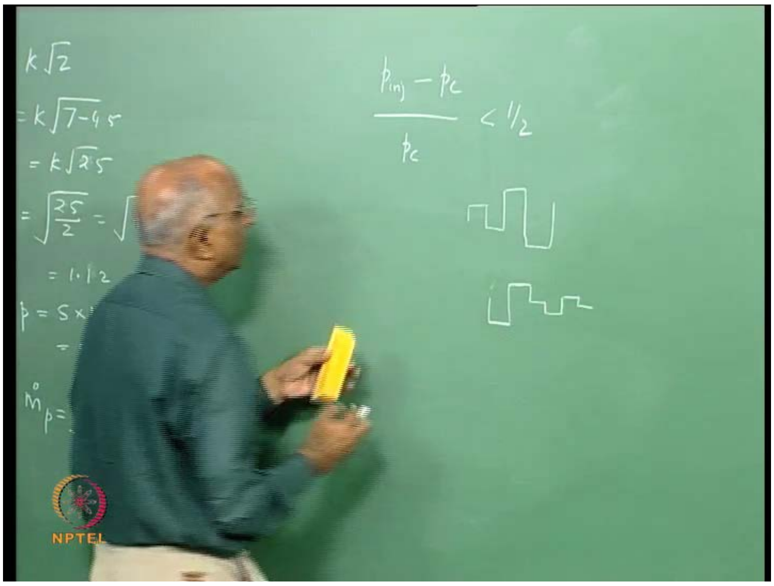

When I have p injected minus the value of chamber pressure divided by p c, if it is equal to half I get neutral oscillations provided I do a lot of trials, if it is going to be less than half I get the case wherein I get diverging oscillations, and if it is greater or if it is greater than half then I get a stable system, wherein it comes like this maybe it keeps decreasing and falls back to 0.

It is possible to do it numerically, let us try to solve the equation. After all, we know how to solve a differential equation; let us try to solve this equation. But, as I told you solving is not going to be straight forward, I have mass generated t c earlier is what leaves the nozzle, how do I do this? Now, we must recognize one or two small things as a precedent before I solve this equation.

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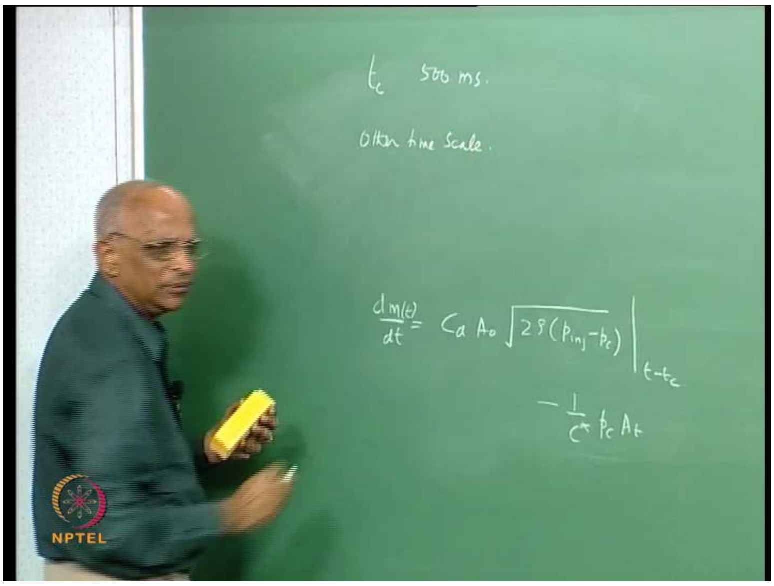

See, when I say combustion time delay t c, let us say it is equal to 500 milliseconds, I just choose a number a large number. See, in a large rocket 500 millisecond maybe different than in a small rocket therefore, I must say I must compare the delay time or the time taken for reactions to occur with respect to something else, what is that other time scale which I should have in mind? Like for instance, now let us take what are the other times which could probably enter into our system or into our computations.

