Rocket Propulsion

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Lecture No: 07

Review of Rocket Principles: Propulsion Efficiency

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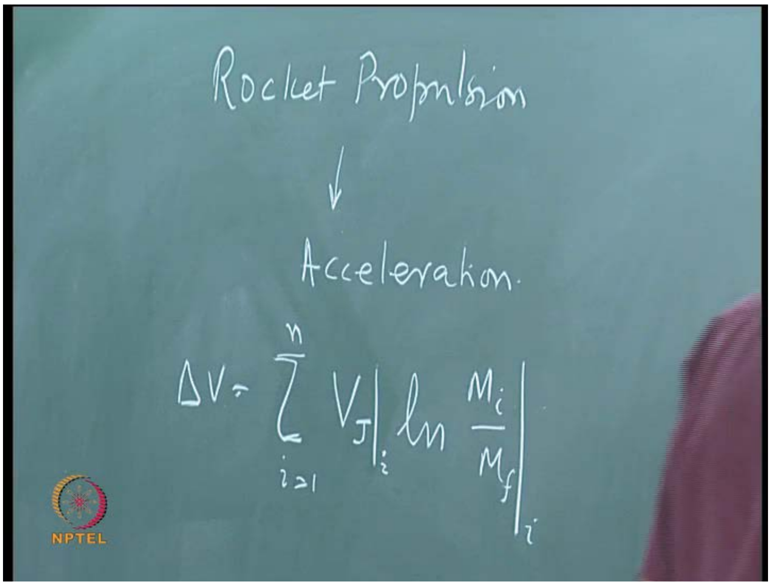

Well, Good morning! In today's class, we will look at the following. We will look at the theory of rocket propulsion again. We derived the rocket equation in the last class. We also looked at staging; we looked at clustering of rockets and also the strap on rockets and what function they do. But, we did not really calculate what is the type of acceleration what we can get from a rocket at takeoff; let us say. See, you know this will illustrate yours in a better way; why we need additional straps or additional clustering when a rocket is going, when the rocket is take off.

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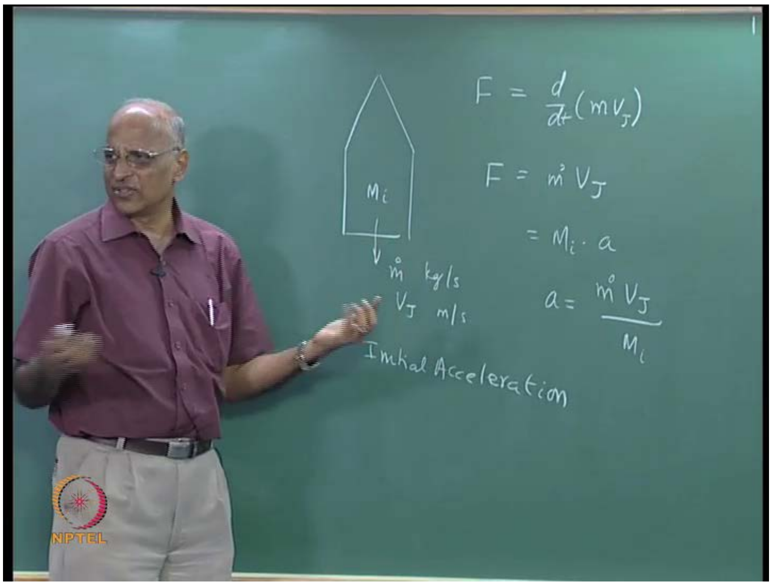

Let us consider this example. Let us say I have a rocket, whose initial mass is M i. And, let us say it allows mass to the out of the nozzle at the rate m dot. Let us also assume that the rate at which the efflux leaves the nozzle is at velocity V J. Let us put the units together; m dot, so much kilogram per second; V J, so much meter per second. I want thrust to be able to calculate, what is the initial acceleration of this rocket. How do I do it? You will recall. In the last class, we told ourselves that the force on the thrust with which a rocket is pushed up is equal to rate of change of momentum on d by d t of m v. And, here it is the momentum is m V J; therefore, m V J. And, therefore is equal to m dot into V J is a thrust over here.

