**Rocket Propulsion**

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**Module No. # 01**

**Lecture No. # 36**

# Combustion Instability in Solid Propellant and Liquid Propellant Rockets - Bulk and Wave Modes

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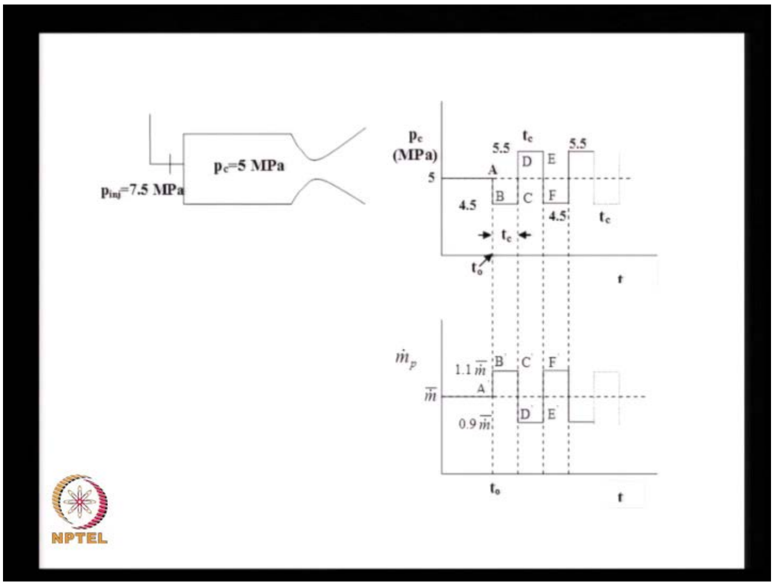


In the class today, we will first review what we learnt in the previous class through a through this power point presentation. In the first slide what I show here, and this is what we learnt in the last class namely, if the chamber pressure of the rocket is 5 Mpa and the injection pressure is 7.5 Mpa and if there is a sudden dip in pressure by 0.5 Mpa, then what happens is this sudden dip in pressure allows more fluid to come inside the chamber. And when more fluid comes inside the chamber, it does not burn instantaneously, but it takes a small time t c is what we defined in the last class namely, it takes t c time to burn. And when it burns since more fuel comes inside or more propellant comes inside the chamber, there is an increase in pressure after this decrease there is a time decrease and thereafter the pressure increases; in other words during this time additional propellant has flown in the chamber.

And because when the pressure has increased, when the pressure has gone up the pressure drop across the injector has now decreased therefore, less quantity of propellant now flows and it burns after some particular time little less pressure is generated, and we got a pressure oscillation like this. In other words, when the injection pressure was 7.5 Mpa and the chamber pressure was 5 Mpa. We progressed and found that when there is an initial drop in pressure by 0.5 Mpa then after sometime the pressure increases to from 5 to 5.5 again decreases from 5.5 to 4.5 and this sequence of oscillation continues.

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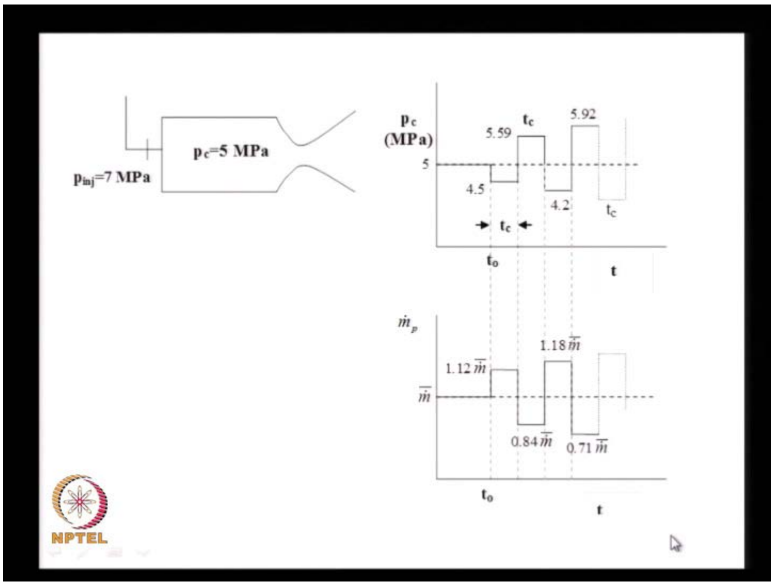


We would like to see, what happens when the pressure change. We change the pressure injection pressure from 7.5 to 7. And when we did that, what did we find for same value of the combustion delay time; we started with a pressure drop from 5 to 4.5; when this happened, we had additional propellant flow which was 1.12 times the nominal value of flow corresponding 5, and for this value since the flow rate is higher and it took some time to burn, when it came over here, you got additional gas getting generated in the chamber, and this led to an increase in pressure, this higher pressure led to starvation of propellant inside it. And therefore, after sometime the value that means, the propellant flow rate decreased when this came up, you had a net decrease in pressure; and what did we find that the pressure amplitudes kept on increasing with time?

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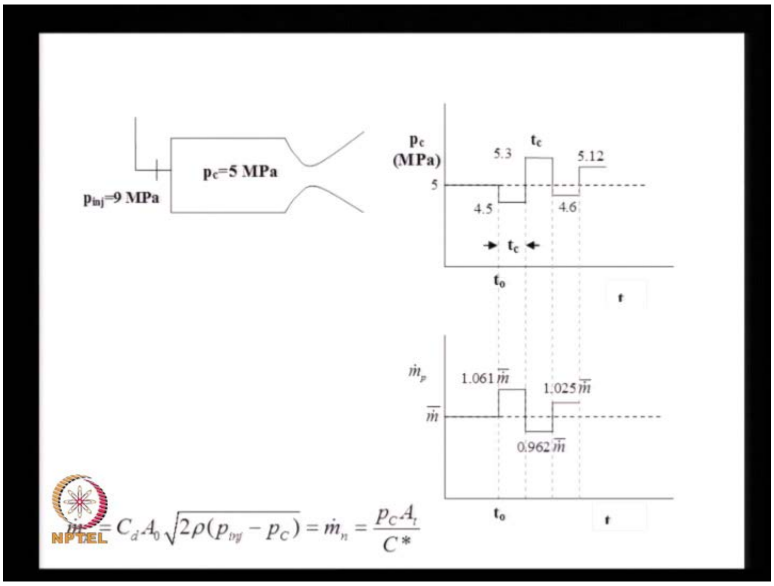


We also did 1 exercise wherein we had the injection pressure at 9 Mpa and what did we find in this case instead of repeating the whole thing, we started with a pressure drop if up to 4.5 after a delay time, because of the decreased propellant flow or because of the decreased pressure increased mass of propellant in the chamber, pressure again increased, but the pressure increase to a smaller value, the successive values kept on decreasing. In other words, in this case you had the pressure amplitudes decreasing with time; in the earlier one, you had pressure amplitudes increasing with time; and still in the earlier one you had neutral level of oscillations that means, pressure amplitudes were constant.

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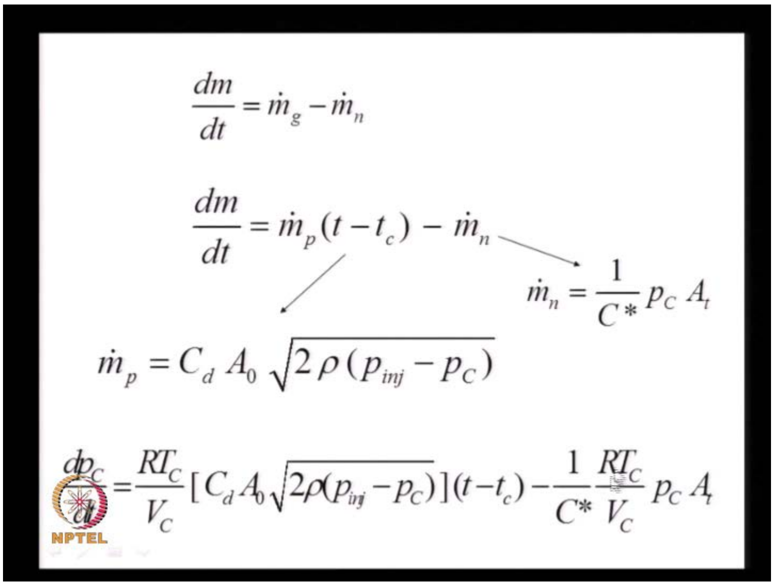


We also went ahead and try to find out, what is the reason for these changes whether we could find it out through some expressions. And therefore, what did we say we said that the rate at which mass increases in the chamber rather d m by d t is the rate at which the propellants gasify that is m dot g in the chamber minus the value at which the gases leave the nozzle m dot n. We wrote these expressions in terms of a delayed combustion time; in other words what is it I am trying to tell you the gases, which are generated are not corresponding to the propellant, which is getting injected rather the mass which gets generated comes from propellant which is injected t c earlier as you will recall if I go back to the previous slide now when the pressure drops the propellant flow increases, but this propellant flow has increased over here, it takes a t c time to gasify in other words there is a delay in time and therefore, the quantity of propellant which has flowed t c seconds earlier is what contributes to the pressure and therefore, d m by d t or accumulation of mass in the chamber for of the gas is corresponding to liquid propellant flow is d m by d t is equal to m dot p that is liquid propellant flow t c seconds earlier minus m dot n.