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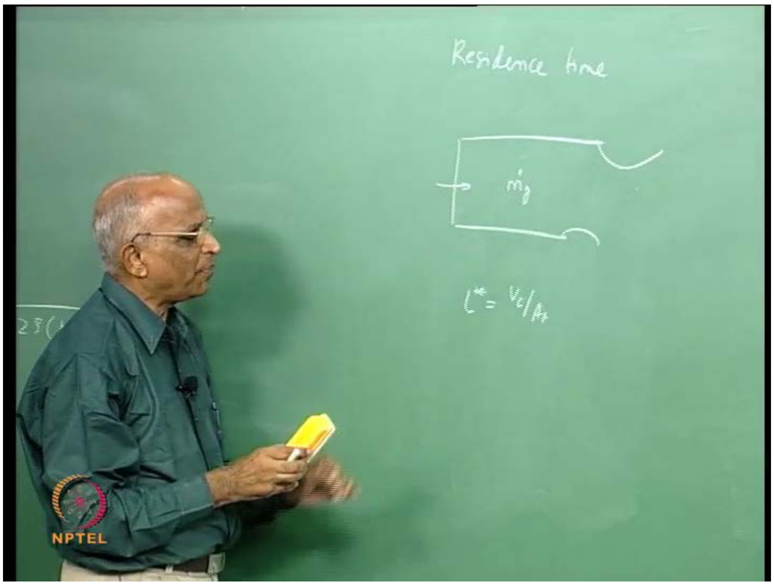

We let us say what is the residence time, what is the time over which a propellant stays in the chamber; propellant vapor stays in the chamber? What is the time which is available for the propellant to stay in the chamber? Maybe, I inject something here, I form gases here m dot g, the m dot g the stays in the chamber for sometime before it is evicted, we said maybe L star is equal to V c by A t gives us the time over which a propellant if the length is large, the gases are going to stay for a longer time, maybe the chamber may require a it can still accommodate a large value of t c before it becomes un stable, why is it I am telling this? Maybe, I should go back to that problem again and ask you one more question. If t c was 0,

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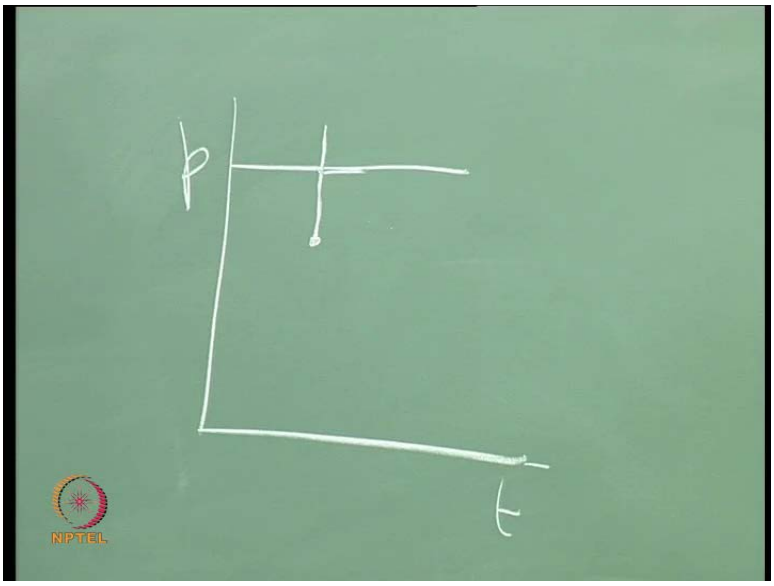

Let us say, if t c is equal to 0 would you have got that phenomena? And if you are going to get that phenomenon y, if it you do not get that phenomenon y, let us try to imagine this again. Let me say yes my chamber pressure with respect to time, originally I had 5 bar it drops to 4.5 bar, but there is no time delay immediately it burns and immediately it goes off, what is going to happen? It goes back here and adjusts. goes back here adjust Therefore, without a time delay I cannot think in terms of these oscillations, and what is the role of this time delay? Well, the same thing there is a lag and that is what causes it. Now, can I compare the time delay with respect to some other time here? Because, if I have a very large chamber, maybe my time delay which is still maybe the same value, I can still accommodate it and it may not give me that problem.