Now, can you tell me what will be the initial acceleration of this rocket? Therefore, the initial acceleration should be equal to the initial mass of the rocket into the acceleration a. That means, force is equal to mass into acceleration or rather acceleration is equal to m dot into V J divided by the value of the initial mass. And, what did we tell ourselves in the last class? As I keep on adding more and more mass to the rocket, the initial mass increases and it becomes impossible for me to take it up.

We will work out an example, a numerical example to be able to figure out of how we calculate the acceleration and how we decide what must be the level of acceleration; because you know acceleration is also important. Suppose some human beings are sitting in a rocket, it cannot takeoff at a very high acceleration. The man will go. Therefore, there has to be some control on acceleration. Maybe we will have to address some things. Therefore acceleration, we calculate like this. And therefore, what did we tell now? the delta V is equal to summation of, I now used the simplified system and said I have V J depending on the number of stages and I have lan of the ratio of the initial to the final mass of the corresponding stages That is, i i, i as i goes from first stage to the other. And, this is how we calculate.

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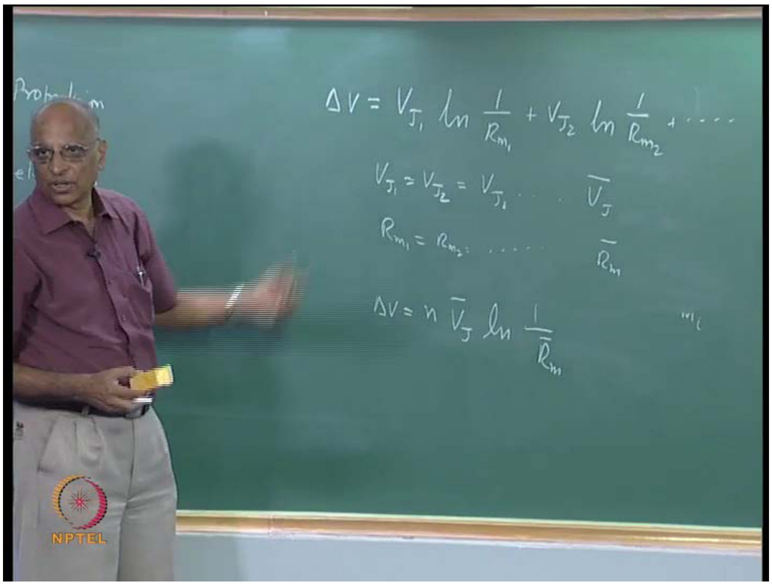

Itry,d…I,try to simplify and get the terms together. And… to simplify things and write things as delta V is equal to V J 1 lan of 1 over the mass ratio one. Does it make sense to you? We said mass ratio of a rocket R m is equal to the final mass divided by the initial mass. Therefore it is R m 1 plus, I have V J 2 lan of the second stage 1 over the mass ratio of the second plus and so on. Supposing I have rockets in which the jet velocities are the same for all the stages, V J 1 is equal to V J 2 is equal to V J 3 and so on.

And, I have the mass ratios of each stage is also the same for all the rockets. Supposing, if each stage has a same mass ratio and we say R m 1 is equal to R m 2 is equal to and so on. Then, what is it I get? I get delta V. And, this I say is the mean jet velocity and this I say is the mean mass ratio or the mass ratio are all the same. Then, I get delta V is equal to n times the value of V J lan of 1 over the mass ratio. right.

That means, by increasing the number of stages I am able to increase the delta V over value. But, this is not possible because the mass ratios of the individual stages may not be the same and the jet velocity of all these stages may also not be the same. You could relax these things by considering V J to be different, you could consider R m 1 to be different, and I can keep on getting different ideal velocities. May be we will do our homework problem a little later and try to find out how to calculate the jet velocity taking into consideration a number of stages of rocket together. Well, this is all about the rocket equation and the number of stages, clustering of rockets, may be putting straps in a rocket.

