Rocket Propulsion

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Lecture No. # 04
Velocity Requirements

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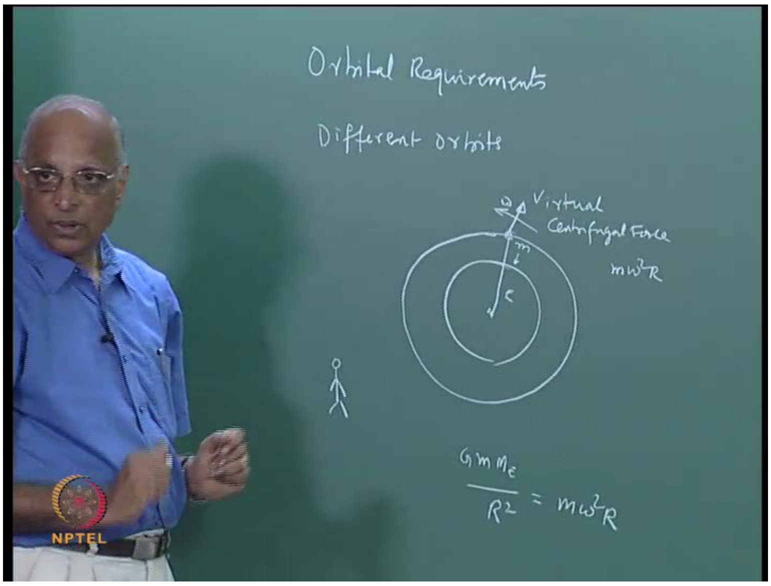

Good morning in today’s class, we will be looking at the orbital requirements and also examine the different orbits. To be able to do this, let us just briefly recap where we were in the last class. We told ourselves, you know you have the earth as it where and you have something going around the earth, an object going around the earth, we told ourselves we would like to have the frame of reference of this on this body itself.

That means, I am sitting on this body as it were and looking at my rotating body, it is not that I have an inertial frame where in, I sit here and watch this. But I have what we said is the rotating frame of reference and what did we find. if I have a rotating frame of reference it is necessary for me to put a psuedo force and this psuedo force we called as a psuedo or virtual force which we called as centrifugal force. We told ourselves that this centrifugal force is equal to M omega square into R, where omega is the rotational velocity of the body as I am sitting on it and R is the radius from the centre over here right. So far so good.

The mass of this object is m. You know this force was required to correctly predict the motion of the body. In the frame of reference of the body itself, how did this force come. Well, in the perspective of me sitting on the body the body is not moving and therefore, I had to put a force to correctly define the motion of the body. Namely, we said x star is the coordinate of the body there is no change in x star and therefore, we had d 2 x star by d t square was equal to 0. Using this psuedo force, we wrote an equation which we said is, the earth is attracting this body due to the universal law of gravitation; we wrote it as G mass of the body M e. That is the mass of the earth if I am considering the earth into R square is equal to the psuedo force which is acting namely M omega square into R.

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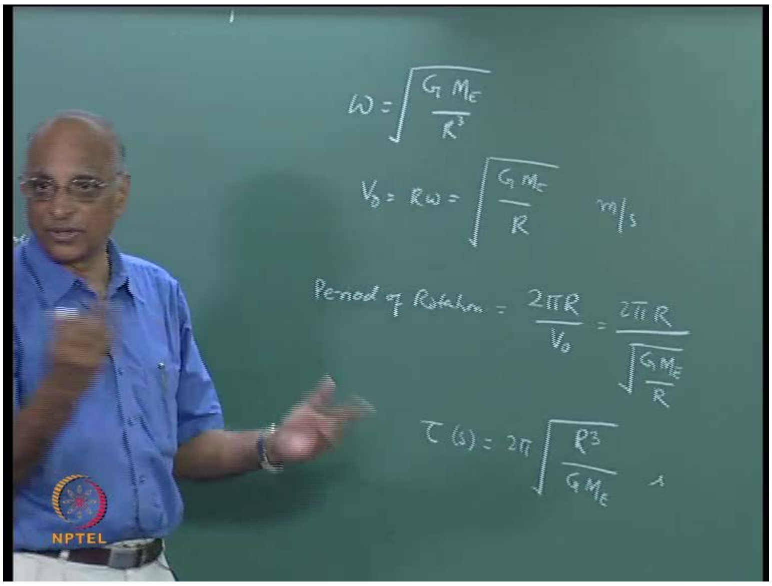

And therefore we were able to get the angular velocity of rotation, omega is equal to under root G M e by R cube right, so, many radians per second this is what we did. We also went one step further wanted to find out what is the velocity v0 of this object as it is rotating around. We told ourselves v0 is equal to R omega and therefore, the v0 is equal to G M e. I have R square coming over here, into it R square therefore, it is equal to G by R. If I express G in Newton metre square by kilogram square mass of the earth in kilograms and R in metre, the unit we got was metre per second. This is the orbital velocity right.

Now, we go one step further ask ourselves supposing this body is rotating what is the period of rotation, what do you mean by a period, what is the time taken to complete one rotation, how will I say period of rotation. I find that it travels through 360 degrees or 2 pi radians and therefore, I say the period of rotation must be one, the distance it travels is, I have the radius R it is 2 pi R is the total distance it travels divided by v 0. Therefore, the distance travelled in one orbit is equal to pi d or 2 pi R divided by v 0. And therefore, the period of rotation will come out to be equal to 2 pi R divided by under root G M e by R. And what does that give you, the period of rotation which I call let us call tau is equal to so much seconds is equal to R is in metres therefore, have 2 pi into under root R comes on top R square that is R cube by G M e so many so many seconds is the period of one rotation. Therefore, what what is it you find from this let us let us do one or two small examples. Let us find the period of rotation for two two distances let us let us take two distances

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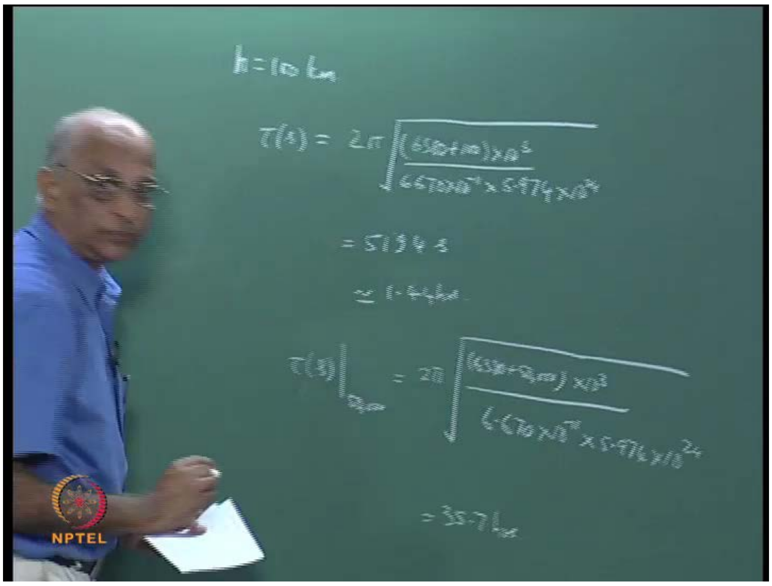

Let us take the problem which we did the other day was. When the height above the earth is 100 kilometres we found out the value of the orbital velocity, we said it is 7 kilometres per second 7.4 or something right. I want to find out what is the period and therefore, the period in terms of seconds tau seconds is equal to R cube R cube is equal to how much is it? We said that the radius of the earth is how much and the height above the earth is some some value therefore, R is equal to 6380 is the radius of the earth. Let me write it 2 pi into radius of the earth is 6380 plus, I have 100 over here, so many kilometres into 10 to the power 3 divided by G the value of G gravitational constant is 6.670 into 10 to the power minus 11 and the mass of the earth is equal to 5.974 into 10 to the power of 24 kg.

