Rocket Propulsion

Prof. K. Ramamurthi

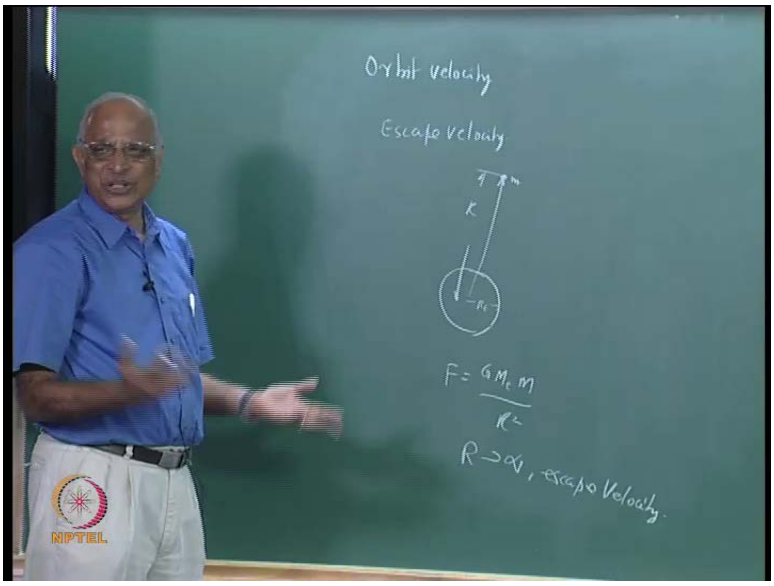
Department of Mechanical Engineering

Indian institute of technology, madras

Lecture No. # 05

Theory of rocket propulsion

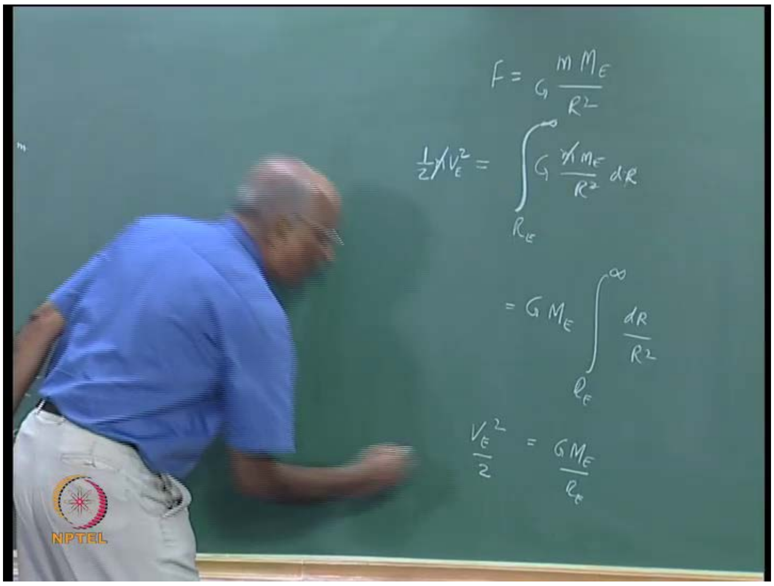
(Refer Slide Time: 00:20)



You know in the class today, what we will look at is, typical Orbit velocities may be illustrated with some problems, and since you all mention something about Escape velocities. Can we find out, what is the velocity required to escape from let say the earth or from some other planet? What is it I am talking out? Let say come back to the Escape velocities first, let say I have the earth, and I want to escape from this earth it way as it where. That means, I do not I want the force with which the earth is attracting me to vanish; that means, I escape. In other words, I am looking at may be the force is given by GM e may be the mass of the body, which is M divided by R square that must go to; that means, this force must vanish.

In other words, I am looking at maybe this is radius R E may be I want to find out the radius at which I can escape from the earth right and I have to give the corresponding velocities; and that velocities becomes the Escape velocities. Therefore, Escape velocity is the velocity required such that I escape from the attractive force of the earth. Therefore, what must be the value of R to escape? I want the force to be 0. I want to escape from the attraction therefore, it becomes infinity you are right; therefore, I am looking at R must be infinity in order to have the Escape velocity. How do I do this problem? It is same like what we are doing.

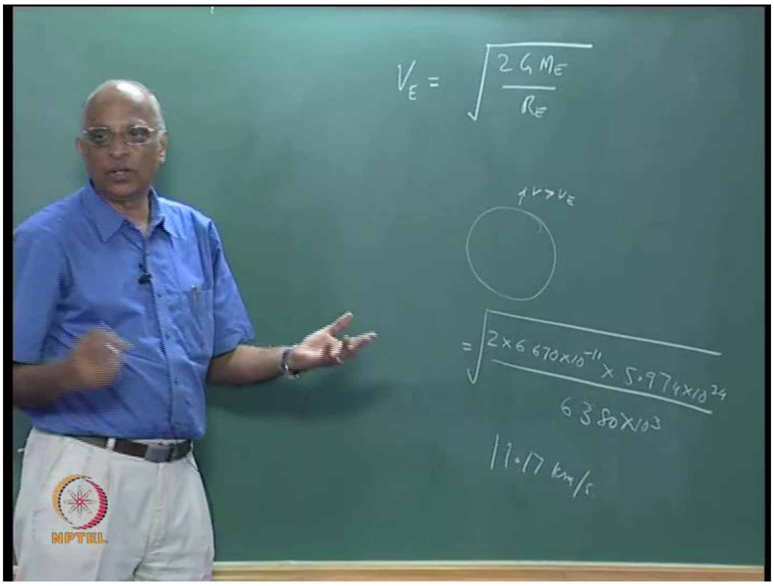
(Refer Slide Time: 02:11)



Let say, I am looking at the force is equal to M the mass of the body, mass of the earth divided by R square into G, and what is the work which I must do to take it from to infinity. The work, which I must do is equal to G m m E by R square into d R, this is the small amount of work. And now I want escape from let us say the surface of the earth having radius R E; I want to go to infinity and therefore, this must be the type of velocity what may if give. And how do I give the velocity? I give the kinetic energy to the body therefore, I give half M into V escape square; and this is what, is the escape velocity.

And let us find out what this is, we again the find that mass of the body cancels out; and when I say GM E by R square, which is again equal to GM E, and here I write R E to the value of infinity of d R; I am sorry this should have been the small increment in the radius divided by R square. And this you know, if I integrate out is minus 1 over R, and at infinity I get 0 minus 0; now I get minus of minus over R e and this become equal to GM E by R E. And what is the value I get this equal to V escape square divided by 2.

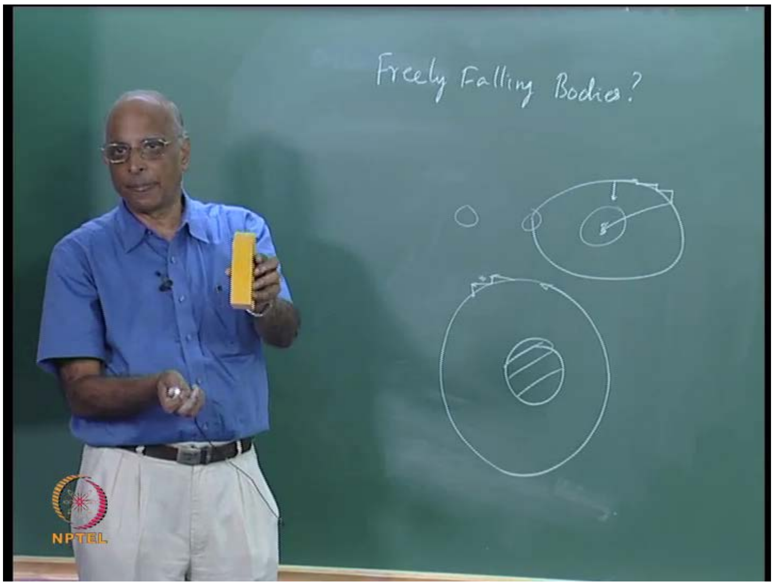
(Refer Slide Time: 03:56)



Or rather I get the Escape velocity V E is equal to I get 2 G M E by the value of the radius of the earth just over here that is 2 G M E by R E V E to the power (( )) and this is the escape velocity. In facts it is very interesting you know, we must remember of few things; see when the earth was born, we had lot of hydrogen, which was available around the earth. Hydrogen is the very light gas and therefore, when the light gas is there as we shall see when we get into theory of Rocket propulsion little latter in this class. We know the velocity at which hydrogen escape if it is hot is greater than the Escape velocity and that is how we lost hydrogen; whereas on the surface of the earth, we do not get (( )). The gas goes which such a high speed that is greater than the Escape velocity and that is how we lost out the hydrogen. Anything which travels at a velocity greater than this keeps on going up and you escape from the surface of the earth.

