Rocket Propulsion

Prof. K Ramamurthi
Department of Mechanical Engineering
Indian Institute of Technology, Madras

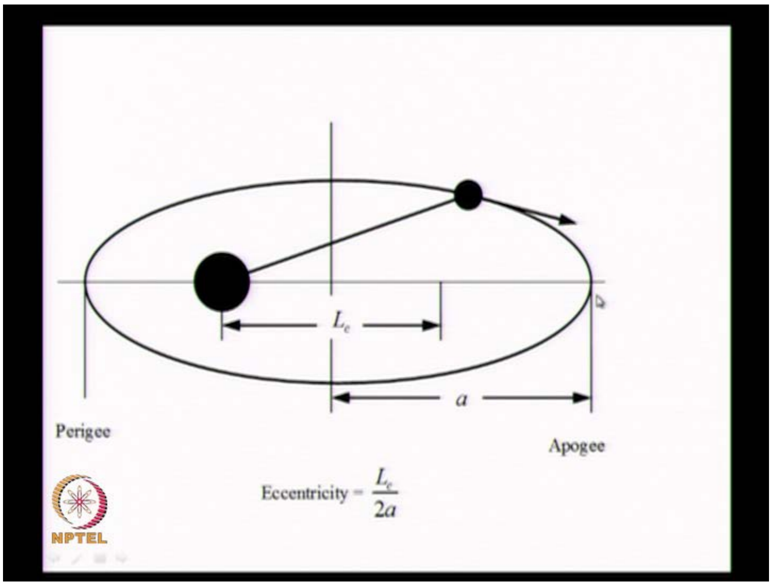
Lecture No. 06

Rocket Equation and Staging of Rockets

Well good morning. I think in today's class we continue with where we were yesterday namely the theory of rocket propulsion. You know, in order to set space and just make sure we are on the right track; I will go through one or two slides which will illustrate where we were yesterday and we will also illustrate that not only rockets do something in space wherein we eject momentum in nature itself. We find lot of lot of things which make use of the same principle.

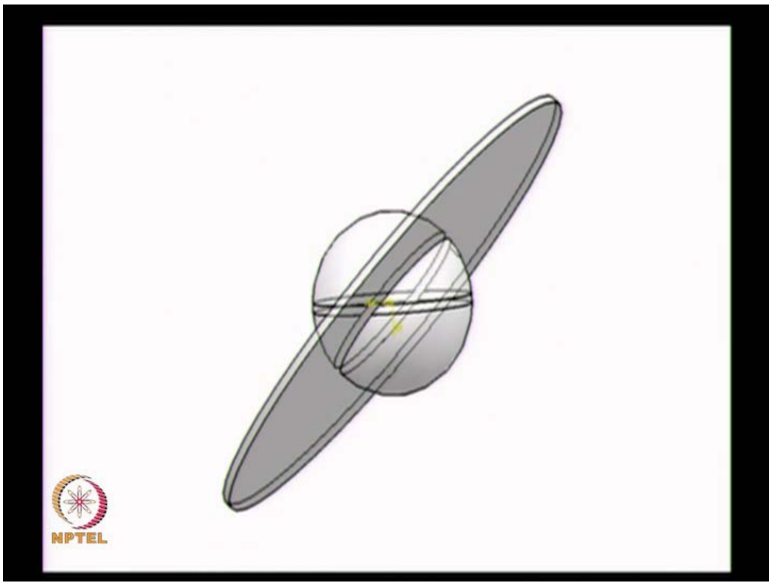
So, having said that; let us briefly go through some some of the things. You know we we told the ourselves that rockets are used to launch satellites may be objects in space. We talked in terms of circular orbits, we talked in terms of geosynchronous orbits, polar orbits, sun synchronous orbits and retrograde orbits, different orbits.

(Refer Slide Time: 01:18)



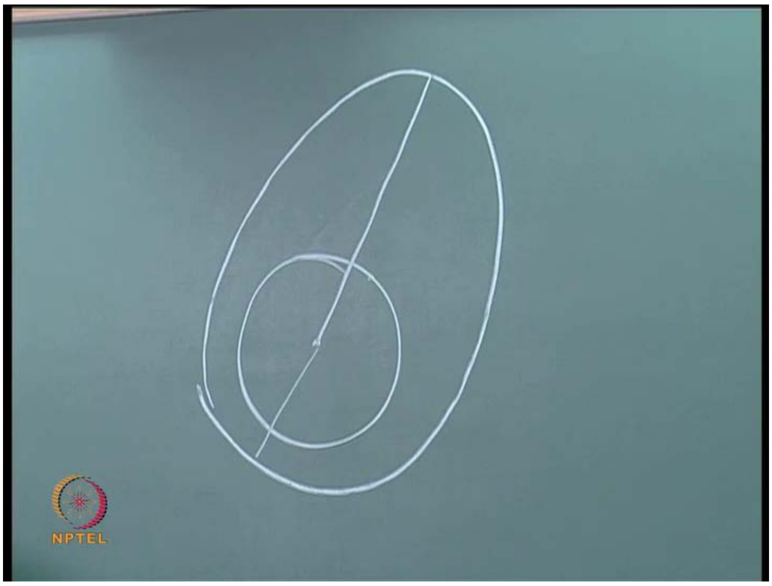
We also talked in terms of elliptical orbits and we said elliptical orbit is one wherein you have two forces. This is let us say the earth and this is the spacecraft going round. We defined something known as eccentricity which was the distance between the two force and the major axis which is 2a over here.

(Refer Slide Time: 01:38)



We also said for an elliptical orbit you could have an inclination, even a circular orbit could have an inclination and the inclination is between the equatorial plane and the plane of the orbit. I want it to make it clear because we define one orbit known as a Molniya orbit and we told ourselves in the northern hemisphere country like Russia or something wherein the satellite has to stay in the orbit above the northern hemisphere for a longer time; we have highly elliptical orbits wherein the apogee is at a distance of something like almost like 60000 to 70000 kilometers and the perigee is quite small of the order of 6000 kilometers.

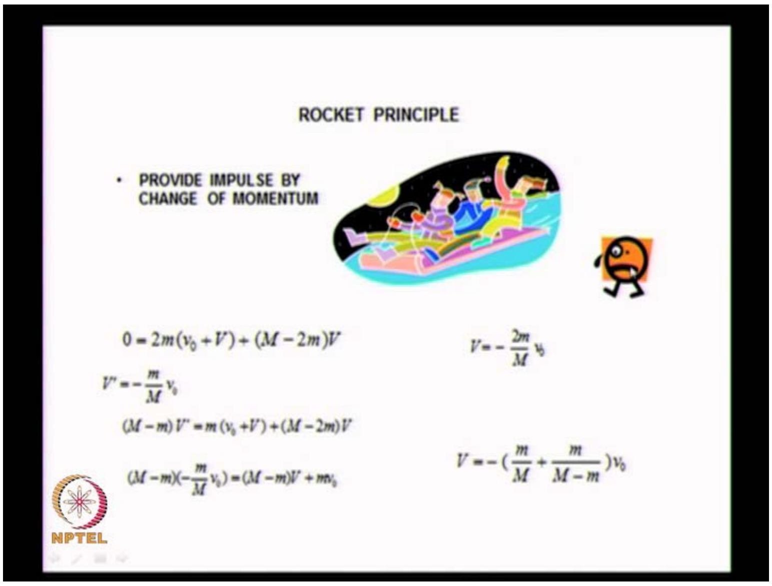
(Refer Slide Time: 02:23)



With the result what we told ourselves? yes I have the earth over here this is the northern hemisphere, it is in the northern hemisphere for a very long time. Something like 11 hours it is in this period and only one hour here so that the country in the north can view the satellite for a longer time. This Molniya orbit is particularly important because it is something which was developed by in Russia and we told ourselves this is the apogee which is of the order of 6 70000 kilometers. We told ourselves this is perigee and the inclination of this orbit we told ourselves is something like 63.4 degrees.

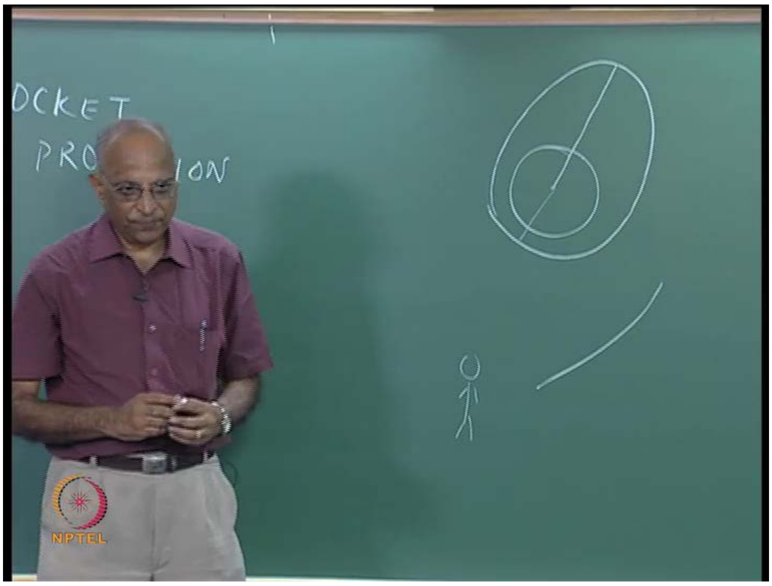
Having said that let us goes forward. We we also talked in terms of the rocket principle and we told ourselves if we when we had this sled and we had these two boys who were throwing stones together.