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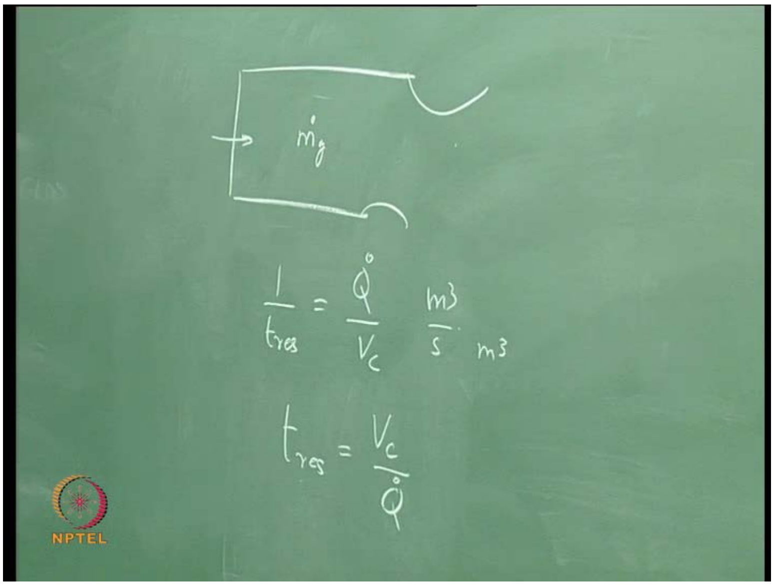

Therefore, I define something known as residence time, which is that time with which the propellant stays in the chamber, and how do I define residence time? Maybe, the rate at which gases are generated in the chamber divided by volume of chamber, if I now put it down it is meter cube by second divided by meter cube no, it must be 1 over t residence right? Or rather, I now write t residence is equal to V c by Q dot, rate at which mass is getting generated in the chamber divided by the volume of chamber is the time the gas is spend in the chamber. I want to express this, how do I write this equation down?

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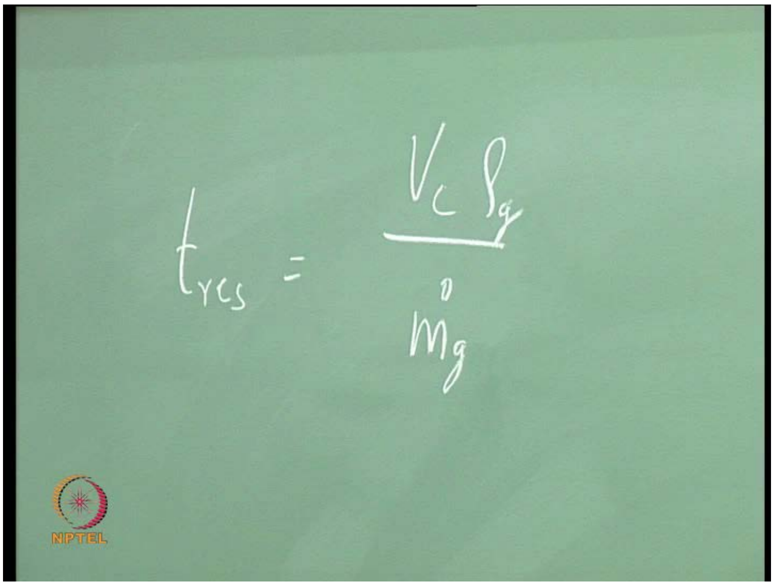

Now, let us put it on t residence volume of the chamber, the volume of the chamber I mean this is the volume over here into mass of the gases in the chamber divided by the density.