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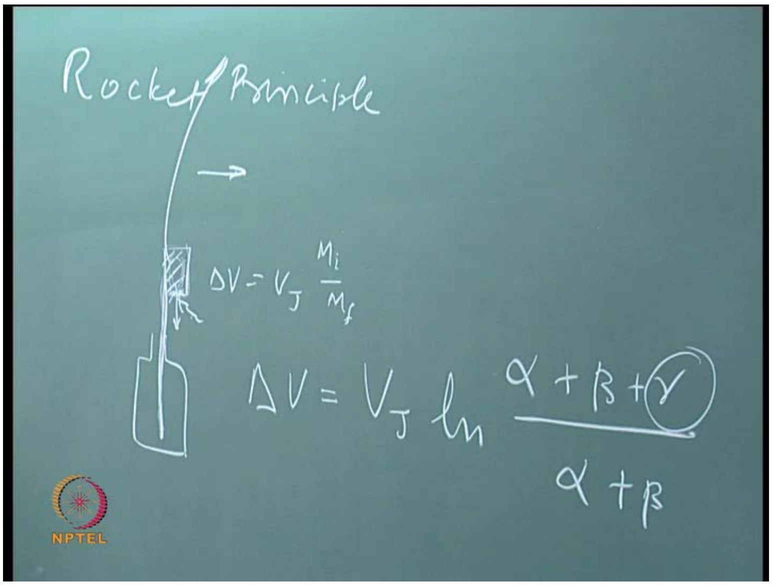

We will look at some examples. But, before looking at examples, I thought can we extend the rocket principle to what we have. You know, see during Deepavali, we will fire crackers. Something known as rocket is also there. What do we do? We have a launcher. Let us take a look. We have a bottle which is used as for launching rockets, we put that stabilizer which is the wooden piece of a rocket and here you have this cracker, which is filled with some black powder. We will look at its composition a little later. And, you have a small squib over here and you light it and zoom goes in this particular direction. Actually a fire cracker? Right. Supposing I want to write the equation for this one, it should be the same equation. Right.

Therefore, here also I should say, well, the initial mass of the rocket will include the stick, the paper which bound binds together, all this together, the mass of the gun powder or let us say a black powder which is available. That is initial mass. And, when the rocket is all consumed and left with this wood and left with little bit of powder or they the powder is all burnt anyway, the paper which has still not got burnt that will be my final mass. And therefore, what is the value of V J? The I have, they have given a small hole here through which the gases are escaping. Through that it escapes, and this is going to be the final value.

But, in cases like this, what happen is the value of the mass of powder we use is very small. And rather, if I were to go back and write the equation what I wrote in the last class namely delta V is equal to V J lan of alpha plus beta plus gamma divided by alpha plus beta; wherein alpha was the payload mass fraction, structural mass fraction and propellant mass fraction. The amount of gun powder or the amount of black powder which I keep is very small compared to the weight of the stick and all this. And therefore, gamma tends to be a small number. And, in fact it is not only in the in the Deepavali rocket that it is small.

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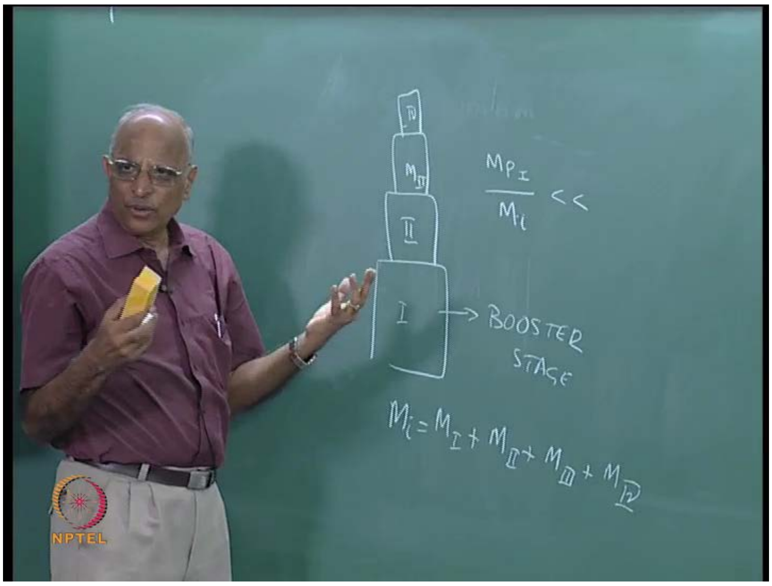

