Rocket Propulsion

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Module No. # 01
Lecture No. # 37
Wave Modes of Oscillation

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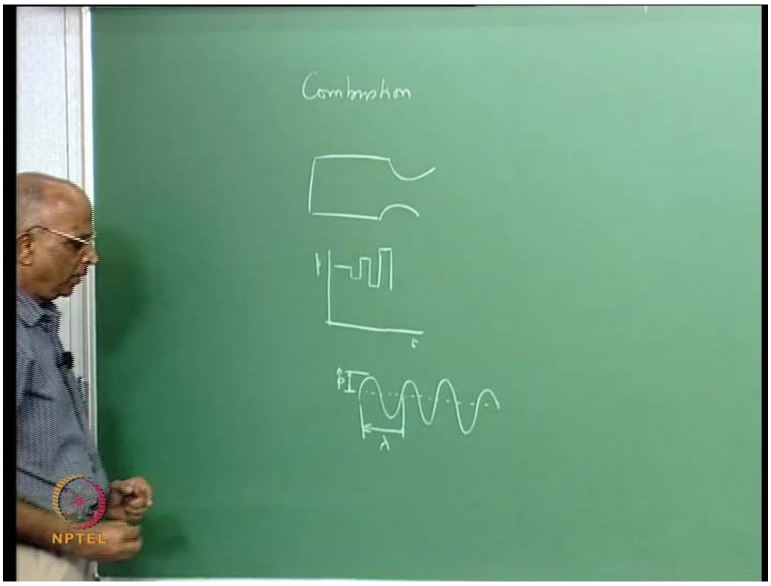

Well, good morning. We will continue with combustion instability. Let us quickly recap where we were last time. See we talked in terms of a let us say a liquid propellant rocket we told ourselves if by chance the steady pressure in the rocket may be pressure versus time if it falls in some way and if the pressure drop across the injector is less than some value relating to the chamber pressure; it is always possible for the oscillations to grow with respect to time. We also told ourselves well you know, this is something like a diverging and pressure could reach a level where in the case may not be standing and it is a problem and whenever we talk of instability; we mean there is a particular frequency at which the events happen and also may be the pressure amplitude keeps growing. And how do we represent it? We said you know to say something is square like this is difficult. May be we take a sinusoidal oscillation and therefore, if we have a sinusoidal oscillation we can always say well the cycle of oscillation starts here, goes to a maximum, comes back here, starts again may be this is the wavelength of the oscillation and whenever we talk of a wavelength; that means, one cycle of osculation correspond to the wavelength and this is lambda. Therefore, we tell that lambda centimeters or lambda meters is equal to your wavelength of your oscillation and the amplitude of oscillation, the maximum value is above the mean value and this level is what we say is the maximum amplitude p hat.

If I have to show this cycle of oscillations; well I say it could be something like a sin theta or a Cos theta.

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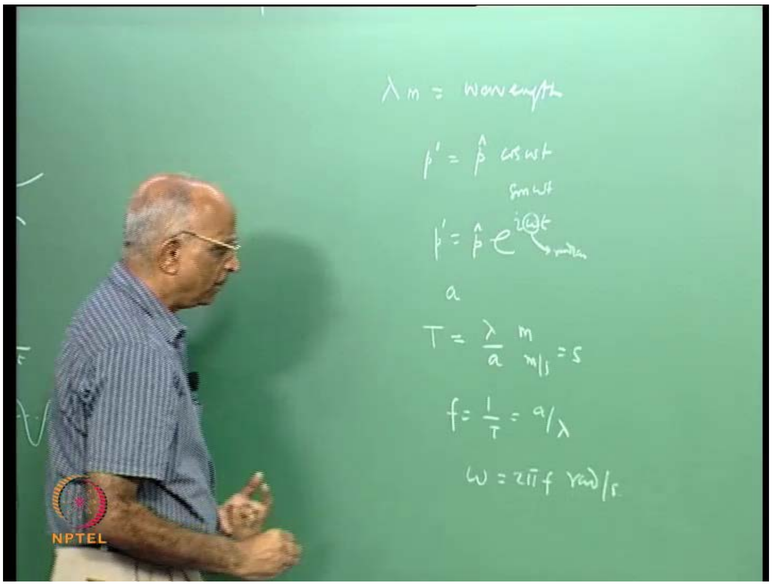

And therefore, I can write the oscillation at any particular point. Let us say over here I say its p prime, may be over here its p prime. I can always write it in the form may be a a maximum value into something like Cos omega t or sin omega t because I presume that this square one which I show as a T c could have been represented something like this, going like this because nothing is going to be sharp in nature. And therefore, if I were to show it as Cos theta and sin theta; I can always represent it by e to the power I omega t right where know omega becomes something like the frequency of the oscillation. How do I define frequency? I have a wavelength lambda and if my wave is travelling at a speed a then, the time of 1 oscillation is equal to, I have time of an oscillation is equal to the wavelength divided by the speed of sound namely, I have meters divided by meter per second that is the wavelength. And therefore, I tell myself this lambda corresponds to a time T and therefore, what is the frequency of oscillation? Frequency is equal to 1 over time. So, many cycles per second or this is equal to a by lambda. Or if I were to put the frequency in terms of radiance per second I say omega is equal to 2 pi into f so much radiance per second. How do I know show the pressure amplitude in terms of both frequency and in terms of let let us say this value here? here Now, when I say omega omega now refers to. So, much radiance per second and I say the value is p prime is equal to p hat which is the maximum amplitude e to the power I omega t.

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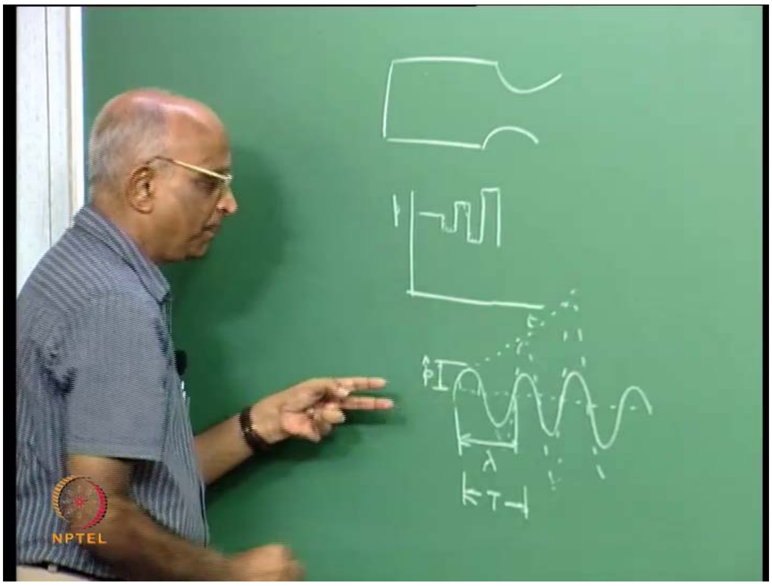

I also wanted to tell how whether the oscillations are going to grow with respect to time. If it is growing the value of the p hat will keep growing in other words it grows with respect to time.

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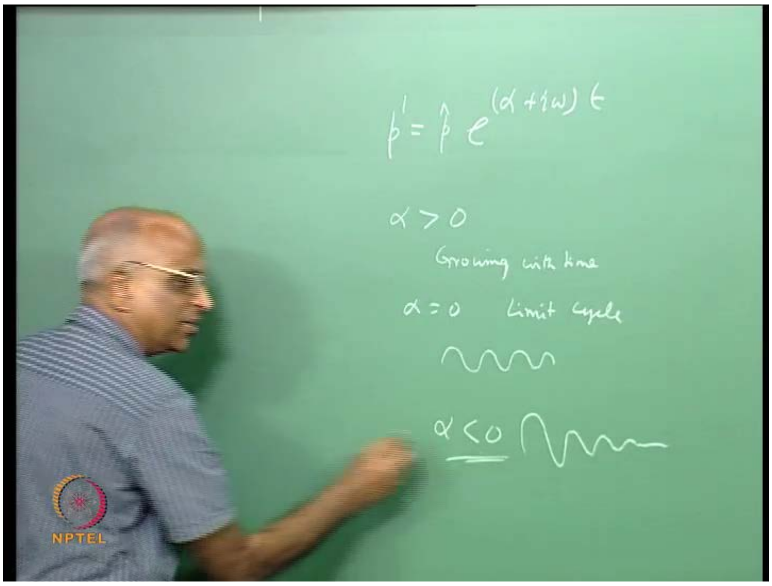

And therefore, I add 1 more term and tell tell that p hat or p prime can be written as some value, some nominal value into e to the power alpha plus I omega into t where alpha shows you the rate at which the oscillations keep growing and if know I say if alpha is greater than 0; it means that the oscillations keep growing with time and the system is unstable, if alpha is equal to 0 what does it mean? The oscillations are steady there is no question where of whatever be the peak, it keeps going. It is something like neutral or we say it is a limit cycle oscillation right, it goes into a limit cycle mode, it keeps oscillating at the same amplitude. If alpha is less than 0; all what I say is I start with an amplitude it keeps decreasing with amplitude and therefore, alpha less than 0 means it is a stable system.

Therefore, we had defined alpha as the growth rate of the oscillation and what will be the unit of growth rate? Well, I have T over here therefore, if I were to take the logarithm of both sides; I have Lon p prime is equal to Lon p into alpha T. Therefore, I have 1 over second must be the unit of growth rate.

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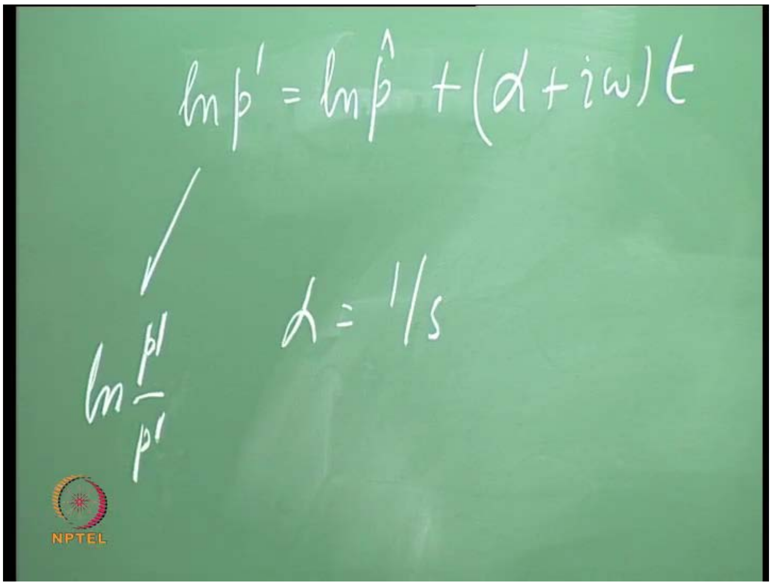

Let let us put it together. I want the unit at which the waves grow. I have Lon of p prime is equal to Lon of p hat and therefore, and then I get alpha plus I omega into t because Lon of e is 1 and therefore, the unit of growth rate should be 1 over second because this is Lon p minus Lon p. Therefore, I can I can take it as a Lon p prime minus Lon Lon of well the Lon p prime minus Lon p I can write as Lon of p rime by p hat and therefore, alpha should have unit of 1 over second and anyway frequency also has units of radiance per second over here and this is the way we normally show the oscillations.