(Refer Slide Time: 03:11)



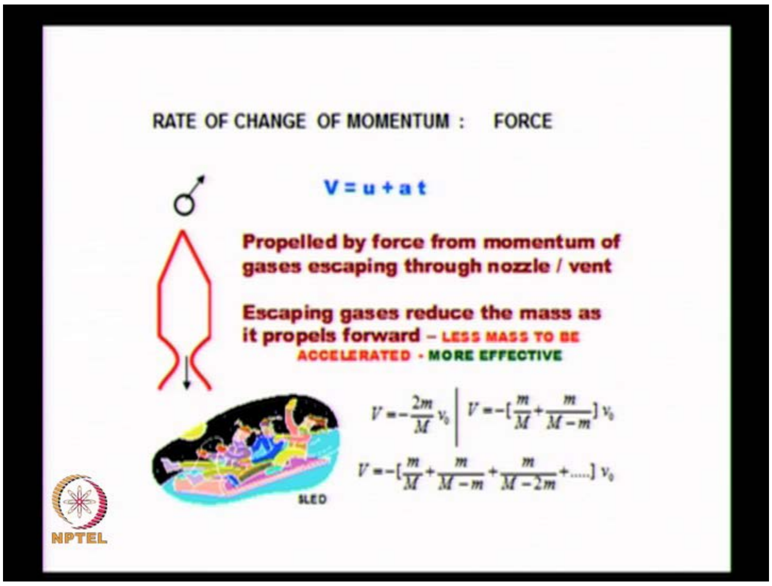
We got the net velocity of the sled was something like the mass of the stones thrown divided by the total mass into the velocity with which the stones were thrown. The important thing here is when we are looking at the sled which is moving, we are talking with the inertial frame in mind. I look at the sled. I am standing here namely there was the sled here, there was this boy.

(Refer Slide Time: 03:33)



You know, I am observing it from the inertial frame of reference and writing the momentum conservation equation.

(Refer Slide Time: 03:44)



When the two stones are thrown independent of each other; that means one after the other; we got a higher value of velocity because we got in the denominator 2m by M was what we got when the stones were thrown simultaneously. Now, we got m by M plus m over M minus m since M minus m capital M minus small m is less than small m I get and increased velocity and so if I have a series of stones thrown one after the other; well I can get a higher velocity than both the stones thrown together. The question is why is it? Because when the net mass is decreasing and I throw the same and I throw one stone, I accelerate a lesser mass and I get a better one. Therefore, we find that when when I throw one after the other, I do not have something like a stone being thrown in which case I have the net velocity is equal to u plus a t; but, since I am accelerating or I am I am I am sort of forcing a mass which is lesser in one, I get much better velocity. This is what we said.

(Refer Slide Time: 04:52)

Having said that let us go back and look at some examples. I chose some examples you know this this is from the national geographical magazine. What is shown here is something known as a squid. This is a is known as a squid. Squid is something something like a large fish something like 10 meters long is what it is it is found of the coast of Japan may be new Zealand. It periodically visits these coastal areas and it is an endangered endangered specious and it propels something like the rocket principle itself. What what it does is let let us look at the parts of this particular squid what is available here. You know this is something known as a man fluid, there is something like a funnel here.

(Refer Slide Time: 05:39)

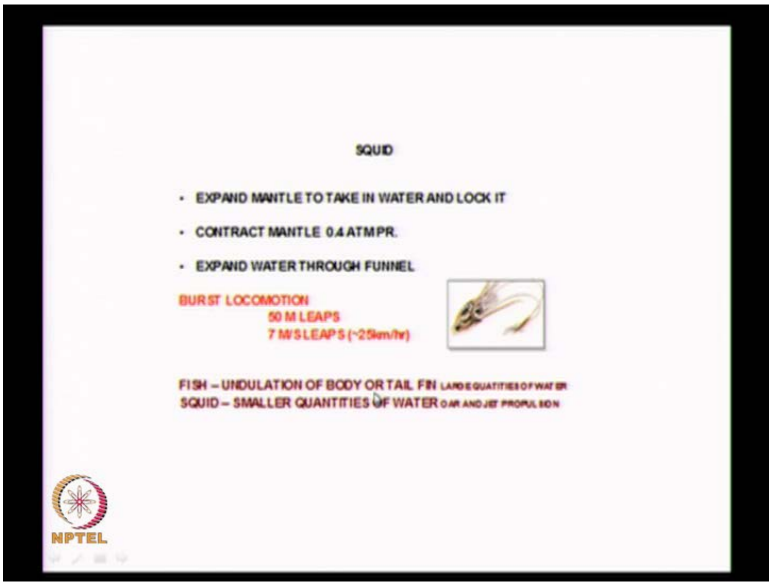
And what it does is let us go to the next one. See, this is a squid which is caught, which was which was picked up from of the coast of new Zealand and you see the length compared to this man it is something like 10 meters long.

(Refer Slide Time: 05:50)

Let us go to the principle of it. You know there is something like a funnel here. You know what what it does to move itself is; it opens its naught or it open this part of the funnel here, may be gulps in water. As it gulps in water it gulps in some sand may be eggs may be small fish whatever it is gulps in here. Then it closes the funnel over here it closes its gate or the mouth as it were and then it through its muscles it contracts itself such that it builds some pressure of water within this mouth what what available here.

And then when it wants to move, it just opens the gate again and spouts out the water in this direction and when it spouts out the water in this direction it moves in this direction. It keeps on scouting the water and it propels itself. Therefore, what it does it moves by shooting water out from its funnel and whenever it wants to move; it expels water, waste eggs and all that and that is how it moves. It is very similar to a rocket thing. It collects water, pressurizes the water, releases the water gradually and it moves out.

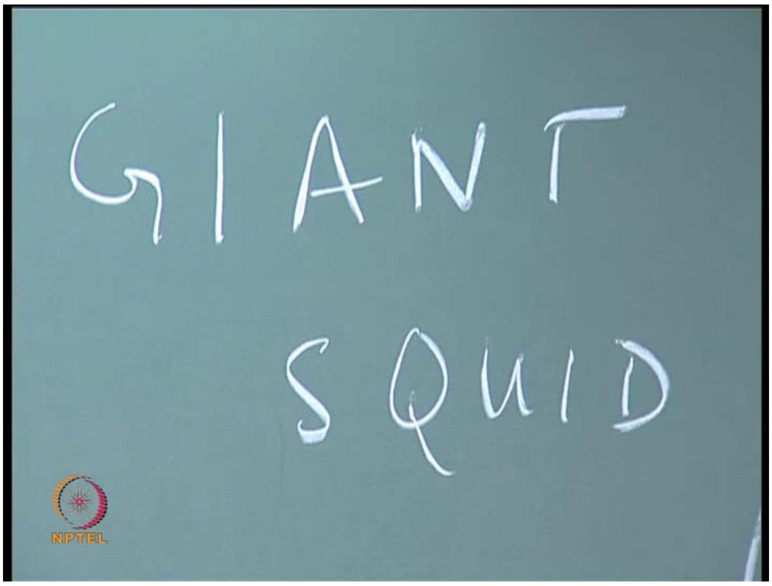
(Refer Slide Time: 07:05)



And the type of velocity what it gets if you really see is; it expands, it pressurizes to around 0.4 times the atmosphere and then in one in one scot once when it pushes out it is able to leap something like a 50 meters and gets a velocity of something like 2.5 kilometers per hour. It is quite phenomenal you know that is that is something in the water which is viscous itself. It is able to go at that speed because of the of the gradually or the release of water pressurized water from its mouth.

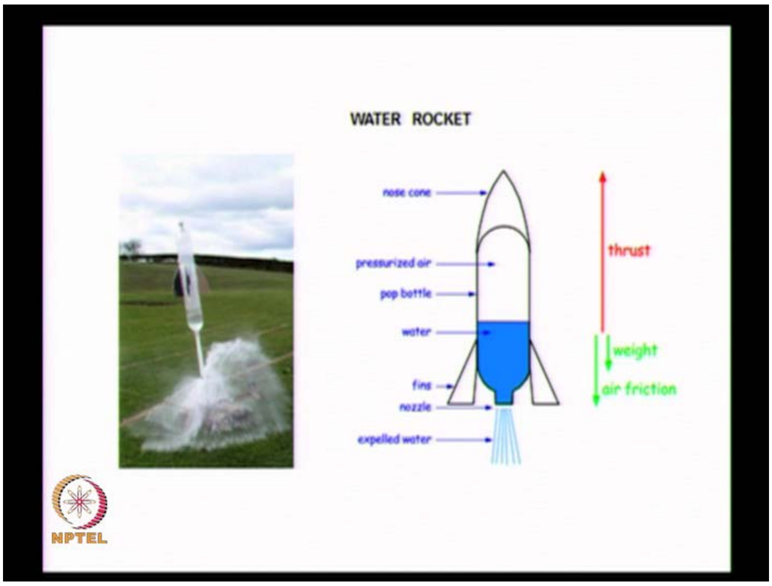
Now, if you really see this is where I show the funnel again. It opens its mouth, water enters, it contracts itself and then it opens it, closes the gate, compresses it, releases the pressure, then the then the water and whatever is available you may get it is scouted out and it moves.

(Refer Slide Time: 08:01)



And this is the principle of what we call as the GIANT SQUID. In fact in US we had projects known as squid project. Squid project was after this because that is again a rocket project and it was named after this particular a creature which lives in the sea. In what way is it different from from a fish?

(Refer Slide Time: 08:24)

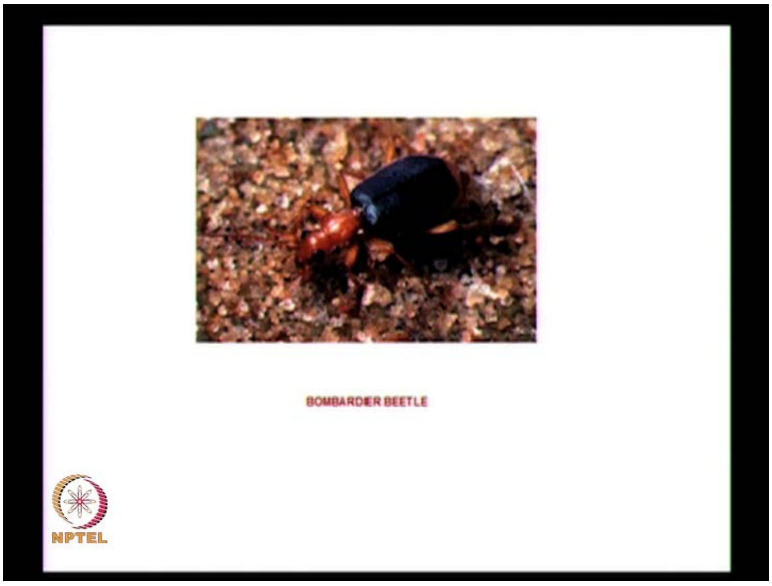


Let say you know a fish what it does is; it has fins it displaces the water. That means it slowly displaces the water like I go on a boat let us say. I am sitting in the boat I have a ore. With the ore I displace water. I again I i I displace some water but, the mass of water displaced is large but, the velocity I give is small.