But, if I have to look at another example, you know when I consider a rocket, I say that we put stages; one stage after the other. I say let us have a four stage rocket. The initial mass of this rocket is going to be M i, is going to be of first stage M I plus the weight of the second stage M II plus the weight of the third stage which is M III plus the weight of the fourth stage which is M IV. Now, I have some propellant in the first stage, I have some propellant in second stage and all that. But, whatever be the propellant I put here in this first stage, since the mass of the total thing is so much, the mass of the propellant of the first stage divided by the initial mass of the rocket M i. This is going to be small because I carry so much mass that it is going to be small.

The case is something similar to this; wherein the structural mass and the inert mass are and the payload mass, whatever it be. You know in a rocket, nowadays you give good payload. What it carries is, get some metal powder and once it reaches the height, it releases the metal powder and you see all these things going around. The mass of the payload plus mass of the stick… the stabilizer plus mass of the paper is going to be much higher than the amount of propellant in it.

So, also in these cases which we say that first stage, which we sometimes call as "Booster"; what is the "Booster"? It helps the rocket; it allows the rocket to takeoff. It boosts the rocket. Therefore, it is known as "Booster stage". And Booster stage, the mass of the propellant divided by the initial mass is the small number. If it is a small number can we find out whether I can simplify the equation in some way, when it is going to be anything different from; let us say the final stage for which also we have derived the same rocket equation should hold good.

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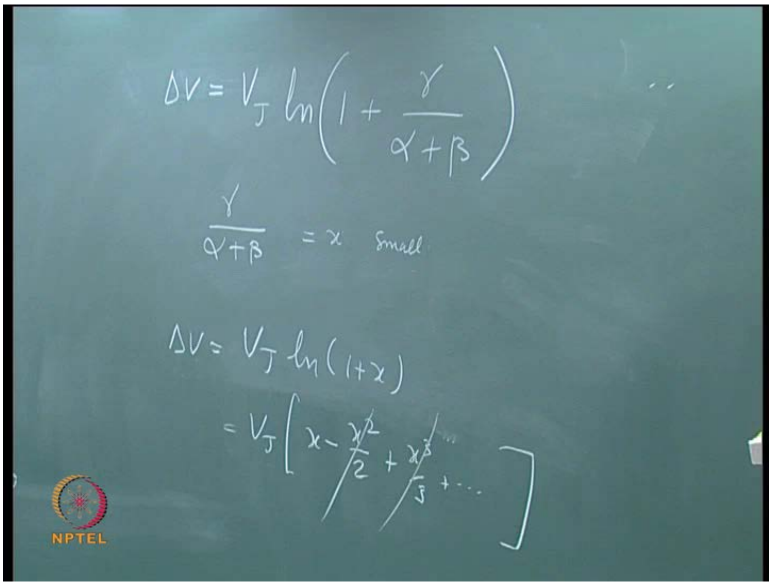

Let us go back and see what happens in that case. Let us go back and write that equation again. Delta V is equal to V J logarithm of, now I simplified, please make sure it is alright. Gamma divided by the structural mass payload mass fraction plus the structural mass fraction. This is this is the general rocket equation, which holds good for each of the stages.