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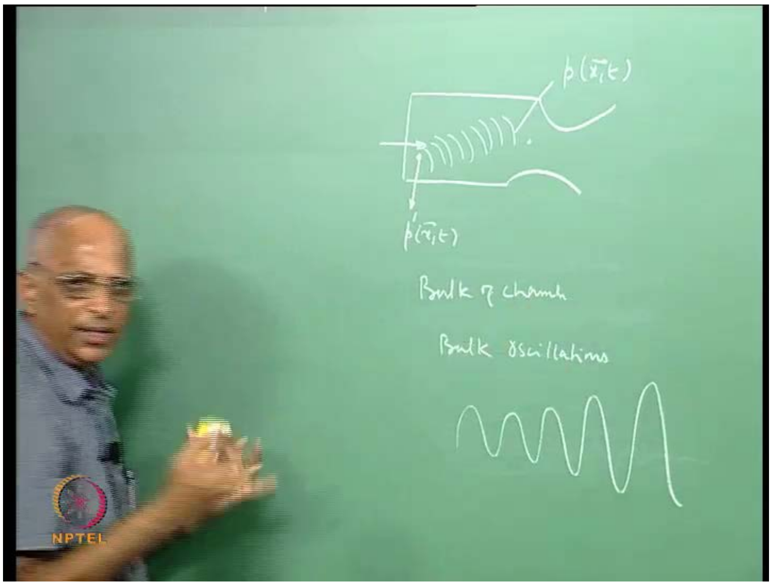

We also talked of one more thing which was fairly important. We told that in some cases when we considered this particular rocket chamber; we never really differentiated the pressure at this point from the pressure at this point. We told ourselves a liquid fuel is injected. It gasifies and produces hot gasses and mixes, burns and all that and the pressure in the entire chamber is the same throughout. But, we also know if I were to create a disturbance at this particular point, the disturbance should travel with respect to time and therefore, the pressure here if I were to write as p x over t should be different from a pressure at this particular point may be p x t. Let us say p prime is the disturbance what we are getting it will it'll be different at the different locations x; where x could be a vector, may be it could be along the axis. It could be anything as the positions changes may be my pressure should change provided the waves are travelling in this when we considered the question of injector and something being built up we assume that the entire bulk of the oscillations or bulk of the chamber oscillates in unison. That means, whatever be the pressure here, is the pressure here, is the pressure here and this is valid only under certain conditions which we have to see why why did we make these assumptions? Therefore, we talked of bulk oscillations. What did we mean by bulk? The entire mass of gas in the chamber is as if it were at a point; that means, a lump mass assumption and lump all the things together and we call it as a bulk oscillation. And for this we developed a theory based on T c and we were able to explain under what conditions in a liquid propellant rocket and in a solid propellant rocket we can get oscillations to happen and these oscillations could grow. Having said that, we also came to the question of supposing I have a wave; how do I describe a wave?

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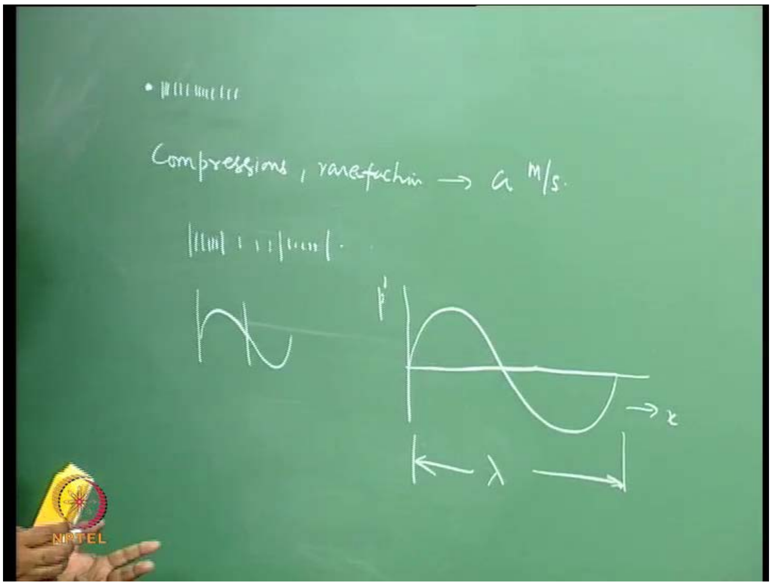

And what did we get at? We said well, I could have a wave and a wave an acoustic wavelength. Consider may be what are the different types of waves? We can talk in terms of an electromagnetic wave, an acoustic wave or shock wave, may be a blast wave may be we will have to look at some of these waves. When I say an acoustic wave is means something like a sound wave and that is how a disturbance propagates I make some noise and it comes to you. Therefore, what is it I am talking of maybe I make some disturbance it and how does the disturbance comes to you may be it compresses the medium my vocal cord. Compresses, then it releases then again it compresses, again it releases the medium. That means, I we can represent it as a series of compressions followed by rarefaction followed by compression. That is how a wave travels and it travels a sound wave travels at a speed a meters per second. Under room temperature conditions room pressure conditions we know that the sound speed is around 330 meters per second. But, it need not be the same speed in a chamber we will have to see that. Therefore, all what we said is well if it is going to travel as compression and rarefaction; that means, I have a compression over here followed by expansion, followed by compression, followed by expansion. I can as well show a compression something like this, rarefaction like this and therefore, a wave can be schematically shown as a compression followed by expansion and this is the wave. A wave travels and therefore, this is x and this is your value of p prime.

Now, we wanted an equation to this and we told yes this point and this point are in the same phase. I can show this by wavelength lambda and a wave which consists of compression and rarefaction can be described by the equation. Let us write an equation for this. We say x is the distance along the along the direction of propagation.

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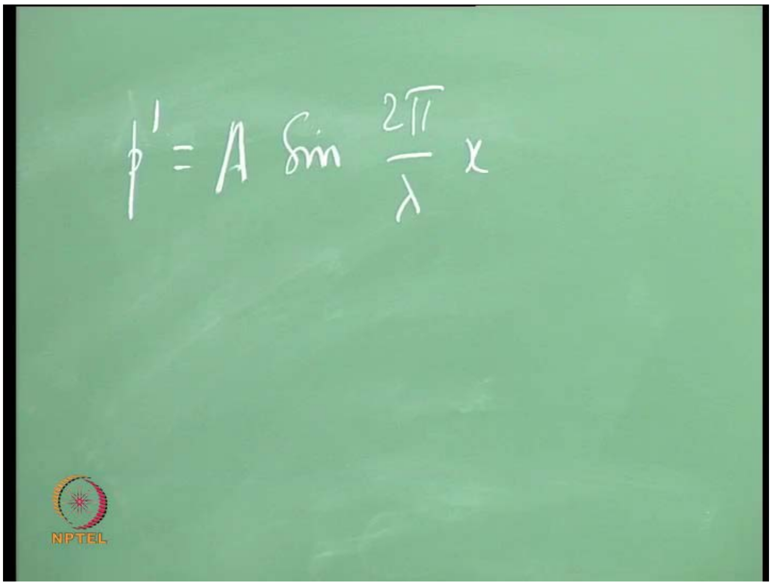

We say p prime is equal to p amplitude; this amplitude I call it as a. A is the amplitude and its a single wavelength I am taking. A into sin of and how do I say sin of? this corresponds to 2 pi corresponding to lambda and I want to find out the amplitude at x. Therefore, may be x by lambda is what gives me a particular position therefore, I say 2 pi by lambda into x is the equation to this. But, we also said that this wave is apparently travelling.

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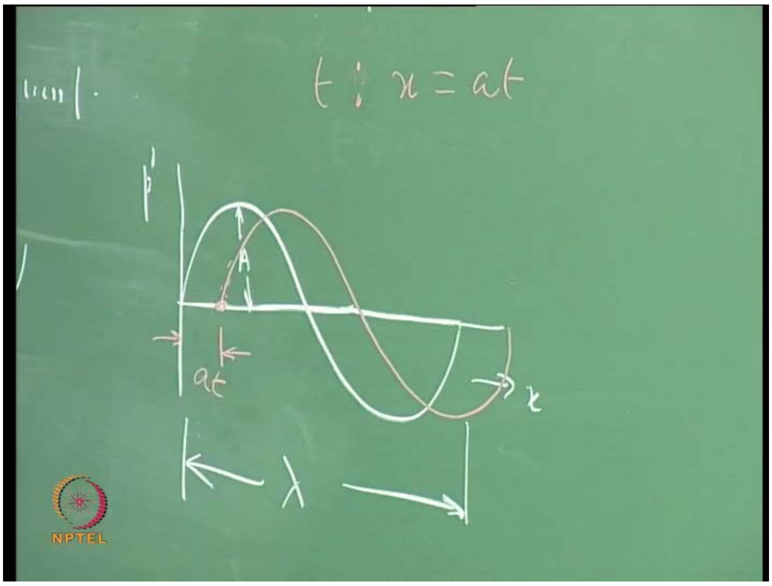

And therefore, if it were to travel at a speed a as we said, at a time t the wave should have travelled a distance x is equal to a t and therefore, at time t it would have come over here and therefore, the wave is now at this position and this distance what has travelled is equal to a t. Therefore, the equation to this travelling wave; that means, it is travelling therefore, if I want at any x here I have to subtract the value of a a t over here.

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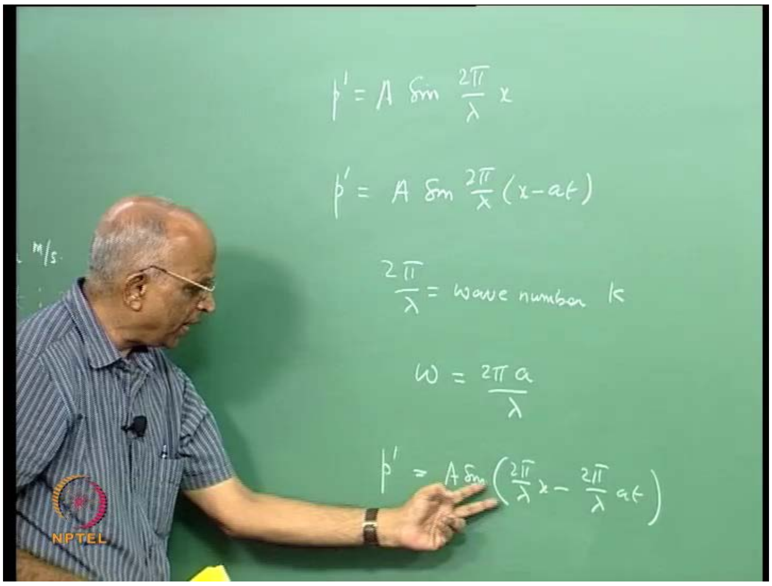

And therefore, the equation to a travelling wave therefore, become a sin of 2 pi by lambda into x minus a t o here. This is what we wrote. Now, we want to simplify the whole thing again and put this equation in a slightly simpler form such that it is more amenable to some analysis and therefore, I can always say 2 pi by lambda is what we call as a wave number and this I call as k capital K. Capital K tells me, tells the wave number and that wave number is defined as 2 pi by lambda and we also just sometime back we said that the frequency of oscillations is written as 2 pi into a by lambda. We defined the value of time T as lambda by a capital time T for 1 oscillation. Therefore, frequency is lambda by a by lambda and therefore, omega is equal to 2 pi a by lambda. Now, I substitute these 2 into this equation and I get the oscillation p prime. I made a correction over here p to p prime is equal to a sin of 2 pi by lambda into x minus 2 pi by lambda into a T over here. Therefore, now I know the wave number is 2 pi by lambda. I can write it as a sin k x and 2 pi a by lambda is equal to omega.

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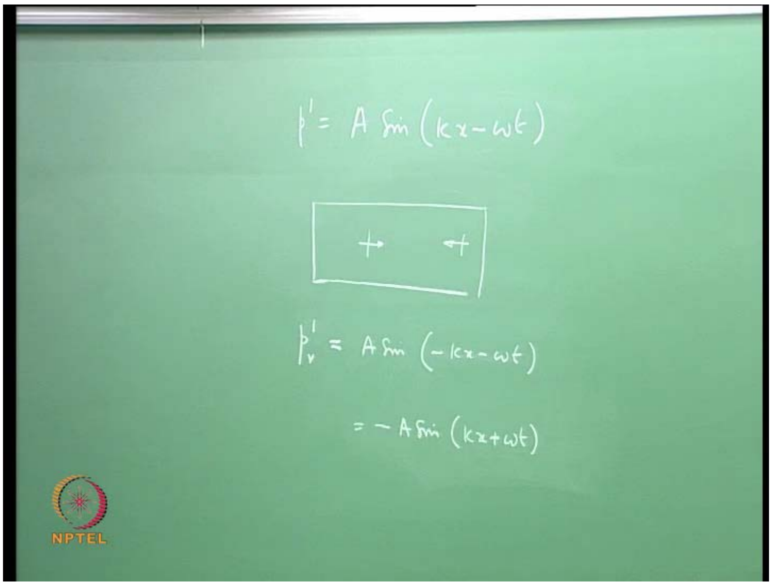

And therefore, the equation to the which is travelling like what I am rubbing out, I showed a travelling wave is equal to p prime is equal to a sin of k x minus omega t. Let us remember this form because k is the wave number wave number goes inversely as lambda 2 pi lambda is known as a wave number and omega is the frequency what we have. We are interested in the frequency of oscillation, we are interested in the wave number and therefore, we can write it in this particular form.