In these cases what it what does the squid do? It takes the amount of water is small but, it pushes it at high velocity and its able to do a much better job. I think this is all what I wanted to tell and I just illustrate this you know if I have a water rocket; if I want to make one, all what I do is I take one of these these may be bottles like this, may be I invert it, I put at constraint here, I pressurize it with gas over here and then when I pressurize it water scouts out and this can go up and this is precisely the principle by which may be the the squid propels itself and this is where I show may be a rocket here, may be water is here high pressure gas here water getting scouted out and the rocket moving up.

The principle of any rocket is exactly the same. Only thing is that you need higher velocities. Therefore, you put more enthalpy into the into the system which expands over here. The thing which is expands has higher enthalpy. Therefore, I get higher velocity may be you need something which will increase the velocity still further and that is what we are going to learn in this particular course.

(Refer Slide Time: 10:02)



Having said that you know, I came across some other example. You know I was I was teaching at IISC a course on propulsion and at that time, one of the teachers there told me that there is one one insect which is available in nature which uses the rocket principle. I tried to get some information on this and this is what is known as a bombardier beetle. You know beetle you know in Tamil we say (( )) or Malayalam it is known as (( )) and all that what does it really do you know it is a harmless creature. It goes round and all that and whenever it comes we take a card board we take it and throw it out. That is all what we do.

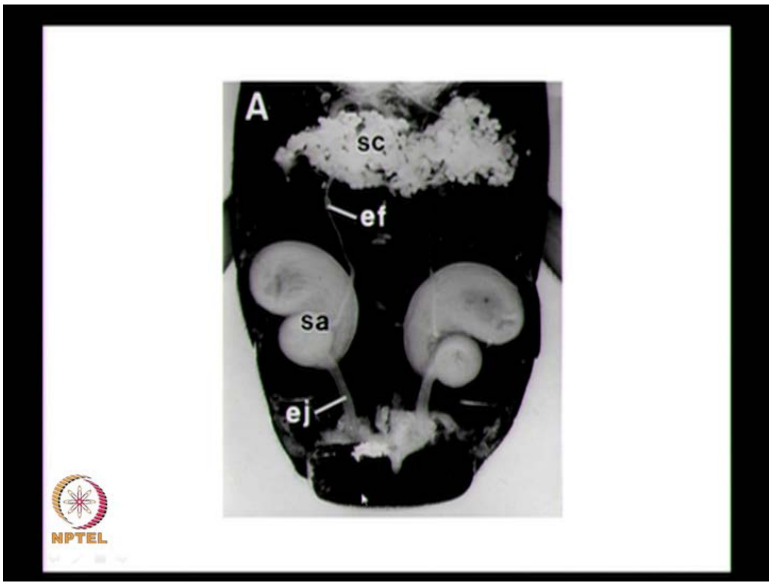
(Refer Slide Time: 10:39)

You know this bombardier beetle is one wherein you know it is a harmless creature but, you know being harmless, you know these ants and all go and sting sting its legs therefore, its always in a in a state of alertness but, nature has given its some features.

(Refer Slide Time: 10:56)

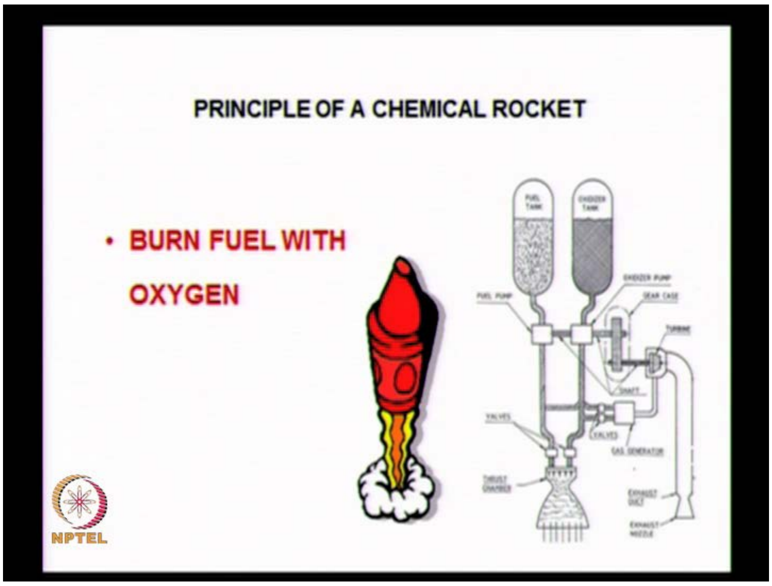
Let let us see what exactly it is. You know you know it has something like two stomachs, after two stomach there is a another receptacle here, a third stomach and in one of these stomachs what it generates is its secretes hydrogen peroxide. In the second stomach it secretes something known as hydroquinone which is a fuel. Hydrogen peroxide is a oxidizer, hydroquinone is a fuel. The the oxidizer hydrogen peroxide and hydroquinone when it is chased or when it is chased or when it is bitten by hands; it immediately scouts it into the third stomach where in both the things react generate hot gases and these hot gases it it it it sort of scouts out and when it scouts out we have a jet coming over here may be it kills its thing and it also moves forward. Let let us see the principle again.

(Refer Slide Time: 11:49)



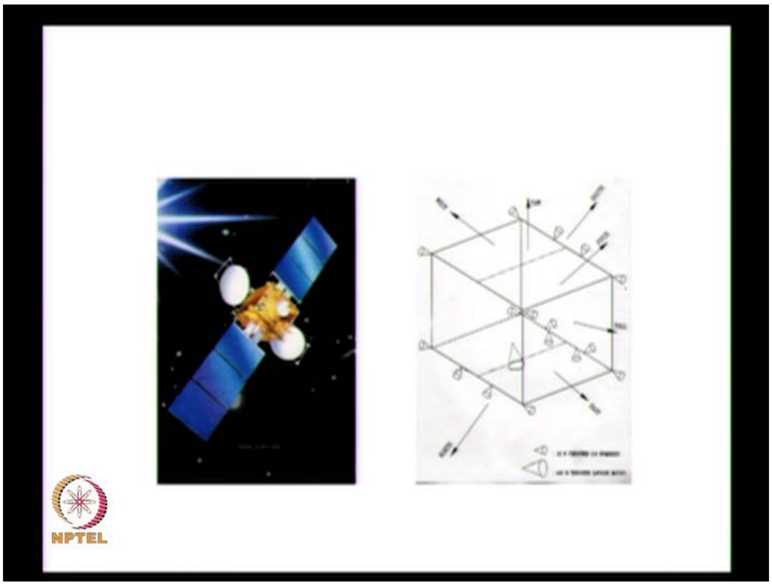
Here I show the two stomachs in one hydrogen peroxide, the other hydroquinone. They are secreted here, this has a lining of mucus which acts as a catalyst. Catalyst it promotes the reaction and whenever it wants to move it just scouts it out something very similar to what we say is a liquid propellant rocket.

(Refer Slide Time: 12:07)



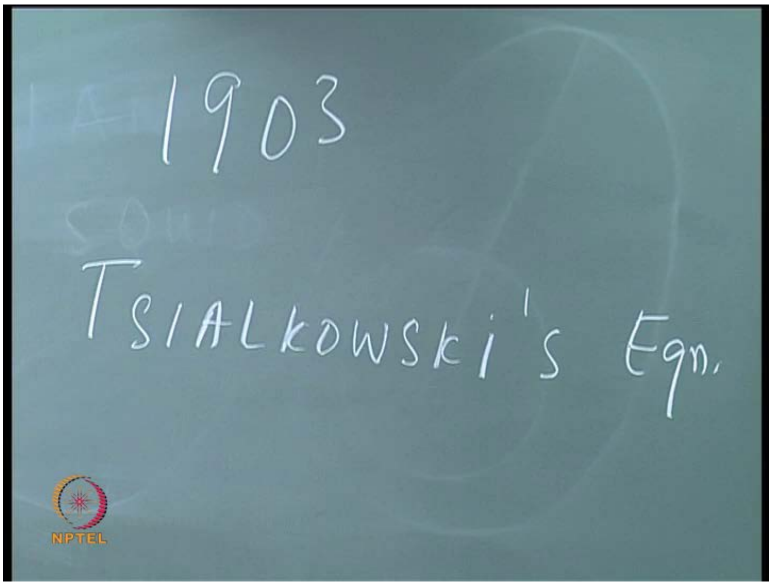
What does a liquid propellant rocket do? Let us say they are still to get into it. You have a fuel tank, you have an oxidizer tank, you pump the fuel and oxidizer into it, you ignite it and you push the gases out. So, does this small insect which is available in nature.

(Refer Slide Time: 12:33)



I think we should look at nature to understand many of the things what we are studying. Everything is there may be we have to be more observant. Having said that let let us get into some details but, having said this you know I thought I will also illustrate because we told ourselves whenever a satellite is there in space in geostationary orbit sometimes we have to push it out and therefore, any satellite has something like like a 16 rockets which are there at the edges of this and these are used for correcting the attitude. May be station keeping of the satellite and whenever the life time of the rocket is over, we fire some of these things such that we remove it from the geostationary orbit and push it into deep space and this is an (( )) satellite.