Let us calculate the value again we say Escape velocity from the surface of earth is equal to 2 gravitational constant 6.670 10 to the power minus 11 Newton meter square by kilogram square mass of the earth 5.974 10 to the power 24 kilogram and the radius of the earth 676380 in to 10 to the power 3 meters. And this is what, is the Escape velocity from the surface of the earth. You can just calculated what out be something like 11.17 11.17 kilo meters per second and therefore, if you want to escape let us say be we are talking of a the mission to the moon. I will I will talk about the orbits; you know supposing you want to go to the moon you first go around the earth, you have to escape from the earth; that means, have to the Escape velocity to get out of the earth, then I get into the gravity of the moon and supposing I want to come back, I need the Escape velocity from the moon and come back over here. And this is how we find out the total orbital requirements plus the total velocity requirements to put space craft sorry any bodies in orbit this is how we do it.

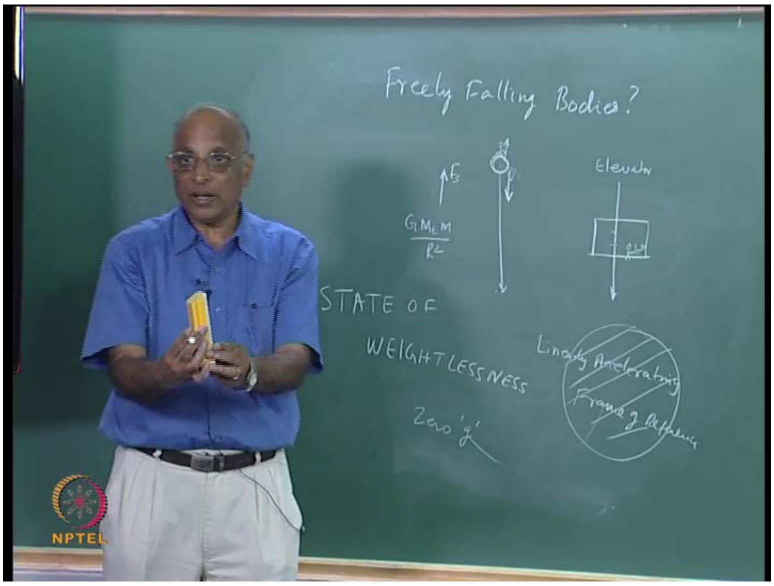
(Refer Slide Time: 06:57)



Let me let me take one or two small examples. We will do one or two small problems such that we are very clear. But however, before doing that I also want to tell a something about we were talking in terms of freely falling bodies. What do we mean by freely falling bodies? What do you meaning by this freely falling bodies? What did we tell ourselves? I have the sun I have the eight planets, which are going around the sun elliptical orbit. We told ourselves all these planets are just falling freely on this surface of the sun, falling towards the sun, but why is it just not falling, because by the time its falls it goes through some distance again it falls again it goes through some distance, again it falls, again it falls; that means, it is just falling toward the centre of the sun. Therefore, all planetary motions, all of us are freely falling on the sun as it where.

And how does space craft now; I say this is the earth let us for easiness consider something like a circular orbit. If I say this an orbit, it is going at a constant speed over here, it falls towards the centre of the earth again it goes because I give an orbital velocity it goes like this again it falls. Therefore, all the bodies which are in orbit are something which we call as freely falling bodies. It is as good as I drop a stone and it falls freely. So, also all the bodies, which are in orbit are freely falling, and what is the function or what is the thing, which we understanding by freely falling bodies.

(Refer Slide Time: 08:32)



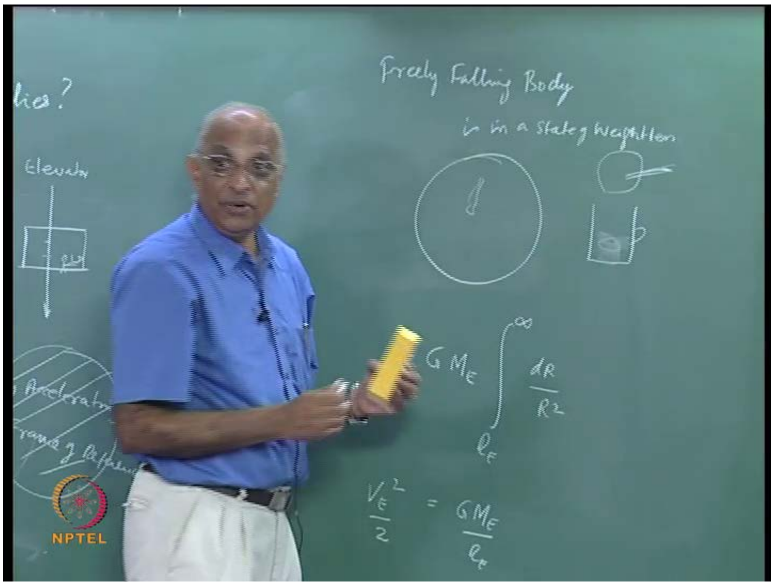
Let take the less take two examples, let us assume I have a stone it falls freely, but let us assume that there is no resistance due to air, because we are talking of space therefore, it just falling freely. I also take an example of an elevator or lift know be claim to this room through the elevator, and let us say the elevator falls freely. Let us take an example, I am in the elevator may be this is the elevator which is falling freely, I am going down, it just falls freely may be I am standing over here and in my hand I holder cup of tea; we all would have seen this. Now, what happens to the teacups, when I am holding? See, what is it? This stone is freely falling, this is freely falling this is not an inertial frame of reference, because it is picking up acceleration. Therefore, it is something like a linearly accelerating frames of reference; it is different from the rotational frame of reference. I am just taking this body.

Now my point is supposing, I am standing over here on the stone or I am sitting or am standing in the lift, and I am having the tea cup in my hand. Now, what is the force, which I will experience on this tea cup? Can somebody tell me? All what I am talking is yes, I have may be the earth is over here, earth is attracting me. And therefore, I have to correct, because it is not a initial in a not an inertial frame of reference. I have to put some corrective force over here and therefore, what I have to put is, may be the stone is freely falling; I have to put some force here, because for me, I am not moving at all. Therefore, I have to put a pseudo force opposite to the attractive force of the earth. Attractive force of the earth is GM E M by R square, and I have to put a pseudo force here, which is equal to this.

And the moment I put a pseudo force over here my motion is taken care of I am able to describe my motion correctively because I am not moving with respect to the stone, I am not moving with respect to this lift, but when I am put a force equal and opposite to the force with which I am attracting, the net force on me become 0. And when the next force on me become 0, I am weightless, I am in a state of how would I will take a look, I am in a state of weightlessness. Why is it? I am not I am not having any weight. It is not there. I have my weight, but to be able to correctively define me, because I am dropping along, I am sitting on the stone; the stone is coming down, but I have to correct my motion because I am not moving with respect to the frame of reference of this stone, because I am not moving. Therefore, I would to put the pseudo force, vertical to it, and I am now attractive by gravity, but I put the force equal and opposite to the gravity, then only I can say, I am not moving.

Therefore, the moment I put the pseudo force, I do not have any force or weight, I am in a state of weightlessness. Some people call it as 0 G; actually it is not 0 G; G is gravitation; gravitation is always there, but a body in orbit which is also freely falling bodies, is always in a state of weightlessness.