You will also recall that m dot n we expressed as p c A t by C star and therefore, and therefore, now we would like to solve this equation, but this equation as we noted was rather clumsy, because time effect came over here and therefore, we want to simplify it and what did we do? We wrote m p dot t minus t c namely, m p dot is equal to in the case when the chamber pressure is p c the discharge coefficient of orifices and the total area of orifices is A 0 discharge coefficient of orifices is C d m dot p is C d A 0 into 2 of the density of the propellant into p injected minus p c this is the injection pressure drop. And therefore, I could write d m by d t m is equal to p V by p V by RT therefore, I could write m is equal to p V by R T assuming t as a constant is equal to t c, I get d m d t is equal to d p c by d t is equal to here I get R T and V c I bring it on the right hand side, I get d p c by d t is equal to R T c by V c into this value this value of propellant injected t c time earlier minus the 1 leaving the nozzle as 1 over C star RT c V c into p c by A t.

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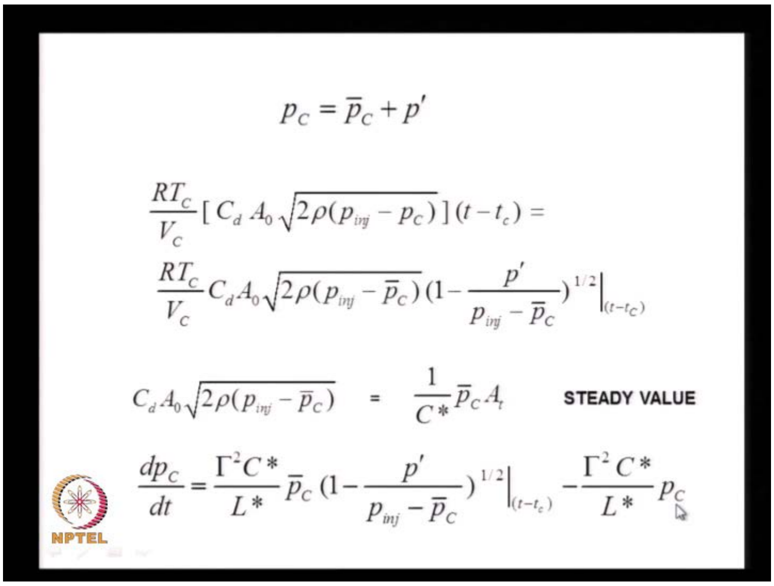


We want to solve this equation and how did we go ahead with it? We told ourselves we are interested in finding out whether the oscillations grow or decay therefore, express the chamber pressure as the mean chamber pressure that is p c bar plus the perturbation rather we had the term in which propellant mass is there, we had R T c by V c C d A 0, we had p minus p c therefore, this p minus p c is at t minus t c, we could write it as p injected minus here we said p c is equal to p c bar plus p bar rather p injected minus p c into 1 minus p bar; that means, p injected minus p c minus p bar and that is what this expression tells you R T c by V c C d A 0 into 2 rho into p injected minus p c and minus p bar over here at time t minus t c.

Now, we note that this value is corresponding to the steady chamber pressure, and this steady chamber pressure corresponds to the steady value of mass flow rate through the nozzle namely, 1 over C star into p c over A t. And therefore, the expression which we had for d p c by d t could again be written as may be we have we are saying that this is equal to 1 over C star p c A t. And therefore, we also had the expression for let us say that the mass that this c the value of chamber pressure is here the A t is here we could write it as gamma square C star by L star into p c bar that means, you had this value and this value over here into one is the steady value which is p c bar minus the value of p bar into this to the power half minus the value leaving through p c over here.

How did this come over here? We had the value you will recall if you go back to the old slide over here, we had C star is equal to under root R T c by capital gamma therefore, R T c is equal to C star square divided by gamma square, and this is what is substituted subsequently to get this particular expression over here. We wanted to solve this equation, and how did we solve this?

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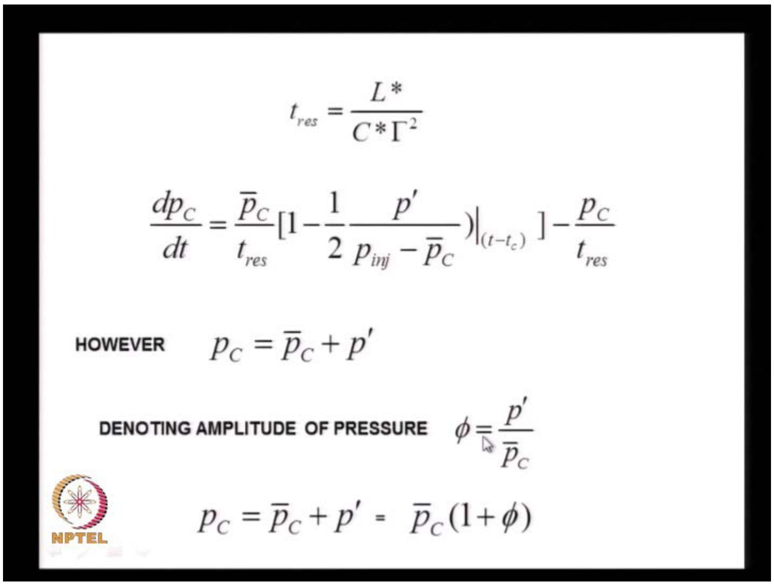


We said well we will define residence time we derived the expression as a function of L star; that means, the value of V c by A t divided by C star by gamma square and therefore, we got the final expression as d p c by dt is equal to p c bar by t residence time that is the time in time spent by the gases in the chamber into I take the under root sign, I take the first term I know, my p bar is small therefore, I write it as 1 minus half of p bar by p injected minus p c bar minus what I get on the right hand side namely, the value which I took here gamma squared C star by L star I put it in terms of residence time and get p c by t residence.

However since p c is equal to p c bar by p bar and I can also denote the amplitude of pressure oscillations in a non-dimensional manner as p bar by the value of p c bar I substitute instead of writing p bar and p injected minus p c bar and p c bar. I substitute the value of p c over here in terms of p c bar plus p bar or 1 plus phi into p c bar. And what is the expression I therefore I get.

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$$\frac{dp_C}{dt} = \frac{\bar{p}_C}{t_{res}}[1-\beta\,\phi(t-t_c)] - \frac{p_C}{t_{res}}$$

$$\beta = \frac{\bar{p}_C}{2(p_{inj}-p_C)}$$

$$\frac{d\phi}{dt} + \frac{\phi}{t_{res}} = -\frac{\beta}{t_{res}}\phi(t-t_c)$$

I get the value as d p c by d t is equal to p c bar by t residence into 1 minus the value of beta and this beta I substitute in terms of p c bar divided by 2 into p injected minus p c bar; let us go back to the old one. I denote this particular expression or rather p injected minus p c bar I take p c bar by p c and therefore, I get p c bar on top therefore, p prime p c bar is equal to phi and I get p c bar on top therefore, p c bar by p injected minus p c bar and that with a 2 here is what constitute my value of beta and I get a value of d p c by dt as equal to p c bar by t residence into 1 minus beta into the value phi which is p bar by p c bar at time t minus t c minus p c by t residence.

In this way, I am able to incorporate the value of time into the equation that means, the value of t minus t c corresponding to this. And now I solve this equation and therefore, I simplify it and I get the value of now I get p c is equal to p c bar into 1 plus p prime by p c bar, which is what we had earlier p c bar is equal to p c is equal to p c bar 1 plus phi. And now, if I substitute it here I get the value of d phi by d t plus I get phi by t residence is equal to minus beta of divided by t residence into phi. I need to solve this equation; and now, I have been able to incorporate the earlier time of fuel injection into this equation and then I solve it

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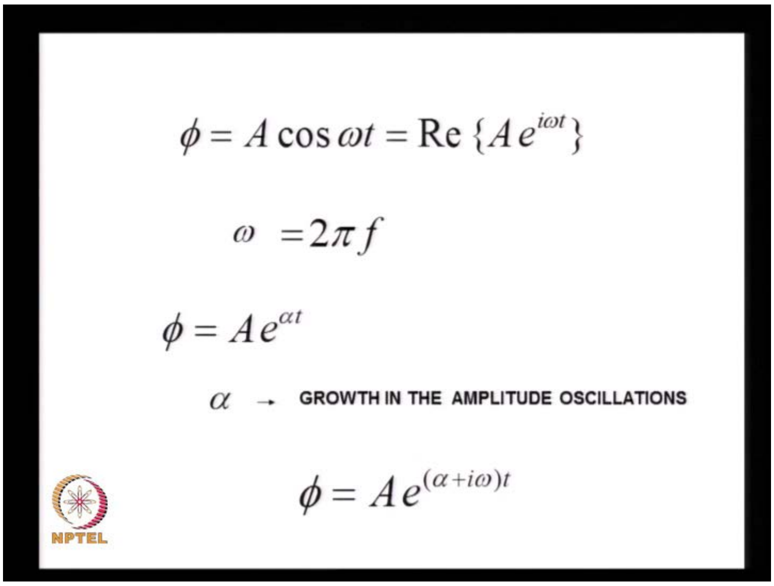


And how do I solve it? Well I assume a form phi as equal to A cos omega t which is real part of A into e to the power I omega t and I know omega is equal to 2 pi of frequency. And therefore, if I say the oscillations grow with time well phi is equal to the real part becomes A e to the power alpha t if alpha is greater than 0 well the oscillations grow in amplitude and therefore, if I denote phi as equal to not only in terms of frequency omega, but also in terms of the growth of oscillations, I can write phi as equal to A e to the power of the growth rate of oscillations and the frequency of oscillations namely phi is equal to A e to the power alpha plus I omega t I substitute this expression in the earlier expression namely, over here.