This is the time taken to complete one orbit right at a distance of 100 kilometres above us. When I calculate the value I find this comes out to be something like 5194 seconds or something like equal to 1.44 hours.

Therefore, now I say instead of 100 kilometres I go I go to an orbit, instead of being 100 kilometres height instead of being 100 kilometres above the earth, I go to a distance let us say 50000 kilometres above the earth. And now, I ask what is the value of the time period and if I do the same thing, tau at seconds at a distance of h equal to 50000 kilometres will give me the same thing I again put 2 pi into what I say 6380 plus 50000 kilometres metres 10 to the power 3 divided by the same value again 6.670 into 10 to the power minus 11 into I get the mass of the earth which is equal to 5.974 into 10 to the power 24 right. And what does this come out to be, this comes out to be something like 35.7 hours is it all right? All what we we are doing is, we want to find the time for going through one orbit.

At a height of 100 kilometres we find it is something like an hour and odd and if have the orbit which is at a height of 55000 it is something like 35. And if I want to plot this lets plot it in and see what what we get over here. I keep this figure and erase this part.

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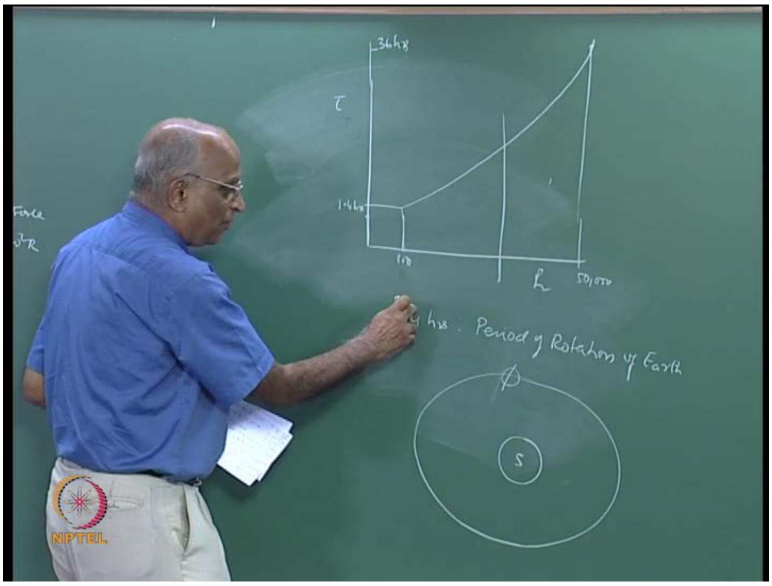

Now I plot the radius or height above the earth as a function of time period. On the surface of earth which is something like 6300 and something, I will get little bit lower at a height of 100 kilometres I got a value equal to 1.44 1.4 hours or so. Excuse me at a height of 50000 kilometres I got it as equal to something like 35.7 or 36 hours or so.

And I have something like, the graph shows that it is R to the power 3 by 2 therefore, the graph shows this somewhere in between 1.4 hours and something like 35.7 hours we have the value corresponding to a single rotation of the earth and that is 24 hours. That means, I have 24 hours is the period of rotation of the earth. How do you define the period of rotation how will you define it. We define, may be the sun is vertical today and sun is vertical the next day; that means that, is the period and that we say is one day or 24 hours.

But actually what is happening, let let us put things together. Actually when you see what is happening, you have the sun at the centre of the solar system, you have the earth rotating in an elliptical fashion and the earth is also revolving on its axis as it is rotating on this. Therefore, the solar day is going to be different or the period of one rotation is going to be slightly less than 24 hours and therefore, because it is revolving and as it is moving and therefore, the period cannot be 24 hours.

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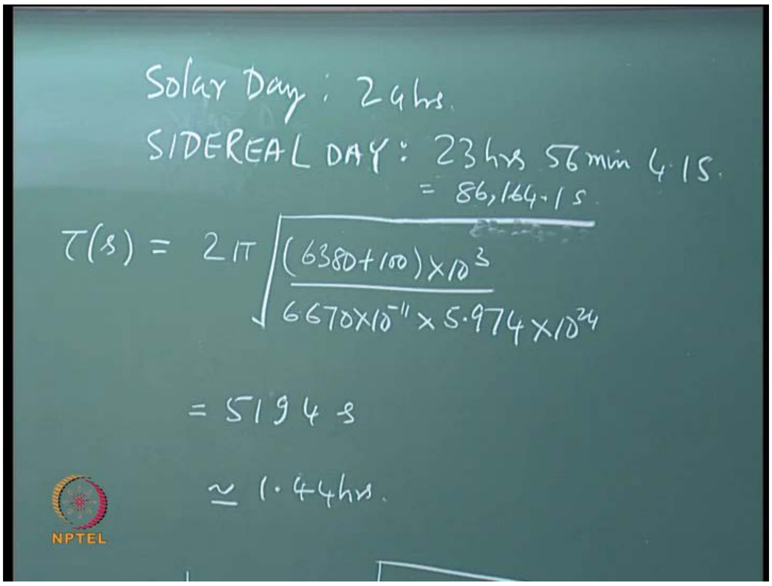

And therefore we define another one, namely we talk in terms of solar day; that means, that is 24 hours period of one rotation and we said the real actual time taken for a rotation which we call as sidereal day slightly less it is something like 23 hours 56 minutes and 4.1 seconds. Therefore, whenever we say one rotation what is happening is the earth is rotating on its axis and as it is rotating it is also revolving and therefore, one rotation corresponds to not one solar day. But one work we call as a sidereal day which is, 23 hours 56 minutes and this works out to be something like equal to 861 64.1 second.

You know, I do not think we are going to get into that depth of trying to find out the difference between the solar day and the side real day and we will assume that the earth rotates ones in 24 hours. And having said that you know somewhere in between 36 hours and 1.4 hours we will find that you have 24 hours.

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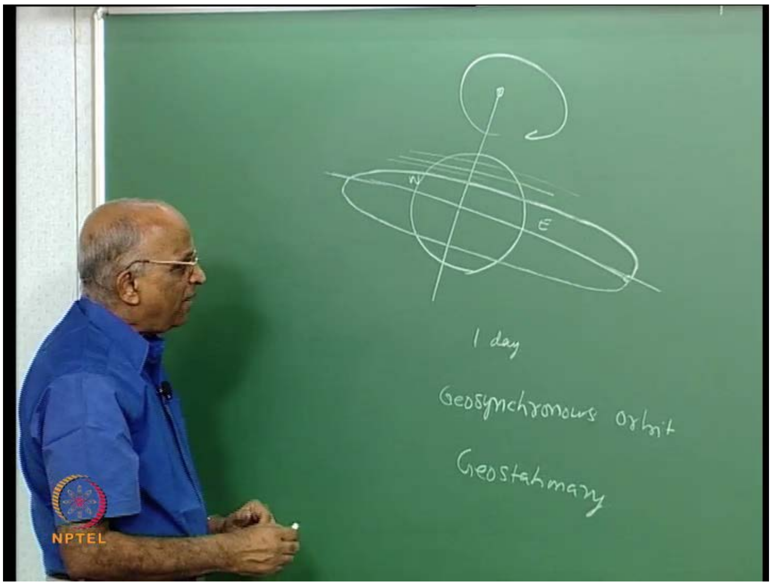

And at this point what is going to happen. You know we showed the earth over here the earth rotates on its axis and you have the body. When the body of the satellite moves with respect to the earth and the rotation of the body with respect to the earth is the same, in other words the time taken for one revolution let us say the body is on this plane the body is moving, let us say from east to west over here may be going around over here the time taken for one orbit as it goes around is one day which is the same rate at which the earth is rotating on its axis. In other words the period of rotation of this body is synchronous with the rotation of the earth and we call this orbit as geosynchronous orbit.