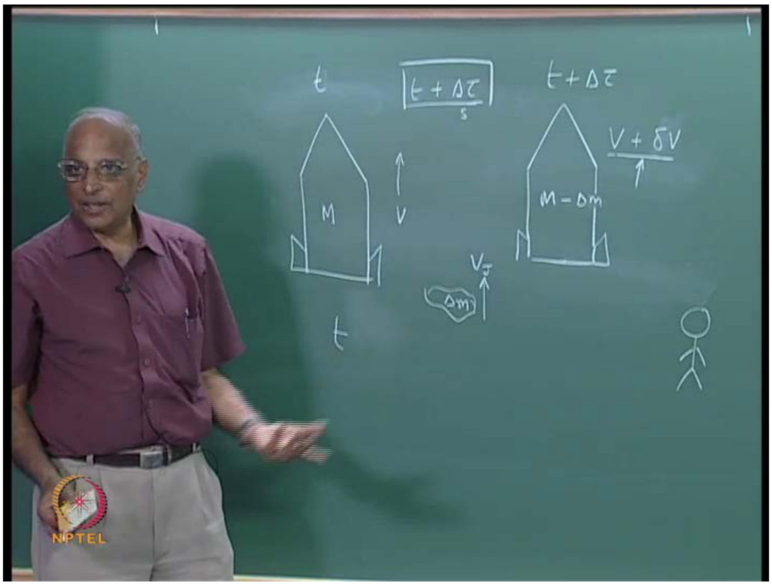
(Refer Slide Time: 13:20)



I think this is all I want to illustrate at this point of time I will come back to the slides a little later. Let us with this background ask ourselves how do I develop the theory of rocket or what we call as the rocket equation? Let let us get into this. What what do we mean by rocket equation? See it is so simple but, you will be surprised to know that the rocket equation was developed quite late, only in the year 1903 and that also by a Russian school teacher by name TSIALKOWSKI. Now, let us let us see how this and therefore, this rocket equation which describes the propulsion of a rocket is also known as TSIALKOWSKI equation.

Let lets us derive this. Let us go back. We will follow the same procedure what we adopted while finding out the velocity gain by the sled, these two boys standing on it, throwing one stone after the other. We will make some simplifying assumptions.

(Refer Slide Time: 14:31)



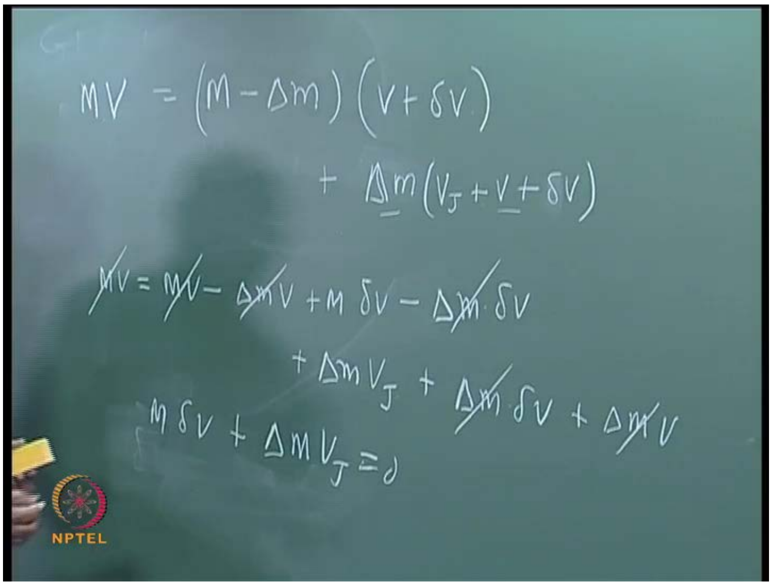
Let us assume I have a rocket for the present. We will assume rocket has a shape something like this. It need not really be the shape may be as we evolve may be you will be come out with better configurations of rockets. Let say at time t at time t it is moving with a velocity let us say V. Let its mass at time t with capital M. After a small time t plus a small time let us say delta t, delta tau a small time period delta tau seconds it is again.

Now what does it do? It is moving forward and then during this small time, a small mass as coming is coming out. It is ejecting matter therefore, a small it has lost a mass. Let us say delta m during this period and let us say it is also ejecting mass at a at the velocity. Why why should I put an arrow? We will make it even more general. It is ejecting mass at some velocity V J. I do not know whether it mass is moving upward or downward. I just presume that a velocity V J some mass has left the rocket.

And therefore, the final mass of this rocket is going to be M minus delta m over here. Let us presume that the velocity of the rocket at time t plus delta tau seconds. This is at time t this is at time t plus delta tau is V plus. Let us say del or small delta e. Let me repeat the assumptions again. A rocket is flying with a mass M at a velocity V at time t. How does it fly? It keeps on releasing mass at a velocity V J and during a small time delta tau, it releases a mass delta m with a velocity V J and therefore, during because of this the velocity increases. Because we we presume let us find out whether it really increases or not. We we think that the velocity would have increased from the original value of V to V plus delta V but, since it has lost a mass delta m in this, the final mass of the rocket is M minus delta m. This is the problem.

Now we talk in terms of inertial frame of reference therefore, I am here. I am watching the fun. I watch it. I am in the inertial frame of reference. I am looking at the velocity rocket go with a velocity V and then after a time delta tau I am looking at it going with a velocity V plus delta V. Now, I do the momentum balance in the inertial frame of reference, what is it I get? The initial momentum of the rocket is equal to what? yes It is mass into velocity is there.

(Refer Slide Time: 17:50)



Now what is the final momentum of this rocket as a I am seeing from here? I have M minus delta m into V plus dv plus you know you also find that delta m has gone out. Therefore, I have delta m and what is the velocity? How do I see it? The rocket is flying at V plus delta V plus V J over here. That is the velocity of this stone, the relative velocity V J plus V plus delta V. This is how we wrote the equation for the sled. Therefore, we were exactly in the same way and therefore, we find this is the initial momentum. In that case the sled was stationary, initial momentum was 0. In this case, this is the rocket plus whatever is left behind therefore, for this system this is the momentum equation.

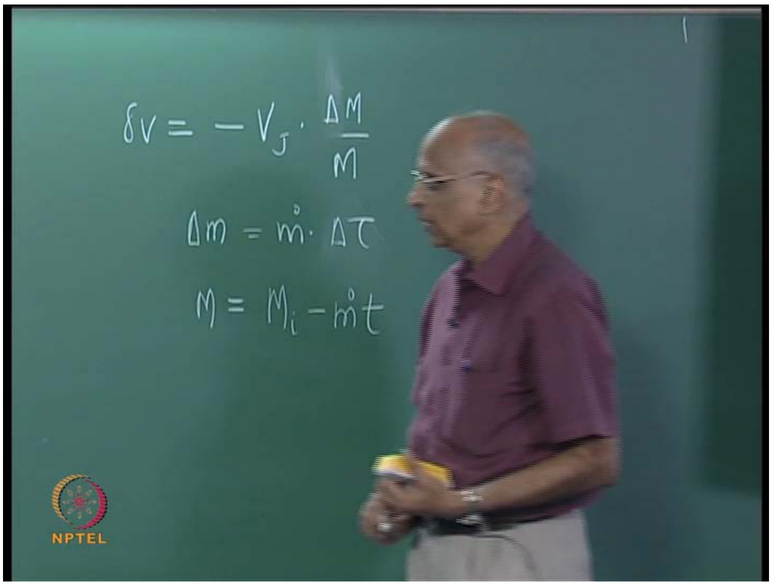
If it is, let us lets simplify it what do we get therefore, MV is equal to MV minus delta m into V. then I get plus M into delta d delta V minus delta m into delta V. Now, from this set of equations I get plus, from this term I get delta m into V J plus delta m into see I should have written here small delta that is the value which I said because we could have been independent because I want to reserve the capital delta for something else. Therefore, delta m into delta V.

And now I find that this delta m delta V and delta m delta V cancels MV MV cancels. Therefore, I am left with I can write delta V is equal to yes what will I get? Let us write it. Delta V will be equal to minus plus delta m are the signs all right? This I get delta m into V divided by yes is it all right? See what did I get from this equation? Let us write it again. We get minus delta m V plus M delta V is equal to 0 and therefore, delta V is equal to delta m into V divided by capital M.

Just see if there is any any any error in this equation. Can I go forward? Which one? Let me write. Let us derive it yeah see MV is equal to MV plus M delta V minus delta m V minus delta m into delta V plus delta m into V J plus delta m into dv. I get this and this term cancels. This one term is missing. How can I get so many terms? Let us lets see we have made some mistake somewhere. MV is equal to MV plus delta m V yes and I have M delta V I have 4 terms. I that means I have I have not I have I have what what happened to a term here? Delta m V J plus delta m plus I should have got I have forgotten this namely delta m V and this delta m V and this delta m V should have canceled. See, when I expanded this out I have delta m V J plus delta m V plus delta m delta V. So, thank you for pointing it out.

See I have 3 velocities here; I have V J, V and delta V. Therefore, I should have got delta m V J plus delta m delta V which I wrote. I will skip this portion and now I get delta M into V over here. is it therefore, what is it you are telling? Let us write it out therefore, M delta V plus delta m V J is equal to 0.

(Refer Slide Time: 23:12)

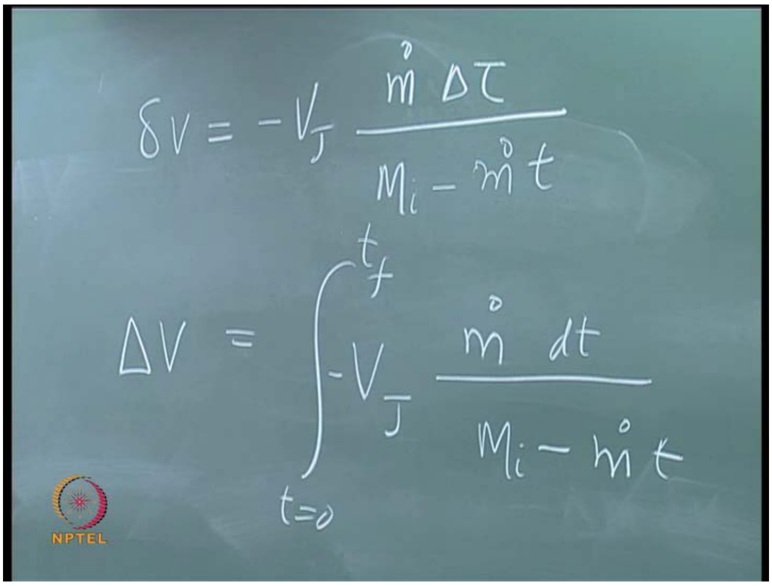


Therefore, we get delta V is equal to minus V J into delta M by M or let us say delta M if we use the capital delta M here.