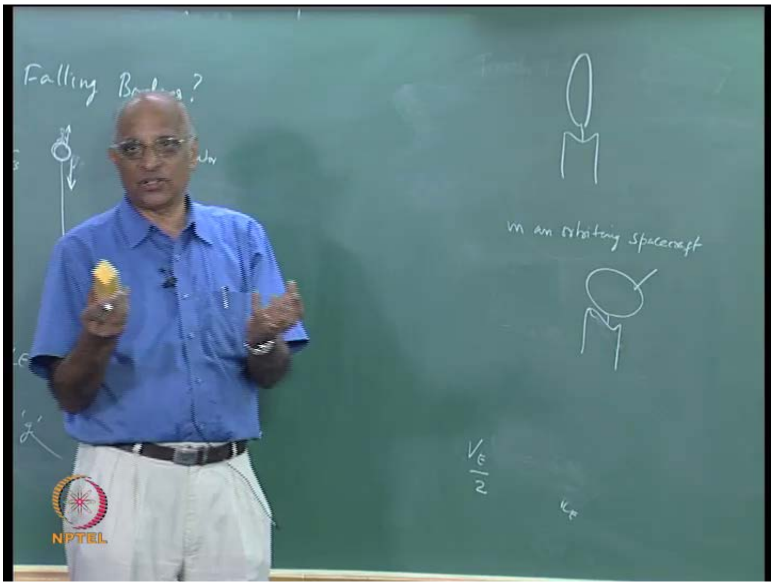
(Refer Slide Time: 12:16)



That is freely falling body is in a state of weightlessness. It does not have any it is not to able to see any weight. And so also if in this in this lift itself suppose I hold a tea cup and come down, you will feel it is not heavy at all, it is as it fit 0 G, because I have took I am I am standing here, I have to correct my motion using a pseudo force, and that is why whenever you see the mo the picture of may be astronauts in space, you see they are all floating around, there is nothing to really hold them, because you have to correct their motion by a pseudo force and you see, whenever you see the astronauts in a space capture all they at just floating. And you know to be able to drink a cup of water, supposing I go there in a deep mission, I am very thirsty is not possible for me to drink water from a cup, because it is they does not settle to the bottom of the of the tumbler or the cup.

Therefore, it will be freely floating; that means, I have water in a tumbler, but it will just be floating, therefore what want has to do may be one has to arrest it somewhere, may be put a straw suck it through then only even drinking little bit of water in space, when I am orbiting become a problem. This is what we call as a state of weightlessness or we call it as some people call it as 0 G b.

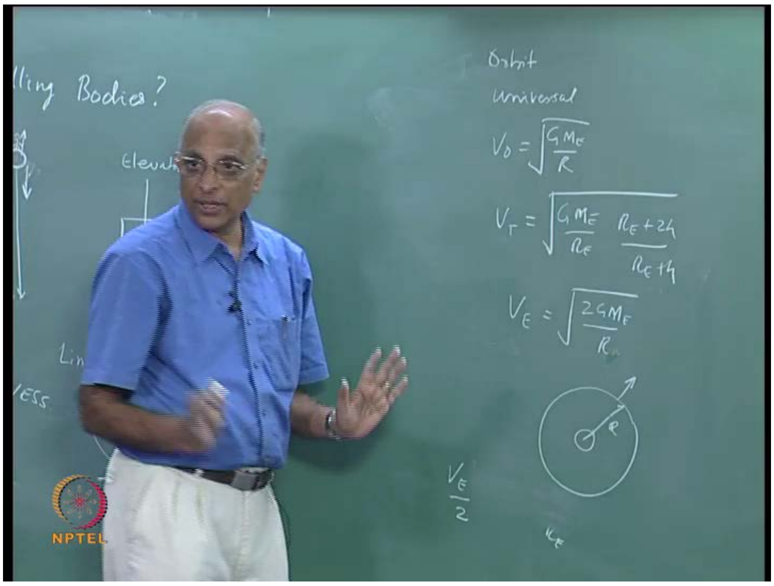
(Refer Slide Time: 13:57)



But let us not confuse G is always there in fact you know since, many of you are from mechanical engineering and working in combustion, you know if you have let say a candle flame is like this in, there are some experiments done in orbit what does the flame look and how will a flame look in space. Why do you have this shape on ground and how will at look. When the same candle you strike in orbit in orbiting (( )) spacecraft how should it look like any guesses and this is very powerful you know what is happening the gas become lighter (( )) force and that is have you know the lighter gas raises and you have the candle flame like this if I way to look at it at spacecraft. I will have the candle again it will be pure sphere because there is no no weightlessness. It does not have any light weight or strong weight; it just a perfect sphere and that is where my equations which I describe, I can have a spherical frame of reference.

I can solve for the diffusion equation and they can solve for the energy release equation and I am able to find out what is the mechanism of diffusion flame, much more strongly than if I can do a experiment on the ground with a candle. And you know in fact, may be little bit later on, I will bring some picture on show how may be an astronauts drinks water in space may be, how does a plant, how does a flower, which is growing in space how will it look, how it will be different, and there are pictures which are all available on this. Therefore, what is it we have done so far? Let us quickly summaries and do one or two small problems, which will make sure that we have understood this subject.

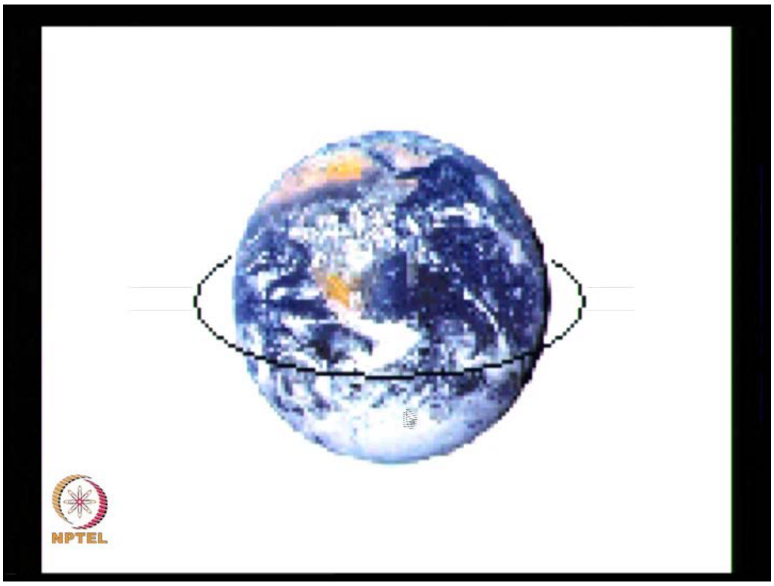
(Refer Slide Time: 15:44)



We talk in terms of orbiting of the different planet in our solar system. We went ahead, we formulated the universal law for gravitation as determine by new term. We told ourselves well, all planets are falling, an apple is falling, and he said a heavier body attracts a lighter body and you have the universal gravitational law. We use a gravitational law and we found out V 0 is equal to GM E by R, where R is the orbit; V is orbital velocity. We also found out that the total velocity required to the orbit for a circular orbit is equal to GM E by R E in to R E plus 2 H divided by R E plus H and a root.

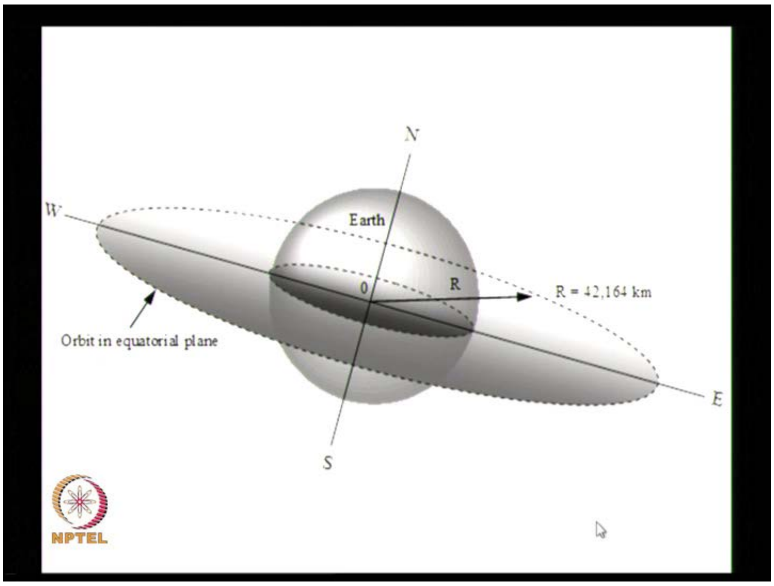
We also talked of geo synchronous orbit, polar orbit, maybe we talk be told ourselves for remote sensing go around, so that I can see entire earth as it where, for communication and weather prediction may be geo synchronous is better suited. I can also have lower eth orbit around the earth at different altitudes and all that and we found out the total velocity requirements. We also talked in terms of the Escape velocity, we said it is equal to 2 GM E by the radios of the earth from the surface of the earth. If I have something which is orbiting at it distances R from the surface of the earth and I want to push it to infinity then in this case it becomes R right; and this is all what we have done so far. Let us take one or two small problem, but before doing the problems, I will quickly revise what we have done so far.