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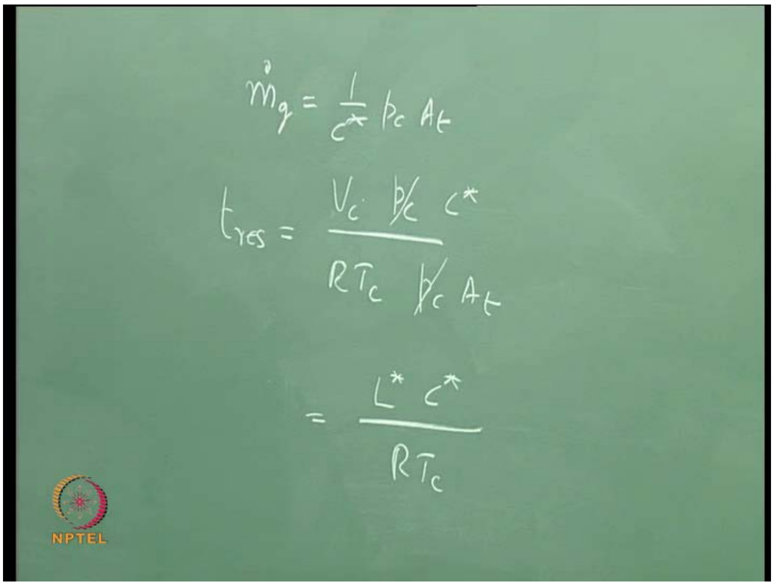

Now, can I write an expression for m dot g what is generated? Whatever is generated goes out to the nozzle 1 over C star into p c A t. Therefore, I can still write t residence as equal to V c, what is rho g? P V is equal to m R T, m by V is density therefore, rho is equal to p c by R T c divided by m g C star comes upstairs so here, p c A t over here. And therefore, now I can cancel the p c out, it is a not a function of chamber pressure anymore and therefore, I get V c by A t we have defined as L star. Anyway, we know it is resonance time into C star divided by R T c. And do you remember, what is R T c equal to in terms of C star? We have done it earlier. We said C star is equal to, yes. See, some of these things are we must keep on our finger tips, so that we can relate it.

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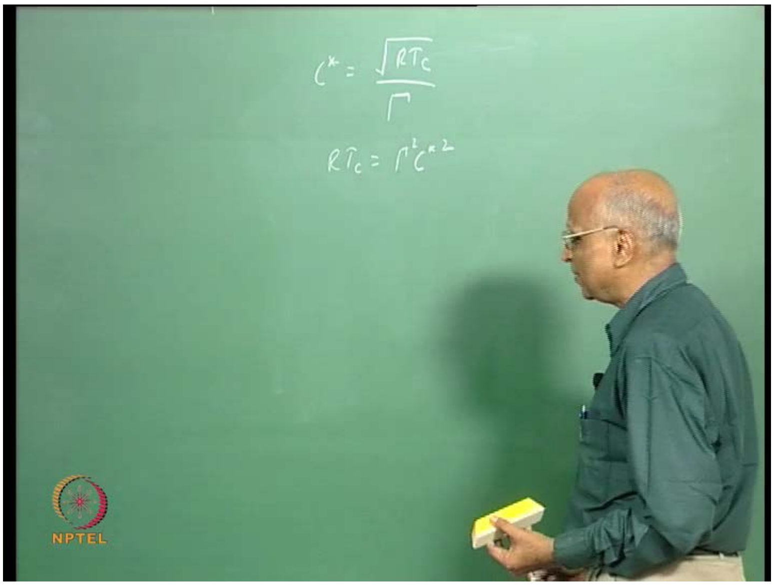

We said C star is equal to under root R T c by capital gamma therefore, we get R T c is equal to gamma square into C star square, substitute the value of R T c there.