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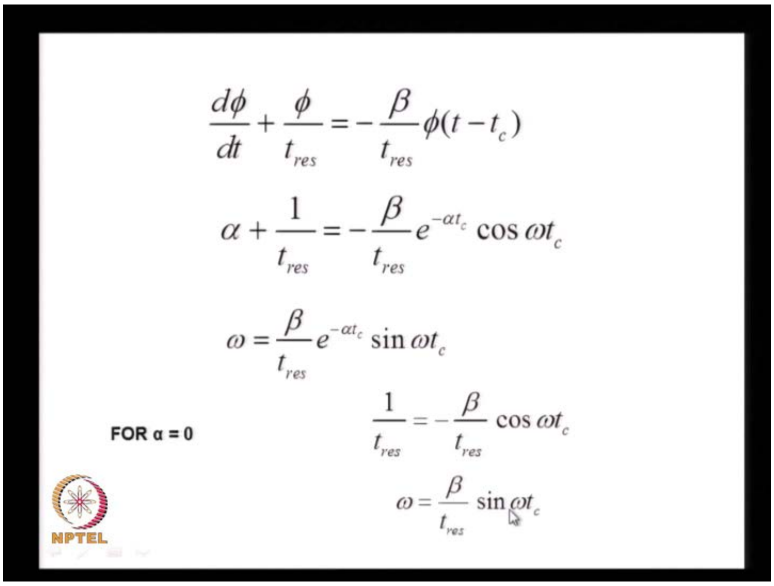


And now I get the value as d phi by d t plus phi t residence is equal to beta t residence into phi t minus t c. I substitute the value of the expression which involves phi in this particular from over here. Phi is equal to A e to the power alpha plus I omega t and I separate the real and imaginary parts this is the real part of the equation and this became my imaginary part of this equation now I need to solve for the real and imaginary parts and what did we do? Well let us not solve it for the total let us solve it for the critical case for which the waves do not grow; that means, alpha is equal to 0 I substitute alpha is equal to 0 here and I get this equation becoming 1 over t residence is equal to minus beta over t residence into cos omega t I put alpha is equal to 0 here I get omega is equal to beta over t residence into sine omega t c. And this I further simplify, since I have cos omega t c and sine omega t c cos square omega t c plus sine square omega t c is equal to 1

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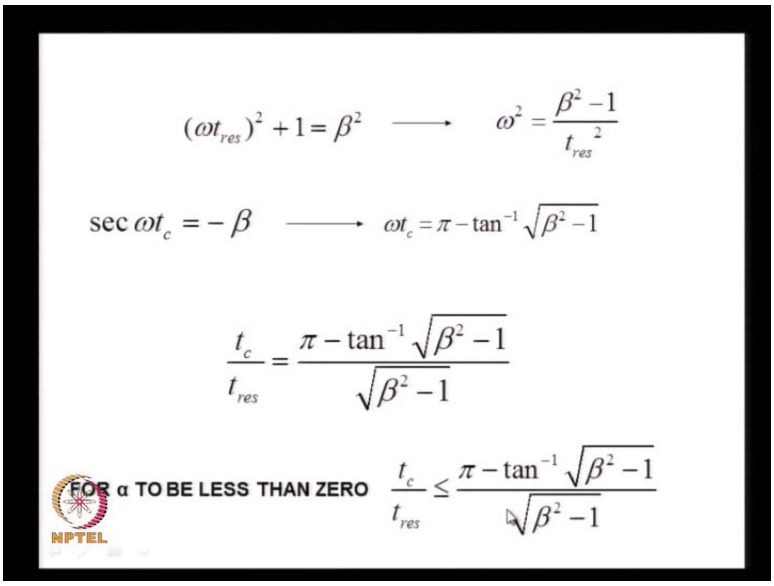


I can write this expression as equal to or on the right hand side sine square omega t c plus Cos square omega t c is equal to 1 or rather I get omega t residence square plus 1 is equal to beta square or rather the circular frequency square is equal to beta square minus 1 divided by t residence square. I also get from the second equation here, I can I can now write the value at t c; that means, I get the value of cos omega t c as equal to 1 over t residence t residence cancels it becomes 1 over beta minus 1 over beta which means, it is minus it is in the third quadrant. And therefore, I can write cos or the value of secant of this as equal to secant of omega t c is equal to minus beta t or the value of omega t c is equal to pi minus tan inverse beta square minus 1. Now I i combine these 2 equations I get the value of t c from this particular expression t residence from this particular expression and then I put the value of t by t residence, because it gives me the value pi minus tan inverse under root beta square minus 1 divided by under root beta square minus 1 over here. Now, what is it I find, I get this critical value this came for alpha is equal to 0 in for alpha to be less than 0. Well the value should be less than this and based on this.

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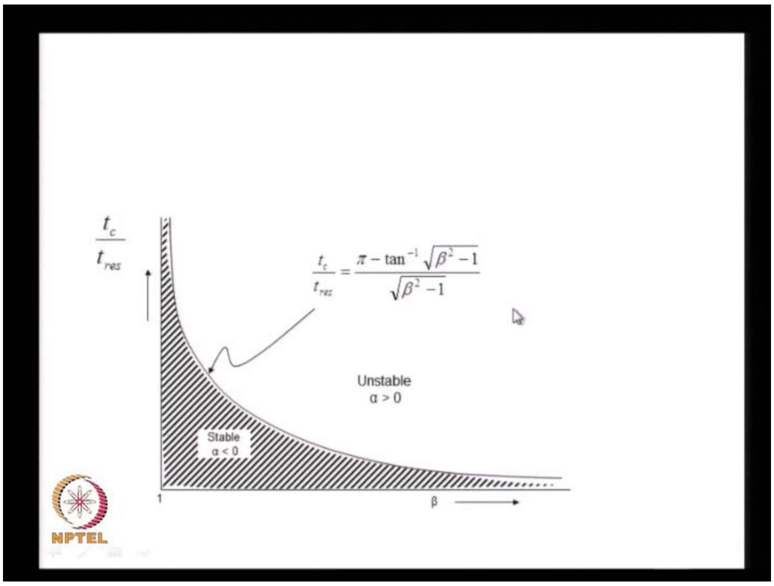


We plotted the stability diagram and what was the stability diagram? We got this expression for t c corresponding to alpha is equal to 0. If alpha is less than 0, the value is over here that means, it is stable if alpha is greater than 0, this corresponds to the unstable part, and this is what we learnt to do.

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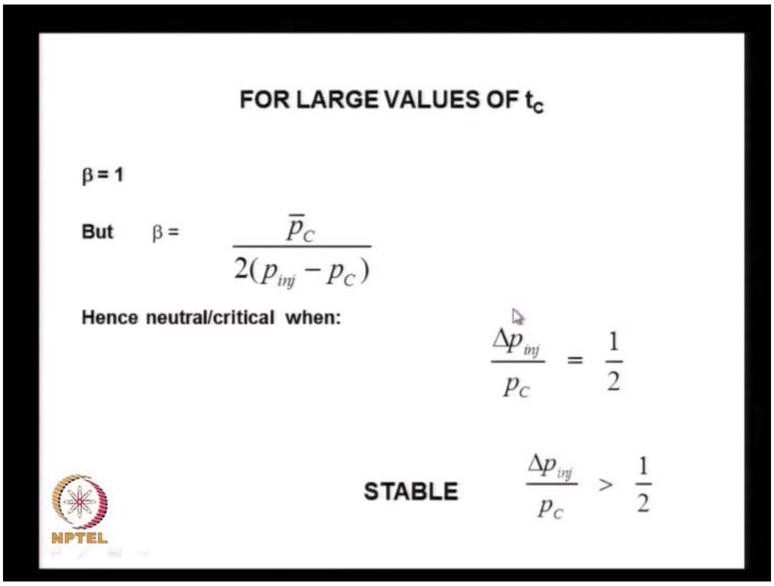

Now when I want to get the value of the injection pressure drop as a function of chamber pressure drop, let me go back to the previous equation for large value of this, if I go over here and if I see my combustion delay is infinity rather t c by residence is very large the value of my beta is equal to 1 this is under the critical under the condition that the combustion delay is very large if I want the value A t c for a very large value; that means, beta is equal to 1 then I get, but I also know that the value of beta is equal to p c bar by this therefore, I will get neutral oscillations. The condition under which pressure amplitudes do not grow corresponds to the value of p injected minus p c bar, which I denote by delta p injection divided by p c is equal to half it will be stable, when it is greater than 1, because I took the inverse for beta therefore, I get delta p injected by p c must be greater than half for stable and if oscillations have to grow delta p injected by p c must be less than half.