Now now we want to solve this equation. How do I solve this equation? I get delta m. What is the value of delta m? Let us assume that the mass which gets exhausted from the nozzle is something you are constantly pushing out, mass at the rate let us say m dot. Therefore, the value of delta m should be equal to m dot into the small time what I said here, delta tau. You know the rate at which mass is leaving the nozzle is m dot over a small time delta tau it is equal to m dot delta tau and what should be the value of the capital M? Capital M must be equal to the initial mass of the rocket at time t is equal to 0 minus m dot. We are telling that is the mass of the rocket at time t. This is equal to m dot into t.

In other words at time t is equal to 0; the rocket had a mass initial mass it continues to eject mass at the rate m dot constant value and therefore, at time t its value M is equal to M is equal to M i minus m dot into t. Now, please please keep checking.

(Refer Slide Time: 25:13)

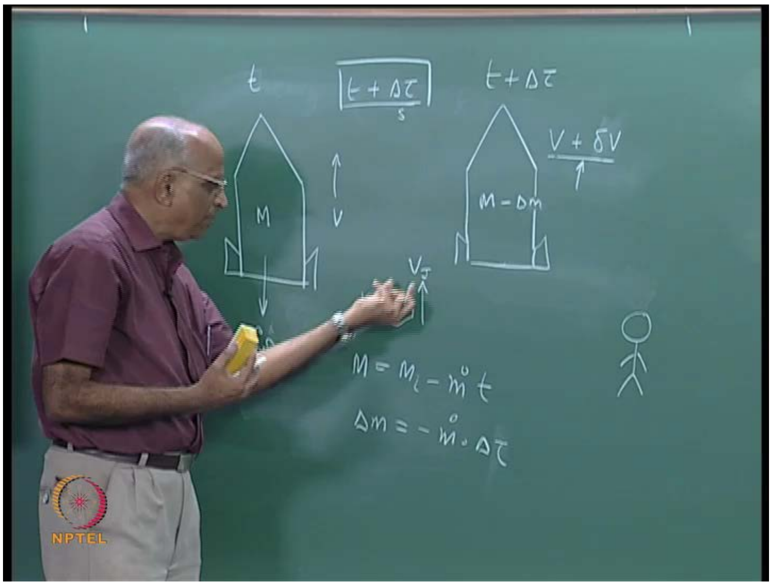


And therefore, now I can I can erase this part and now I can write the value of delta V is equal to minus V J into delta m delta m is equal to m dot into delta tau divided by M i minus m dot into t.

I want to integrate this equation and if I have to integrate this equation, I get this is equal to I get, I i I i have to integrate with respect to time, I have to integrate this equation from the time t 0 or t is equal to 0 let us say to the final time at which I get a total velocity increment of delta V is equal to V J into. Now, I get the value of m dot. I call it as dt into M i minus m dot t. Of course, I had a negative sign somewhere. But, you know one thing I have missed somewhere. What is it I have missed? I think that is very important. I think one of you must find out. I should not get this negative sign here and what is the mistake we have committed? What did I write here? I told you mass at any time is equal to M i minus m dot V.

Therefore how would I look at the value of delta m? Delta m is equal to what? I think I think this is important you know. Let me let me put that on this equation. I do not want to rub this out.

(Refer Slide Time: 27:16)



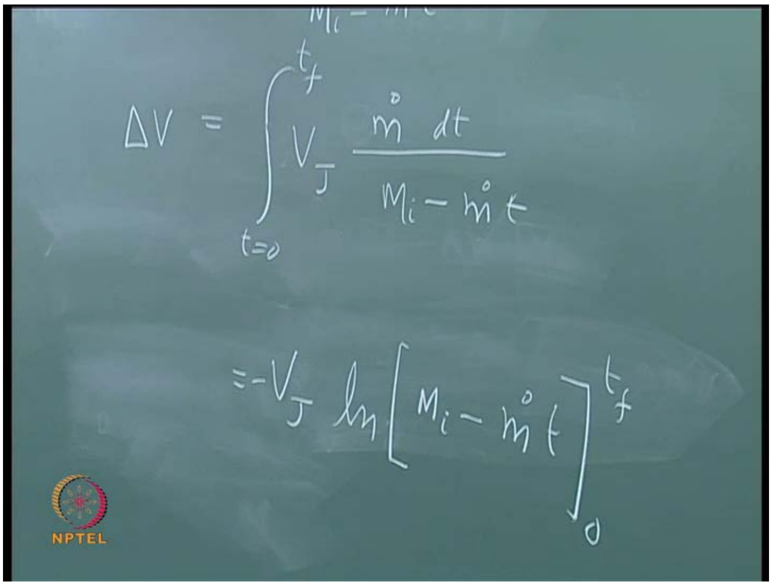
Let me again put this here. I think this is important is equal to M i minus m dot into time t. Therefore, what is the value of delta m? Is equal to over a small period delta tau what is the value? Is equal to minus m dot into delta tau. That means if I consider I call it as delta tau in a short period.

Therefore what is this minus sign? Minus sign shows that the mass has left the system and therefore, what when I when I when I substitute the value of delta m. I should have substituted minus m dot delta tau that means this should have been a negative sign and this negative sign and this negative sign would have given me a positive sign and the expression should have been plus V square. Please lets go back, check it again.

Now, this is important you know because the frame of reference we must be clear, we must know what is leaving what is gaining. Therefore, what is it we have done? We took the momentum equation and when we took the momentum equation, we balanced MV with M minus delta m into V plus delta V plus delta m into V J plus V plus delta V and then we got an expression which gave us delta V was equal to minus V J into delta M divided by m and we find delta m is equal to minus of this particular expression and therefore, we got it as V J m dot del d tau into M i minus m dot V.

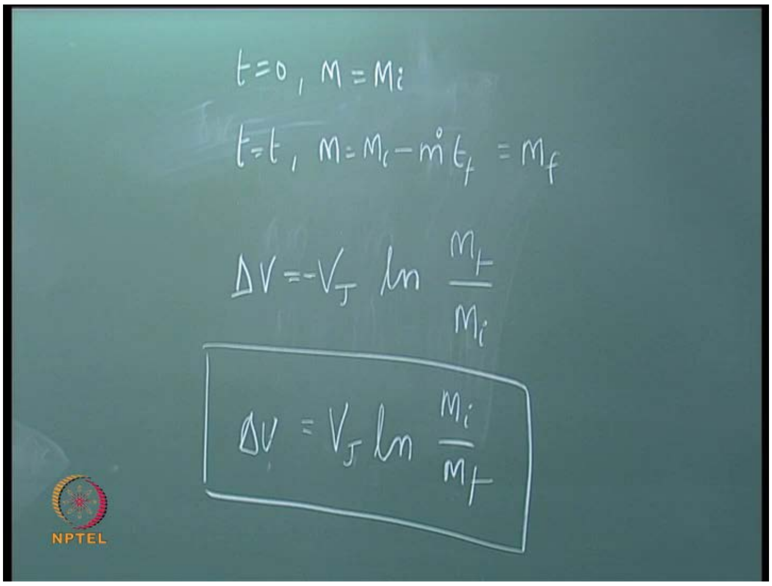
Let us ensure this is a absolutely clear because this is the basis of all the TSIALKOWSKI equation or rocket equation. We must be absolutely clear about it. I want to integrate this equation from the start of the rocket at 0 velocity to the final value at t f and therefore, I say I want to integrate it V J from this to this.

(Refer Slide Time: 29:28)



And what is the expression I get now? Is equal to V J let us assume that the velocity with which its leaving is a constant V J into now, when I do this, I get long logarithm of now I get another negative sign that is 1 over m dot m dot cancels and therefore, I get lon of M i minus m dot t going from 0 to the final value of t. And of course, I get a negative sign because this is over here. Is it all right? Please check it. Anything wrong on the board or anything? Can I proceed?

(Refer Slide Time: 30:29)

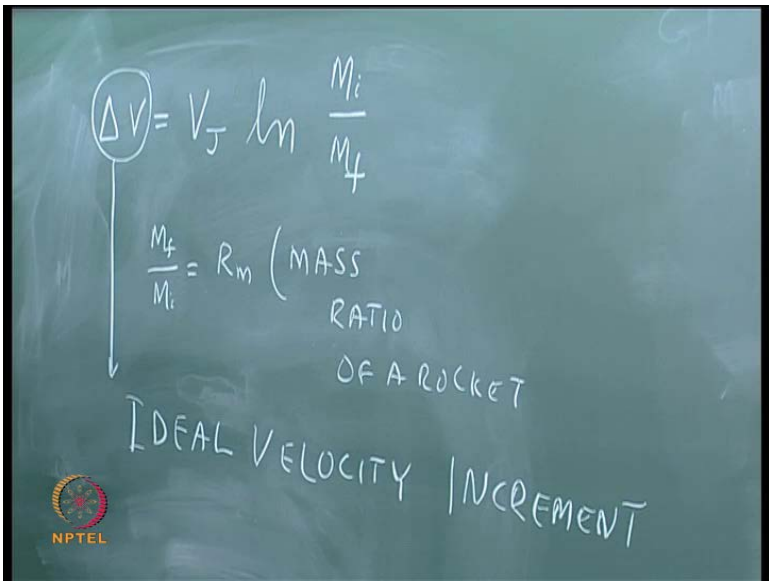


Now I will I will ask you what is the value of mass at time t is equal to 0? The mass is M i at t is equal to 0 the mass is equal to m i which is what we specify. At time t is equal to t the mass is equal to M i minus m dot into the final value of time which is t f I call it which I call as a final mass of the rocket. And therefore, what is this value delta V is equal to V J into lon of minus V J is the lon of I get M f by M i initial mass or if I want to take the negative sign into the logarithm over here I get it as a V J lon of initial mass by final mass which is the incremental velocity which I get from a rocket or the velocity what I get from a rocket and this is what is known as the rocket equation.