Now, I consider the wave is travelling. Let us say a tube. Let us assume that the tube is closed at both ends. The wave is travelling forward. This is the way it is travelling. The wave comes and hits a solid surface here and let us say there is no attenuation of the wave the wave does not lose any energy it is reflected back and comes over here. Now, when it is reflected back the same wave it is travelling in the opposite direction x. Therefore, the equation to the reflected wave should be equal to a sin of minus k x minus omega t and therefore, this becomes equal to minus a sin of k x plus omega t.

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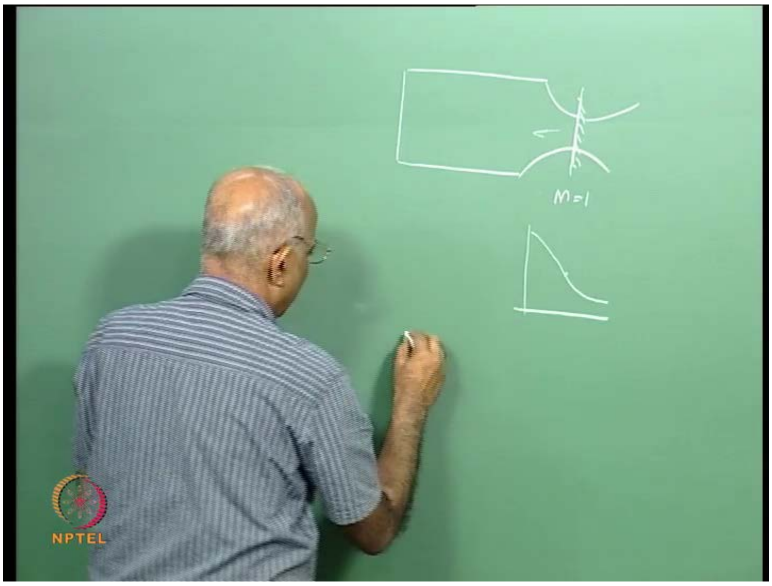

Now, we want to find out see after all a rocket chamber we have been repeatedly telling that a chamber consist of something like a hole, a cavity and what does the cavity consist of? Here it is blocked. You have you have the injector, nothing can go in this direction and you have the nozzle in which high velocity gases are flowing the throat is sonic, Mac is equal to 1 and whenever we plotted the density variations and pressure variations in the nozzle; we found that the density keeps decreasing, it becomes very rapid at the throat and therefore, there is very rapid change in density at the throat. Therefore, this is as good as a solid surface. There are so much changes taking place in throat region that it behaves something like a solid surface itself.

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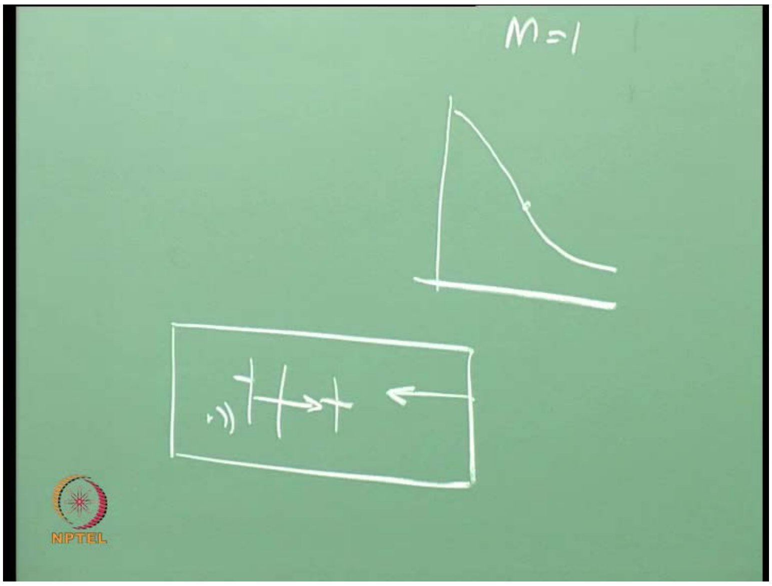

There are so much changes in density that the way when it sees sees it like a solid surface. And therefore, reflects back therefore, the chamber of any rocket chamber is like a cavity closed at both ends. Now, you have a wave which is travelling forward and the forward travelling wave is given by a sin k x minus omega t, the reflected one is given by minus a sin omega t and therefore, you have these periodic disturbances which are created. Therefore, the waves are travelling forward and the waves are coming back, there is interaction between these waves which are going like this and the one's which are reflecting and the net wave what we get is a combination of the incident and the reflected waves.

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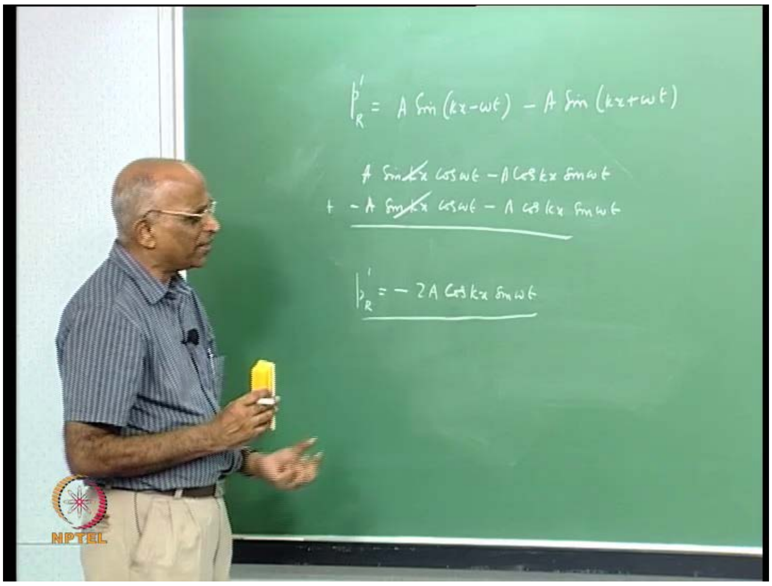

Therefore, I can now write p which is the resultant wave is equal to the incident wave plus the reflected wave. Incident wave is p prime. Let us say the resultant fluctuation is equal to sin k x minus omega t. Now, plus I have p prime or let us keep the same notation a sin k x minus omega t and the reflected one we just now see is equal to minus a sin k x. Therefore, it is equal to minus a sin k x plus omega t. This is the resultant.

If this is the resultant, let us try to solve this. Sin k x minus omega t is equal to sin k x Cos omega t minus sin minus Cos k x into sin omega t and since I have a a and a is for the first term; the second term becomes minus a sin k x into Cos omega t and now I have plus here. Therefore, it is minus a Cos k x into sin omega t and therefore, I add these two things together I find this term drops off and therefore, the resultant wave is equal to the value of minus 2 A into Cos k x. That is wave number into sin omega t is the resultant wave. Therefore, what has happened? I had a series of waves going like this. It will bounce back came like this and the net effect of this and this put together is the resultant wave and now when I look at this I have some strange things happening. The wave was traveling with a speed a, now I do not have anything traveling and what is it what is which is getting reflected here let let us ponder on this resultant wave for a couple of minutes. Let us see what is happening. Now, what what does this represent? May be at some location x at some location x that Cos k x gives me the magnitude of 2 A Cos k x, I have p prime which is this or rather;

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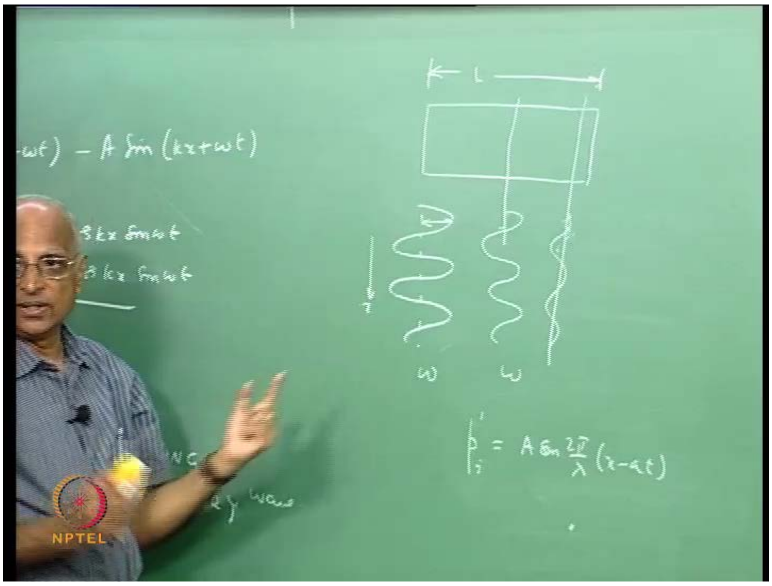

If I were to say well my chamber which now I say is as good as close cavity and this close cavity is of length L; all what I am trying to say is may be at this point x is equal to 0; I have Cos k x which is one I have minus 2 A that is 2 A sin omega t. That means, the amplitude of oscillations here is quite large. May be it oscillates like this. This is my time axis and the frequency of oscillation is given by omega may be at some other location x where x is not equal to 0 I have the Cos k x which is less than 1, the magnitude may be at some other location over here may be what is going to happen is this is the value, this is the value of two a what I had and now this is going to be much lower, smaller magnitude. My my my amplitude will come down and I have the oscillations at the same frequency omega. But, my magnitude gets changed. Over here, may be the magnitude may be still smaller I have to find out, but, my frequency will be the same and therefore, what is it I get? I get at fixed points oscillations of a certain amplitude and these are not traveling any more. That means, the combination of the incident wave and the reflected wave leads to something like a standing or a stationary wave which no longer travels. Therefore, what is going to happen to my combustion chamber? My poor combustion chamber if the waves are traveling forward and backward at some point I will have a value of p prime. At the another point I will have a smaller value of p prime may be at second third point I may still get may be a still different value of p prime. This p prime is oscillating at some frequency. It is also oscillating at the same frequency only the amplitude is different. Here may be the amplitude may be so small that it is like this. Therefore, different points have different pressures in this, but, each point has the particular thing things are not traveling behind it. That means, it becomes a standing wave and therefore, the combination of the instant wave and reflected wave leads to a standing wave and the characteristics of the standing wave are such at a given location the amplitude remains the same throughout the period. At different locations the amplitude changes, but, at a given point the amplitude is same.

Where as if I talk in terms of a traveling wave what is equation to an incident wave? p incident p prime, we said is equal to a sin 2 pi by lambda into x minus a t and what is going to happen to a traveling wave? May be the point is here. The point sees the maximum amplitude a, then it sees a slightly smaller amplitude, then it sees a negative amplitude, then it sees a positive amplitude whereas, when the combination of this at a particular point I get the same value. The amplitude does not change at all and this is important. In a wave, in a cavity because you have forward and backward waves which are interacting you are let to a standing wave or a stationary wave and the frequency at the different points remains the same. But, the amplitude at the different points are different. Is this clear? If this is clear; I can now calculate the frequency and how do I calculate the frequency of the wave? Well I have the equation to the net wave.