In other words when I have the earth rotating earth rotates from east to west and. So, also if the body rotates in a plane which also rotates once per day we say that the rotation of this object or body or satellite is synchronous with respect to we call it as geosynchronous orbit. However, if now the body is rotating on the east west axis namely on the equatorial plane and then what happens is, it also rotates rotates once in 24 hours just the same way as the earth rotates in 24 hours. Then, any point on the surface of the earth since the earth is also rotating once in 24 hours and this is rotating in the equatorial plane at the same rate as once in the 24 hours, the satellite will always appear stationary to all points on the surface of the earth. And such an orbit is known as geostationary we call is as geostationary because for all points on the surface of the earth the satellite appears stationary. But if by chance the orbit is not along the equatorial plane, but it is in some other latitude or in some other plane, we call it simply as geosynchronous, but not geostationary. Therefore, the distinction between geostationary and geosynchronous is, geostationary is in the equatorial plane having a rotational period same as the rotational period of the earth whereas, geosynchronous could be at any any of the plane.

Having said that, it is not necessary that, we should have Geosynchronous and geostationary orbits only for the earth. It is possible to have the geostationary and geosynchronous orbits for any other planet so long as the planet rotating. Any other heavenly body if it is rotating and the the satellite or object is moving at the same rate as the rotation of the body. We could have a geostationary or geosynchronous orbits.

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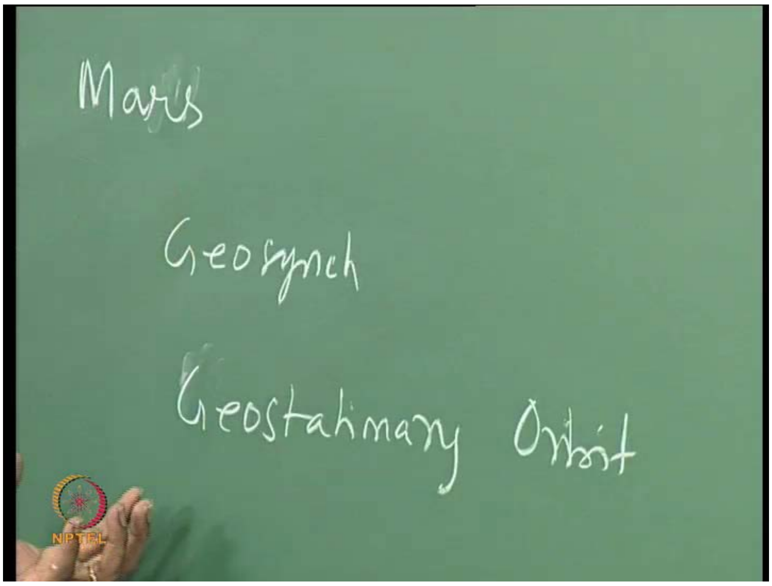

For instance for mass also for mass we could have a geosynchronous and a geostationary orbit any planet any object heavenly body which rotates we can always have a satellite moving around it at the moving around it which could be in geosynchronous or geostationary orbits. You know, all what I am I am trying to say what is the value of the height and I call this height as radius of geosynchronous orbit which is equal to the radius of the earth plus the height of the orbit at which, the period is going to be 24 hours. I also qualify by saying 24 hours is the solar day and I must distinguish it from the sidereal day which is slightly smaller, but for us you know it really does not matter. May be for a person who works on machine he must take into consideration the sidereal day of 23 hours and may be 56 minutes and 4.1 second. therefore, lets lets straight to put this together we will therefore, tell ourselves

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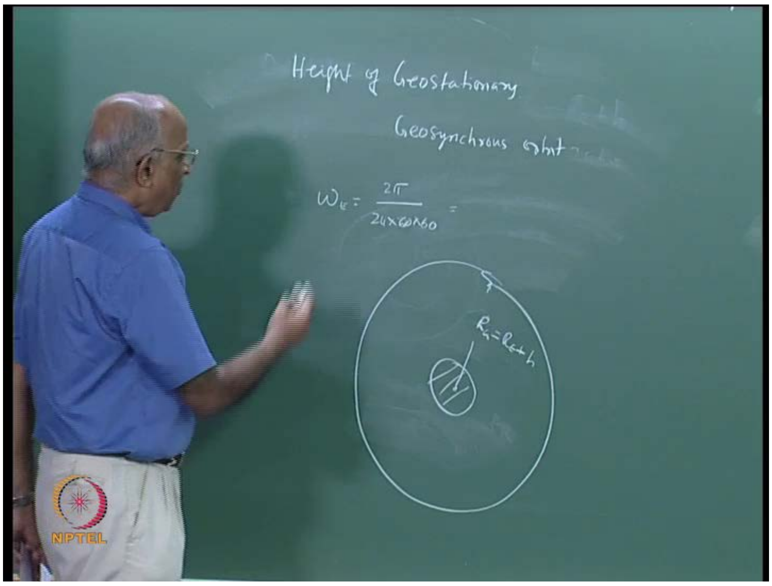

Now, I want to find out the height of the geosynchronous orbit or geostationary orbit and what is the difference between the two geostationary is in the plane of the equator equatorial plane whereas, geosynchronous could be in any plane that is the difference. We want to find out therefore, we do the same thing rotational period of the earth it rotates through 360 degrees that is 2 pi radians in 24 hours that is, 60 into 60 seconds is the angular velocity and what is the angular velocity at a height which will be R g; that means, I am considering the earth over here I am considering orbits going round over here may be on the equatorial plane as it were. I am considering the height as h G and the radius if I consider the radius I say R G R G is equal to the radius of the earth plus the value of h corresponding to the geostationary orbit. And therefore, we just had the expression for the omega e if now I were to calculate the value of the height of the orbit at Which I am going to get 24 hours what is it I get?

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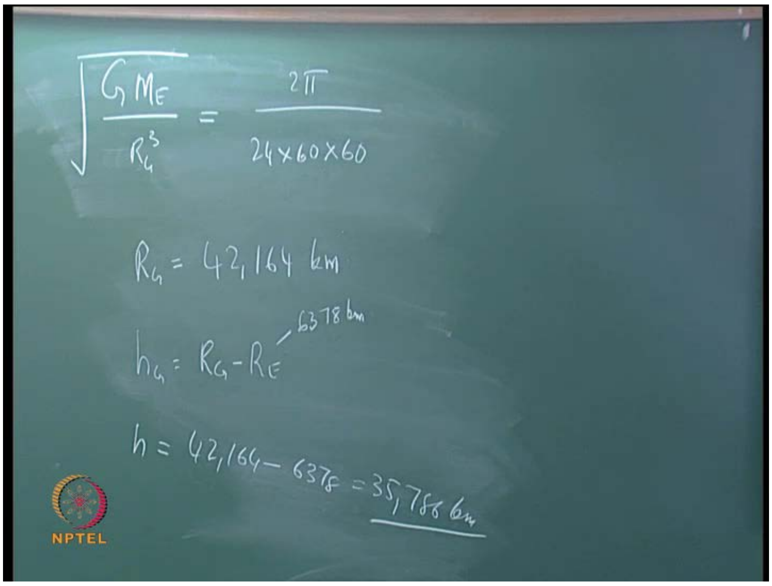

I get G M e by R G cube under root is equal to I get 2 pi divided by 24 into 60 into 60. And you know the value of the gravitational constant is 6.670 into 10 to the power minus 11. We have, we know the mass of the earth and the mass of the earth we said is 5.974 into 10 to the power 24 kg and therefore, if now I find the value of R G, R G works out to be something like 42164 kilometres I tell myself yes I have the earth here I want to find out the period of rotation or the angular velocity of rotation the angular velocity of rotation we derived as equal to G M e by R G cube. That means, we had 2 pi R we we had derived this. In fact, we put M omega square R which is equal to the psuedo force and we said its balanced by the by the friction by the attraction due to the earth and that is where we got this expression. And therefore, we got the angular rotation of the of the orbit is this much and the angular rotation of the earth which was equal to it goes to 360 degrees or 2 pi radians in the same time and therefore, we get the distance at which the period of rotation of the object and the period of rotation of the earth as same as equal to 42164.