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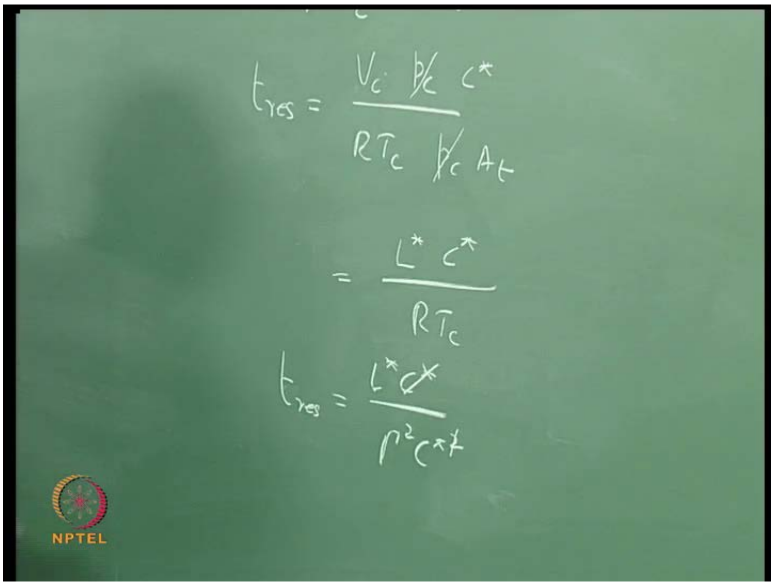

We get t residence is therefore, equal to L star C star by gamma square C star square goes off.

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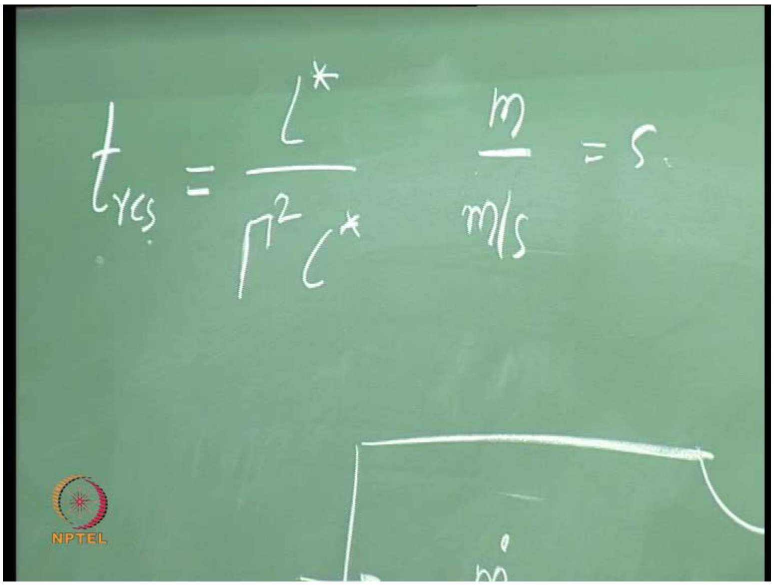

And the value of t residence is therefore equal to residence time t is of the gases in the chamber is therefore, equal to I just get L star by gamma square, you know gamma is a function of small gamma into C star. Well, it makes sense, L star has units of meter, gamma has no units meter per second, this is second and this is your value of residence time. You are able to get the residence time. And now I will say my t c that is the combustion delay, should have something to do this residence time and therefore, I would like to bring in the residence time and combustion time into this equation. Combustion time is already there, only thing I do not know how to solve this. Therefore, let us let us try to solve it in some way, let us try to put some things down and try to solve it. We will first try to bring down or bring in the value of residence time

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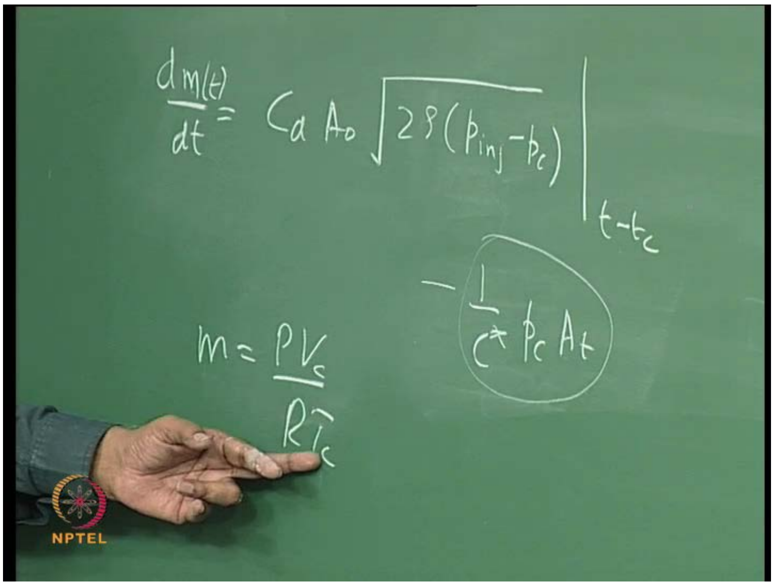