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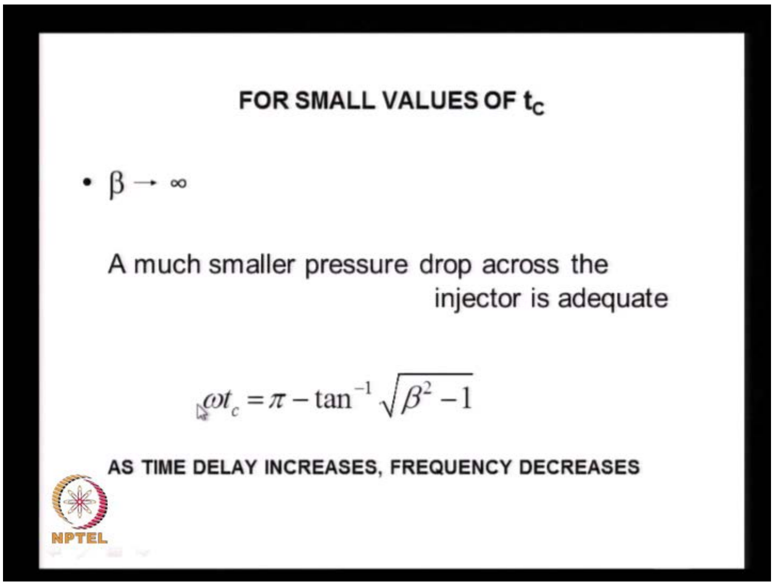


And this I follow up and now I say if the value of t c is not infinity, then what happens to the value of beta you go back yes this is for infinity. When beta is equal to 1, when t c has a finite value and keeps on decreasing the value of beta keeps increasing. If the value of beta keeps increasing, well for large values of beta. Now, I find that the injector pressure drop, which is very much smaller than the value what I specify, is adequate and that is what I will be examining further in the course of my talk today. But I also find that the value of omega t c from the previous expression comes out to be pi minus tan inverse under root of beta square minus 1 rather as the time delay increases my frequency of oscillation decreases.

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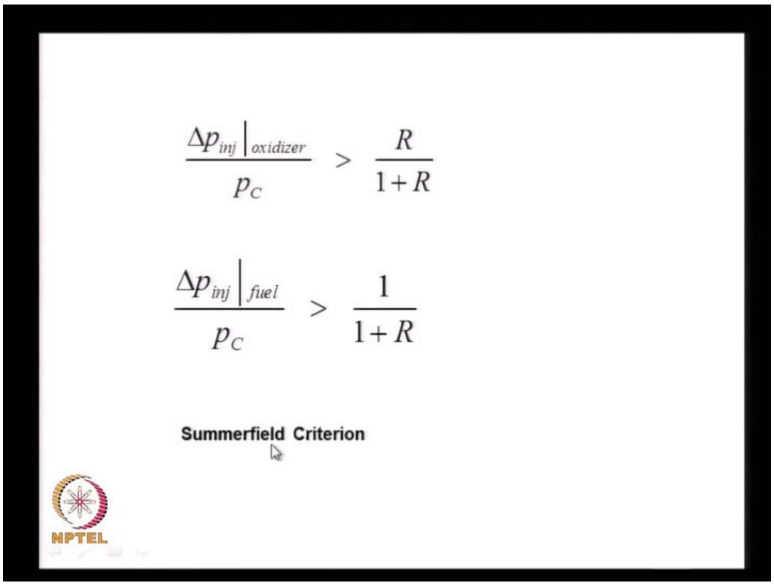


To summarize then, if I have instead of 1 propellant flow, if I have propellant flowing at a mixture ratio R, well for the condition under which is t c is infinite I have delta p injector for the oxidizer divided by p c is R by 1 plus R and for the fuel, I get 1 over 1 plus r; that means, these are the conditions, which must be satisfied for stable performance, when the value of t c is infinity. When the value of t c is finite, well a much smaller pressure drop is adequate to ensure stability; this particular criterion was developed by summerfield, and it is known as summerfield criteria.

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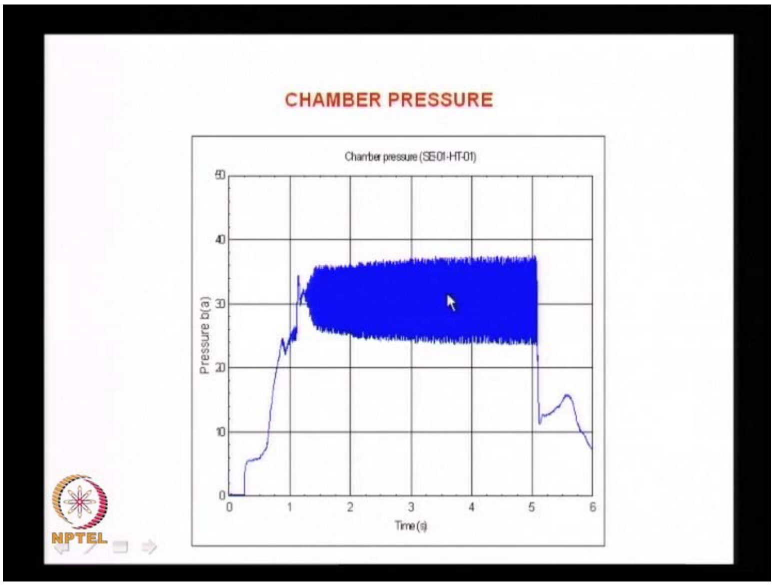


So, let us quickly summarize where we were. All what I am trying to say is well under certain conditions a liquid propellant rocket shows severe oscillations and these oscillations in chamber pressure and therefore, the thrust is what we call as combustion instability the plot shows the chamber pressure. As a function of time you see that the chamber pressure instead of being constant at a about 30 bar fluctuates between 24 and 36 bar the magnitude of the pressure oscillations increase with time and thereafter get saturated.

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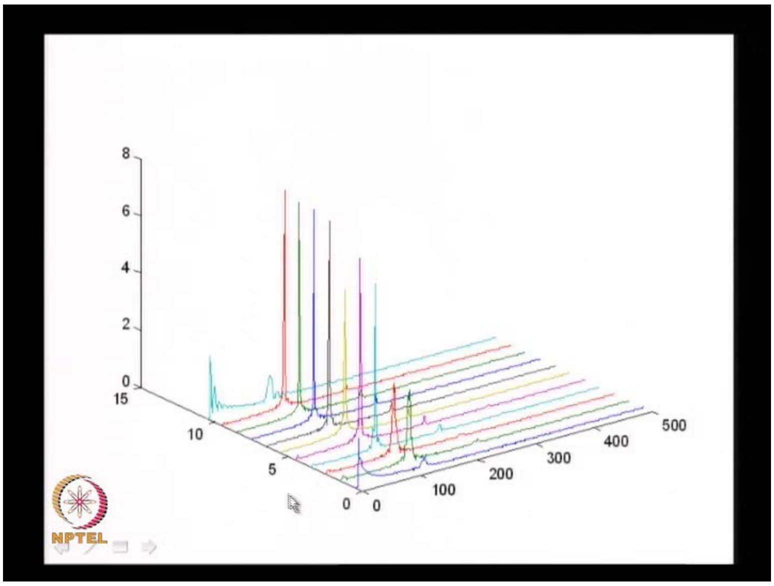

And when you look at these oscillations, you find in the in the previous plot, if I were to take a look at this I find this is the duration of my this is the frequency over here, this is the time of different instance over here, I find the this is frequency of the oscillations this is known as a water fall plot in which I plot the amplitude of the oscillations as a function of the frequency and as a function of the time this is at 0 time motor has not ignited it is just got ignited I get an ignition peak and I get a small oscillation here after some time may be at 1 second. I start getting some oscillation here and then it at a frequency of around 100 hertz and then for higher frequencies I get no oscillation and this hundred hertz oscillation is seen throughout till the motor sort of dies down over here; that means, I have frequency of hundred hertz and which these are typical ways of representing the instability, wherein we plot the amplitude of pressure oscillations, as a function of time going in this direction for different frequencies over here.

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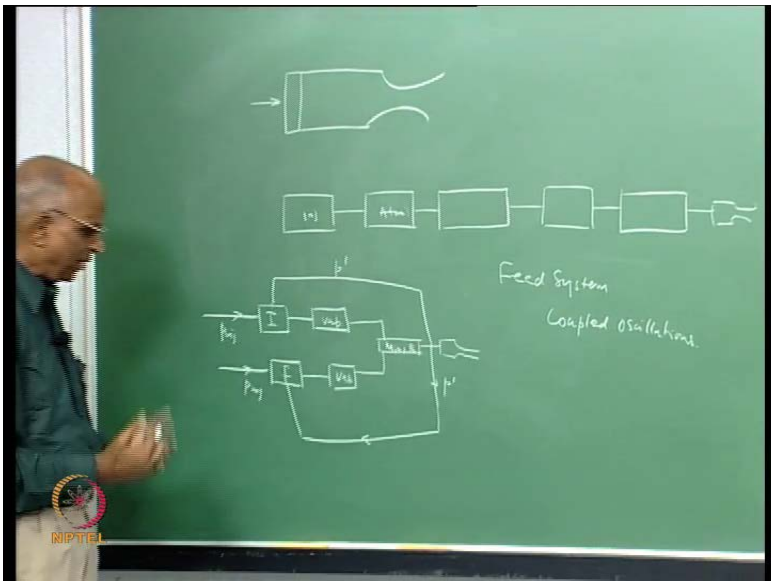


We have an injector, we have a combustion chamber and the nozzle you are injecting something over here and what is it if I were to denote it. I say I have an injector over here and what happens after injection. I have the process of atomization taking place atomization then I have vaporization of the propellant taking place after vaporization I have mixing taking place after mixing I have chemical reactions taking place and then the gases leave through the nozzle. Now, in a bipropellant injector; that means, when I have fuel and oxidizer may be I have fuel injector fuel holes, I have oxidizer holes over here I call it as injector over here I the fuel injector may be creates the fuel drops and it vaporizes first, it vaporizes the oxidizer drops vaporizes. The vapors come and mix in the chamber may be this is also in the chamber, but thereafter you are mixing and reactions taking place over here, and the mixed one is what indicates injected out of the chamber.