Now the height of the geosynchronous orbit is therefore, equal to R G minus R G minus R E and the radius of the earth we said that the radius of the earth is 6378 kilometres rather the height at which a space craft appears stationary is equal to 42164 minus 6378 which is equal to 35786 kilometres. What is it I am telling you?

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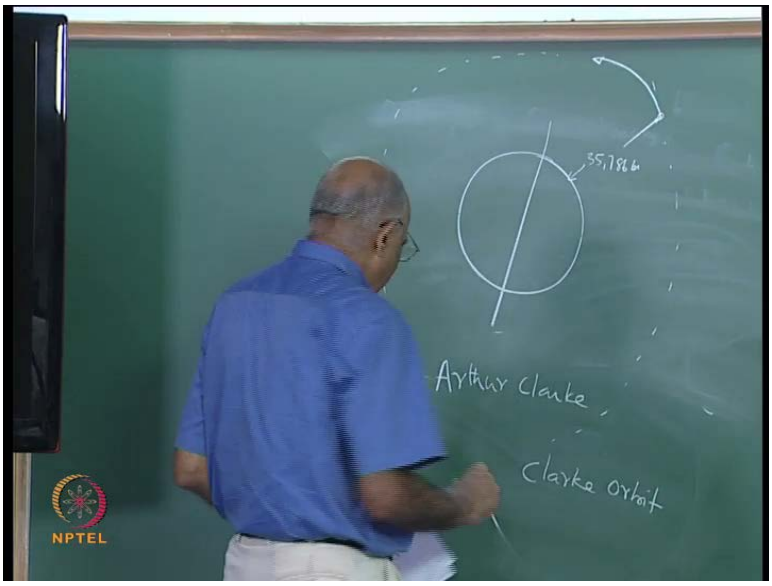

All what I am trying to say is, well you have the earth. Let us draw the earth and the look at the rotation of the earth may be the axis of the earth, may be the earth as well. The earth is rotating once in 24 hours the angular velocity of rotation is 2 pi divided by 24 into 60 into 60. And may be the period at which if I have a height above the earth which is 35000 and we say 700 and 86 kilometres above, the period of rotation or the period of this orbit and the period of rotation of the earth is (()) provided that the orbit is in the equatorial plane. And therefore, this object will appears to be stationary.

This concept was not developed by rocket engineers or people in mission, but was told by the famous science fiction author Arthur Clark. Arthur Clark has been writing a lot of science fiction books. In fact, he settle down in cylone. I think he passed away several years ago, very prolific writer. And he proposed this orbit in the year around 1945 and therefore, the geostationary orbit is also referred to as Clark orbit after him. Therefore, let me just repeat this again, once again. All what we we are telling ourselves we have the earth rotating east to west once in 24 hours.

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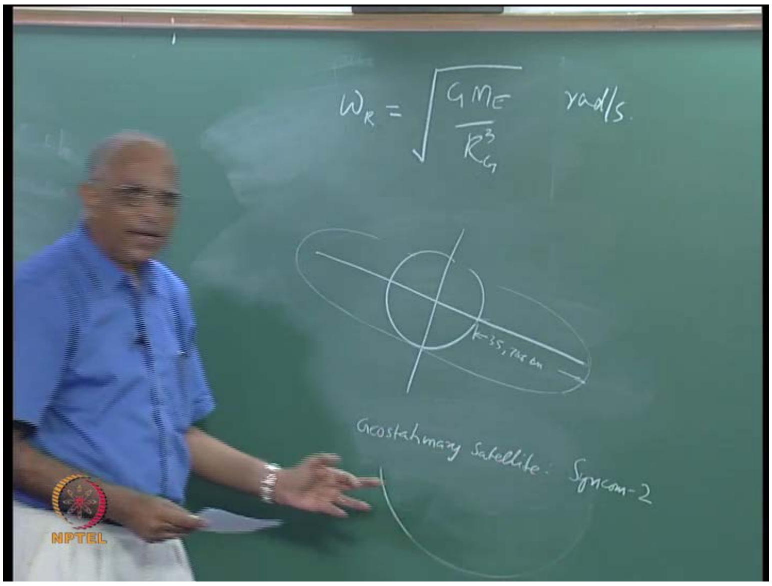

And now if I have a plane which is on the equator, equatorial plane and around this I have an orbit a circular orbit which rotates once in 24 hours, the height above the height we are talking is something like 35700 and 86 kilometres. And at this point the rotating object will appears stationary. In fact, this was recognised and it has been the effort to have such orbits and the first orbit or the first geostationary satellite; that means, satellite is the body rotating around geostationary satellite which was developed as a geostationary satellite was something known as sincom 2. It was launched by US in 1963 I think July 26, 1963 and that is when, you know we had this geosynchronous satellite there at that height may be looking at the earth always there and that was the year when Tokyo Olympics were held and it was the first time we had TV communication from Tokyo and people could watch it in the US and may be in some other countries was. Therefore, the geostationary satellite is one for which the period of a single rotation is same thing as the period of the rotation of the earth. Therefore, we we what we have done so far is we started with the velocity of the orbit. We told ourselves… let us just again put down the equations clearly such that we are very clear.

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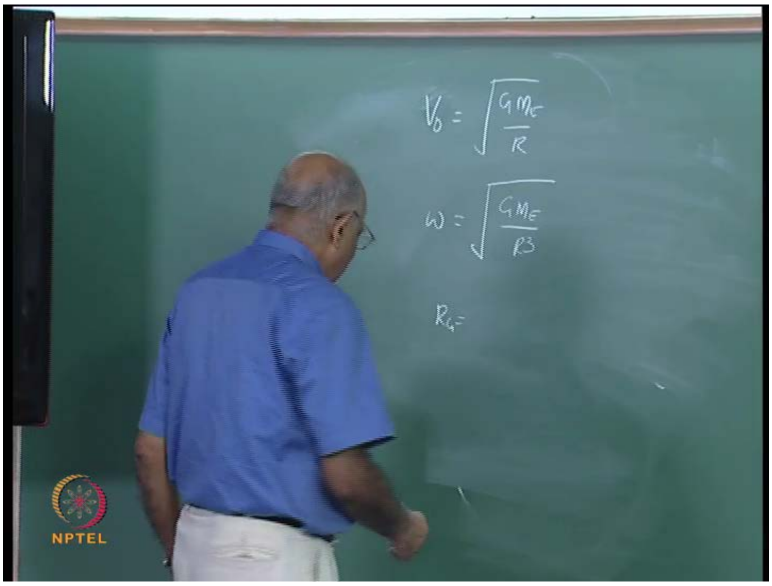