Now, I come back to the Booster stage or to the case of a Deepavali rocket. For that, I say the propellant mass fraction divided by alpha plus beta should be small. It is small because there is so much of mass above it. The mass of the propellant is going to be small over here. Therefore, let us say this is equal to x which is small. Therefore, now I get the equation; delta V is equal to V J lan of 1 plus x, where x is the small number. And, what is lan of 1 plus x? x minus x square by 2 plus x cube by 3. But, x is the small number. Therefore, I can as well forget about square, cube and four and all that. And, now I get the delta V for a rocket, like a booster rocket or the bottle rocket which we used for Deepavali time.

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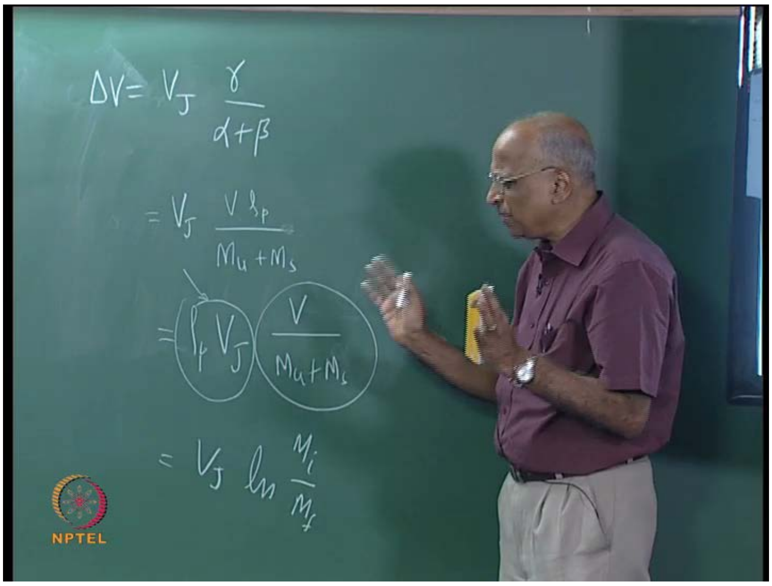

It is something like delta V is equal to V J into; I now, I plug back the value of the number gamma over alpha plus beta. Let us put it in terms of the dimensions. Therefore, I get V J into mass of the propellant divided by mass of the useful part plus mass of the structure. Ok. I am here just playing that terms. And, what is the mass of the propellant? Mass of the propellant is equal to the volume of the propellant into density of the propellant. And therefore, this becomes equal to density of the propellant into jet velocity into the volume over which the propellant of device divided by M u plus M s over here. In other words, you know what is the comparison between the two? You find that in the other expression what we had for delta V, it was just V J… let us write it out again. It was equal to V J into lan of M i by M f. Whereas, in this case I get the figure of merit of the propellant as coming as the density into this.

Therefore, for a Deepavali cracker or for a booster stage, instead of V J being a figure of merit, it becomes that the density into V J becomes a figure of merit and that is the distinction. In other words, one of the equations gets slightly simplified in that. Instead of meter per second V J being a figure of merit, it becomes rho density of the propellant into V J become becoming a figure of merit. And, of course this becomes the volume proportional to mass of the propellant and M u by M f. This is the this is what we must keep in mind when we become when we design the boosters. I will come back to this point, where I show some slides. But, this is something which is important, which comes very simply by looking at the expansion of this particular… or what I am trying to tell you is the following.

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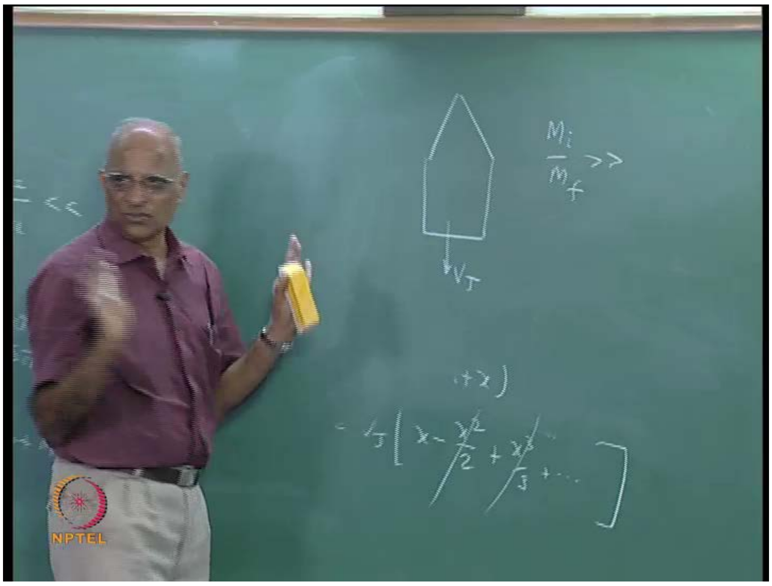