During this process of combustion, if there is some oscillation namely p prime it gets back into your injector over here this is where you are injecting p injector you are injecting p injector over here this p prime gets modulated in your injector and therefore, you are getting some changed thing, which is coming into your chamber again. Your vaporization gets effected and this becomes something like a feedback circuit. In other words a change in the downstream value of p prime due to mass generation, gets coupled to your intake to the injector; that means, it is a feed system which gets effected and therefore, this type of combustion oscillations is known as feed system coupled oscillations. Let us now physically try to again go through what little we have done so far in some other way. We will tell ourselves well, I have something like a tank which supplies the propellants into the liquid propellant combustion chamber.

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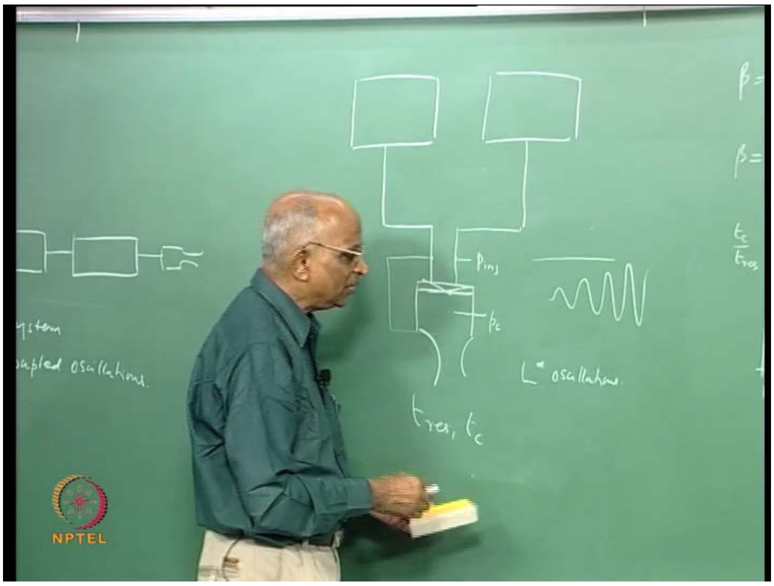


Fuel oxygen I do not draw any of the intermediate things all of us know the differences. I have an injection pressure here, I have the injector here spraying it, I have the p c over here I have the injector pressure over here. I take the same values on both the sides and now what is happening any pressure changes which is happening creates a differential flow over here. In other words, this is what gives me the feedback that the pressure changes here reflects on the flow of propellant into the chamber from the feed system; that means, from the feed system; that means, feed system is influenced and such type of oscillations are known as feed system coupled oscillations. Therefore, it is known as feed system oscillations or rather the oscillations depend terribly on t residence and time of delay t residence depends on the L star of the motor.

Therefore, some people also call it as L star oscillations. In other words all what we have seen is it is quite possible when I have the injector pressure drop, which is less than some threshold value then it is quite possible for me instead of having a steady value of pressure to get started and have a diverging pressure and these oscillations are link to the feed system because the feed system because the feed is what gives you the pressure drop over here. If I have a very high pressure here I will not get these oscillations and therefore, it is known as feed coupled oscillations or L star oscillations maybe we should go through it again in some way maybe, we should go and look at the derivation and what we derived the results, but something I want to caution you many people wrongly use we derived the expression that may be the value of injector pressure drop.

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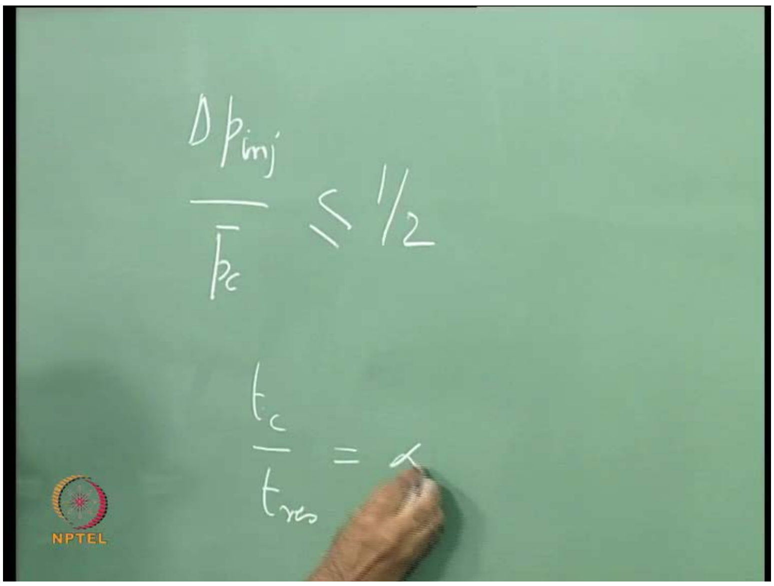

It is delta p injector to the value of p c bar must be less than equal to half for oscillations to occur, but this condition is true only for the condition that t c by t residence is equal to infinity how do I explain this.

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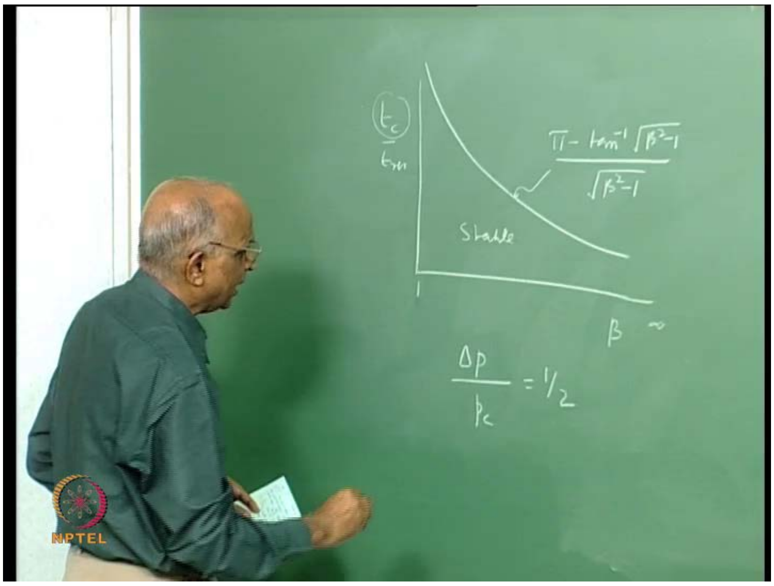


Let me get back to our stability diagram, in which we had t c divided by t residence as a function of we had the value of beta going from 1 to infinity and initially for very large values of t c infinity we had this value. And we had this boundary which was given by pi minus tan inverse of beta square minus 1 divided by under root beta square minus 1 what did we try to get we found out the value for the condition when t c is infinite or rather it is the value beta is equal to 1 for which we said beta is given by the value, which was equal to injection pressure drop or rather 1 over beta. And therefore, we had the expression for expression as delta p injected divided by the value of p c was equal to half for this particular case or rather we said that, when t c is very large this becomes my stability criterion.

However, when t c is less the value of beta is larger and when beta is larger well the value of delta p which I can take is going to be much lower than alpha this was why my stable region and therefore, a much lower value of injection pressure drop is will give me stable operation. When I have smaller values of t c corresponding instead of the infinite values corresponding to beta is equal to 1. Let me repeat this again all what I am saying is when t c goes to infinite.

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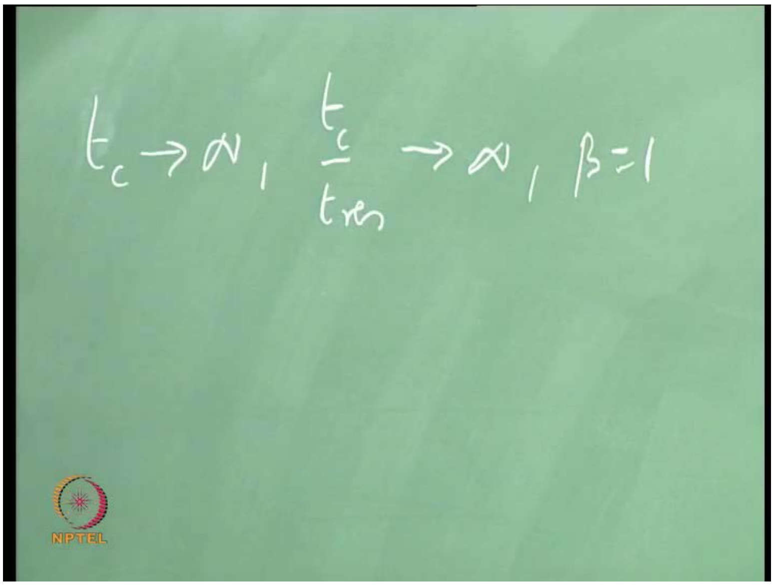


We are talking of t c by t residence also of infinite going to infinity or rather beta is equal to 1. When t c is smaller, well the value of beta is larger and therefore, a smaller injection pressure drop will be sufficient for me to provide stable operation.