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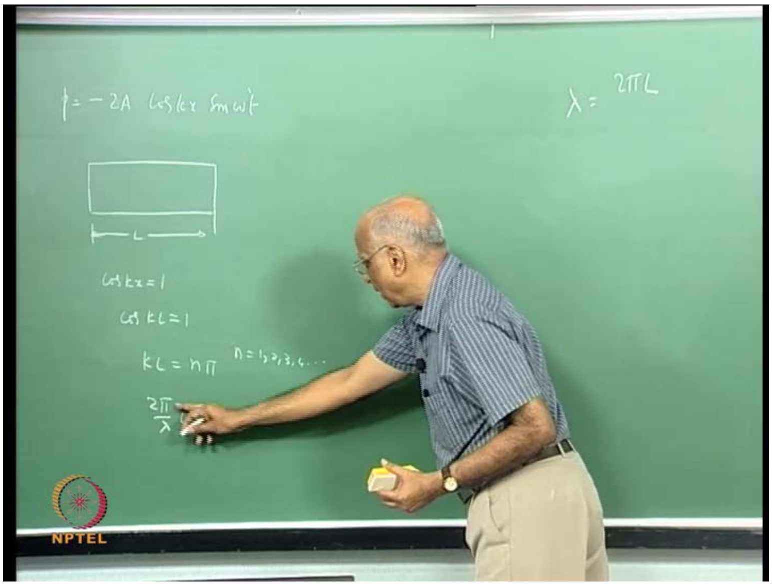

And the equation to the net wave let us write it again. P is equal to minus 2 A and this is equal to Cos k x into sin omega t omega is the frequency. I want to calculate the frequency. Therefore, I again may be make the same plot again well this is my cavity. I have I have simplified my cavity as a cylinder of length L let us say and my wave is traveling in this particular direction. It interacts and therefore, I now form a standing wave and now what is going to happen? I take this particular term. I tell myself this is the closed end. At a closed end the velocity perturbation should be 0 because it is a solid surface. If velocity is 0 the pressure should be a maximum and therefore, the value of Cos k x should be equal to 1 at this end and at this end. At this end, the moment I put x is equal to 0 indeed it is satisfied at this end also it is a closed end therefore, I say Cos k L is equal to 1. If Cos k L is equal to 1 k L is equal to n pi where n could be may be 1 2 3 4 and so on because we have no control. It could be 180, it could be 360 could be any number and therefore, if k L is n pi and what was k wave number was equal to 2 pi by lambda into L is equal to n pi.

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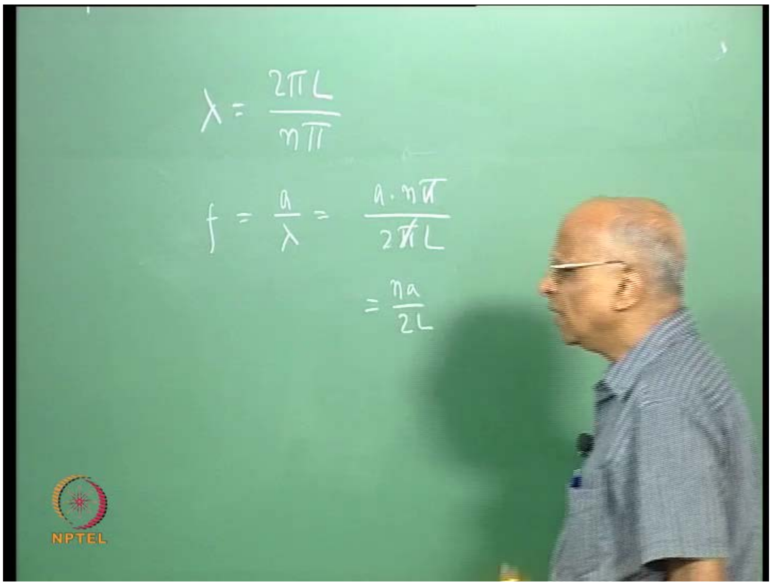

And therefore, we can now now simplify the equation to read as lambda is equal to let us put it together lambda comes on top over here and therefore, we have 2 pi L 2 pi L divided n pi is the value of lambda and what is the frequency omega? Is equal to let us put it together it was a by lambda or frequency let us let us put it as frequency itself by carry f over here frequency is equal to a by the lambda sound speed by the wavelength that is distance traveled in one wavelength and therefore, this becomes equal to; if we were to substitute the value I get the value a into n pi divided by 2 pi L which is equal to pi and pi get canceled have n a by 2 L is the frequency.

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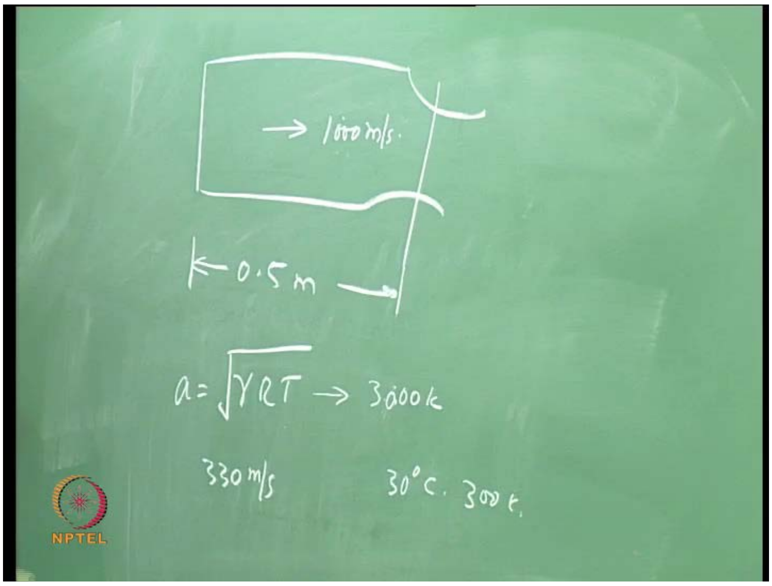

Let let us let us sort of internalize this a little bit by by working an example. Supposing I have a chamber of length let us say half a meters and half a meter looks to me to be reasonable. That is the length of the chamber is 0.5 meters and the velocity of sound in the chamber is let us say 1000 meters per second. Why did I write 1000 meters per second? The velocity of sound a goes as a gamma R T under root; I think all of you would have derived it, a square equal to under root dp by d rho. And therefore, this becomes gamma R T and temperature in the chamber is quite high of the order of some 33600 Kelvin the ambient temperature at which it is 330 meters per second is of the order of 330 3 330 meters per second at a temperature around let us say 30 degree centigrade and therefore, if I take the mean temperature in the chamber to be something like 3,000 into the 36,000 3,000 and if I take the ambient temperature as 300 Kelvin well I get the speed of sound to be much higher may be under root ten of 338 and this gives me around 1000 meters per second I just take a number it could be between 900-1200.

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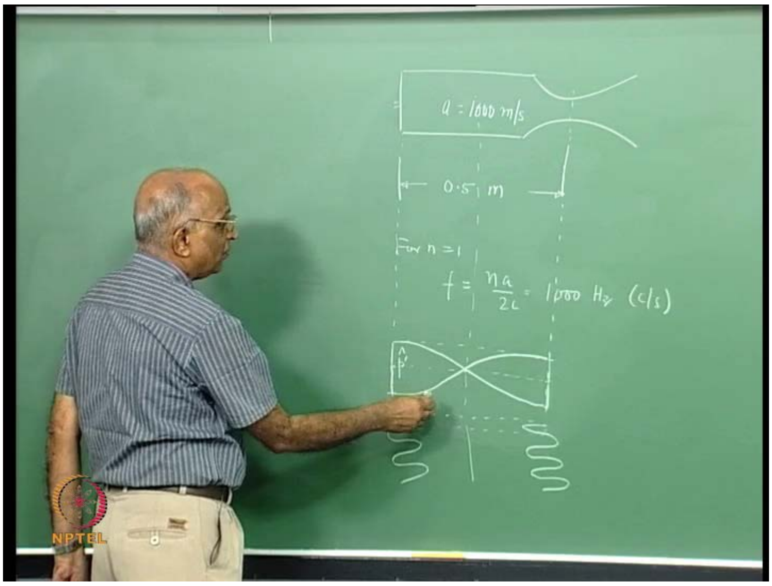

And therefore, what is that? Typical value of frequency for n s equal to 1 we have the frequency as equal to n a by 2 L and in this case I have n is equal to 1. That means, fundamental a is 1000, one is 1000, 1000 divided by 1 which is equal to 1000 hertz or 1000 cycles per second. This is the frequency in the fundamental and now if you are looking at the wave structure, the way the waves are formed; well at the point over here that is on the left hand side well it is the pressure antinode. At the nozzle portion where you have steep gradients it is again a pressure antinode. Therefore, I have large oscillations over here. I have similarly, large magnitude oscillations over here and in between at the center I have a node. In other words the oscillations are maximum and the left hand side and over here rather the value; that means, I have oscillations which are of this magnitude over here. Similarly, on the right hand side I have oscillations of this magnitude over here and now the oscillations keep decreasing until at the pressure node the oscillation is 0; I have therefore, the way form like this.

In other words all what I am trying to say is on the left hand side wherein you have a pressure antinodes the oscillations are like this of large magnitude. The oscillations here in pressure are 0 being a node and over here the oscillations will be of exactly the same amplitude, but, of a different phase. Rather now I will get the value coming out to be here it if it is positive here it will be negative. And therefore, the oscillations are like this therefore, at the fundamental you have large oscillations in the amplitude of the pressure at the antinode regions and therefore, what is it we also find over here; when the oscillation in amplitude p bar is maximum and it is it is positive in this case; the amplitude of oscillations leaving the nozzle is negative. And therefore, the thrust of a rocket motor depends on the pressure at the nozzle and therefore, the thrust oscillations could be out of phase with the pressure oscillations as you are measuring at the head end of a rocket motor. You know our aim was to find out the frequencies for the different cases.

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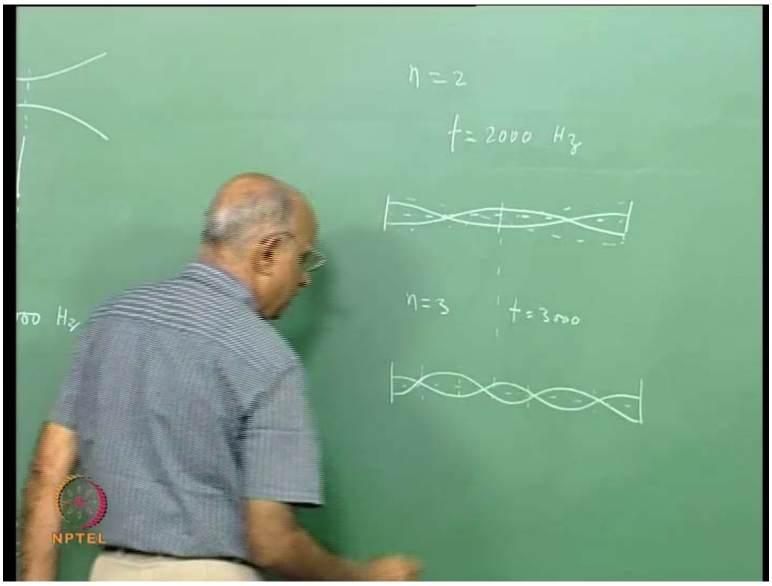

And therefore, now if I consider n is equal to 2; the frequency is equal to now 2. 2 into 1000 2 into 1000 that is 2000 cycles per second or 2000 hertz and what will be the type of the wave structure what I get? Well I have something like an antinode here. I had at the nozzle an antinode over here. In the first case I just had these points over here; that means, this was my waveform at n is equal to 1. Now, I what I start getting is now I start getting a node here, I get an antinode over here, I get a node here and therefore, my magnitude of oscillations will go like this, come back over here, go over here, come over here, go over here, again come back to this. In other words I have n is equal to 2 which gives me 2000 in this case my pressure node corresponding to n is equal to 1 now becomes my pressure antinode and this is my magnitude of oscillations. The frequency is 1000 when I have n is equal to 3. Well again, I need to have maximum pressure over here in other words I need to get one more waveform. In other words I come over here this again becomes may node here. I get I start getting more waveforms. In other words I get something like this coming back over here, coming over here, coming over here, coming over here and similarly, the waveform goes like this and and so on for different values. Now, n is equal to 3 gives me a frequency of 3.000 and what is it I observe in all these cases?