Let us be very clear about our mission. You know the subject is simple and we must not feel diffident at any time. When I want to make a rocket there are two important parameters as I told you. One is V J. It must be very high. The ratio o M i by M o must be large, M F right must be large. But, when we make a booster like this or when we are making a fire cracker, then in that case what is going to be different? Instead of V J, I would like the density of the propellant into the V J to be large. And of course, since this fellow carries so much mass, you know that the density of the propellant also plays the role. And in fact, as we go along we will see, since the density of hydrogen is small to be able to use cryogenic rockets, which use hydrogen and oxygen as booster stages; it is not that advantageous. Whereas if I use solid propellant, which is the dense material, may be it is better for the booster stages. When we go subsequently, we will we will come back to this, this particular point.

Well, this is all about rocket propulsion, rocket equations, staging and all that. But, I think that at this point in time, let us go back and refresh ourselves on what we have learnt through some slides. And then, we will come back and see what we mean by propulsion efficiency and then we will solve one or two problems.

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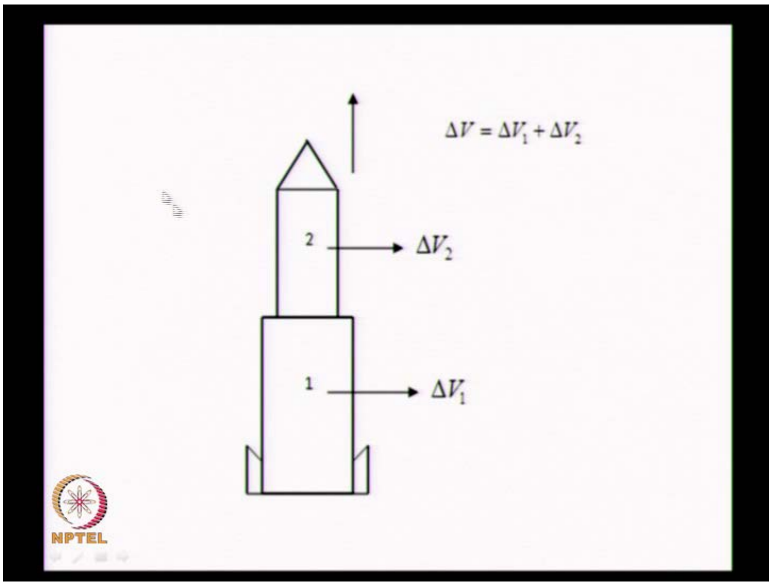

Let me come back to these slides. You know, here we see a two stage rocket. May be the first stage gives you a velocity delta V 1, the second stage gives you a velocity delta V 2. The total velocity of this combination of the first and second stage is delta V is equal to delta V 1 plus delta V 2. This is two stage rocket.