All what the rocket equation tells is when I burn a quantity between the initial value and the final value and I am exhausting it out at a constant velocity V J, the final velocity is equal to the velocity with which I am ejecting matter out into logarithm of the initial mass to the final mass and this is what we call as the rocket equation. And it will call for this Russian school teacher to come and derive it and he said with using this we can go wherever we want. We can go into interplant remissions because all what we want is we want a jet velocity and the mass should keep getting depleted and this is what is the theory of rocket propulsion or which we say is the rocket equation is it.

Can I can I repeat it to some extent see because this being the the basis and you know why why I keep harping on this is you know there are some limitations we have with the existing rockets.

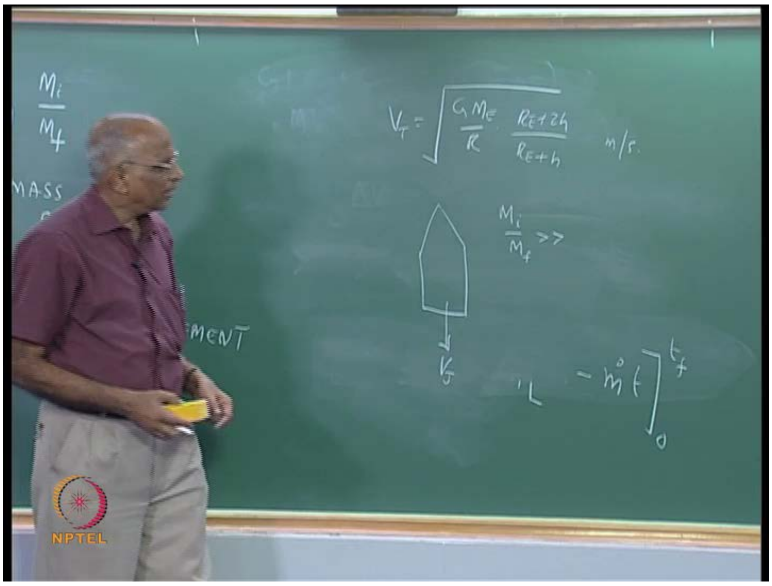
(Refer Slide Time: 32:45)



And if you want to go any further to make better ones we have to look at the way things work namely we said that jet velocity or the velocity which with mass is being removed into lon of the initial mass divided by the final mass and the value of final mass of a rocket to the initial mass of the rocket is also called as the mass ratio of a rocket. You know when I when I say this is the velocity which a rocket gives, I did not consider the gravitational forces. I did not consider a drag and all that.

Therefore, delta V is also spoken of or it should be called as ideal velocity increment. Now, let us go back and see what did we learn in the earlier 5 or 6 classes. What did we tell ourselves to be able to put a satellite or an object in space? I have to overcome the potential energy. I have to give the orbital velocity and the total velocity we derived the expression by the term.

(Refer Slide Time: 34:21)



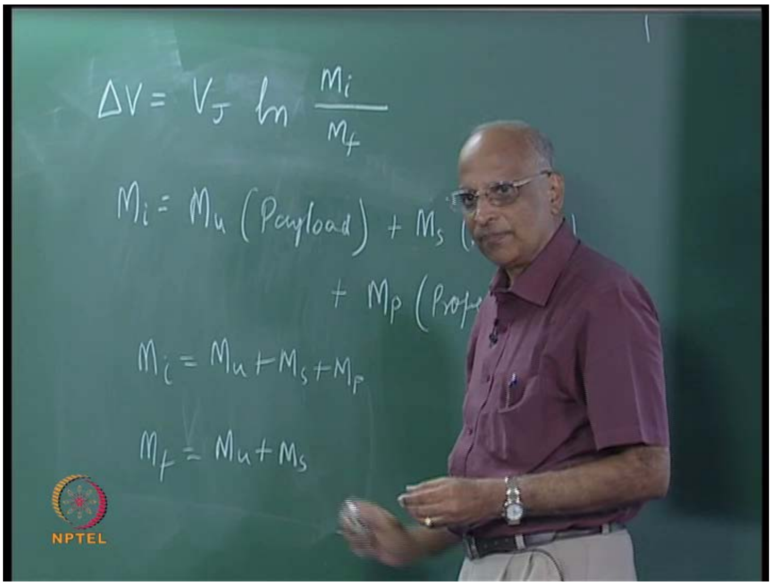
If I can write it correctly, let us say V T is equal to under root G M E by R for the for the case of the earth R E plus 2 h divided by R E plus h so many meters per second is the velocity what was a required.

Now, if I have a rocket and on top of this I put whatever I want to launch; I give it a velocity I can put it in orbit and but, we told ourselves I need a velocity of the order of when we did some homework problems, we found we need something like 10 to 12 kilometers per second. Therefore, what it is required in order to give high velocities? See, you have to have mass ratios which are that means the ratio of initial mass to the final mass must be a lot good number. That means the the the difference between the initial and the final must be large and also the V J must be a large value. That means the jet velocity becomes a controlling parameter and a higher the control parameter higher this jet velocity you can have higher delta V what you require in machines and in additional you need a large difference in this.

Therefore, the figure of merit of a rocket if somebody were to ask us we must say well one is the jet velocity, the other is something related to the masses. Therefore, lets go back and look at this term because this is fairly clear to us. What am I telling? Let us again put put a figure across over here. Supposing, I were to exhaust at some jet velocity and I can get as high a value, see apparently you cannot get the speed of light, you cannot get more than some amount. But, I have some limitations I will come back to these limitations plus in addition I am talking in terms of M i by M f should be a large number.

Let us just examine this number before I come back and see how best I can make a rocket because as of today you cannot get more than something like 3000 to 5000 meters per second as jet velocity. We will find out where what limitation are there but, let us first examine this and we come back to this. Let us let us therefore, go forward. Can I can I say yes the rocket equation is understood and we are in position to now go back and analyze what are the parameters, which are the drawbacks? Well, let let us go for this.

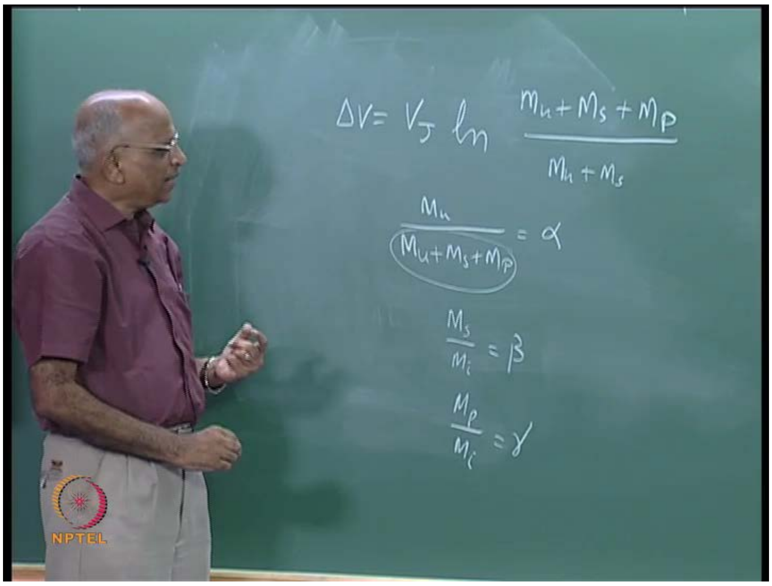
(Refer Slide Time: 37:02)



Let us therefore, write the ideal velocity increment is equal to jet velocity into lon of the initial mass to the final mass of the rocket. What does the initial mass of the rocket consist of? Any any thing you take, will have something like the like the useful part of it. What is the useful part of a rocket? The object which which is going round and round that is the useful part of the rocket which is we call as payload correct. Then they then the second one what we have is the structure of the rocket that means it must have some metal and all that which is all there the structural part of it, it will have some inert something to glue something to contain something. That we call it as structural mass let say structure.

Plus it should also have some have some fuel and we said fuel is used for propeller propellant and it is known as a propellant. Therefore, we have something known as a M p which we call as propellant. That means a rocket initial mass or the mass of a rocket compresses of the payload, the structure plus the propellant and we say M i is equal to M u plus the structural mass plus the fuel or the propellant. This is the initial mass. When the rocket has done its job, what must be the final mass of the rocket now? It has done its job that means all the propellant has burned out. Therefore, the final mass will be the useful one plus the structure and all will still remain because it is still there. But, you know we will come back and see whether we can also remove the structure and throw it out. But, otherwise the structure will remain therefore, the initial mass is M u plus M s plus M t the final mass is M u plus M s over here.

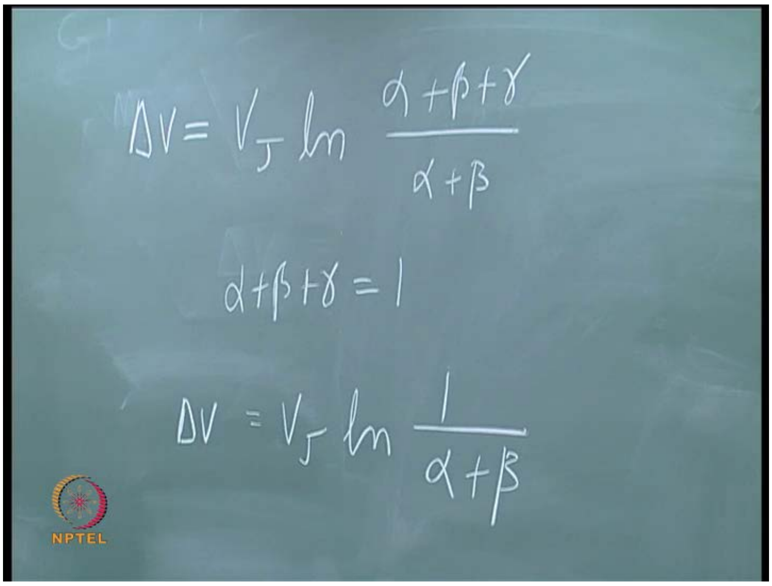
(Refer Slide Time: 39:33)



Now, let us go back and analyze the masses let let us let us erase this out. V J lon of M u plus M s plus M p initial mass. Now, we will instead of working with these kilograms of mass; let us put it in terms of non dimensional or mass fraction and let us say I call the useful mass of a rocket M u divided by the total initial mass namely M u plus M s plus M t as equal to alpha. This is the initial mass of the rocket. I non-dimensionalize that payload mass. Similarly, I non-dimensionalize the structural mass by this particular one which is the initial mass as beta.