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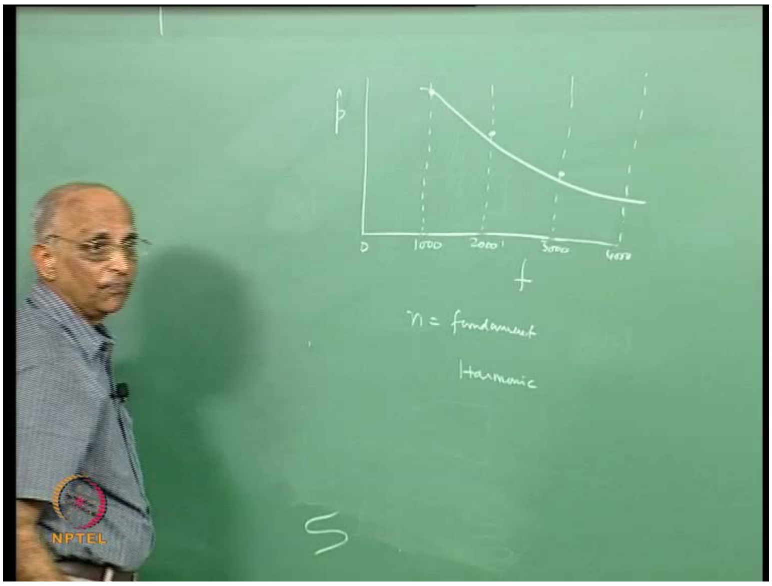

What what I observe is; I get discrete frequencies corresponding to frequencies as a case of pressure amplitudes. Let me plot the maximum pressure amplitude at the antinode portion. I get at 1000 hertz, then again I get a 2000, I get a 3,000, I get at 4000 and so on. These are the discrete frequencies at which I get pressure oscillations. May be this is my magnitude here. May be this might be my magnitude here, this might be my magnitude here. Therefore, I get oscillations like this and what is important to note is only at these discrete frequencies of let us say 1000 2000 3000 and 4000 hertz; I get amplitudes and in between these frequencies in between the frequencies between 0 and 1000 or 1000 and 2000 there are no oscillations at all. It is all discrete at these particular values of frequencies. We call n is equal to 1 has fundamental and n is greater than 1 2 3 4 as harmonics. N is equal to 2 is the first harmonic, n is equal to 3 is the second harmonic and n is equal to 4 over here. That is 2 3 4 4 is the third harmonic, first harmonic, second harmonic and third harmonic.

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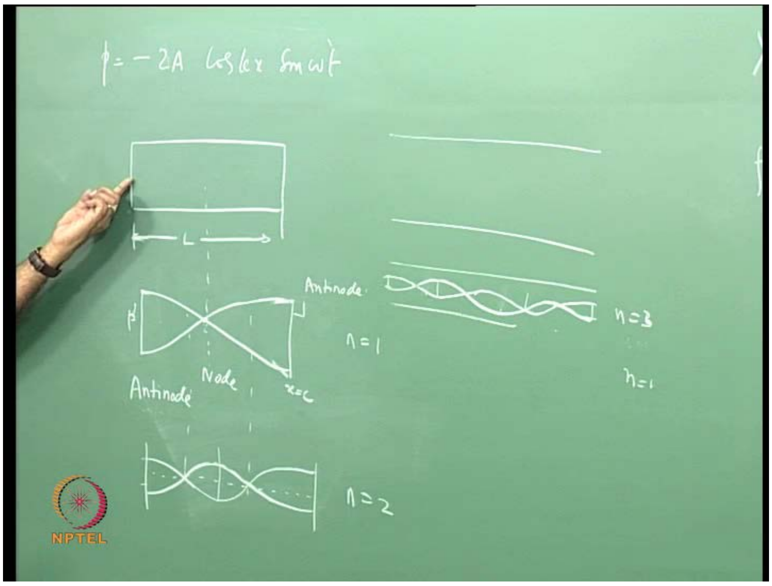

Now, the question comes; we also told you know why should there be only pressure oscillations. It is also possible to get velocity oscillations. After all in a chamber, I am getting flow. I also told this is closed end. Therefore, the velocity here, velocity perturbation is 0. How do I calculate the velocity perturbation from the pressure perturbation? Well, I use my momentum equation and what is my momentum equation? Rate of change of momentum by Newton second law rate of change of momentum is equal to the impressed force.

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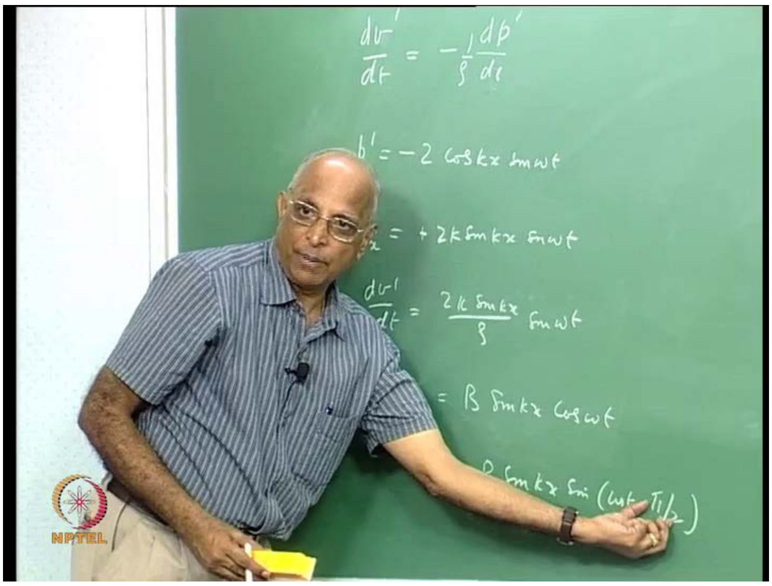

And now I therefore, write dv by dt is equal to minus the pressure difference. What is this mass into acceleration? Therefore, if I have a distance dx well I have dx over here dx into a dx and area combines over here and therefore, I brought dx down. Therefore, this is there and now I have the total mass is rho dx. Therefore, 1 over rho this is my momentum equation what is momentum dv by dt is equal to minus 1 over rho dp by dx and instead of looking at it let us just say mass into acceleration is equal to the impressed force and rho dx into a is the mass and I have minus dp coming over here and therefore, this my momentum equation.

What was value of pressure? Let us say my velocity perturbation, my pressure perturbation over here. Therefore, but, my pressure perturbation was equal to minus 2 Cos k x into sin omega t. We have derived this was 2 A, this was my pressure perturbation. How did this come? It came from the incident and reflected and this was my value. I want to find out my velocity perturbation. Therefore, I differentiate it with respect to x I get the value of dp prime. Let us say y dx is therefore, equal to plus 2 k sin k x into sin omega t and now I have to get the value of v prime and this is equal to divided by rho is equal to the value of d b prime by dt 2 k sin k x by rho into sin omega t and now if I wait to integrate with respect to time and I take all the constants including 2 k by rho and now I will get divided by omega when I integrate with respect to time. I get something like b into some value let us say sin k x into Cos omega t and now I find that now I get Cos omega t where is my basic pressure signal sin omega t my velocity signal v prime is sin omega t. And therefore, I can write this as b sin k x into sin of omega t minus pi by 2. In other words the velocity perturbations lag behind the pressure perturbations by pi by 2 d it is.

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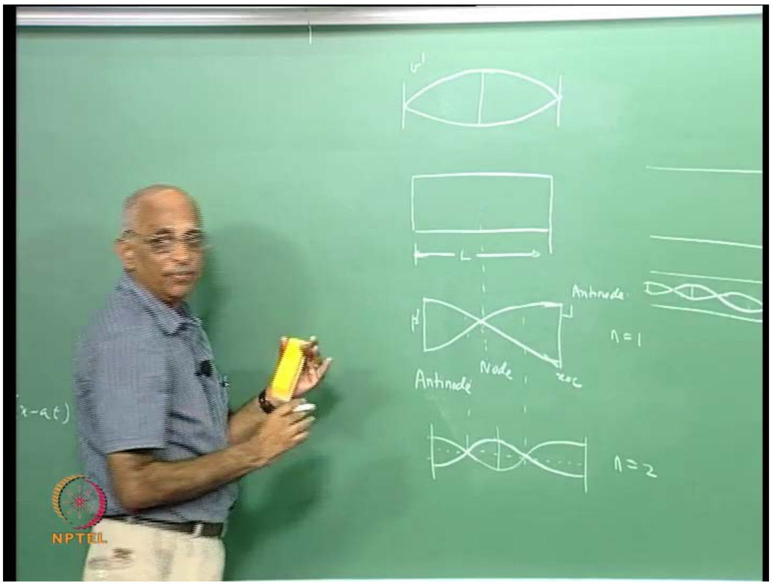

And therefore, I can now plot since it is lagging behind for the same case. If I where to plot my velocity perturbations; I show it on the top lag by ninety degrees. I get velocity 0 here, velocity node here, velocity node maximum and I get my velocity perturbations here it is. So, also I can keep plotting wherever I have a pressure perturbation or pressure antinode, I get a velocity node. Wherever I get a pressure node, I get a velocity antinode and this becomes my velocity perturbation. This is all about standing waves, but, standing waves are important because they tell you at a particular location in the chamber. I know what is the value of the p prime and therefore, it is possible for me to get the fluctuations. If this part is clear let us put the composite whatever we studied earlier and today what we studied for standing wave together. What is it we are telling?

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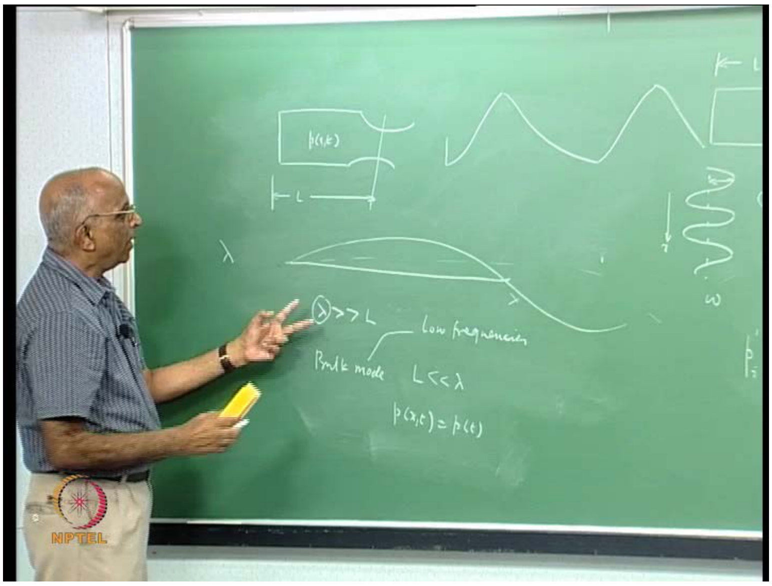

Supposing, I have a chamber and this is the length of my chamber if I have a wave of very long wavelength. What is it I am saying? Well, my distance is x my wavelength is very long in other words the wavelength is so long that now I get a wave like this and now because the wavelength is so long that there is not materially much difference in the pressure perturbations. That means, compared to the length of the chamber if my lambda wavelength is going to be very much larger than the length of the chamber well my value of p x t need not be a function of this. If it is very much longer, may it is going to go like this and come over here. That means, the pressure in the chamber can be very small or rather the bulk mode of oscillations corresponds to the condition when the length of the chamber is when the wavelength is so large that my length of the chamber is very much shorter than the wavelength or rather I can assume the value of p x t for all x to be same as p t or rather the bulk mode of oscillations will happen only for large wavelength. That means very low frequencies. Therefore, whatever we studied for bulk mode of oscillations coming from the injector or from a solid propellant coming from the response of the surface giving you the oscillation wherein you brought down the pressure again it went up it will went up and then again it came down like this. Again it went off, again it came down. These have to be essentially at low frequencies. Therefore, L star oscillations which we studied yesterday are essentially low frequency oscillations and the wave mode of oscillations must essentially be at higher frequencies. I think this part must be clear to each one of us.