The velocity of the orbit v 0 is equal to under root G M e by R correct. The angular rotation of a orbit is equal to G M e by R omega therefore, it is equal to R cube. When we are looking at the period of rotation, when the period of rotation of the earth and this is same we get the value of R G as equal to 42000 or the height above the earth as equal to something like 35000. And this was postulated by the science fiction author Arthur Clark it is also known as the Clark orbit. Now, all countries have their geostationary satellite in India we have insat satellites in all countries we have this satellite we had etsf and different satellites may be we will take a look at these satellite, but the first one was sincom in the year 1963. Having seen the geostationary orbits let us go to some other orbits

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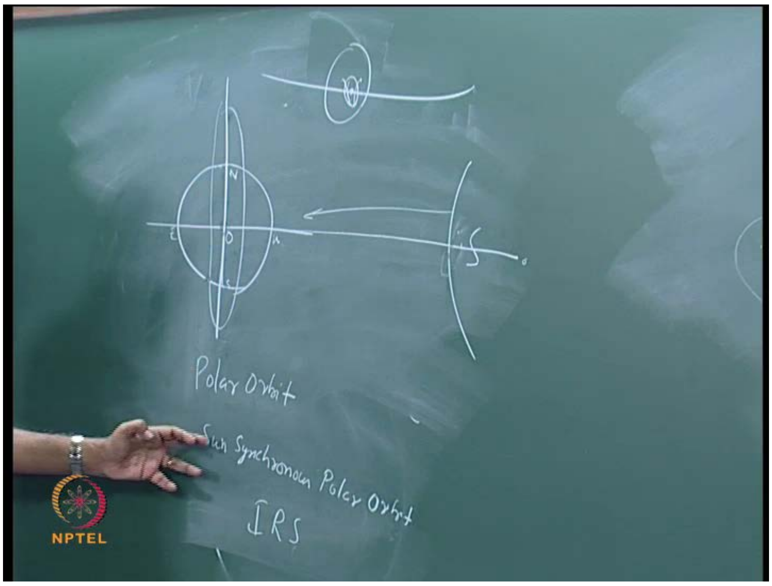

Let let let us see if we can have a satellite which goes from north to south; that means, this is the earth, may be the north pole, the south pole, the satellite to go round from north to south. And there is one advantage, if a satellite can go round like this in a circular orbit we are still talking of circular orbits, the earth, this is the east and west, earth is rotating like this, and if I have the satellite which goes round and round here, as the earth rotates this particular satellite can see all parts of the earth. Therefore, this is known as from pole to pole that is the north pole to south pole, this is known as polar orbit. The equations are exactly identical. Supposing, I want to put it at a distance R from the centre of the earth, I calculate the value of omega and I find out the period or I calculate the value of the velocity of the orbit.

Therefore, we have polar orbit, but then, we know as it is rotating we have the sun which is over here; let us say the sun is over here and you know what is happening, the polar orbit is really not 90 to 90 because what is what happens is, you know the earth is little chubby not not really a sphere and it is necessary… Supposing you want the sunrays to come; that means, we are talking of the centre of the earth o to the centre of the sun, this is one axis and we have the orbital plane over here. If the angle between the orbital plane and the centre line of the sun is kept constant we call it as sun synchronous polar orbit. Why do we need this? You know if this if the line joining here and the polar orbit makes the same angle the intensity of the sun, which the satellite sees on on the earth or any other planet will be the same. And therefore, I can compare the reading of what I take today and may be some other day and this is known as sun synchronous polar orbit. And we have in our country, Indian remote sensing satellites which keep sensing or keep looking at the earth as it were.

Now let let us divert our attention a little bit and ask ourselves, why why all these why all all these different orbits. You know if I say geosynchronous orbit that means, I have the earth as it were I have along the east west I am at a height of 35000. You know the satellite is always or the object which is rotating is always stationary with respect to the earth. We must do this, may be on a clear day, we go at night and we can see the satellite we can see insat satellite geostationary satellite. And when I look at it, I will see the stars and all that you know we said the galaxy stars are in a state of continuous motion. We will see the stars and other things drifting across, but this satellite will be dead stationary. That is because both are rotating at the same speed and because it is stationary may be I have the insat satellite pointing may be towards the centre of India may be at Nagpur and it is able to cover the entire Indian continent and it is able to provide may be communication telephony may be TV programs and all that is what the insat satellite.

When I talk n terms of polar orbits and talk in terms of sensing the earth why do we need this? You know, supposing, we say some crop is grown in some part of Andhra Pradesh and I want to find out the crop is healthy or not, I can think in terms of crop is not healthy then, it withers. The frequency what I see, or the colour what I see is going to be different. I can find out from this particular satellite the nature of the crop and I can warn the people, look here, your crop is not going (( )) or may be if somebody wants to catch fish or something fish always are in the ocean when the temperatures are little higher may be I monitor the temperature of the ocean and give a message to the to the (()) industries you go here and catch fish. And that is how you use the remote sensing polar orbit and you use which is again if it is sun synchronous I will get the same illumination, I will be able to compare between the different days and we talk therefore, in terms of may be polar orbit and a geostationary orbit, these are the two orbits.

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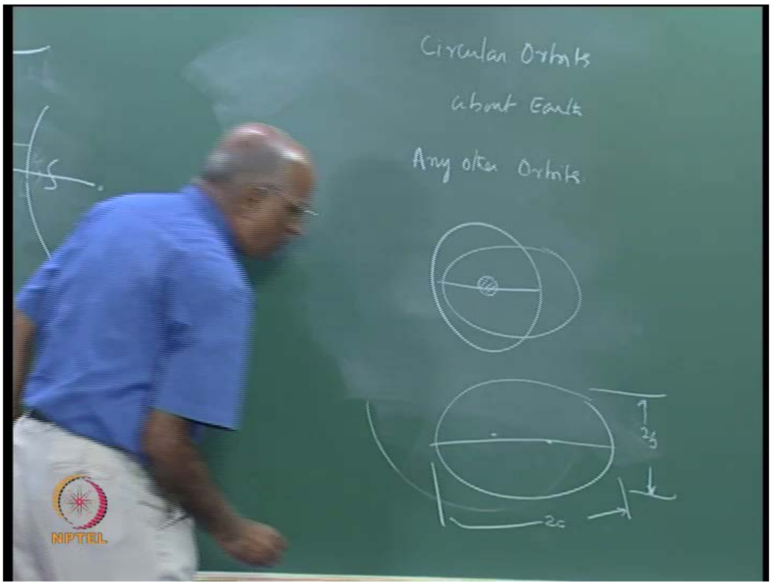

Having said that, let let us go one step further We we we we could as well have some other orbits, we could have a low earth orbit around the earth we could have something like medium earth orbit. In other words, I have the earth may be I go to low earth orbit and low earth orbit are normally used for scientific studies. You all would have scientific studies and what are the scientific studies? May be in the we we we talk in terms of troposphere, we talk in terms of stratosphere. In this stratosphere we have the wind, you would like to find out the wind velocity. We were also told there are something like like charge particles which are available in the stratosphere, may be I would like to measure some of these things and that is how the lower orbit is used. But there is a limit to the lower orbit. Typically, it has to be greater than 300 to 400 kilometres otherwise, the air or atmosphere which is there will cause the (( )).