(Refer Slide Time: 17:33)



And let us let us take a look at this power point presentation. See here, we see the lower eth orbit going around the earth as it where.

(Refer Slide Time: 17:45)

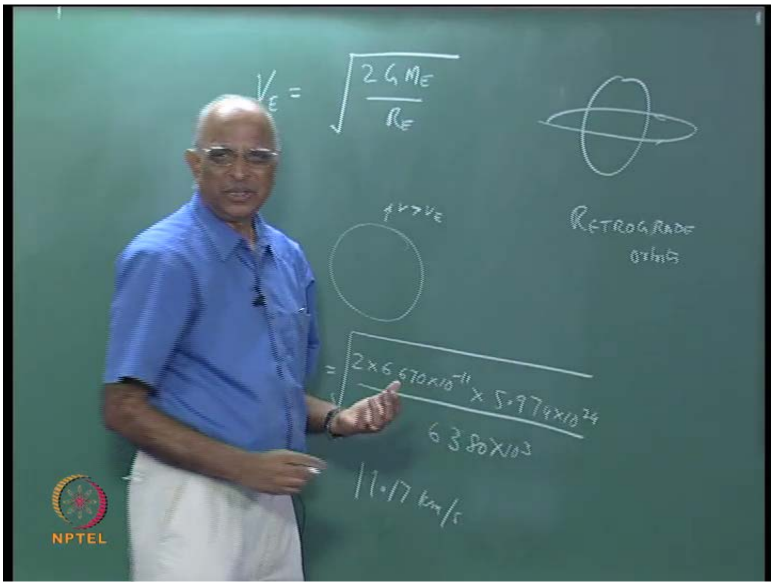


This shows the sun's geostationary orbit, you have the earth as it where, this is the east to west and you find that the spacecraft is going around at a height of 42.164 kilo meters; it is in the equatorial plane; this is the earth and this is radios, what is there, you subtract the radius of the earth and that is the height of the stationary orbit.

(Refer Slide Time: 18:12)

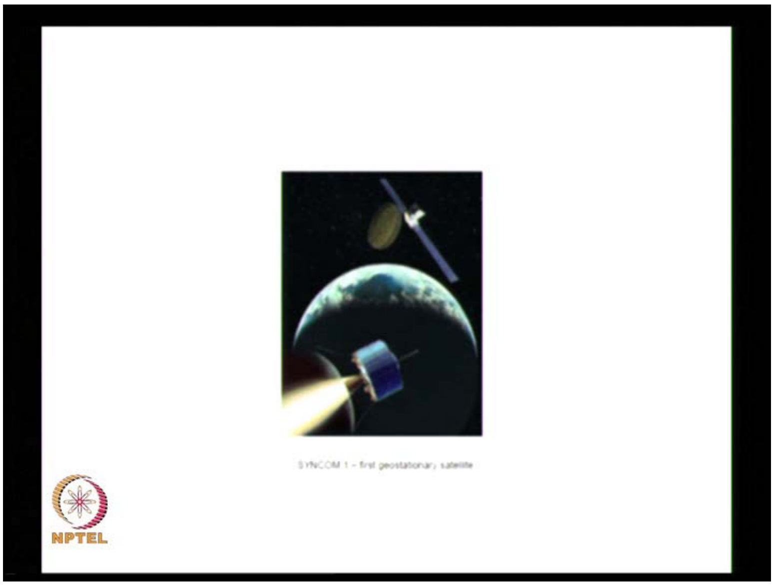
Geostationary, this shows the earth equatorial plane going around east to west.

(Refer Slide Time: 18:23)



And supposing we have an orbit let us say, I have earth here, I am sorry it should have been a circle; instead of going from east to west, I go from west to east. I am going against the rotation of the earth on such orbits are known as retrograde orbits, retro; but there is no were used, because why should I go against and not get any benefit at all.

(Refer Slide Time: 18:52)

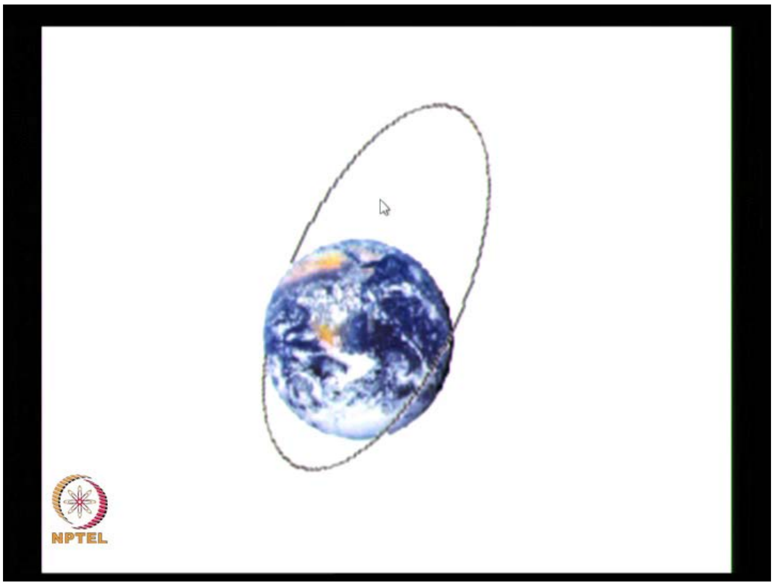


Let us go to the next one; this show the first Syncom 2, the first geostationary satellite, which was launched by US, was not successful therefore, the second one was successful. This was on 26 July 1963, and it relate the Tokyo Olympics.

(Refer Slide Time: 19:09)

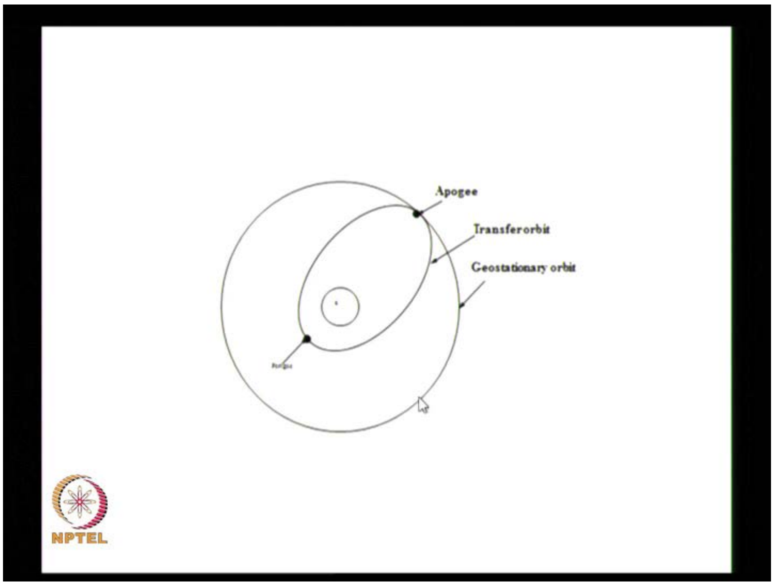
This is the polar orbit you go from north to south, you see this is the north to south; you are going around it like this. And you also find this angle is not really 90, but little more than 90.

(Refer Slide Time: 19:24)



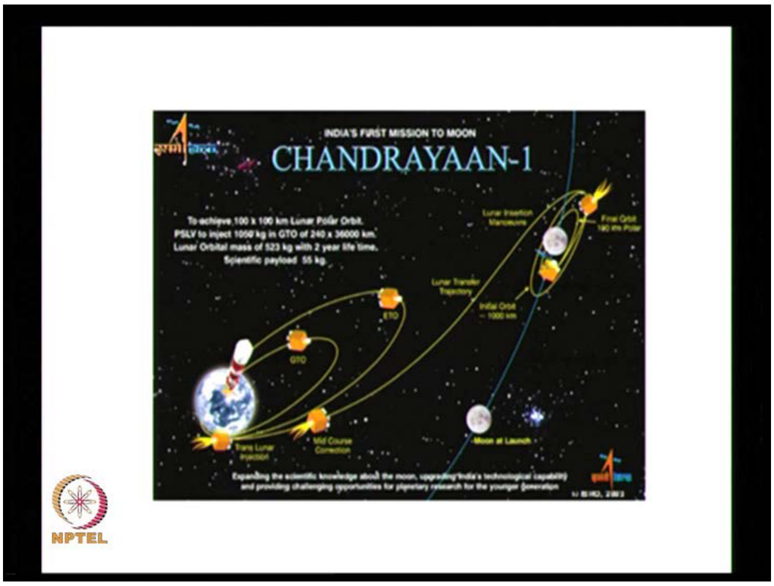
This shows the highly elliptical orbits; this is the perigee, this is the apogee. This distance we set for only orbit it is something like 42000; this distance here is of the order of 6000, and this is an elliptical orbit.