Let let us now go forward let us try to understand a little bit more of what what we wanted to study further in this. If this part is clear I ask myself one question. You know we have still not address the total point when we had the flute and we said it make some noise. Why did the noise come? I therefore, I want to I want us to be very clear about it. Let let us say this is the particular flute what we had? We said this is the length of the chamber. We had a small hole at the point where we are injecting.

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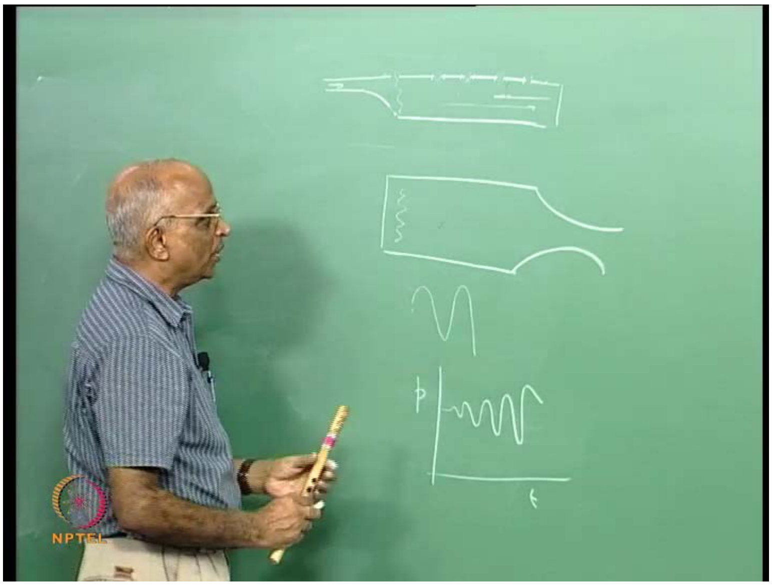

Let let us again draw this figure over here. I blow air over here. I allow it to come. I had a small hole over here and then I have the chamber. We also have for musical purposes different frequencies. I have a serious of holes here and this end was open. I now close this end. I have a closed end here, blow air here still it makes noise. But, if I were to plug this hole which essentially generates the disturbances, I blow it does not make noise. In other words, the disturbances which are created here because of this an interaction with this goes forward, gets reflected at the open end travels back forms a standing wave and this standing wave gets modulated by these things. If I open, it may be my length gets decreased. Therefore, my frequency gets changed. I have a higher frequency and then I keep playing with the frequencies by opening this and I can played music and this is essentially what is happening in a rocket chamber. Also, when I have instability problem, I have the chamber like this. I have the nozzle over here. May be I could get combustion taking place here. The combustion interacts with the pressure mode here increasing my amplitude of pressure at this point and my whole amplitude, the standing wave amplitude increases and therefore, my pressure increases and therefore, may be with respective pressure and time. May be I start with here a small oscillations it goes faster and it creates a problem for me. I end up with very high pressure and this is what we mean by combustion instability and combustion instability happens in the standing wave mode and it is known as a wave mode of combustion instability. Therefore, let us now put together the points what we have studied.

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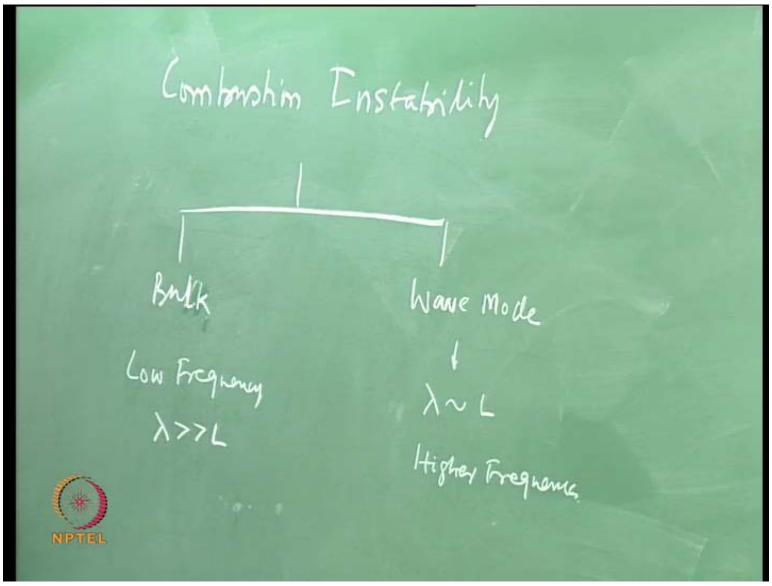

So, far together we tell ourselves well, combustion instability could happen could happen either in the bulk mode of oscillations and these bulk mode of oscillations are essentially low frequencies. Why is it low frequency? Because all points are the same pressure or rather lambda is very much greater than the characteristic length over here or it could also happen in the wave mode. We are still to write an equation and find out what happens in the wave mode. But, we recognize in the wave mode the frequency is lambda is going to be comparable with length. If not going to be less than the length and this happens essentially at higher frequencies. Therefore, we say that well injector induced oscillations or the feed system induced oscillations are essentially low frequency oscillations or because of the heating of the propellant and low L star values. I have low frequency oscillations whereas, for wave mode that need not be true I can have any standing wave and would create oscillations at higher frequencies. I think this part must be clear.

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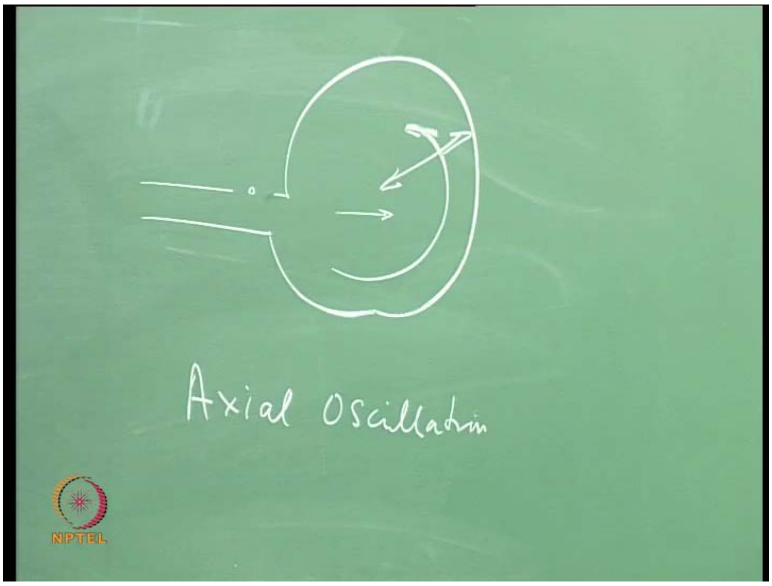

See yesterday, I also showed you the figure of a whistle. The whistle was not long. What was the construction of a whistle? Same thing over here; a pole over here I had a chamber over here, but, then here the motion was not in this direction. Motion was in this direction. It could have been oscillating in the tangential mode. It could be going in the radial mode. But, we have not we have all what we have studied is something going up and down over here. We have studied only in terms of the axial oscillations and let us say in a rocket chamber in a rocket chamber.

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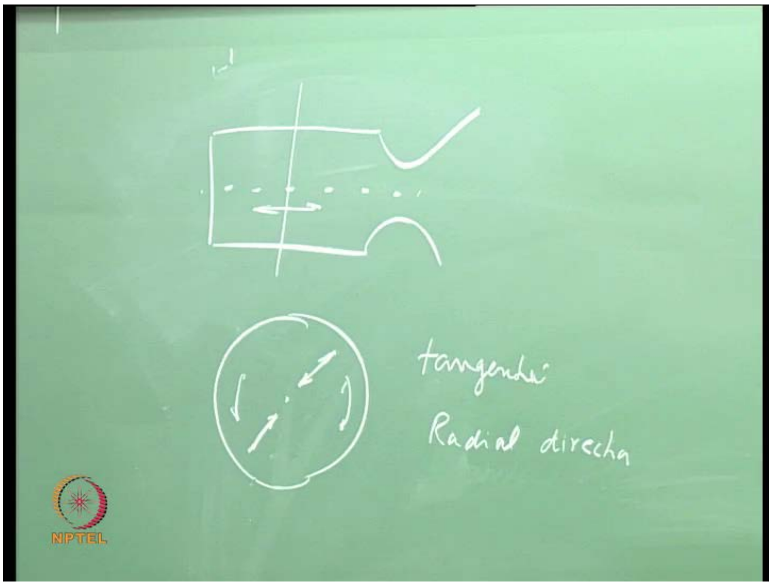

Supposing I have let let us again plot it out and see in a rocket chamber. It is also possible for us with cylindrical in shape any cross section. I take is a circle. It is also possible for waves to go in this direction up and down and form a standing wave in the axial direction. It is also possible that the wave goes in the circumferential direction which we say is tangential. Why tangential? I have a center. It could go in the radial direction. Therefore, I could have oscillations in the radial direction and therefore, my problem becomes even more complicated. That means, I could have oscillations in the radial and the tangential direction just like in a whistle. I have a small thing, but, still it made a shrill sound. That means, oscillations, I induce disturbances. The disturbances travel in the tangential and radial modes and I get that shrill sound of a whistle. So, also I could get tangential and radial modes in a chamber.

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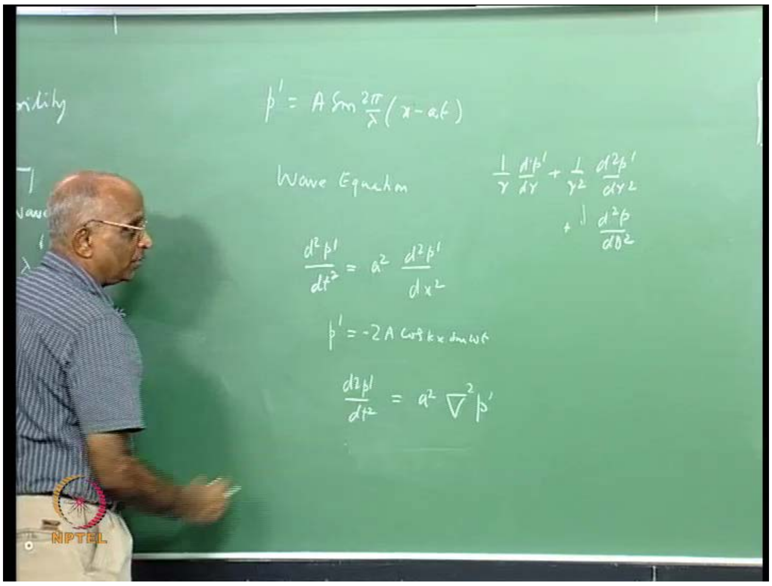

How do I solve for the tangential and radial modes? Well, it is little more complicated. I cannot write the simple equations because axial I could readily write may be p prime is equal to A into sin of 2 pi by lambda into x minus a t for the forward wave from which we derived the equation. It becomes a little more complicated. I have to go back to my basic wave equation and what is the wave equation? I have d square p prime by dt square. For a one dimensional axial wave is equal to a square into d square p prime by dx square let us see if it is clear a square is meter square by second square meter square over here T square over here. Well, this is my wave equation, but, if it is going to be in the other directions; I have to change this equation to the d square p prime by dt square is equal to a square into I have nabla square p where nabla square is equal to may be 1 over r I have dr by dt. If it is cylindrical, d d 1 1 over r into the value of dp by dp prime by dr plus I have 1 over r square into d square p by dr square plus I have the value of a value of d square p by d theta square where again the value of something like 1 over r square will come and therefore, it becomes a little more complicated. But, I can readily solve it using the method of separation of variables. may You all should do it.