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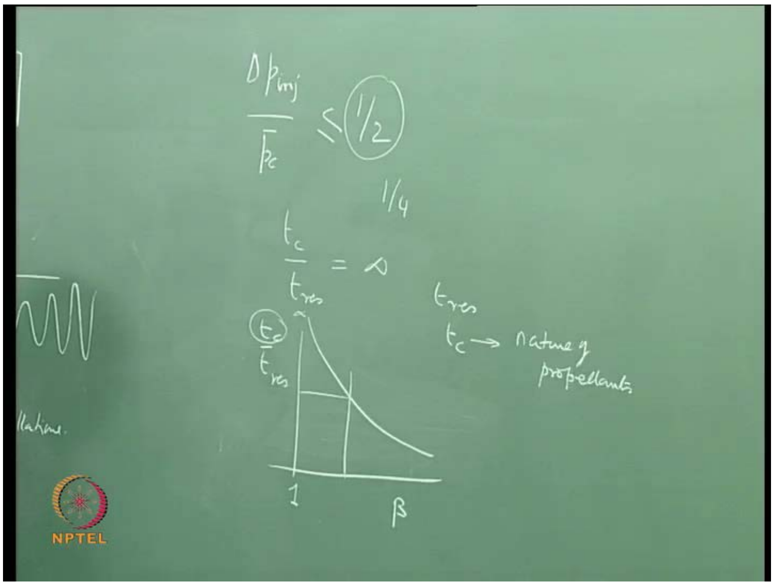

If you have chemical delay is a small number well, I need a smaller injection pressure drop to avoid oscillations and this we also saw we saw when t c was 0 well there is no question of oscillation at all therefore, the feed coupled oscillations or L star oscillations are a function, we must remember of not only the residence time and the value of t c it is, because it is a function of A t c it becomes a nature of propellants may be if I have propellants, which are sluggish like aniline and red fuming nitric acid it is more susceptible, because they have a larger delay I think this is all what I will do in this feed coupled oscillations with respect to liquid propellant rockets. Now, let me see whether I can get similar things for solid propellant rockets let me do a simple argumentative explanation.

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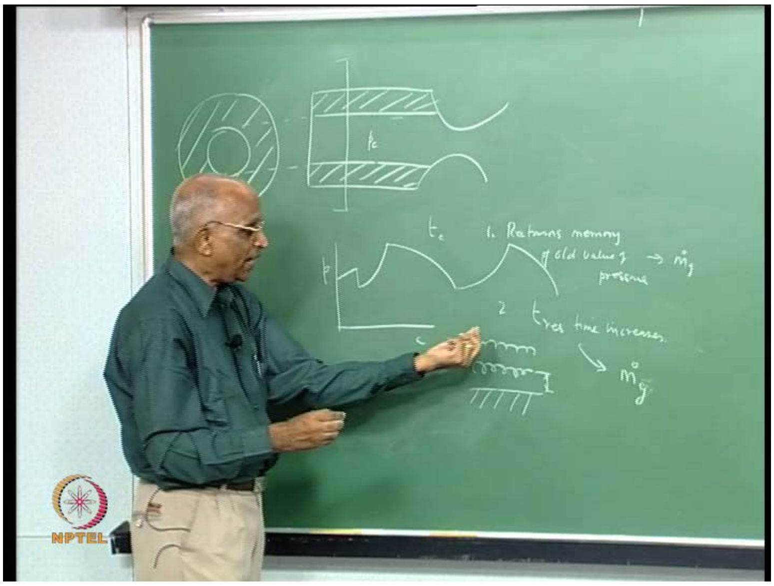


Let me consider the case where in I have let us say a solid propellant rocket, I have a propellant over here, which is burning we will try to go through the same arguments I put the for the liquid propellant rockets let the chamber pressure be p c this is let us say a section over here is the propellant grain outer diameter let us say the chamber pressure is the high value steady value burning is progressive may be it is going to go up, but it is the progressive study value at some point of time let us see that the pressure drops some way pressure drops we have said if the pressure drops what is going to be the effect would I have an cascading effect like what I had in liquid propellant rockets.

If I have A t c over here a delay let us see what happens when pressure drops the heat from the gases is still seeing the propellant; that means, the propellant surface is still hot and it has a memory of higher pressure over here. Higher pressure means the flame is nearer to the propellant surface therefore, more heat is generated and therefore, propellant surface gets more heated than at lower pressure. Therefore, I find yes at a when the pressure is higher may be the heat flux or the heat load on the propellant is higher.

Even though the pressure drops it still retains memory of the old pressure and it does not immediately relax to a lower heat flux at the surface; that means, I say point 1 retains memory of old pressure old value of higher pressure old value of pressure number 2, the pressure has dropped if the pressure has dropped what is going to happen the flame front that is the that is the distance of the flame from the surface goes a little bit further and since the pressure depends the pressure the 1s the number of molecules the rate of reaction decreases. If the rate of reaction decreases well the rate of generation of hot gases decreases or else the velocity decreases and if the velocity decreases the residence time increases. Let me go through this a little more in little more detail all what we said was whenever I have a propellant surface I have a flame which is standing of at a certain depth.

I reduce the pressure therefore, the flame standoff increases, but the surface still has memory of this is point 1. Now this is gone up over here the pressure has decreased if pressure has decreased the value the chemical reaction rate has decreased. If chemical reaction rate has decreased the rate at which mass of the gases is getting generated has decreased the velocity has decreased if velocity has decreased for same distance I get more residence time residence time has increased. If residence time has increased the time taken for chemical reactions to get completed is faster therefore, residence time increasing results in more value of the rate of the mass generation.

Now, the surface still retains memory of the past it still hot therefore, it is still producing gases at the old rate and therefore, even though the pressure has fallen it takes some time for the surface to come back to it the present state and therefore, what is going to happen is the pressure increases, because I increases, I have I still have more mass of gases which are going up I have higher mass flux rather than this value and therefore, I get a higher value of pressure at higher value of pressure well what happens is the memory of the surface is with respect to the older value of pressure for which the mass generation rate is lower the residence time is less if residence time is less I have a smaller chemical reaction time to generate gases and therefore, it again falls back again I have this and therefore, the same thing is possible even in a solid propellant rocket namely whenever there is a delay due to the thermal lag at the surface. And we have the residence time getting change I can get this oscillation.

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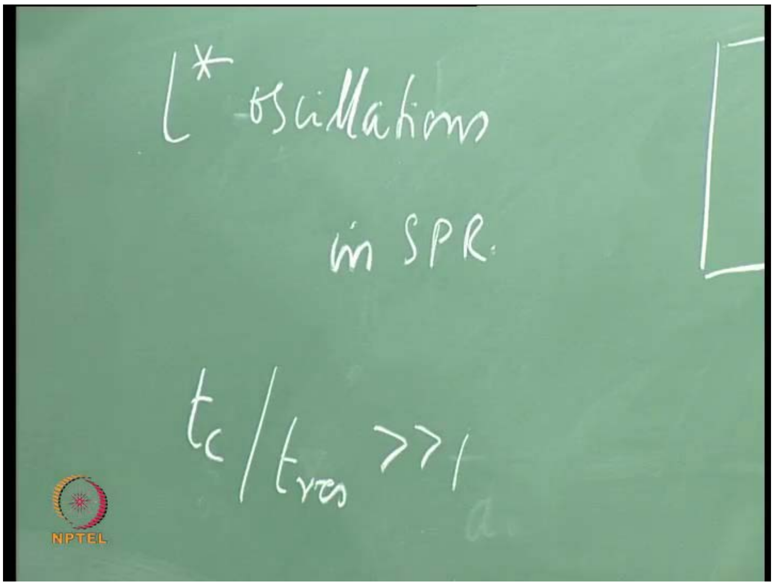


And this known as L star oscillations in solid propellant rocket. Why do I say L star it depends if I have a very small rocket in which the value of volume by throughout area is small well the value of t c compared to residence t residence. Will be larger and therefore, the oscillations are more profound and that is why it is known as L star oscillations.

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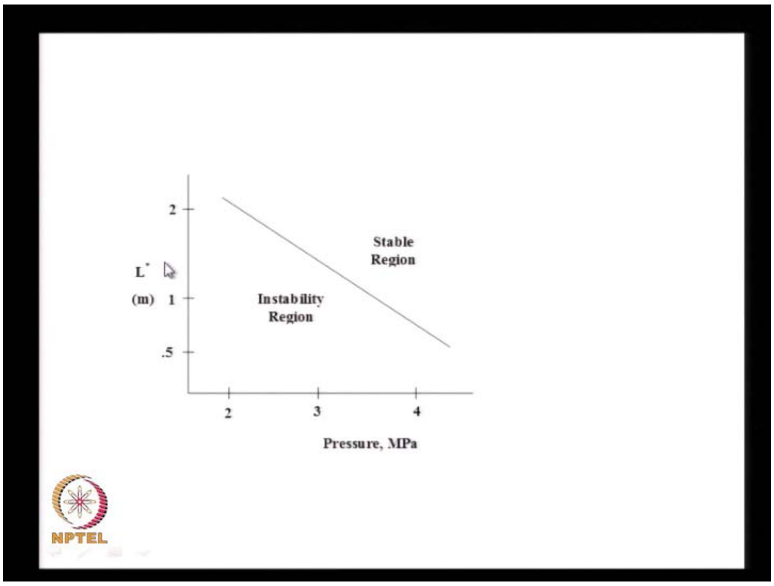


Let us focus on this particular slide over here, we in this slide I show the value of the L star as a function of chamber pressure and I define the regions of instability that is L star mode of oscillations and the stable region what is it we are telling if the value of the L star is small. Well the time of residence of the gas is t residence of the gases in the chamber is small when t residence is small the value of t c by t residence is larger and therefore, the combustion is more likely to be unstable and therefore, I have the instability region corresponding to small value of L star. If the chamber pressure is smaller the chemical reaction time is larger chemical reactions take more time to go to completion t c is larger. And therefore, I have an instability region which is over here and a stable region corresponding to larger values of L star and larger values of chamber pressure therefore, we find that for solid propellant rockets it is necessary to provide either a large value of L star or a large value of chamber pressure for stable operation to take place.