And therefore, let me first take a look at this particular thing the mass which is leaving the nozzle, and before that let us try to find out what is the variation in chamber pressure, we say m is equal to P V by R T, we keep doing the same thing P V c by t c is a mass in the chamber, we assume t c is constant because small changes in pressure or oscillation is not going to change this substantially therefore, let us simplify this equation in terms of dp.

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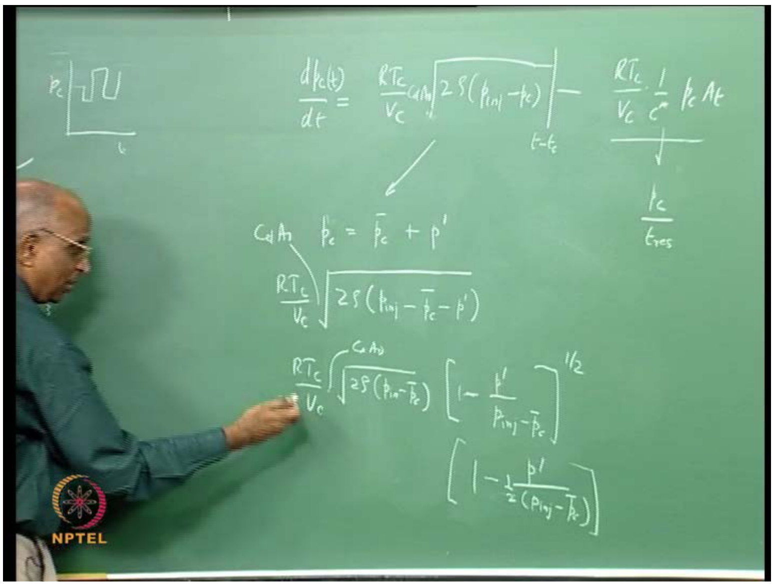

We get dp in the chamber divided by dt is equal to R T c by V c into under root 2 into density p injected minus p c minus, I write the value again R T c by V c into one over C star into p c A t. This should be multiplied by C d into the area of orifices A 0 C d A 0. Essentially, what is it we are looking out? We are looking at the variation of p c with time, we are trying to see whatever we studied just now with numerical example, we found under some conditions it goes like this, what is the rate at which pressure change with respect to time? Is what we want to predict using this equation and precisely that is what I am solving for? Therefore, but we have to also note something here I must not forget that which is very primary this happens t minus t c before this.

Therefore, this is as a function of time therefore, p c is also at time t over there. Let us simplify this part of the equation, let us let this out in terms of maybe residence time, let me go back here… Precisely, what did I write? R T c was equal to gamma square C star square V c by A t is L star into I still get a value of C star into p c over here and therefore, the second expression over here I can write it as equal to p c into L star divided by gamma square C star and this is equal to residence time and therefore, the last part of this gives me a value p c by t residence. I still need to solve this, I do not know how to do this. Let us see how I can manipulate this in some way, and this is quite interesting quite simple.

Well, let me say the chamber pressure at any instant of time p c at any instant of time is denoted by p c which is equal to the steady value, in our example the steady value was 5 MP a plus I have p prime some oscillation, some disturbance in it. Therefore, I can write this expression now as R T c by V c into under root of 2 rho into p injected minus the value of p c, that is the steady value minus the value of p prime, or rather I take the steady value outside I take it as R T c by V c into the steady value that is 2 rho into p injected minus the value of p c bar over here, and then I write the balance as 1 minus p prime divided by, I have to divide it over here p injected minus p c bar to the power half. All what I have done is I have taken this outside, I took this again outside and therefore, I have p bar minus this over here.