Therefore, can I assume that we are we should be fairly clear at this point in time relating to circular orbits above the earth or for that matter why should it be earth? If I go and have a geostationary satellite about Jupiter, it should be the same. I take the mass of Jupiter, I take the radius of Jupiter and I can find out at what height I must have. Therefore, I think at this point let let us ask ourselves, are there any other orbits other than circular orbits? You will recall when we looked at the at the orbital velocities of the planets around the sun, we said all orbits are elliptical. What is the difference between a circular orbit and an elliptical orbit? In other words, instead of the of may be the earth being here, let us say smaller, if I say its circular, circular if I say it is elliptical, may be I am talking in terms of elliptical, something like this. In other words, how do we define an ellipse? We define something like a foci 1 and 2 foci and you have major axis which is equal to 2a and a minor axis which is equal to 2b. Therefore, we have, may be sun. We told ourselves sun is at the foci and earth is rotating around it. Similarly, if I have the earth here and the satellite is in elliptical orbit, I have earth as the foci and the satellite going around this elliptical orbit.

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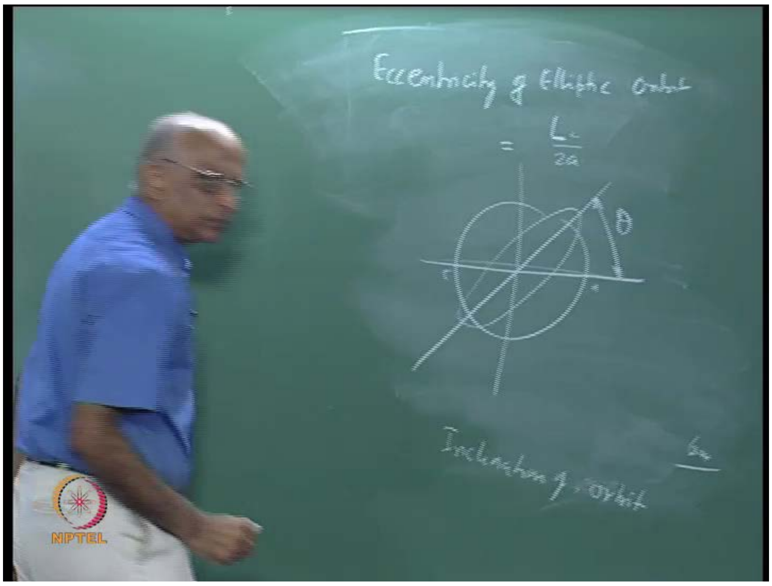

Therefore, you define two terminologies namely, we define something known as eccentricity of elliptic orbit, this is the distance between the 2 foci. That means, this is the L c divided by the major axis 2 a. And you find for a circular orbit, L c is 0 because I have a centre here and the eccentricity becomes equal to 0 for a circular orbit. There is another another point which we must keep in mind and that is, the orbit need not always be east to west, east to west or need not always be polar, we could have in between. May be an orbit could be like this, may be at an inclination this is the orbital plane, this is the east west, I say this is the inclination theta; that means, we say the angle between the orbital plane and the equatorial plane is what we call the inclination of the orbit.

Let us let us let me take you through an example and the example I quote is, see very often let let us.. we are we are very fortunate to be near near the equator and we are not much affected..

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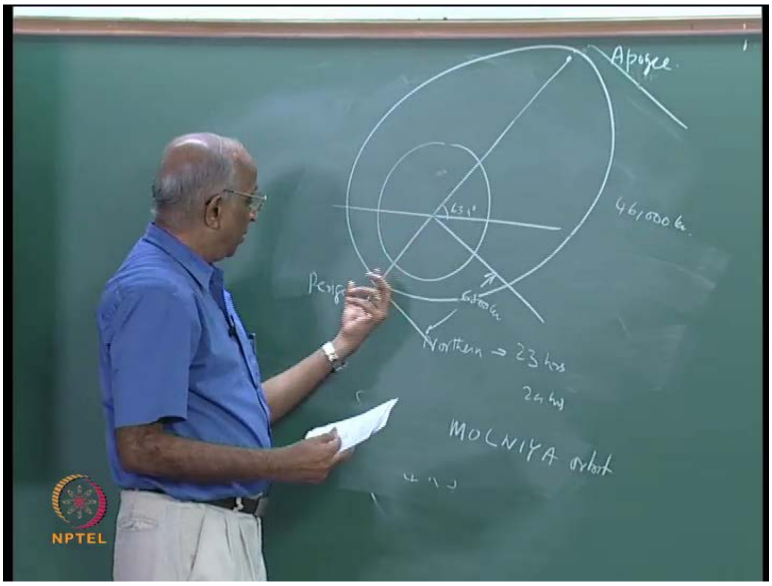

But we take a country like like like say we take a country near let us say near Moscow may be Russia or something. And if we take this particular country it is in the north northern hemisphere and if I were to have something like an equatorial orbit he is not able to see this that distinctly. And supposing what he does is, if I have something like an elliptical orbit at an angle of something like may be like 63.4 degrees I will come back to this little later. And then I have an orbit which has a smaller distance here and a much longer distance here that is, the orbit is something like elliptical over here, I find that these these these space craft or the object which is rotating spends much more time in the northern latitude and correspondingly smaller time in the southern latitude. And what was the second law which we said, when when we talked of the orbital motion equal areas in equal times and that was Johannes Keplers law. And therefore, it spends much more time and they can have, in the northern hemisphere the satellite can spend something like 23 hours out of 24 hours. And this particular type of highly elliptical orbits is what the Russians call as molniya and they operate. It has an inclination let me get the numbers clear.

It has an inclination of something like yes 63.4 is an inclination, the distance to the top most point is 46000 kilometres and the distance from the centre to the nearest point is something like only 6800. That is, you see the the distance between the centre of the earth and and the furthest point from the earth which is the which is over here, and this is the furthest point away from the earth over here. The distance between the centre and this point is what we call as this point we call as perigee; that means, this is the point which is nearest to the earth, this point is, away from the earth which we call as apogee.

Therefore, in an elliptical orbit we also define something known as perigee and apogee. Perigee is something which is the orbital distance nearest to the centre of the earth and apogee is the farthest distance from the earth. These are for the case of orbits which are elliptical. Therefore, I do not think I should go further into elliptical orbits, is something very similar to what we study for circular orbits. We have to put a psuedo force, we have to balance it by the attractive or universal law for attraction of the object to the centre of the planet and then figure it out. But something which we have really not done is, see again telling ourselves about orbital velocity what is this orbital velocity?

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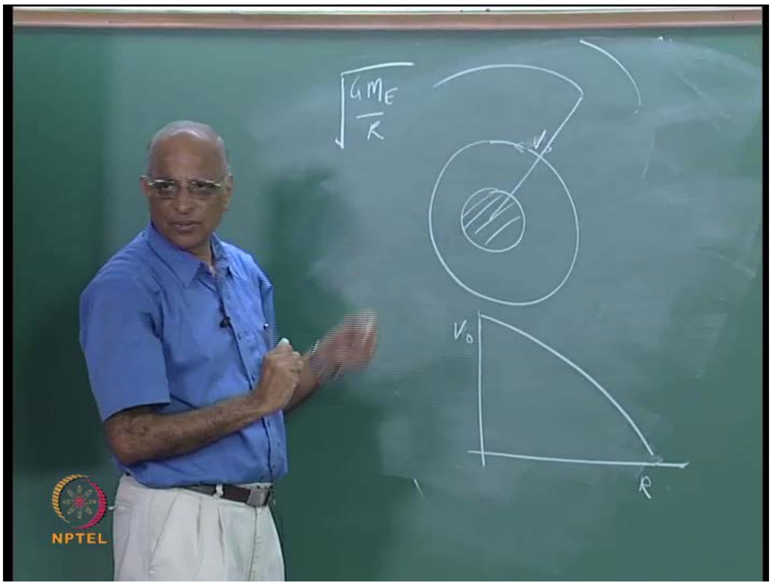

Let lets lets take one more small look at it. We told ourselves we have the earth as it were may be if I consider the circular orbit I have orbital velocity v 0. We also told ourselves if I plot v 0 mind you we did it in the last class and I plot R we got the orbital velocity v 0 in metres per second equal to G M e by R to the power to the square root sign right. We find, if the radius is infinite the orbital velocity is 0, if the radius in the centre of the earth as R increases at the surface of the earth the orbital velocity is higher and it keeps falling 0 as we put it infinity. You know why should the velocity of the orbit goes to 0 to infinity how would you explain? Is there any any suggestion for me? Why why should it be 0? Any any any thinking on it.? See we did tell ourselves, supposing the spacecraft to leave the earth escape from the earth well it has to go to infinity. Therefore, when I talk of infinity you have no attractive force on the earth and therefore, it is not in orbit anymore and therefore, you find that the orbital velocity keeps decreasing.