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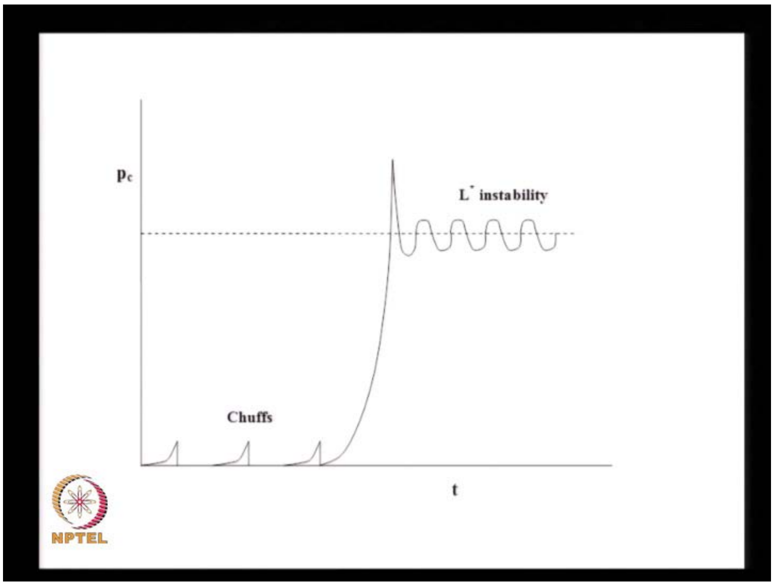


Let me go and illustrate this further may be if I have an L star instability I have oscillations in chamber pressure which are seen over here. This is L star in stability, but very often we also find that when a motor is ignited and the ignition is not that good. We sort of get some spikes like this is the ambient pressure and you get a small spike in pressure the motor gets ignited, but gets quenched. But; however, the motor grain is still hot and thereafter, this heat or the temperature in the grain again ignites the motor and again I get another pressure spike another spike something like a train going chuffing all along chuff. You get these small oscillations in pressure which are due to ignition and hang fire situation rather than L star oscillations or L star in stability, which occurs when the chamber pressure has a finite value this will help us to differentiate between L star instability and chuffs which are essentially due to an ignition phenomenon not the good ignition where as L star instability comes, because of the competition between the chemical reaction time and residence time when the time is large and you have these reflexes from the propellant absorbing energy and delayed getting delayed that is where you get the in L star instability.

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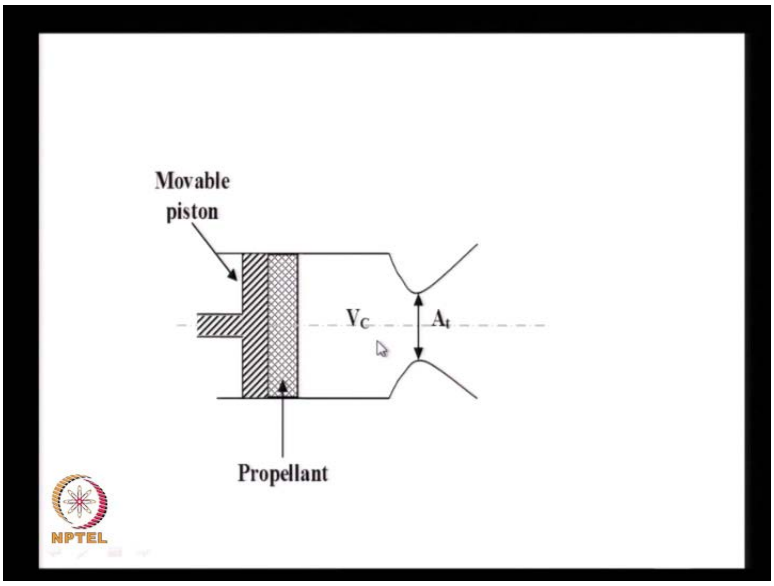


Well I could have a motor in which I can vary my V star V c that is a chamber volume for a given throughout area get different values of a L star and operate it at a different values of chamber pressure I have a propellant here, I have a moveable piston by which I modify my volume for each of the experiment and therefore, I operate the at different values of L star different values of pressure and there after I can get my particular plot and this is how we define the L star boundary this plot is fairly linear for most of the cases and we say well this is the instability region this is the stable region this boundary in the plot of L star versus pressure defines or gives me the boundary between stable operation and unstable operation for L star oscillations. Something which I thought I should at this point in time I should differentiate is may be whenever I talked in terms of feed coupled oscillations in a liquid propellant rocket or in terms of L star oscillations in a solid propellant rocket.

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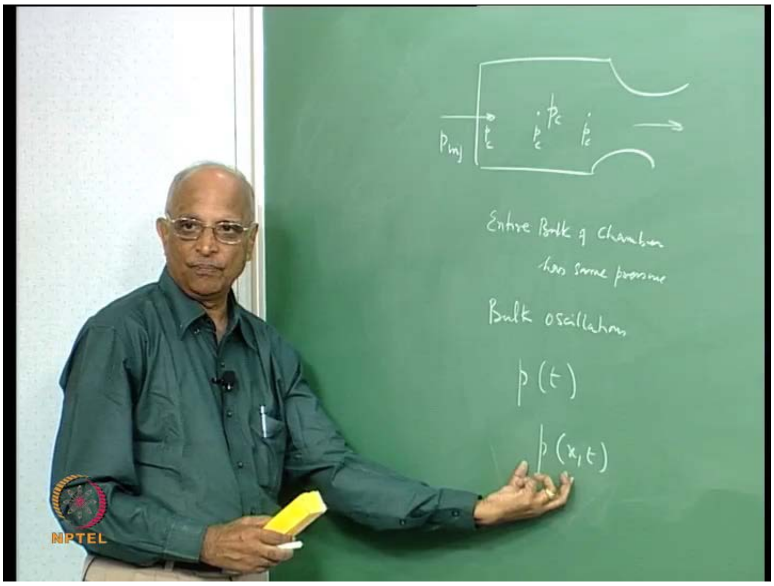


All what I am saying is the pressure in the chamber or within the grain. At this point the pressure is the same at this point the pressure is the same at this point the pressure is the same. Within the chamber the pressure is the same there is no variation in pressure; that means, I am talking of may be the entire bulk entire bulk of chamber has same gas pressure what did I do? I injected something at p injection I have p c over here p c p c p c and it is exhausting; that means, the entire volume or the entire bulk of propellant is at the same pressure and therefore, such oscillations are also known as bulk oscillations. Something very similar to the toy I showed you, the entire body is moving it is not that 1 part not that my hand moves and the other hand is stationary, but in practice what happens I could have a different pressure here I could have a different pressure here.

And that is what happens when I talk I am talking to you and as I am talking you know it takes some time for my signal to reach you therefore; it is quite possible pressure instead being a function of time alone could be a function of distance and time and therefore, whatever we have discussed so far relates to something like a lump mass assumption where in, I take the entire volume to oscillate in unison which may not really be true therefore, we must move to something which will differentiate between pressure at the different points. Let us think of this situation how easy would it be to do that may be we will illustrate it through some physical examples. Let us consider the following.

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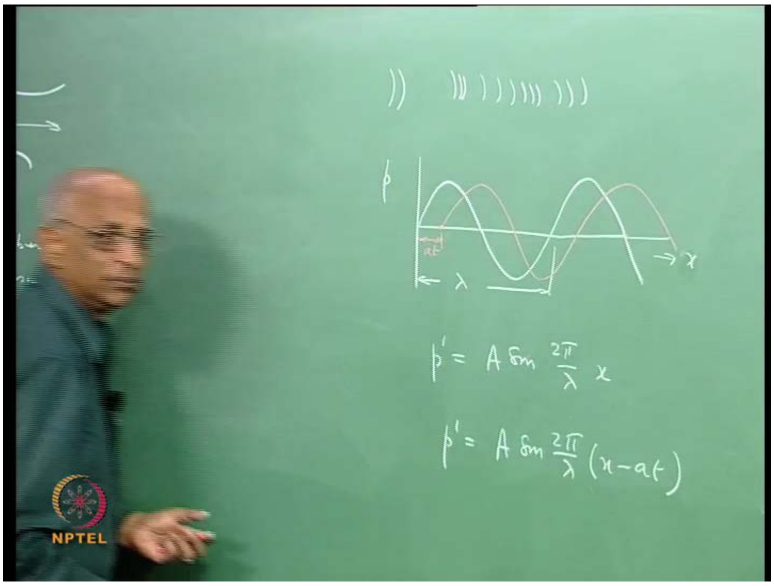


Let us consider let us say may be I am talking to you some acoustic oscillations may be I am talking over here. How does the signal come to you a series of compressions followed by rarefactions followed by compression followed by compression followed by rarefaction right? In other words, if I were to plot it what is it I get I get region where in, I get compression followed by rarefaction compression followed by rarefaction. In fact, this is something is the sound wave and how do I represent pressure as a function of time and I can now write this is a sine wave after all, this is one cycle of oscillation that is one wave length of oscillations and this is my distance. As distance moves my compression rarefaction compression rarefaction travels and therefore, I can write this as p is equal to may be some amplitude a sine of lambda corresponds to 2 pi 2 pi by lambda into x.