(Refer Slide Time: 19:39)



And now you know few other orbits, since we talked of orbits supposing, I want to launch a satellite into geostationary orbit. I first take off from the ground, I put it an elliptical orbit; I put the apogee equal to something like 40 to 164 highly elliptical orbit and then it comes to the apogee; I make sure I fire a rocket and make sure it goes along this (( )) geostationary orbit.

(Refer Slide Time: 20:08)

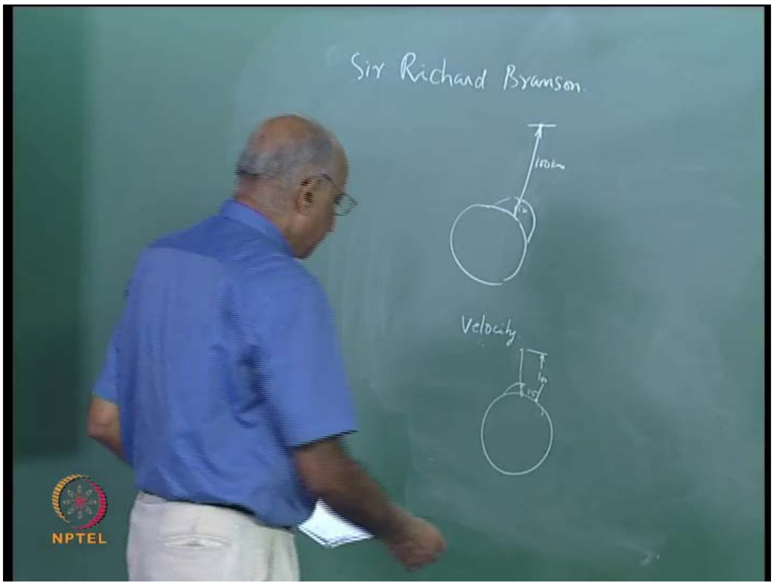


Suppose, I am talking in terms of a moon motion; well Chandrayaan-1 of ISRO, well it is from the earth, I keep going like this, I escape from the earth, I get inserted on to the moon; if I want to come back to the moon, I again come out of the moon gravity, I escape from this and I have to come back, these are the different orbit. Therefore, in the previous slide, I have something known as the transfer orbit; that means I am not talking in terms of the geostationary orbit. But to be able to go to this, I first put my object in a transfer orbit, and then when it is the apogee I circularized. Well these are all the different orbits and (( )) time for us to do our problem.

(Refer Slide Time: 20:48)

And I take a problem, which is something related to a resent rocket. We know you we have a person, I will tell you about it; we know a person by name Sir. Richard Branson with I thought let us do a problem, which has some practical relevant.

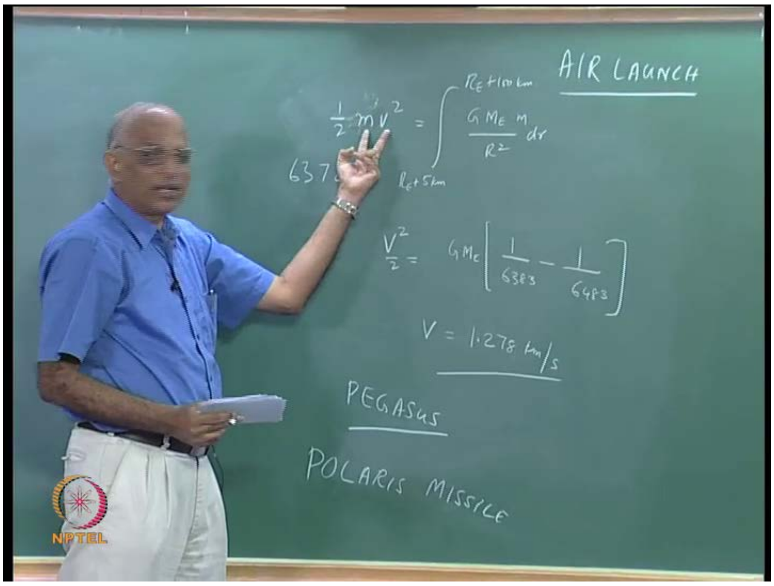
(Refer Slide Time: 21:12)



Sir, he is a commercial person, he wants to take (( )) from the surface of the earth into deep space, no people would like to go to the above the earth and see the earth as it where, how it look and it seems to be very fascinating. Therefore, what he proposes is he talks in terms of and that is what I show in this transparency. He talks in terms of an aircraft; this aircraft is known as White Knight. It is a very little stronger aircraft, you have engines here, you have you have and it carries a rocket and a space capsule. And what it does is the aircraft takes off from the surface of the earth goes to a height to a something like 15 kilometers, and it returns. At 15 kilometers, you fire a rocket, and it takes you to a distance of something like a 100 kilometers.

Therefore, I want to find a problem, where in may be from 15 kilometer above the surface of the earth to 100 kilometers, where a rocket takes you, what is the value of velocity to be provided by the rocket? Because, if I say the velocity required from the surface of the earth 200 kilometers I know, but now it takes me from 15 kilometers to a distance of 100 kilometers; that means, I am looking at this up to 15 kilometers the air the aircraft is taking me White Knight 2. And from here to a height of may be 100 kilometers the rocket taking, I want to know what is the velocity, which must be provided by the rocket. How do I do this problem?

(Refer Slide Time: 23:14)



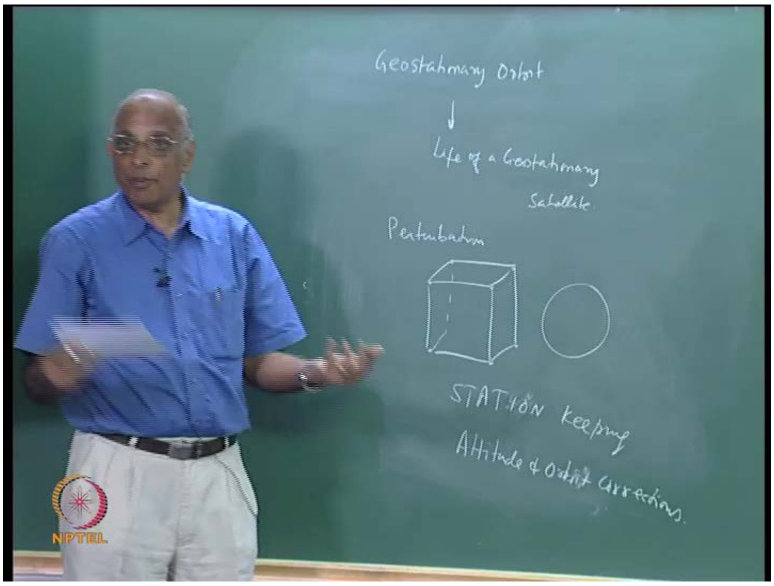
Let us calculate it. We know, we say velocity half M into the velocity half m into v square is the velocity is the net kinetic energy what we are giving. This must be equal to I start from 5 kilometers above the surface of the earth; that means, R E plus 5 kilometers, I go the radius of the earth plus 100 kilometers. And what I do? I give this kinetic energy, which will give me the work done; and what is the work done GM E M by R square in a small distance d R this is the work therefore, as I go from this to this; this is equal to this.

And now how do I get this; I have to solve this equation and find out. What is the value? How do I do? I find M into M gets cancel GM E by R square gives me minus 1 over R. Therefore, I get 1 over. Let us do this V square by 2 is therefore equal to GM E, M goes out I have something like 1 over radius of the earth, we were telling its equal to we just takes this value kilometers therefore, R E plus 5 kilometers is 6383 minus 1 over 100; that is 6483. We substitute the value of G 6.670 10 to the power minus 11, the mass of the earth and I get the velocity. And this velocity will come out to be something like 1.278 kilometers per second. And once you know and what is the velocity, which must be given? I can design my rocket accordingly, and that is what I will be doing towards the end of the class (( )). Therefore, you know you find.