And similarly, I take the payload, I am sorry the propellant divided by the initial mass which is equal to M u plus M s plus M p as equal to gamma.

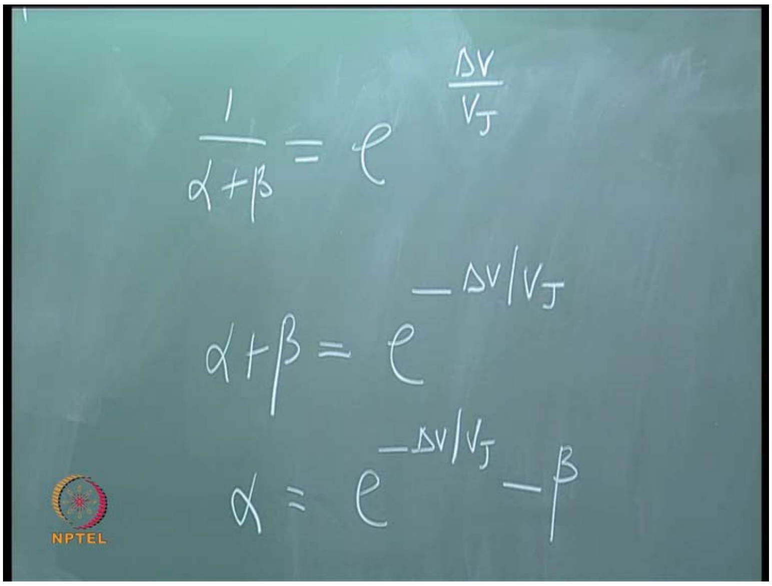
(Refer Slide Time: 40:48)



And therefore, I get this equation simple getting simplified into delta V is equal to V J lon of what will happen? M u I divide I divide M u by M u plus M s plus M p I get alpha plus beta plus gamma divided by alpha plus beta and what is the value of alpha plus beta plus gamma equal to? One exactly and therefore, I get delta V in fact I could have just divided it and got this. But, I thought we should go through this procedure V J lon of into 1 plus alpha plus beta. That means the ideal velocity increment as a function of jet velocity is lon of one by the payload mass fraction useful fraction plus the structural fraction.

Now, what is it what we want to do in a rocket? We want to have as much payload as possible. May be we would like the useful mass to be high. Let us say what is the fraction of the useful mass.

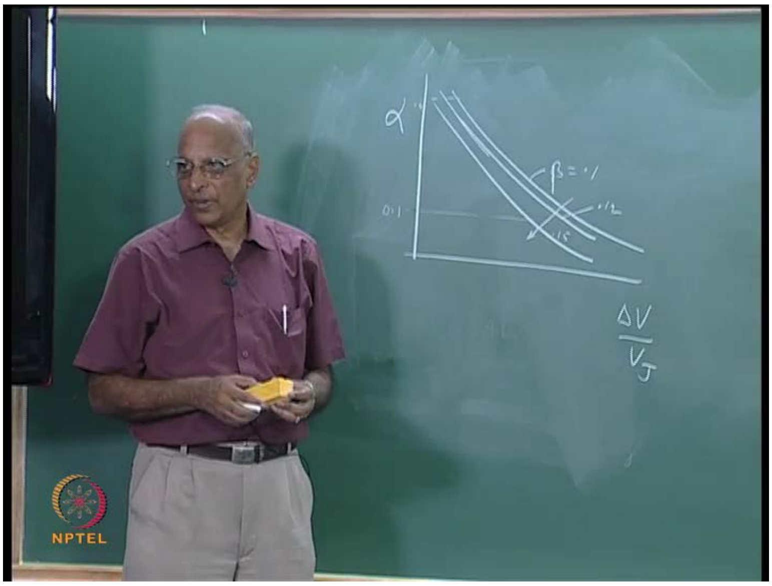
(Refer Slide Time: 42:01)



I get therefore, from this expression I get one over alpha plus beta is equal to e. I take exponential on both the sides the lon term vanishes, e to the power delta V by V J or rather I get alpha plus beta is equal no I should have just got e to the power 1 over alpha plus beta is equal to e to the power delta V by V J. Therefore, alpha plus beta is equal to V to the power minus delta V by V J.

Now, let us examine under what conditions will I get the value of the usefulness of the rocket? Will therefore, be equal to e to the power minus delta V by V J minus beta and let us try to plot it out let us try to plot it out for different values of V J for different velocity increments because we already told ourselves, we need this to be of the order of 10 kilometers per second. There might be some stitches on the jet velocity is of the order of 3000 to 5000 meters per second. What are the values of payload fraction what we get? Let us just plot it out and see.

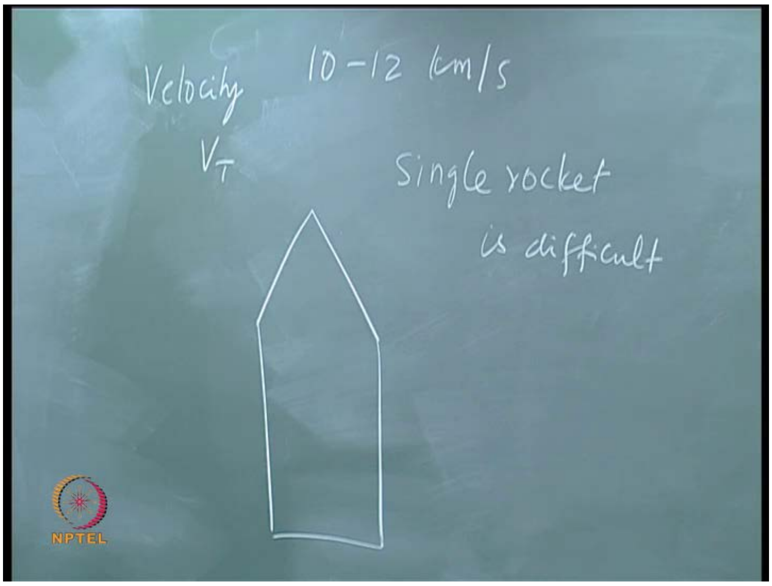
(Refer Slide Time: 43:36)



I will I will show it thorough a slides a little later but, well lets lets try to see what is the dependence of we get alpha as a function of delta V by V J for different values of beta we find that the values are keeps decreasing as delta V increases because its minus over there it keeps falling. And generally this value might be around 0.4 and this keeps falling drastically and in the operable regions, these values mat be around 0.1 to 0.2 or something and as beta increases it it it you find it is minus beta.

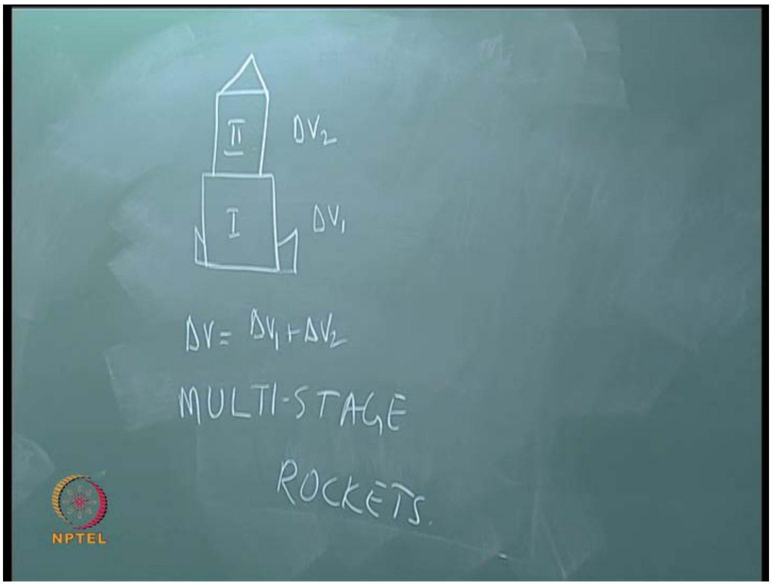
Therefore, as beta increases it keeps falling. I have still not put any numbers here because we do not know what it is the range of beta. But, generally beta should be around let us say 0.1 as beta increases to around 0.12 around 0.15. That means the payload fraction will keep decreasing as the structural mass increases. If I can have a high value of V J then I can get a higher value of the payload fraction. Or if I have a rocket or object which requires more ideal velocity then I get a lower value of payload. You know we are just looking at the rocket equation and trying to draw some conclusions from it. All what we conclude is if I want to put payload of higher mass then I need a structure which much be very light. I must have a very light structure or rather if I all my V J must be large such that I am over here and I get a higher value or if I can have rocket which does not have to go very far away it is orbiting nearby; then I can carry a higher mass. Otherwise subsequently I have to do some tricks to put it out. Well this is all about the rocket equation and we are trying to out at it. May be I will have opt revisit this in some different way.