I also know that the magnitude of p prime may not be very large and therefore, p prime divided by this maybe a small number and therefore, I can write this part of the expression as 1 minus p bar divided by maybe 2 1 by 2 of p injected, p injector that is the injection pressure minus the value of t c bar over here. And what is the value I get over here?

Let us see R T c by V c, we have just now seen what it is. And what is the value of this? 2 rho into p c somewhere we dropped the value of C d, where was C d? You should have got mass which is going C d A 0 here; that means, I should have had here C d A 0, why did we drop this? We should have carried it forward, and if I take the value of C d A 0 into 2 rho is the regular mass which is leaving through the nozzle because and these are under steady state conditions.

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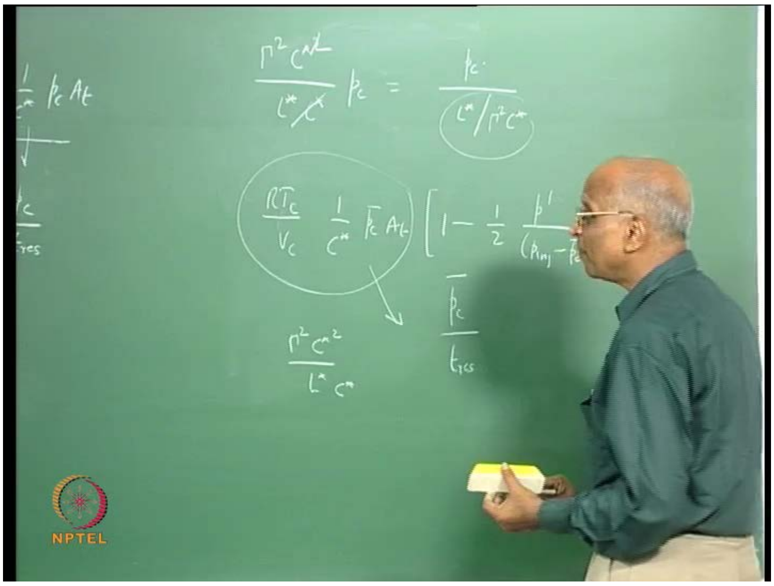

What is it I get I get R T c by V c into C d A 0 into this is the value which is steadily going, that is one over C star into p c bar into A t steady value of pressure, mind you please be clear about it, into I write the same equation here one minus the value of half of p prime divided by the value of p injected minus the value p c bar, is this is that does this make sense?

You have a steady value of thing which is getting injected, and under steady conditions whatever is getting injected is leaving through the nozzle; that means,1 over C star p A t and therefore, I take R T c V c which was a coefficient 1 over C star into p c A t, and if I were to look at this value I again substitute R T c is equal to gamma square into C star square, and I get the value of V c by A t which is L star over here, and I get the value of C star over here, and what is it I am now I am getting? Let us again be very clear. What is it I have got? C star L star by gamma square C that is one over residence time and therefore, this expression now becomes p c bar by the value of t residence and therefore, what becomes of my equation?

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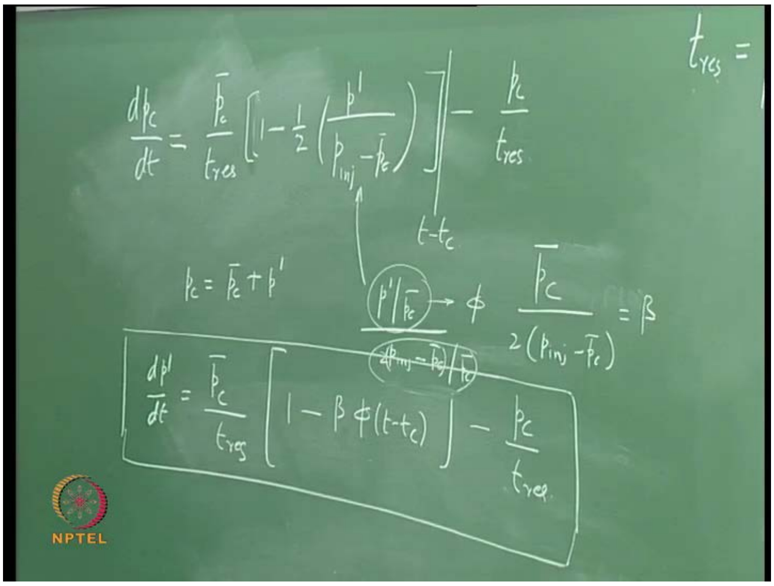