Now this question is had I started from the surface of the earth, which is we say 6378. I find that the difference velocity going to be very small. I really do not see any advantage in launching from an aeroplane and going up. But something which we are forgetting; when air craft goes like this, I also gives you a horizontal component namely an orbital velocity component; and that what make it advantages. You know we have some rockets and one of the rockets please take it down, it is known as Pegasus rocket. And in this rocket, what is done is you take the rocket, and the space craft in an air craft to height of something like 10 to 15 kilometers, where atmosphere is available, you launch the rocket and that way the rocket need not be very powerful, but it can be a small rocket can to do job.

Therefore, we call these things as air launch. We will do some more examples and we will be very clear about it. Therefore, it ne rocket need not always launch from the ground, it can also be launch from under the sea, we have sea launch. We have something known as the Polaris Missile, which is launch from the sub marine may be from under the water, it comes up and it can go. Therefore, wherever we want. All what we want is, we need the value of this to this and we can find out the velocity, what is required; this is all what we learnt in orbital velocity.

(Refer Slide Time: 27:07)



May be I will take a next example which is very illustrated. We know it is told you that we talk in terms of geostationary orbits right. And you know nowadays, we find many countries or wanting to launch space craft into geostationary orbit, because this is very use full. I just go and may be point it towards Nagpur, which is the centre of the India; India gets cover whenever I want TV just switch it on, I beam the program to the satellite and it beam it throughout the country as it where. Therefore, is there is there any problem? Is there something like life of a satellite?

Let say INSAT, some INSAT have been launched as our years may be in the 1980s and all where. Is there any problem? Can I keep on going continuously or is there a life? And if there is a life to a satellite, why should it have a life, because electronics can continue to function for 100 of years therefore why should there is be a life? What is your thinking? Any any any thinking on this, because people keeps telling this satellite is launched only there is the life time of 15 years, some say it has 20 years, some say it is only 5 years.

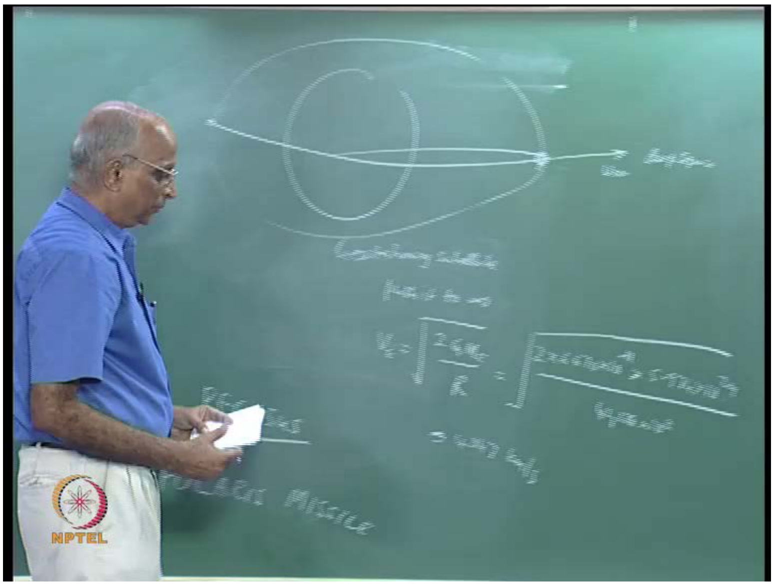
What decides the life time of the satellite, because I keep telling everything is freely falling; everything is vacuum, everything is going around if there is so what, why should there be something like a life of a geostationary satellite or space craft? Why should it be there that you are telling may be the satellite, may de deviating from its part? Why should it deviate? You are right, but why it should it deviate is the question.

(( ))

There are many other forces like for instance, you have the sun, your solar flats the gravity of the sun is changing, may be we have the moon somewhere, we are already something like at a height of something like 43000 kilometers; there is the moons attraction that also variable. Therefore, it is quite possible that there are perturbations are changes, and how do you take care of these perturbations? We have to supply some you will have something like if you have inside spacecraft, something like a box let us say, you have something like 16 rocket put at all the corners. And whenever you find something is changing have to fire these rocket and make it be make it point; that means I have to do something like what we call as station keeping have to keep on. Make sure it as always pointed here; if a drift I have to corrected; that means, the attitude may change attitude means, I am talking of the position, the orbit the orbit can change. Therefore, I have something like for attitude and orbit, some corrections are required.

Therefore, for all these things, I fire a rocket and therefore, I have to take a fuel tank and I use it. And once my fuel is over, the life of the fuel is over; and that is the problem with this spacecraft; and that is why we keep talking in terms of the exotic propulsion like a electrical propulsion, which may not have so much requirement of a fuel, which can be there for much longer time and therefore, we will cover this things has been go alone.

(Refer Slide Time: 30:48)



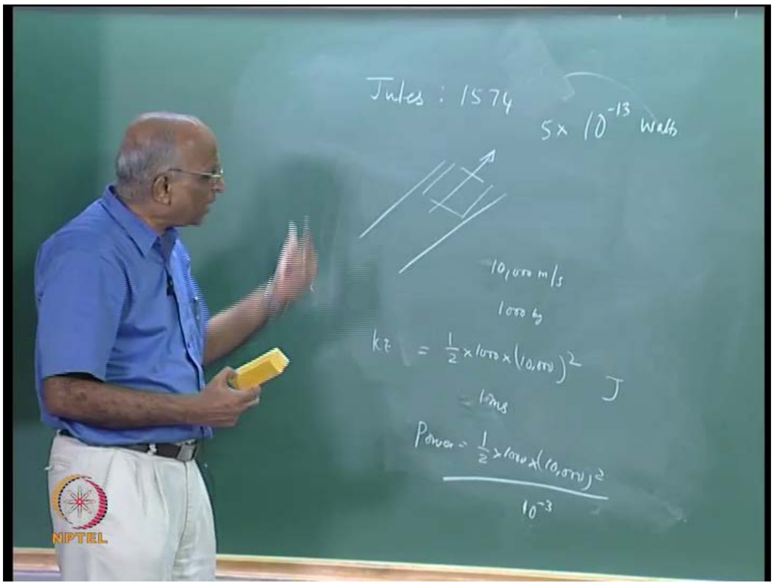
But the point trying to say is supposing we have something we have geostationary orbit. And we told are ourselves the geostationary orbit is over here at large height something like 43000 from the surface. And let us say some of the spacecrafts, some of the geostationary space craft is the life period is over. If it is going to be there, may be the next spacecraft may collide with this; that means, I have the old non-functional spacecraft and therefore, it may colloid and it may be hazard for me; that means, there is the problem even in space even though the space is so much I keep sending another satellite it may collide with this and therefore, my new spacecraft may not function.

Therefore, it is necessary for me to be once the life time is over to push it out with escape velocity such that it goes into deep space and I have no such problems. How I do it? In other words, before the INSAT satellite the life is over, I should make sure that with the remaining fuel, I must push it out such that this orbit which is geostationary orbit is available for me. Therefore, let us do that problem. I deliberately choose this problem. All what I am saying is I have a geostationary satellite; and now I want to push it out of the orbit; that means, I have to push it to infinity that means, it goes up.

So therefore, I am really bother; therefore, what is the velocity required to push a space craft out of the geostationary orbit into deep space that means, I have to escape from the earth let as forget about the pull of the moon, other planets and all that let as assume only earth is attracting. Therefore, I want to find out what is the escape velocity? Escape velocity is equal to 2 GM E by R. What is the R now? I want to escape from this orbit, and what is that R? R is equal to what we said was something like 42000 kilometer. I put the value of R, I put the value of G and M E.

And I find that I still required to push it out something like 4.347 kilometer per second and how let us puts the numbers here is equal 2 into 6.670 10 to the power minus 11 into mass of the earth 5.974 10 to the power 24 divided by you have R which equal to 42178 into 10 to the power 3 meters and this comes out to the 4 points. That means, you know even when I have a geostationary satellite, I must keep some fuels reserve such that with that fuel. I will be able to push it out; and these are all mandatory. Well, this is all about orbits I thing be have covered it in in in some extent; and why do we need rockets? We know be leg in go back and ask ourselves, what did the others do.