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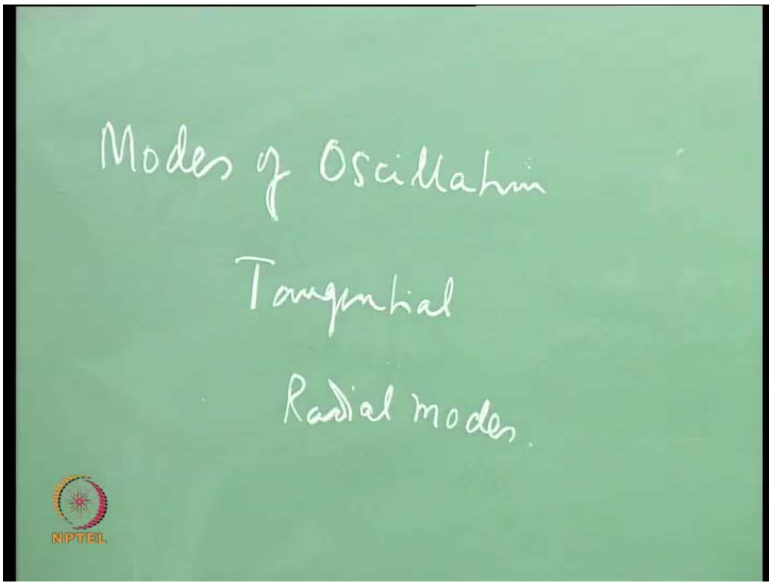

And when I do it I can find out the modes of oscillation in tangential and in radial modes right and now I show this on a power point because I did not want to solve this. Let let us go back just quickly revise through what we have done and where we are now.

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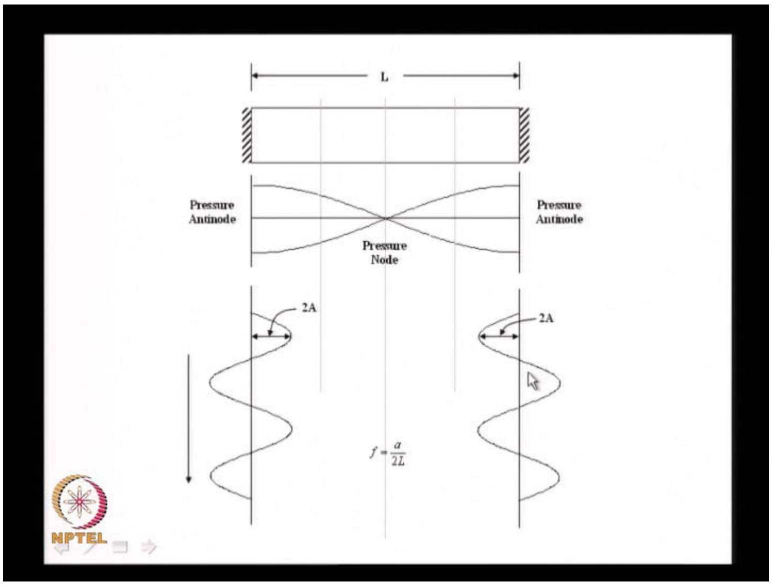

What did we tell? Let let us see is something what we know already. We said the the the reflection of a incident wave from the solid body creates a standing wave. You have pressure antinodes at the ends which are closed and a pressure node at the center. This is the magnitude of the oscillations 2 A and this is my, the value frequency and frequency is equal to a by 2 L.

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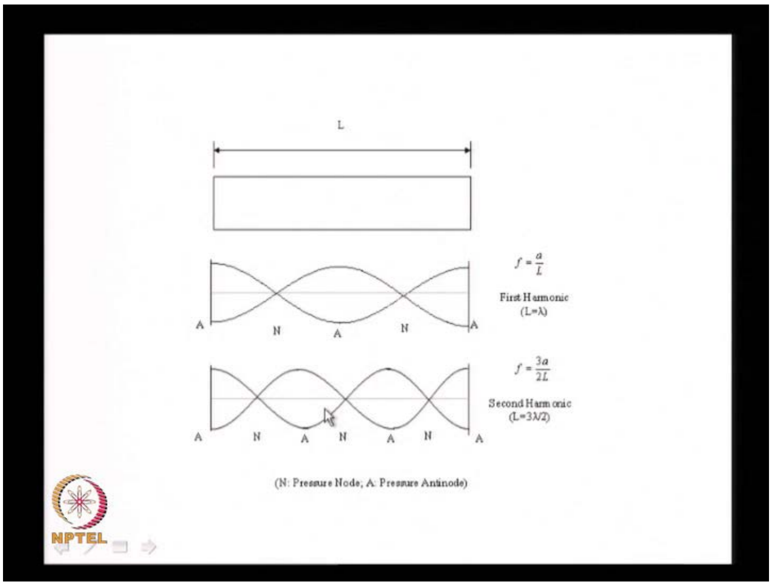

When I go to the second first harmonic that is n is equal to 2; well I have antinodes here. I have another antinode at the center. I have nodes over here. This is the shape of my wave which is going when I go to the third harmonic. That is second harmonic that is put n is equal to 3. Well, I get another series of maximum pressure. Here antinode antinode antinode antinode and two nodes over here.

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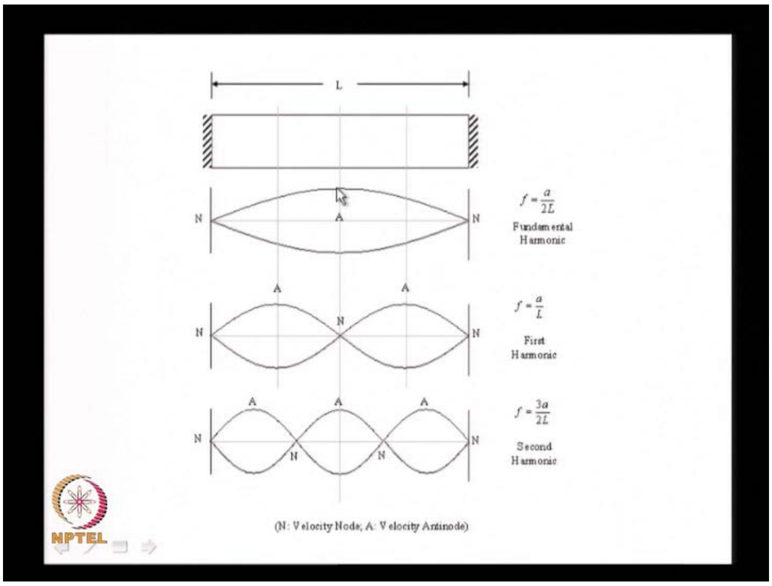

If now I were to put the velocity together; well at the region of pressure antinode here I get a node in the fundamental. I get an antinode here then in the in the first harmonic which is n is equal to 2. I get antinode antinode, a node here, node here, node here well in the next harmonic I get the value I have something like 1 2 3 4 nodes and 3 antinodes and therefore, at these points wherein I have velocity antinode. May be a force could, momentum could increase the velocity. We will will look at it at the next class.

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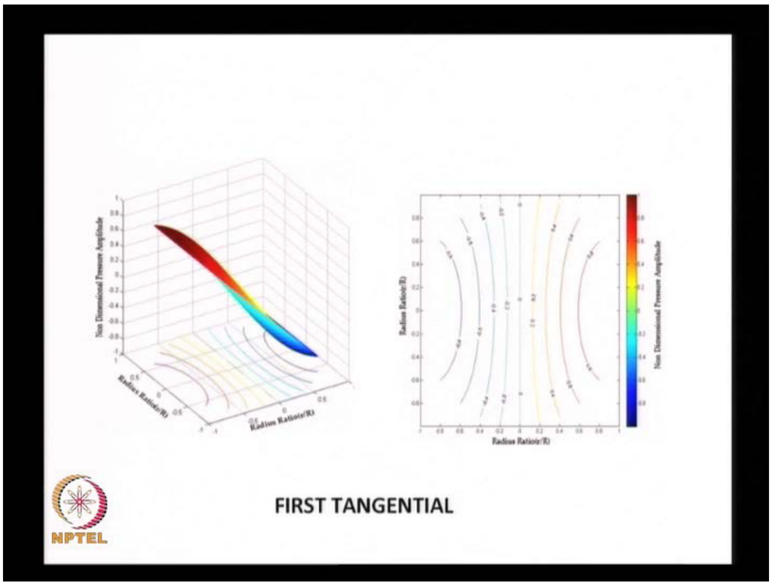

And therefore, now I ask myself what happens in the tangential by this particular diagram? I need to qualify it a little bit.

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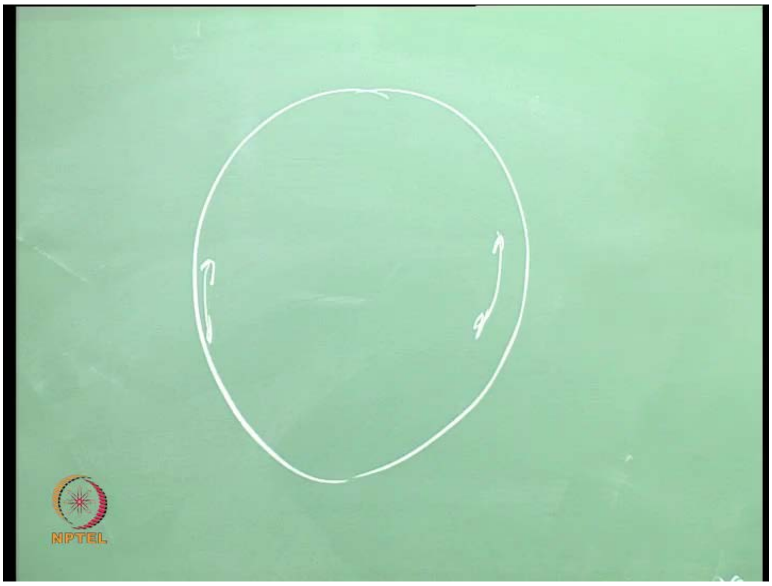

What is it I am talking of? I am talking of diameter and now I get oscillations in this particular direction the first node or the first mode of oscillation is maybe I have an antinode here. I have an antinode here it goes like this and this what I represent here? I have maximum pressure oscillations here at 1 time may be at the next instant of time I have here. Therefore, I have it is oscillating in the tangential direction.

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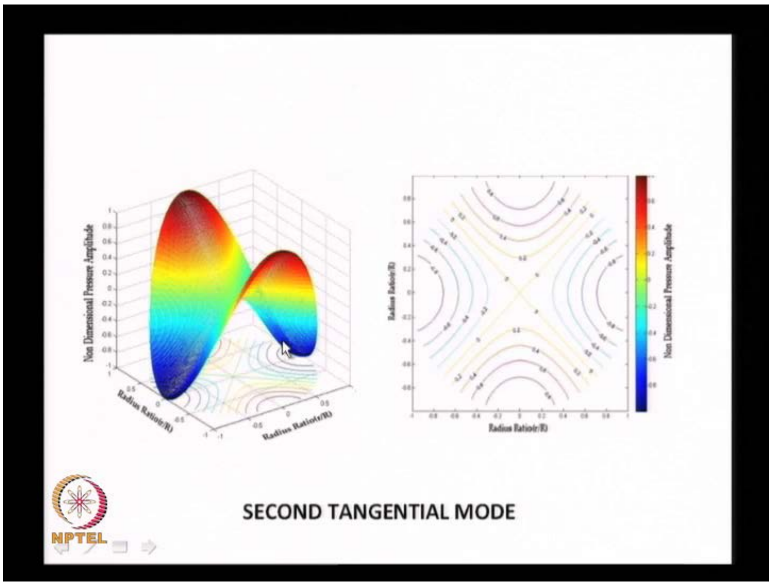

May be when I go to the second mode; well I have two more places where I have the maximum pressure. Therefore, two more places where I have this and this and now I have this and this and I find that the non, the value here shows the value of p prime. That is the local value divided by the maximum value which is possible and this is the non-dimensional pressure amplitude and this is with respect to radius.