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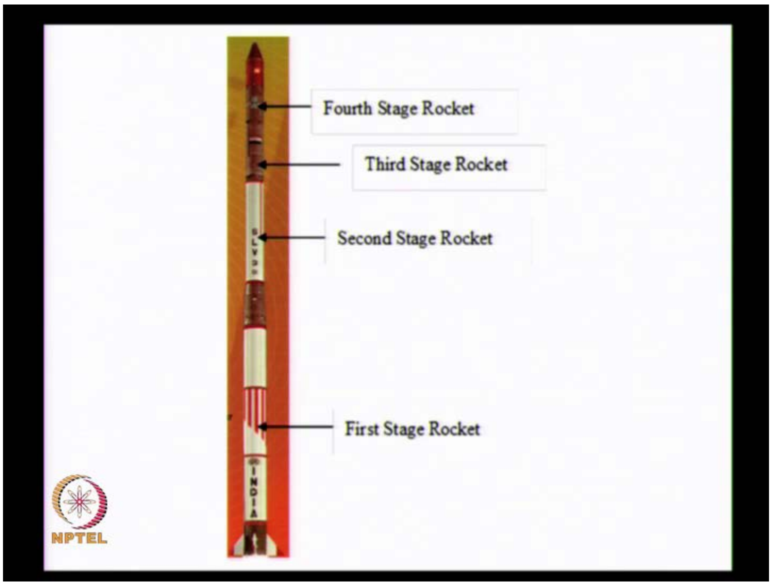

Let us go to the next one. This is the first rocket which we designed in VSSC at Trivandrum. This was in the period, may be seventy to seventy eight when they are working on it. And, the first successful launch we had was in 1980. It was a simple rocket, you know. It consists of four stages. And, the first stage was solid propellant. I will give you a problem involving this rocket, which we will do today. This is the second stage, this is the third stage, this is the fourth stage. The total velocity which the rocket gives is delta V 1 for the first one; delta V 2 for the second one; delta V 3 for the third one; delta V 4 for this one. And, maybe we will work out this problem a little later.

All what I want to say is, well, we had a four staging here. You know it is very deceptive. You know sometimes we feel we can keep on increasing number of stages, may be make infinity stages. I can go back crack and put any velocity what I want. But, it is not that correct. The more stages I have, the more commands I have to give to the rocket. I have to ignite this second stage, I have to separate it out; it becomes more complicated. And therefore, the trend today is to go only for two stage to orbit or three stage to orbit. But, people are still working on a single stage to orbit. Let us see what is the problem.

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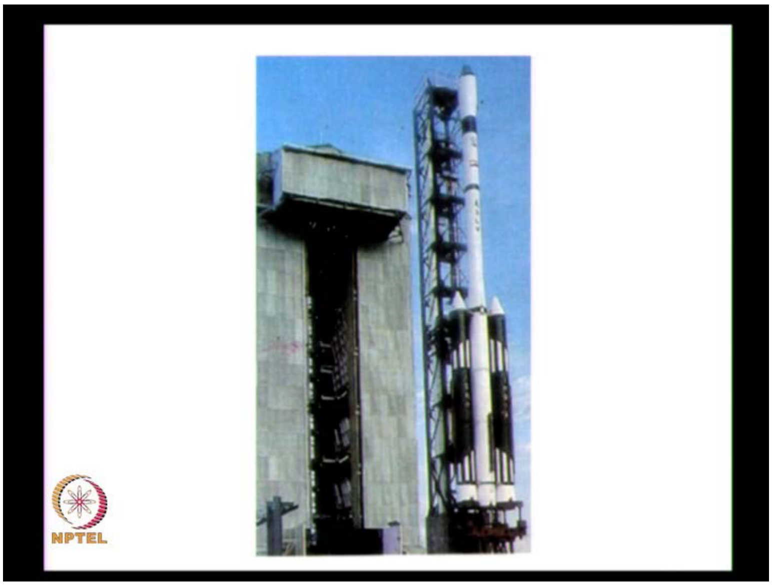

We go on to the next one. See, after the first stage; you know that SLV 3, what I showed you here, it can only take a payload of something like 40 kilogram. It is very small. Therefore, it was necessary to go to higher payloads and you may have a higher payload, you cannot have the vehicle to accelerate; because you need to increase the propellant weight, it was necessary to put two straps over here. Therefore, you have one strap here; I am sorry let us go back, one strap here, the other strap here. The same as the as the first stage here, you have two straps here and then the second stage, the third stage, fourth stage and so on. This is known as "ASLV".