Therefore, this tells me that the orbital velocity v 0 keeps decreasing as I increase the height because as I increase the height above the earth. Now the question coming is, from the surface of the earth you have to go to this height which we have not yet considered. That means, I require a certain velocity or potential energy to be given to go or I have to do some work in taking a mass from the surface of the earth to go to the particular orbit at a distance let us say R or I have to increase the height from the surface of the earth by h. How do I figure that out? See in other word so, far I have talk only of the orbital velocity v 0 I am not told you how much velocity is required to start from the surface of the earth and go to this and then get into the orbital velocity.

Therefore, let us now find out what is the total velocity required for orbiting plus taking the space craft from the surface of the earth or some other planet to the particular radius of the orbit. How do I do it? It is the same thing I think if we if we have understood so far, we must be able to do it very radically.

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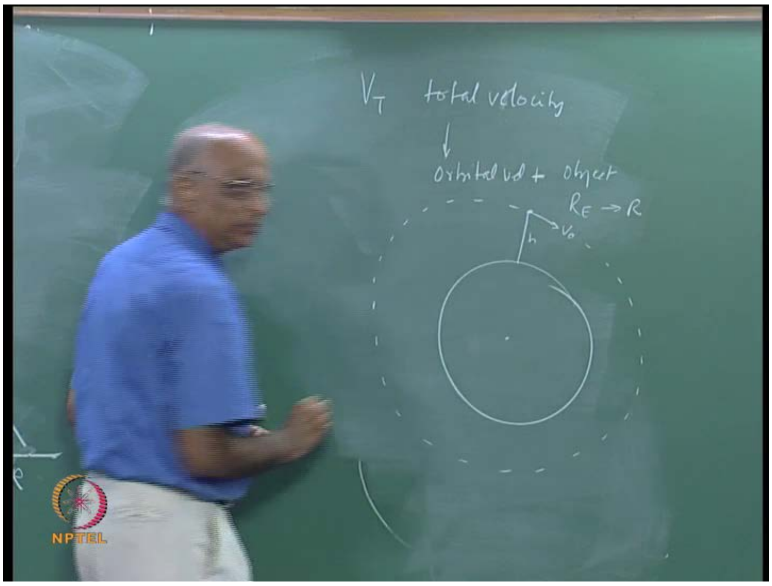

Let let us try to do that. We want to ask ourselves what is the total velocity which includes the orbital velocity plus the velocity required to take the to take the object from the surface of the earth let us say at R e to the particular orbit at R. How do I determine?.

In other words, all what I am saying is I have the earth here I want to take the spacecraft above the earth to a height edge. And I have to insert I have give an orbital velocity and then it will continuously rotate. Therefore, what is the total velocity which I must given how do I determine again we use the same set of equations

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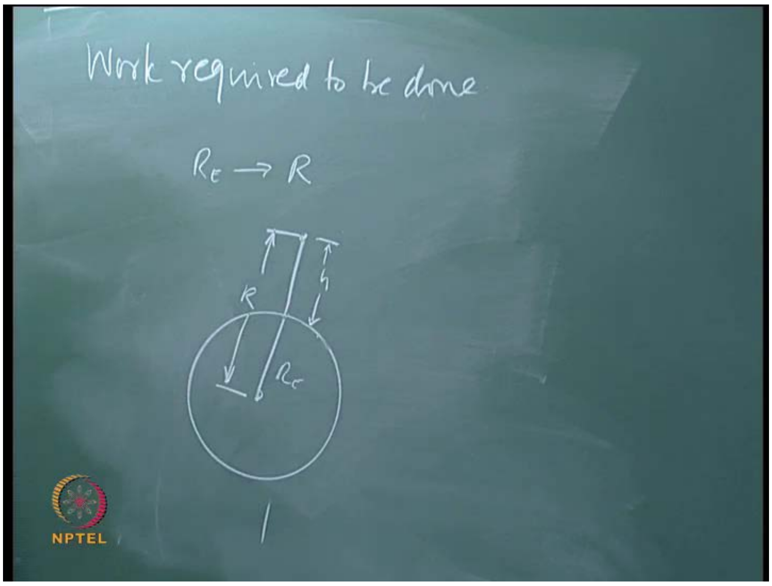

We talk in terms of the the the universal law of gravitations. We ask ourselves what is the work required to be done to increase the height of the spacecraft or to take the spacecraft or the object from R e to R. Let us consider the earth as it were now, all what I am asking is I have the earth here, I have R e here, I want to take it from here to here to a height. Let us say this final height is R and this height is h how do I do this? What is the work required to be done. Let us say that, the mass of the object or mass of the spacecraft is small m how do I calculate the work done? In or the potential energy required to take this spacecraft from the surface of the earth to a height h. How do I do it? Any suggestions yes how how will you how will you solve this problem I want the work which is required and what is work work is equal to force into distance what is the force. yes you have the you have to get rid of the gravitational force.

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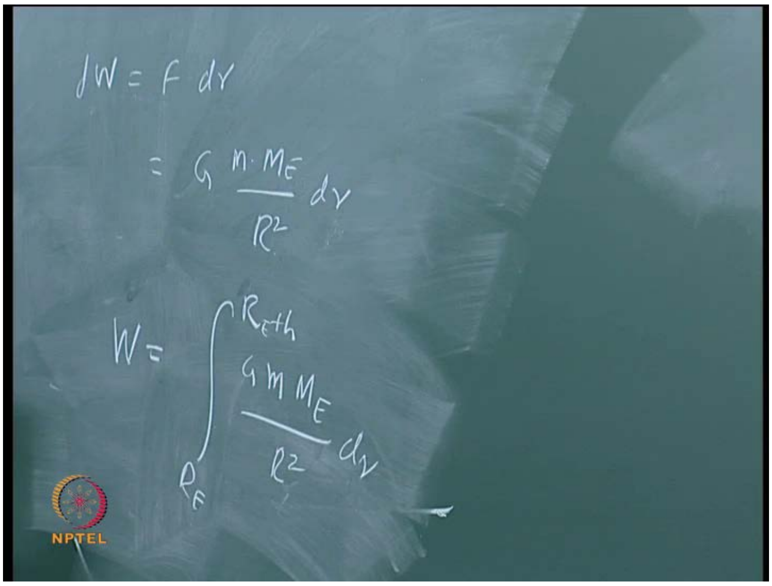

Therefore, let let us put it together we find that the force is equal to G, mass of the earth, mass of the body divided by at any radius it is R r square. What is the value of the work done, when it moves through a distance let us say d r may be may be I assume that it goes may be from a height R to a height R plus d r. What is the small work which is required to be done d w is equal to the force into the small displacement d r and therefore, the d w must be equal to G M m e by R square into d r and what is the work required as I go from may be the surface of the earth having radius R e to a height R is equal to I know I just have to integrate out the total work what is required is equal to integral from the surface of the earth. But mind you let me qualify again and again I am illustrating with respect to the earth I could have all these things for anybody. Supposing somebody wants to go from the surface of the moon to some height I just have to take the mass of the moon the radius of the moon plus the particular height what I consider. Therefore, I have G m M E divided by R square into d r as I go from height, if I want to put in terms of height R eth. Is it all right? And this work must be equal to the potential energy because you have increasing the height therefore, I have higher value of potential energy. But then, we also know that the that the there is a orbital velocity v 0; that means, I have some kinetic energy and what is the total energy at a height I have, this is potential energy plus I have the kinetic energy.