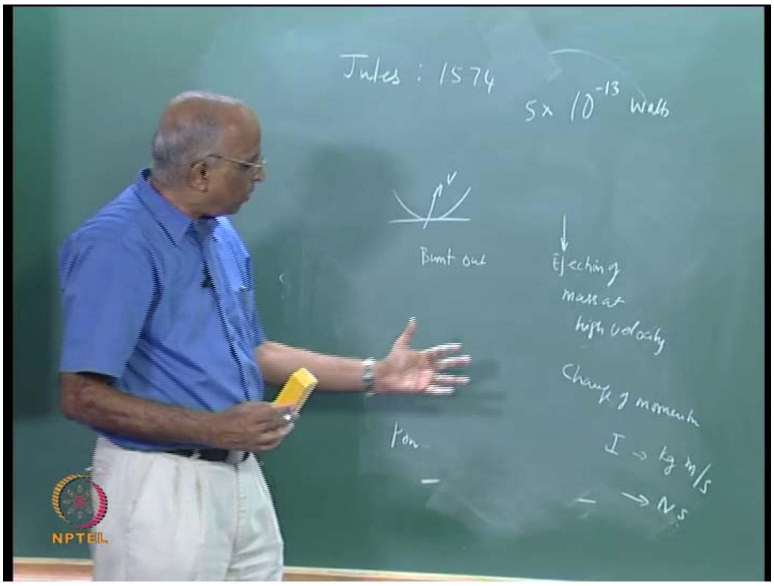
(Refer Slide Time: 34:11)



We told ourselves, well Joules 1 in his book from earth to moon, what is said as you have cannon; in the cannon, you put something like a spacecraft you push it out with extremely high velocities in such that it gets into the orbit. What is the type of typical orbital velocities for geostationary orbit? It is around 13 kilometers per second or let us says we say may be I need an orbital velocity or the total velocity required is around let say 10 kilometers per second at is 10000 meters per second. Supposing the mass of my body, which I want to orbit is around we say 1000 kilogram, because 1000 kilogram is something, where in I can put some equipments may be one or two people can be there; we have something for life support and all that 1000 kilogram seems reasonable.

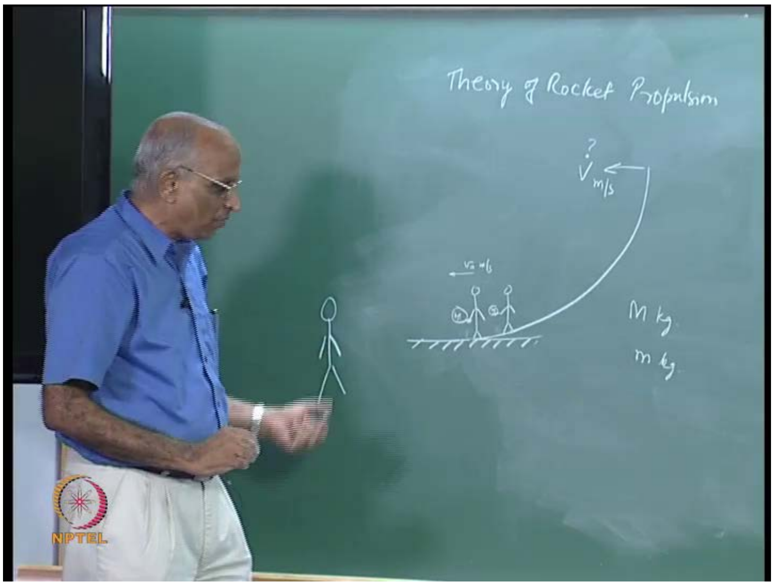
Therefore, what is the energy I must have? I talk energy into 1000 kilogram kinetic energy 10000 square and so much Joules is the energy, what I have to give may be kinetic energy, I have to give. And what is this value? Supposing, I have to launch it instant any as the let say I launch it in something like 0.1 milli second or let us say 1 milli second, because it has to go fast. Therefore, the power required is equal to half into 1000 into 10,000 square divided by 10 to the power minus 3. And what is the number we have to talking of? We talking of 500 5 into 10 to the power 5 10 to the power 5 and I have we are talking of four 0s 10 to the power 8. We are talking of the huge numbers something like something like 10 to the power let say 8 7 8 8 plus 3 11 11 plus 3 14; that means, something like 10 to the power 13 5 into 10 to the power 13. We are talking of a number of this order of so much watts. If you take the entire electricity which is generated in a super thermal power plant, it is very much lower than this; therefore, we cannot use a launch this.

(Refer Slide Time: 36:40)



And if we take a look at what we were telling you know somebody imagine may be a ship is sailing on the sea; there is the joint storm and you get a high velocity. Even if by chance, I get a high velocity, when the high when the body with the high velocity is traveling through atmosphere, it will get burnt out, because you have frictional resistance of the air; therefore, we need some other type of object; and when we say rocket propulsion all what we mean is, you have continuous ejection of mass at high velocities. What it does is, it provides as some momentum or rather some change of momentum, and what do we mean by change of momentum? We call it as impulse I, what is the unit of a momentum; momentum is kilogram meter per second. Therefore, impulse has the same unit kilogram meter per second, but Newton is equal to kilogram meter per second square; therefore, impulse also has units of Newton second. Please be careful about units. Therefore, all what we are saying is, we must gives some impulse to the body by depleting the body of its mass starts slowly keeps rising out; and that is what is the theory of rocket propulsion; and that what I will consider now, maybe we will take an example, we will illustrate; and in the next class, we will go ahead.

(Refer Slide Time: 38:07)



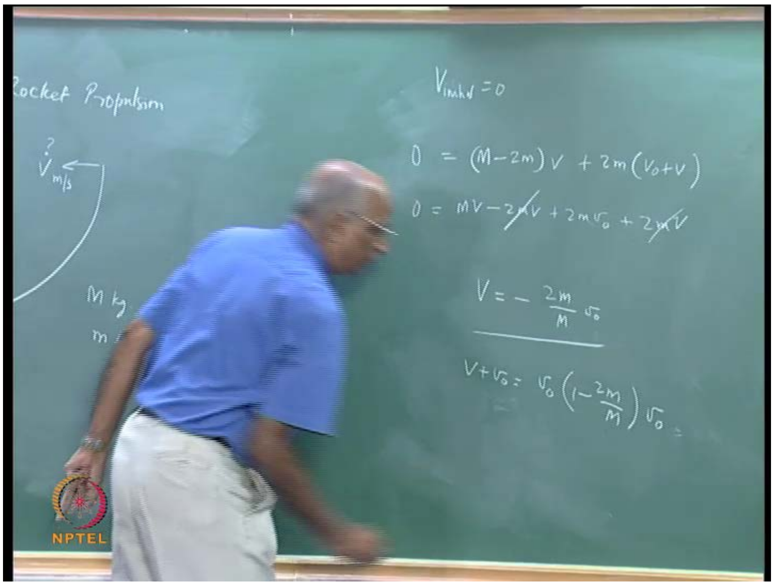
This brings us to the theory of rocket propulsion. All what are telling is I must give some momentum to the body; and how do I give momentum? I throw some mass out of the body. Therefore, let us take an example; we will start with this example; it is a very fascinating example. And what I do is, I borrow this example from my teacher; what he is do? He is to teach the mechanic students and one of the problems used to give is, supposing we have something like a sled; what is the sled? Sled is something you know which may be you go on a mountain its slope and all that you attach yourself and you go down the slope that we called as a sled. And in this sled let us say, there is we are in a place where there is no attraction due to gravity; no external forces.

On this sled may be we have two-boys for standing in this sled; the sled is stationary and these two boys want to move the sled; the sled is on a ground let say the ground is so slippery that there is no friction, let us say there is no gravitational force and all, we idealize this situation. This sled is stationary, these two boys find you know it is all ice here; that means, there is no friction; they do not want to get out; they want to move this sled. Therefore, they say let us provide some impulse or let us provide some momentum.

Therefore, we take an example where each of the boys has mass M of stone, they both carrier stone of mass M over here. Let the mass of the stones, the two boys and the sled be capital M kg; lets the mass of each stone we told small m kg. Now, the boys want to move, how do they move? They say let me throw, let both of us throw this mass out at a velocity V 0, so much meters per second. Therefore, both the boys simultaneously through this mass with a velocity V 0 meters per second.

Now, I want to find out whether this sled will move or not, how do I solve? I go back to my inertial frame of reference. What I do is stand over here; and I am in the inertial frame of reference, because I am moving at constant velocity and therefore, I describe the motion of the body. I am watching these things happen. Now, I want to know at what will be the velocity at which the sled moves? The two boys through the stone in this direction with the velocity V 0; let me assume that the sled also move in this same direction at a velocity V meters per second. I want to determine the value of V. I am looking at it from the inertial frame of reference and therefore, what will be my equation?