This is the equation to a sound wave which is propagating and sound wave means, a disturbance wave p prime is equal to p hat or a sine 2 pi by lambda x, but there is no traveling component in this is just a wave of compression and this, but I was telling you that the wave travels. If the wave is going to travel at speed of sound that is a meters per second where a denotes the speed of sound. Well after a let us say after a time t after I start the wave should come over here, that means, over a time t the wave would have moved a distance A t where A is the velocity of sound.

If that is the case the equation to a traveling wave should be p prime is equal to A sine 2 pi by lambda into x minus A t right. Now what happens now when I look at this particular thing and now I tell myself in a rocket chamber what is a rocket chamber it is something like a cavity why do I say it is a cavity well.

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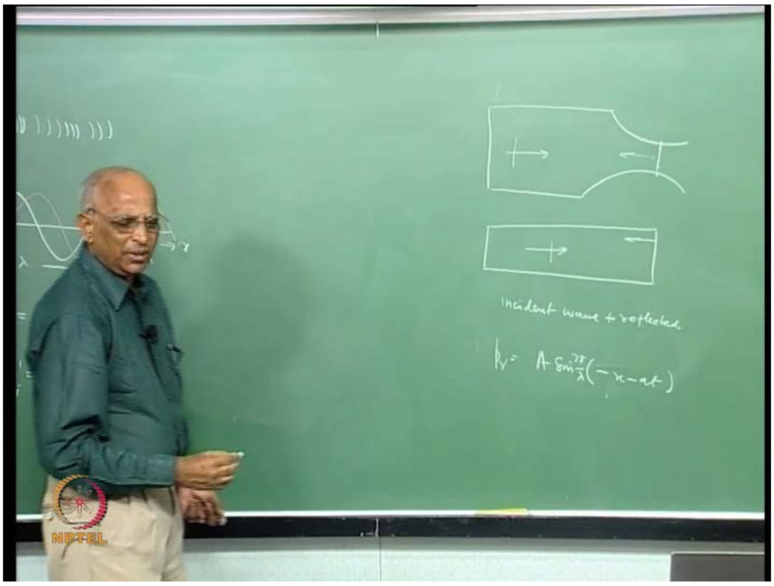


I have the injector side over here or head end of a solid propellant rocket I have the nozzle over here, we found this very rapid changes in density here therefore, the throat acts as if it were a surface itself and therefore, whenever some wave travels up over here this looks at it as it was a solid surface it reflects back the wave is as if it were in an enclosure in which wave moves up over here reflects over here comes back and therefore, there is an interaction of let us say an incident wave plus a reflected wave.

But, I know the equation to a wave can be written like this. Now what will be the equation to a reflected wave if the incident wave I say is p I is given by this. The equation to a reflected wave should be it is moving in the opposite direction therefore, it should be a sine of 2 pi by lambda into minus x minus A t or minus a sine 2 pi into x plus A t and this is what we will be looking at in the next class. We will be trying to see whether some wave motion is possible in the chamber and what it could lead to, but as a prelude to this I want us to think a little bit more and the thinking is maybe I brought a flute I always illustrate combustion instability problem through this.

I have something like I blow into this. Why does it make noise why should a flute make noise? After all I am blowing steady still it makes a noise. Similarly, if I have a whistle a whistle is nothing similar thing at the what is where I am blowing. I blow into this it still makes noise. Why does it make noise? Let us try to understand this problem as a prelude to solving this and if this part is clear. Maybe we will be able to relate it let us schematically show a flow.

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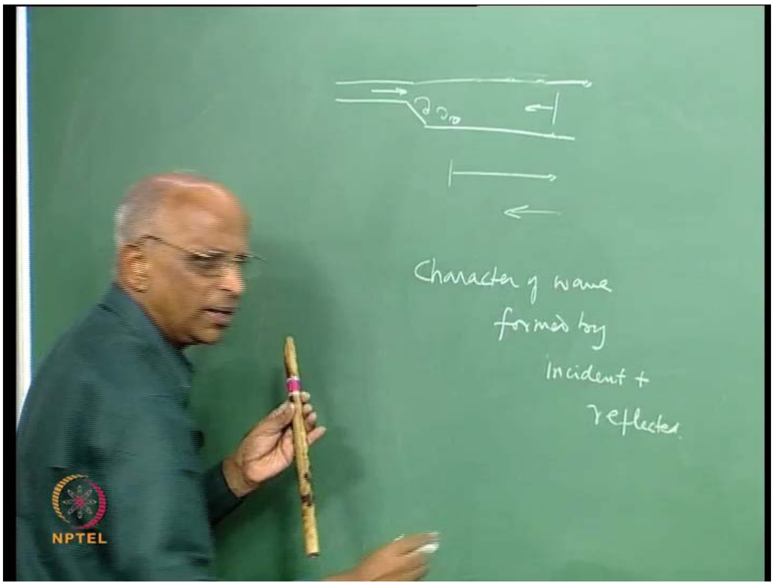


And ask ourselves what happens after all you have something in which I am blowing air I have something like a step over here. Let us be very clear about it I have something like a step over here, I have some holes here what is it I do I blow air here when I blow air I get some eddy's, because all of a sudden there is a change here. Some disturbances are generated and when disturbances are generated they move in a chamber over here it sees an open part here, where it gets reflected back and therefore, a common of interaction between the forward running wave and backward running wave is created and that creates some resultant wave which amplifies this sound and if I were to plug 1 let us say I plug all the holes here I have something like 6 holes I plug it still it makes noise that means, but it makes a different sound noise.

In which case I just have a chamber here in which something is happening when I open something my net reflection is somewhere earlier and therefore, I can change my Character of wave formed by incident plus reflected disturbances and exactly the same thing happens in the case of a whistle. What happens in the case of a whistle let us say you know let us sketch it out.

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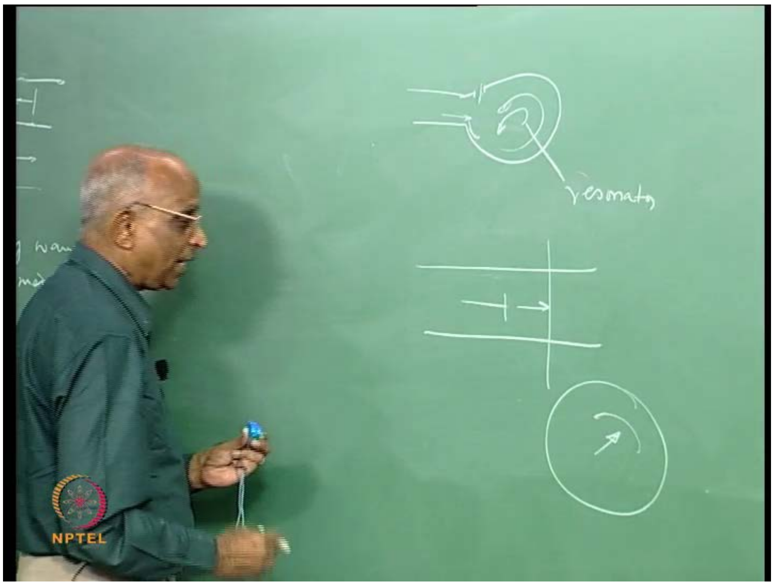

You find the same phenomenon in this, a hole here. What is it I do? I push air it creates some disturbances here some disturbances are generated I have the waves moving in round like this and it is this which functions as a resonator and why does it resonate I have forward and reflected waves coming. In other words, when I have a chamber I could have waves not only moving in this direction, but if I take a section, I have a cylindrical section waves could also move in the tangential direction waves could also move radial direction and this is what leads to disturbances in a chamber and if these disturbances were to couple with the combustion taking place well I could have instability and therefore, in the next class I will start somewhere here may be we will take a look at this reflected wave.

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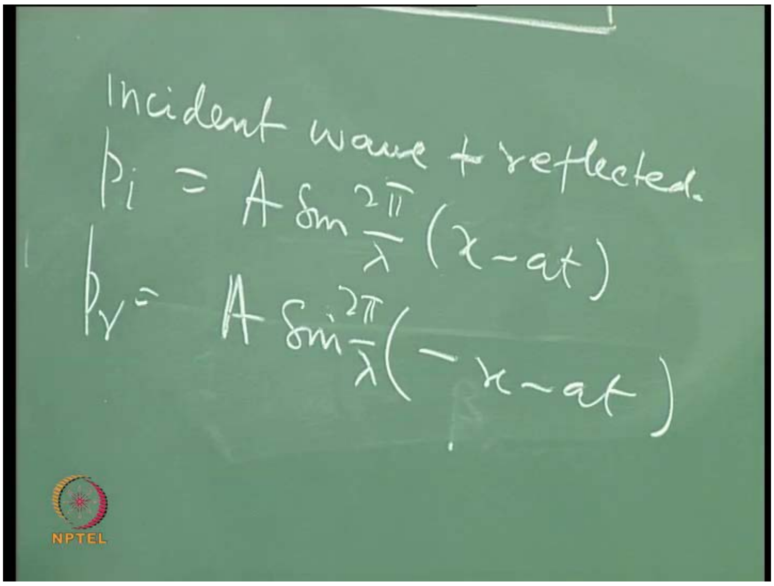


We will take a look at the incident wave, which was equal to A sine 2 pi by lambda into x minus a t we will solve for the resultant wave and based on that we will try to get some more idea of instability for what could could takes place. After this is over, we will go into the process induced instability something like pogo something like slouching and stuff like that well thank you.