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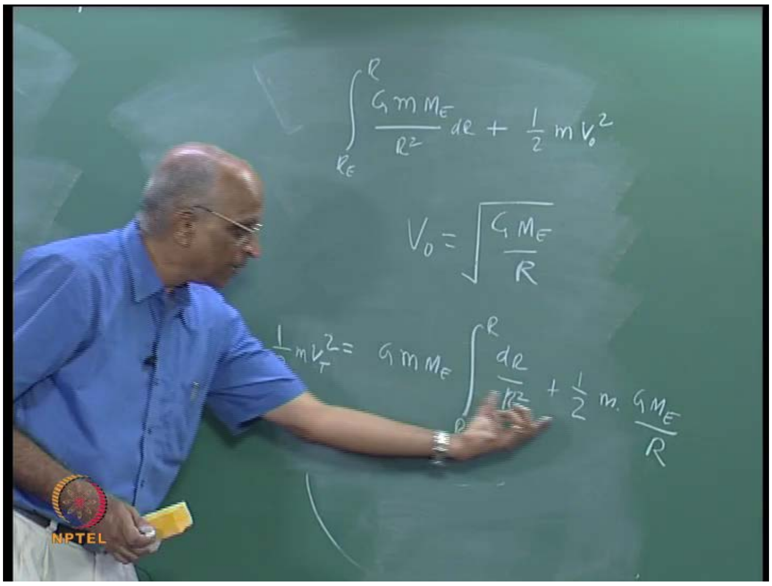

And therefore, I can say that that total energy of a rotating spacecraft in orbit must be equal to I have R e to R G m m E by R square into d r plus the kinetic energy of a rotating body is equal to half m this is the orbital velocity. And what is v 0 square, we have defined it we derived it as, equal to v 0 is equal to under root G M E by R please check it. And therefore, I can say that that the total energy in orbit is therefore, equal to let let us integrate this out, mass of the thing of the object is constant, mass of the earth is constant therefore, I have G gravitational constant m M E into I have integral R E to the value of R of d R by R square plus I have half m what is v 0 square G M E by the, at that particular radius is R is it all right?. And if I say I get started by giving a kinetic energy that is I supply some velocity energy to this and therefore, if I say that the total velocity is v t this must be equal to half m into v t square, that is the total velocity I give to the spacecraft is this, consists of potential energy plus the kinetic energy due to rotation. And therefore, the total velocity what I give to the spacecraft now I can derive the expression. Let us let us simplify m cancels over here m is your constant.

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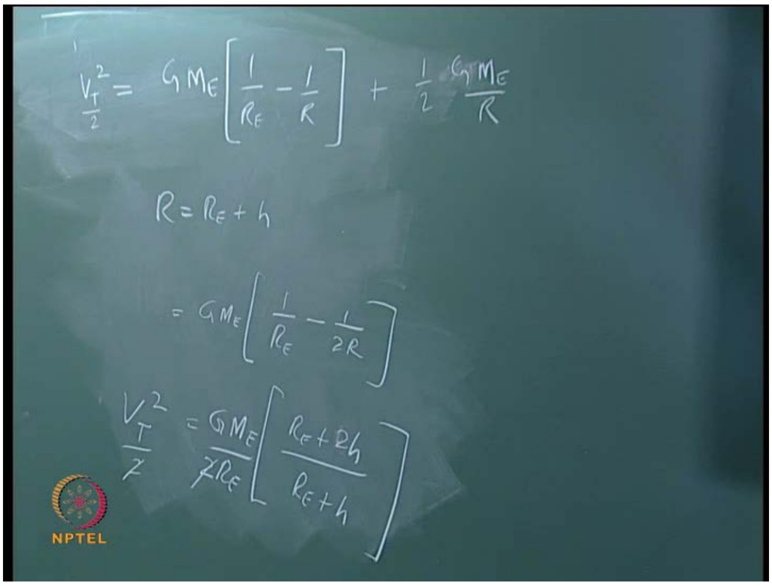

And therefore, I get V T square is equal to lets us get this here half V T square by 2 is equal to G M E into I have minus 1 by R therefore, this become what one over R e because I change the order of integration one by R e minus one by R plus I have half G M E by R therefore, what should be the value of v t let us find out. We say R is equal to the radius of the earth plus h. Now, I have something like I find I have G M E minus 1 over R plus this. Therefore, the right hand side I can right as equal to G M E by 1 over R E minus minus 1 over R plus half that is equal to minus 1 over 2 R please check I am I might may be making a mistake please check if it is alright. All what we said is minus 1 over R plus 1 over 2 R therefore, minus one over 2 R is here and therefore,, but R is equal to R E plus h and therefore, this gives me something like G M E by 2 R minus R e; that means, equal to R E plus h R E plus 2 h and I take R E outside here and I have the value of 2 over here and therefore, this is equal to R E plus h. Please make sure it is alright and this is the total velocity V T square dived by 2, 2 and 2 gets cancelled and therefore, the total velocity which has to be given to a spacecraft or to a body.

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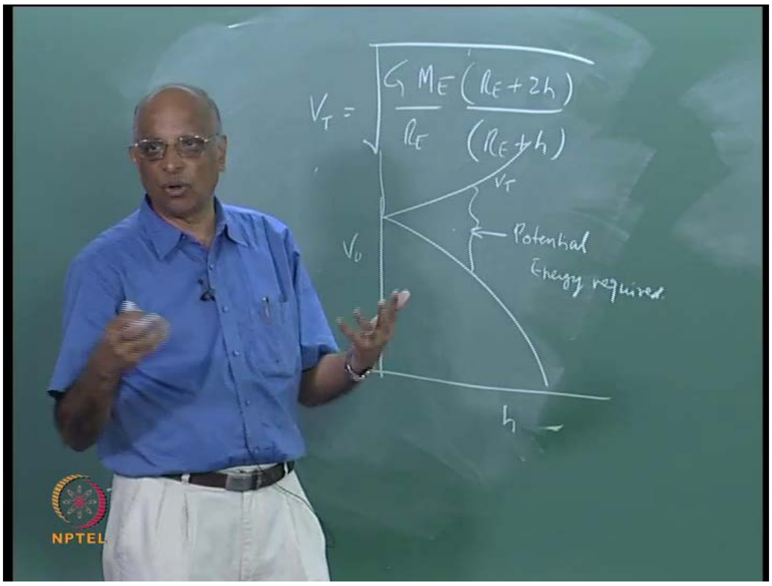

When it is rotating at a height h above is given by V T is equal to G M E by R E into excuse me R E plus 2 h divided by R E plus h. Now, what is it I find here while the orbital velocity we found keeps decreasing as the height above the earth increases or the height increases we find h is multiplied by 2 therefore, this is the stronger compared to h over here the total velocity increases in this fashion. That means, as I go higher and higher I need to give total velocity to the spacecraft which is going to be much higher than the orbital velocity and this difference is what constitutes the potential energy required. Is it alright? Therefore, I think what I do at this point is maybe I stop over here in the next class we will do some problems involving the potential energy may be the kinetic energy we will find out the different orbits and how to solve for the orbits right thank you.