(Refer Slide Time: 41:19)

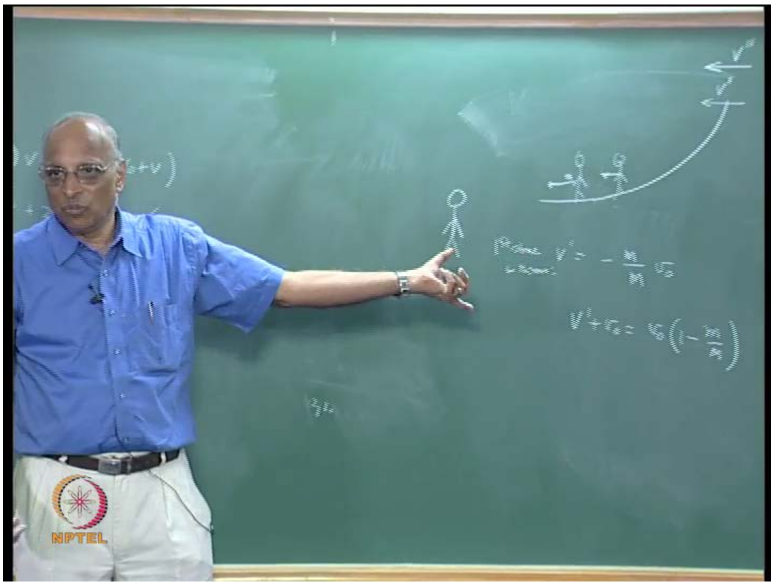


The initial momentum of the sled, the boys the stone put together, they are all at rest V is equal to 0 initially. And therefore, the value of velocity initial velocity 0; therefore, the initial momentum is 0 right. What is the final momentum? The momentum is conserved in the inertial frame of reference. Therefore, what is the final momentum? Let us calculate it. Now the final mass of the sled is M 2m of the mass of the stone has left; therefore, the final mass of the sled is equal to M minus 2m. Let us assume the sled has the velocity capital V anything else, the stones are leaving the sled. What is the velocity of the stone? I am watching it from the inertial frame of reference. The sled is moving with the velocity V, stones are moving with the velocity 0 and therefore, I get 2 m into V 0 plus V in the inertial frame of reference. Momentum is conserved; initially it 0 final must be equal to this, because I am talking of the inertial frame of reference. And therefore, this frame of reference is very important in any mechanics problems the let lets us be very clear. I want to solve this.

Therefore, I say initial momentum is 0 is equal to M V minus 2m V plus 2m into V 0 plus 2m into V this and this gets cancel. I get V 0 is equal to or I want to find out the final velocity of this sled and now (( )) capital V is equal to what is the value I get? V is equal to minus yes what is the value 2m divided by M into the velocity with which the stones are throwing. What is this equation tell us? If the stones are thrown out in this direction, the velocity will be in the opposite direction.

Therefore, just the action of these two boys throwing the stone, they are able to move this sled at this velocity right. Now, I ask the second question. Yes, I have this, what is the relative velocity? V plus V 0 that is, V plus V 0; that means, I say that the value of V plus V 0 velocity is equal to now I say V plus V 0. Therefore, I get V 0 outside into 1 minus 2m by M into V 0. Sorry I have already written this equation that is the relative velocity of the stone with respect to the motion of the sled. Now, let us spend some time on this; we must understand it. And you know, I will just extrapolate this and get you the rocket equation of the theory of rocket propulsion, does it make sense? Let us lets lets go forward, we will ask ourselves…

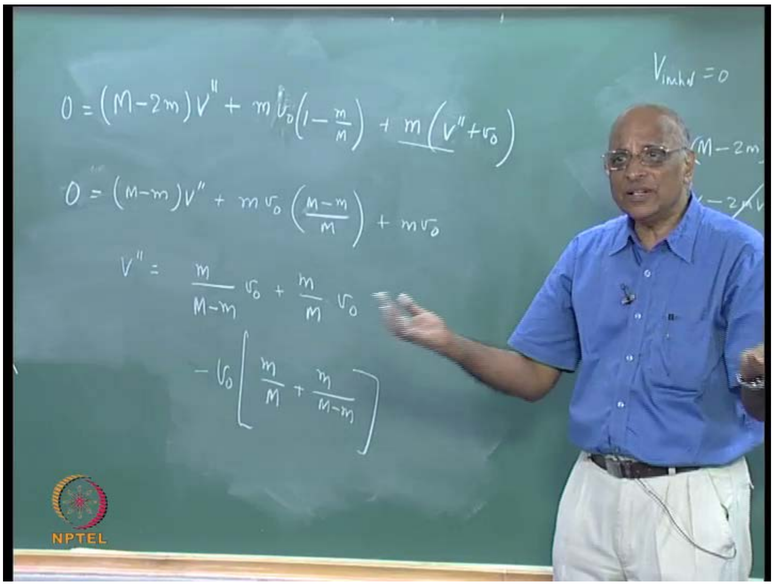
(Refer Slide Time: 44:38)



The next question we ask ourselves is well, these two boys you know they have wise, they know precisely what they want; they want the sled move faster. Therefore, they say yes I have this, this boy first throws the stone and the same stone of mass M, then after some time the second boy now throws the second stone. In fact, they do not throw the stones simultaneously that is 2m mass of stone thrown together, but rather one stone after the other. If the stones are thrown one after the other if the stones are thrown one after the other, what will be the final velocity of the sled I call it as V prime. What should be the value? Let us write the equation of motion again.

Again I am in the inertial frame of reference. I stand over here; watch the fun in the inertial frame of reference. I find that the first boy throws the stone and therefore, the value of V prime that is after the first thrown, first stone is thrown, call it as V prime, and that will be equal to minus only one stone has been thrown, it is equal to capital M into v 0 right because only one stone is thrown. And what is the relative velocity? The relative velocity is equal to V plus V 0 that is V prime plus v 0 is equal to v 0 into 1 minus small m by capital M. At this point in time the second boy throws the stone and therefore, what is it he is getting? Now he is getting, when the first stone is thrown you have been prime, when the after the second thrown is stone, you get V 2 prime. Let us balance the momentum I mean, the inertial frame of reference; I want to balance the momentum change.

(Refer Slide Time: 46:37)



And therefore, what is my momentum equation? Initially they are a trust when the first stone is thrown, you have the velocity is given by the initial velocity is the final velocity is again the same thing M minus 2m into V 2 prime. The first stone goes with a velocity of relative velocity over here, which is equal to M into V 0 into 1 minus M over M. And the second stone goes with a velocity M into the velocity is equal to V 2 prime plus V 0 relative velocity; final is V 2 prime, the velocity of the stone; if it this because my final equation. If I solve this, what do I get? Let let us solve this, I want to find out what is the final velocity? I find I have m V 2 prime 2m V 2 prime m V 2 prime I take it inside I get is equal to or I get 0 is equal to M minus m in to V 2 prime plus I have m v 0 into 1 minus M over m m v 0. I keep it over there 1 minus or I get m minus m divided by capital M and I have M v 0.

What do I get, what do I solve? I want to solve for V 2 prime is equal to now I get m over capital M minus small m into v 0 plus I have m divided by m into v 0 or rather I get v 0 into small m by capital M plus I get small m by m minus m is it alright. Now, let let us takes a look when one stone is thrown after the other, I get this velocity when both the stones are thrown together, we had something like 2m over m over v 0; what is which is more which is again, mind you here also I should had the negative sign, because it is opposite.

And therefore, what is the comparison here? We have m over m plus m over m here; I have m over m plus m over m minus m this is smaller. Therefore, this will be greater; therefore, the art of throwing one stone after the other gives me a higher velocity to this. Now I can generalize, instead of having two stone I keep on thrown one stone after the other, what is going to happen? I will get the velocity, which is much better than the spontaneous throwing of all these stones together. And this is the basis of the rocket propulsion.

What we do is, I have a rocket, I put let say lot of stones in it. I keep throwing one stone after the other, keep on ejecting mass, and that is how I get a final velocity, I will continue with this; in the next class, we will derive Tsiolkovsky's equation, which is known as the rocket equation just following this analog. I will again extend this to more number of stones and all this. But this is basically the principle; and if this principle is clear, we must be able to design new forms of rocket, and that is why I covered this well thank you then.