(Refer Slide Time: 45:32)



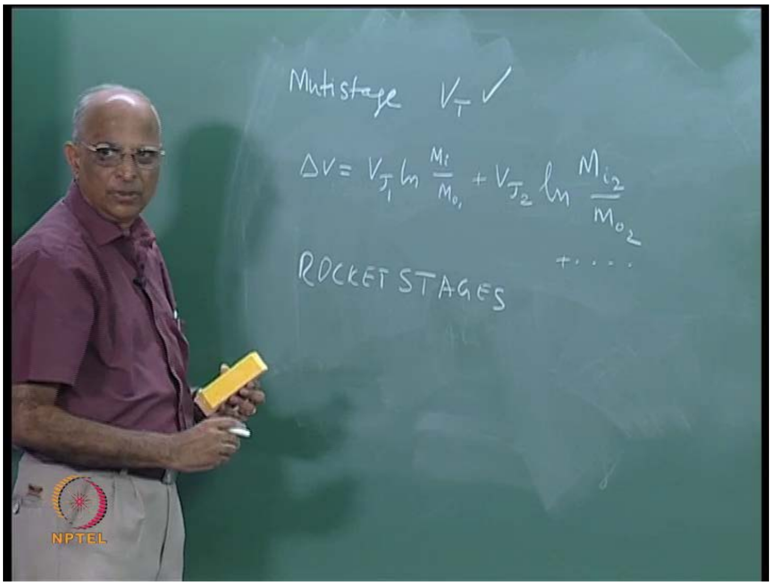
But then therefore, but, the problem is see they are still talking of 10 to 12 kilometers per second which is the velocity what I require. We called it as V T. This is what we said as velocity which is required to orbit V T is of order of 10 to 12 kilometers per second. But, then to get these things, if I have a single rocket I may not be able to get that because I have a definite mass of this structure. I have a limitation on V J and therefore, to be able to launch a payload into orbit using a single rocket is difficult. I use the word difficult because it is still I do not know whether it is impossible. So far it has been impossible but the quest for the rocket engineer is to make a single rocket do the job.

(Refer Slide Time: 46:50)



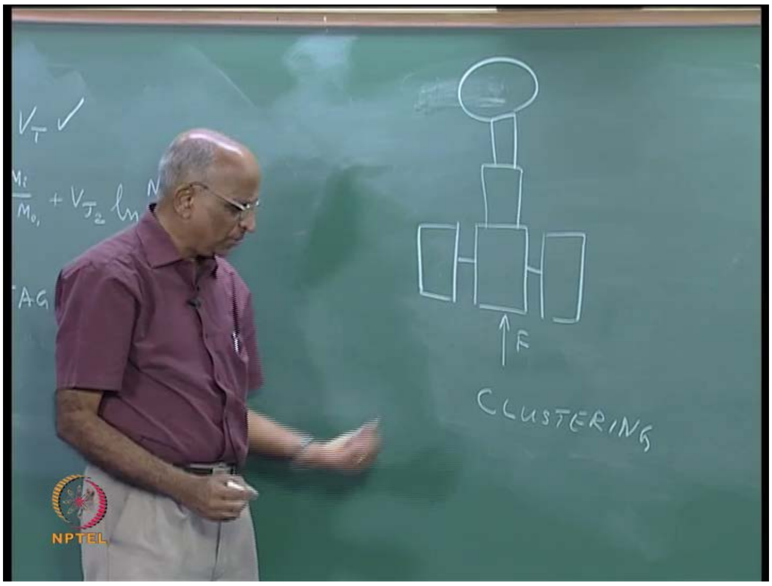
But a how do you make multiple rockets do the job? Let let us examine this point. Let us say instead of having a single rocket, I make two rockets. I put rocket over here and on the top of it I put another rocket. This is my first rocket, this is my second rocket. Now, the first rocket gives a value of the ideal velocity delta V 1. The second rocket already has a delta V 1 when its starts functioning. It gives me a value of velocity delta V 2 and the total velocity of this composite one after the other that is a two stage rocket, gives me a delta V equal to delta V 1 plus delta V 2 and therefore, by putting one rocket after the other that means one stage of the rocket after the other stage you get what we call as multi stage rockets and most of the rockets which we will be seeing in this slides today would be all multi stage. That means you want a velocity increment of something like ten kilometers may be the first one could give you 2, the second one could give you 4 and the third one could be you still higher and therefore, you keep on adding and this is what we say as multi stage rockets.

(Refer Slide Time: 48:21)



And therefore, we tell ourselves using the rocket equation. I now say I have known as multi stage rockets which can give me whatever be the velocity what I want and how do I get it? I have delta V is equal to V J lon of may be the initial mass to the burn out mass of the first one plus now I have V J. May be for the first one, may be I have for the second one logarithm of the initial mass of the second one plus the burnt out mass of the second one plus and so on. I get the final ideal velocity. Maybe we should we should try to analyze this in some detail. But, I think all what I am telling is we are talking in terms of staging or we use rockets as stages. You all would have said the, we as something is the base stage, then on the top of it we have the first stage, then the second stage and so on.

(Refer Slide Time: 49:32)

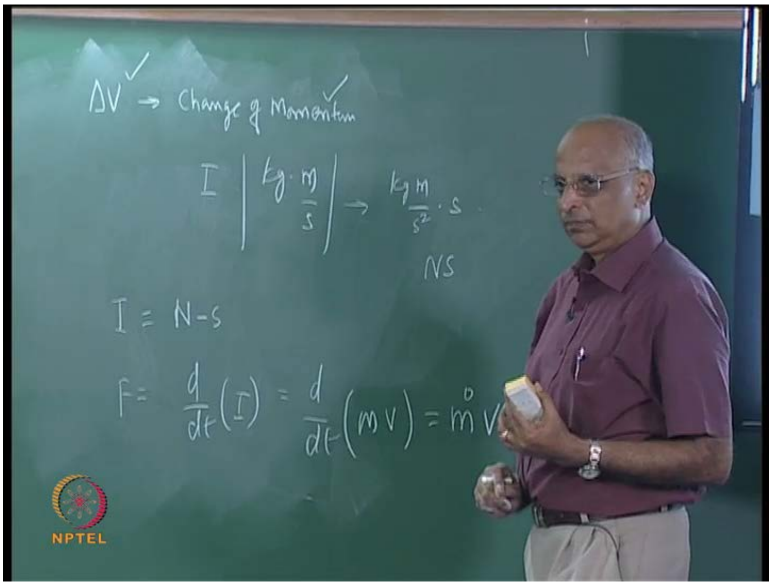


And this is what we say. Let us taken a example, lets plot in terms of our own GSLV vehicle which is very much in the news. We tell ourselves yes this rocket consist of a core, it consist of something attached to it at four things over here, then it consists of the second stage and the third stage on the top of the third stage sits the satellite.

Therefore, you have the first stage, second stage, third stage. This gives you delta V 1 delta V 2 delta V 3 which gives you the velocity to put it into orbit. Therefore, we talk in terms of staging. Staging means one after the other but, what about these things why are they required may be we need to understand a little bit more what is it we have done? We have put one we have put one stage on this, the other stage on this and you have the payload therefore, you have increase the mass. When you have increased the mass and you want this it should generate sufficient force for it go. But, this fellow may not be able to generate that force.

Therefore you need additional rockets so that the force or the thrust is able to take off and that is why we put rockets together and this is known as clustering. Why do we need clustering of rockets? To give you sufficient force. May be to be able to derive something about force, I think I will divert a little bit and look at what are the what what do we mean by force? What do we mean by thrust? Are there any more definitions which we need to keep in mind to understand this clustering of rockets, staging of rockets? Let us just put things. Well let us just summarize what we have learnt so far.

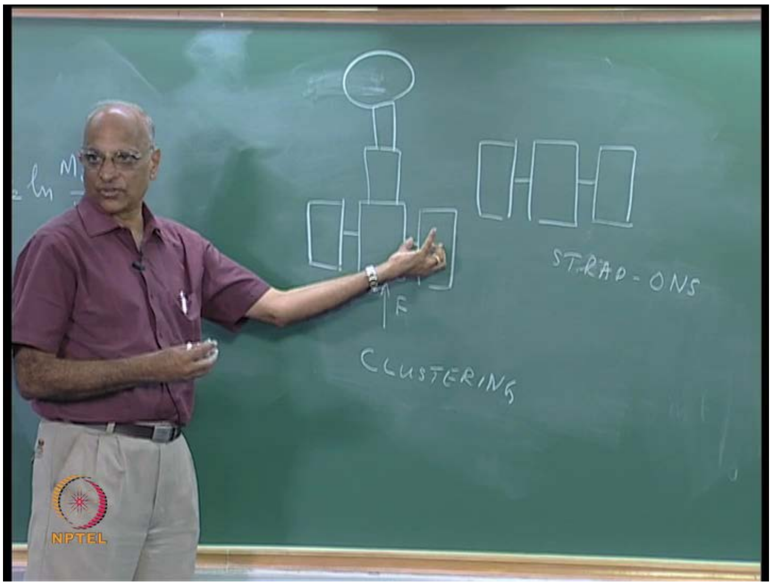
(Refer Slide Time: 51:16)



We told ourselves rocket gives you delta V. How does it give you delta V because of change of momentum and that is because why did I change of momentum. I am standing here. I am watching the momentum change. I wrote the momentum change and I found out the value of delta V. What is the change of momentum known as? It is known as impulse. I What is the unit of impulse? Same as momentum kilogram meter per second. But, kilogram meter per second can also be written as kilogram meter per second square into second which is same as Newton second right. Therefore, I can write the impulse as equal to Newton second.

Now therefore, the impulse in Newton second is what gives delta V therefore, what is the force with which the rocket is pushed up? What is the force with which the rocket is pushed up Rate of change of momentum that means d by dt of mv or d by dt of I. That means it is equal to d by dt. That is I get so much force which is equal to d by dt of m e this is equal to mass of the thing which is going into V which is equal to m dot into V and therefore, I can also write the force is equal to m dot V or compared to momentum which is equal to m V, I write force is equal to m dot V and this is the force with which and pushing the rocket. There is a limit to the mass which can which can be released, m dot which can be released.

(Refer Slide Time: 53:31)



And therefore, I allow more masses to be going through this at a high velocity and I am able to give give a force. That means I need a larger force to push and that is why I allow these things. Some times what we do is, we have a booster rocket we put two attachments over here. What are these attachments? They are like straps. I strap something on to it. I attach two straps and therefore, the side rockets are also known as strapons.

Let me take one or two examples. May be I will show it through slides when when I when we meet in the next class. But, to be able to just conclude at this point of time all what I would like to say is we derive the rocket equation through the rate of through the change of momentum. We looked at the inertial frame of reference, watched the rocket go up and we found out what is the ideal velocity.

We also told ourselves there are some creatures in universe which also make use of the rocket principle. Then we found out that the structure plays an important role just as much as the jet velocity plays a role. Then we found out to be able to get a high value of delta V, we have to operate in stages and to be able to take off we need some additional side rockets which are known as clustering or strapons. Thank you I think we will come, we will continue with it in the next next next class.