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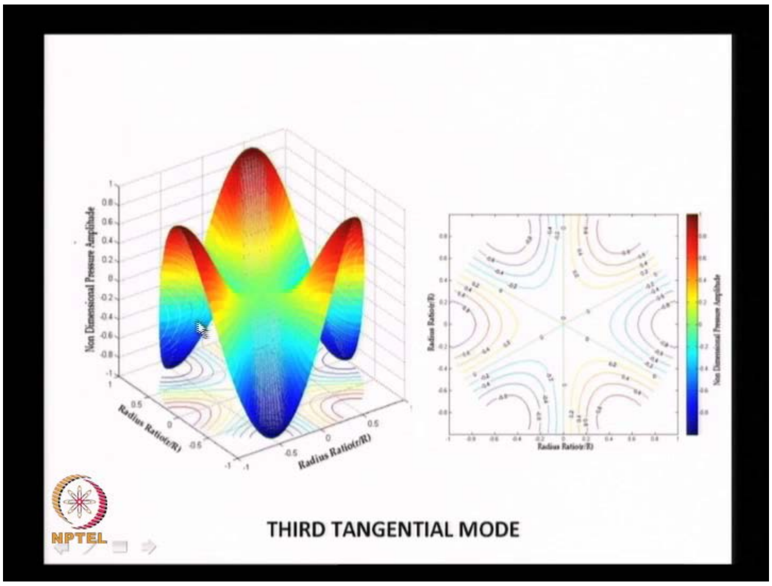

I just want you to appreciate the complexity. This is all done by solving this equation and the figures are shown using Matlab plots and now if you see in the third mode; well I have places wherein I have may be maximum pressure oscillations at the edges 1 2 3 4 5 6 over here and it is still oscillating. You see the red one shows maximum pressure. The green one show negative pressures and in between I have variation of pressure.

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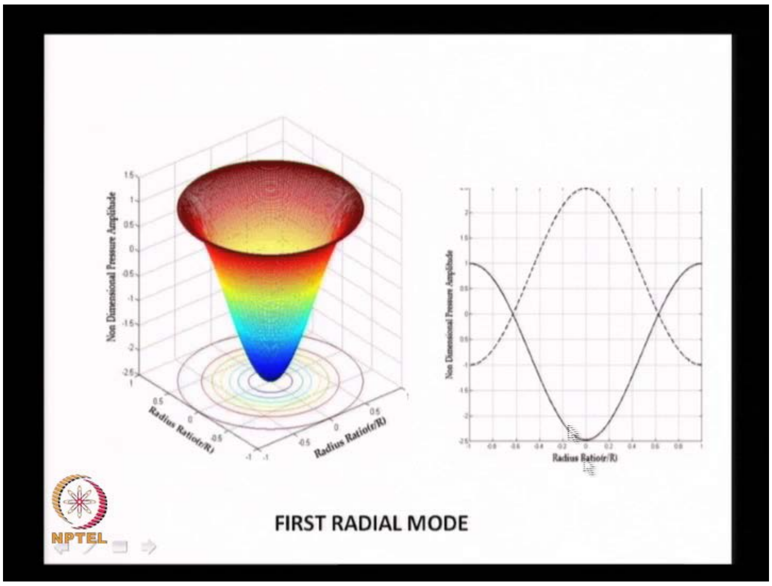

And similarly, in the radial mode well at the wall, I have maximum pressure. At the center I have minimum pressure, negative pressure. At the next instant of time I have minimum pressure here, maximum pressure here and therefore, this is the way I get antinode here, antinode at the wall region, antinode at the center.

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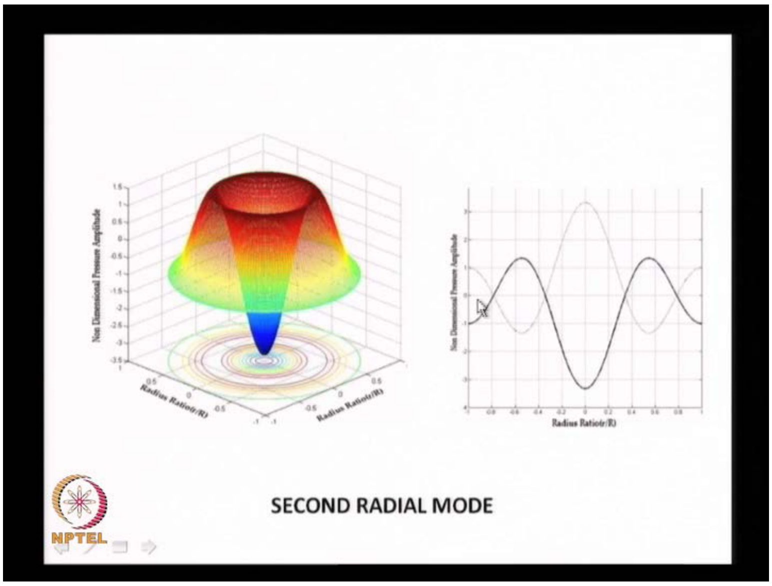

Second radial mode: Well, you see outer I have 1 at the middle of this around 0.6 of the radius. I have another antinode may be at the center. I have another antinode. You see the magnitude of pressure oscillation at the center is much higher than at the walls. Whereas, then we look at the tangential modes, you got just the opposite you had higher values at the wall and lower at the center.

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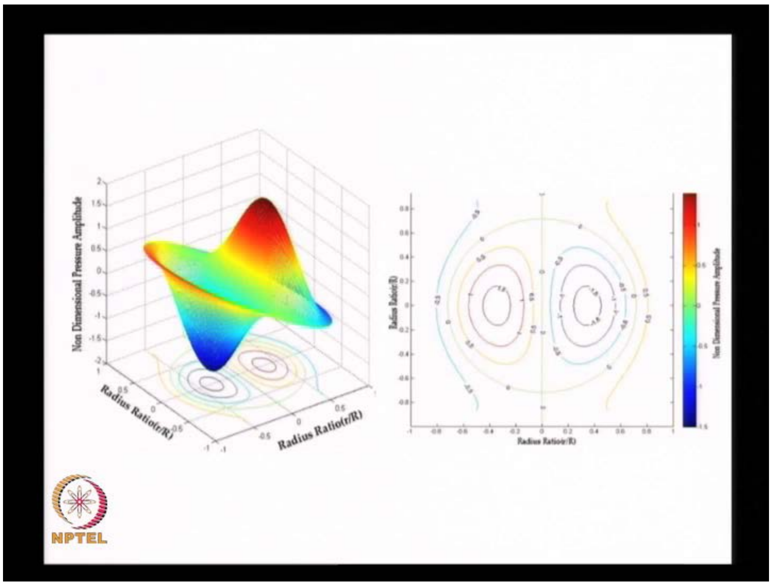

And therefore, if you were to couple after all these; are all oscillating together. It is always possible that some of the radial modes will get coupled with this.

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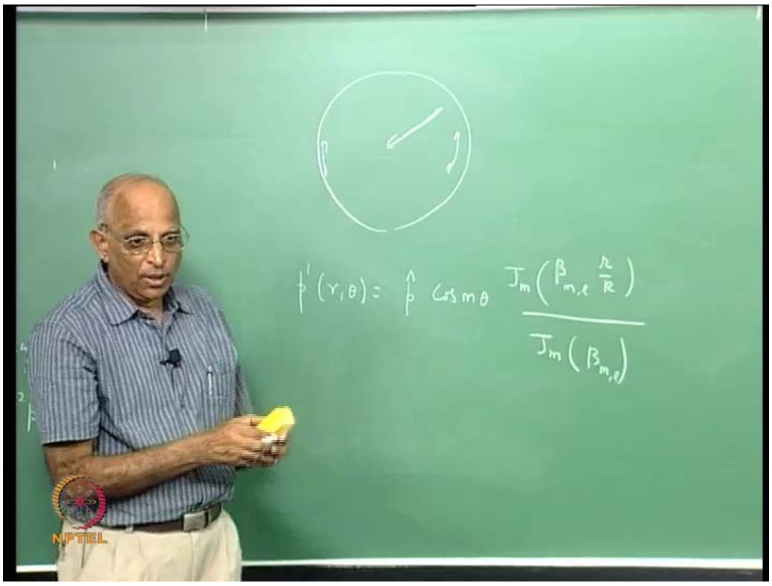

And how does it couple? If I were to write the pressure p prime as a function of r and theta and if I were to solve the wave equation; I will get something like p hat which is the maximum value into something like Cos m theta into I get the value due to the tangential. What happens in tangential? I get divergence the radius keeps increasing and what represents the divergence is the Bessel function. And therefore, I get Bessel function of the order m into something like beta which gives my wave number which is let us say combination of the tangential plus the radial into r by R divided by J m Bessel function of order m into beta m into the value J m b[eta]- beta m over here and beta m L over here. Because I have both the waves together and when I put the things together, I have pressure oscillation in this and this in this form and therefore, if I if I were to say yes I couple both the tangential and radial together, I have different places different values of pressure perturbations. This is in the combination of the first tangential and the first radial.

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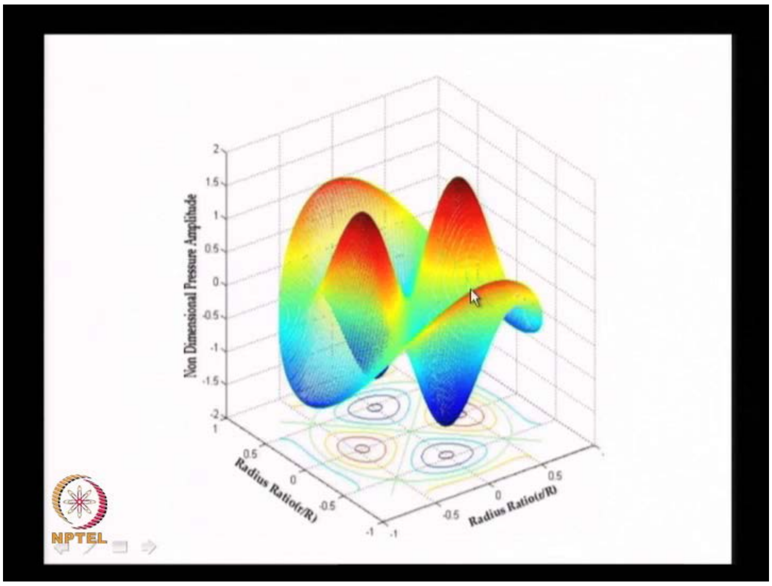

Then, the next one I show the first tangential. No, it is the second tangential and first radial in which I have different values, different places where I get the pressure perturbations are maximum.

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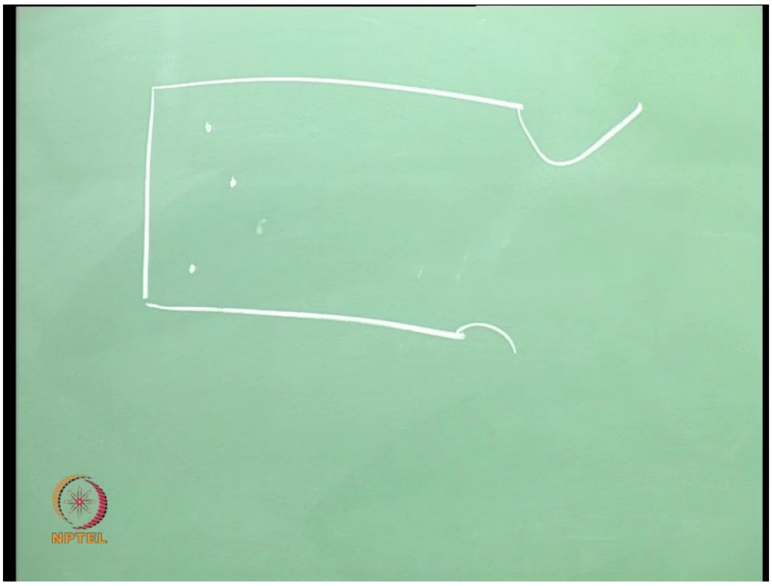

Why did I show all this? The reason being all what we are saying is I can get stationary or standing waves in a chamber, a rocket chamber in which it could happen. This way it could happen. Tangentially it could happen radially and therefore, there are some location wherein I get antinodes some location, wherein I get nodes and if by chance I can excite this maximum values in pressure or maximum values in velocity; well I can get a problem of instability. And therefore, I will take this up in next class and we will try to gain a total understanding of what causes combustion instability in the wave mode and in different modes. Well I stop here. May be we continue with it later. Thank you.