My equation gets terribly simplified, and now I am able to write this equation as dp c by dt as equal to, let us put that down I get the value of p bar by t residence into 1 minus half of I get the value over here as the pressure perturbation divided by this 1 minus half into pressure perturbation divided by the pressure in of injection minus the steady value over here minus the value corresponding to the last term, which we call it as p c by t residence.

Therefore, you see I am able to simplify the equation quite a bit over here. And now, what is it, I have to do to solve this equation. To solve this equation I need to get some way of relating the value of t c, but mind you something which we forgot. And what we forgot was this is at time t minus t c. I need to make some changes in order to understand this and therefore, now I say we have already told well, p c at any time is equal to p c bar plus the value of p prime. And therefore, if I were to change; p c bar does not change, I can write this equation as dp prime by dp as equal to the value p c that is chamber pressure steady divided by t residence into I get the value 1 minus. If I were to make a manipulation, and divide by numerator by p c bar and denominator by p c bar, I can write this thing within brackets as maybe p prime by p c bar divided by p injected minus p c bar divided by p c bar, and this is the magnitude of the perturbation above the min, I denote it by something like let us say if I were to look at this I am looking at p bar by p c bar.

Maybe, let us denote it by a value let us say phi or something, and this value which again has a value 2 at the bottom, I say the value of p c divided by 2 of p injected nominal value, this is what I divided p injected minus p c bar I denote by the value beta, I can write this equation as 1 minus the value of beta which is just the value all steady values, phi denotes the perturbation and I say it is phi of t minus t c is what I get here minus the value of p c divided by t residence, and this is my final form of this equation. I drop the half because I brought half here, because this value was small and this is my well final form of this equation. I just need to solve this equation to be able to get the value of how the pressure changes, and this maybe I will do in the next class what we will have. Therefore, what did we do in this class?

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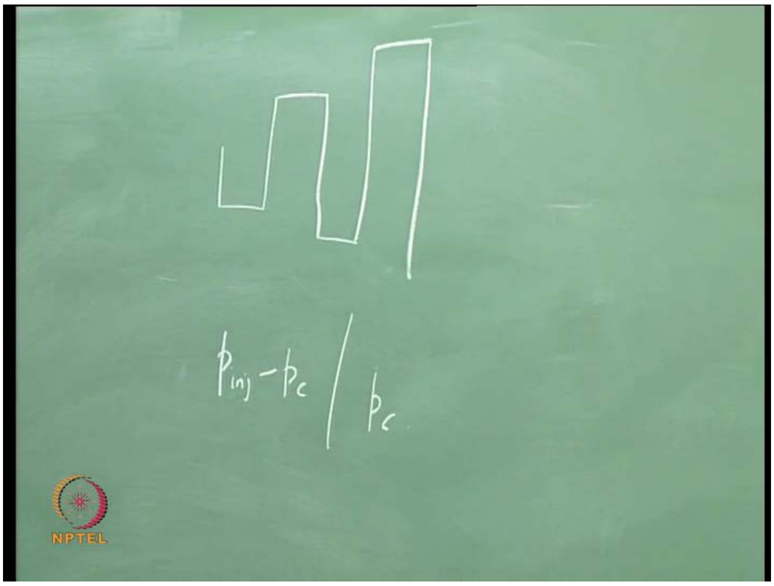

We just told ourselves the moment fuel gets injected, and it takes a certain time delay to burn then there is a problem that maybe any change in pressure could get magnified, and for some values of p injected minus the value of p c with respect to p c, and that is what we are trying to do through this particular equation. We will illustrate the solution of this equation, and then move forward in the next class. Well, thank you then I think.