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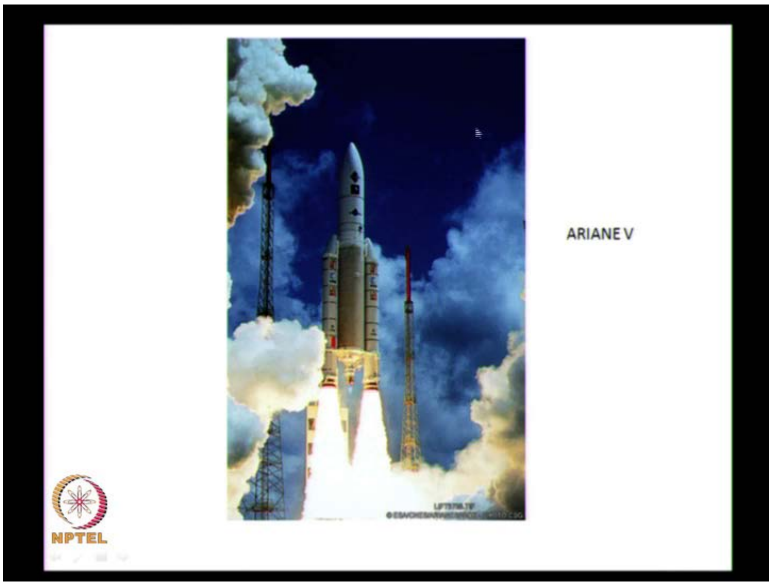

And, now I show some more examples. These are the current rockets which fly; “ARIANE V”, by which we launched the INSAT satellites. It has been used to launch earlier. And, even now we use them. You you have two straps; first stage and the second stage over here, “ARIANE V”.

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This is the “SPACE SHUTTLE”. You know, it has been work host for US and it has been decommissioned now. The last flight of “SPACE SHUTTLE” is over. And, you know here also, you have the space plane, which comes back onto the ground and this has something like three liquid engines, all clustered together so that, you get the high thrust. And, you also have two straps. One strap here, the other strap over here; that means two straps for giving the initial acceleration. It starts off and then you fire this this liquid engine in this and it pushes itself up. The brown thing what you see is the hydrogen tank, which stores hydrogen which is required for particular space plane over here.

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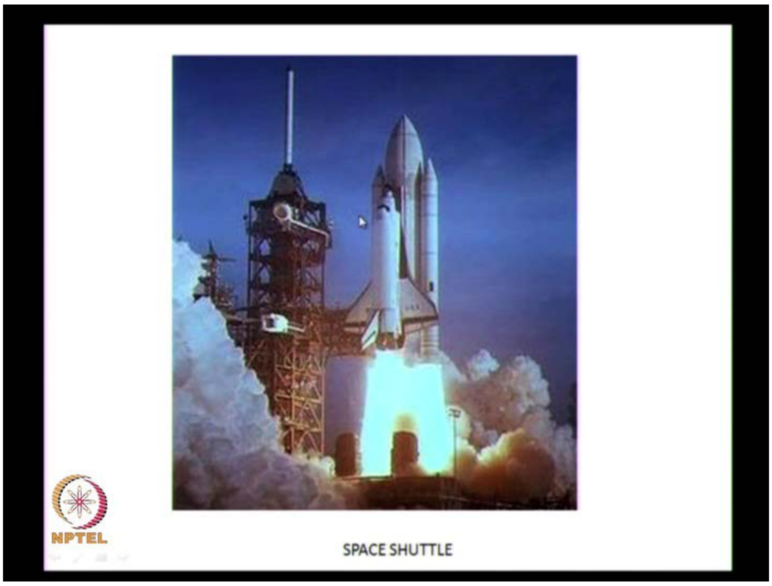

This shows; you know it is always nice to see the SPACE SHUTTLE taking off. These are the two boosters. Solid rocket straps as I said and you have three engines over here.

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Another work host of US is the DELTA vehicle. Again you have a number of straps over here or cluster of engines. I will not get into the details of these things.

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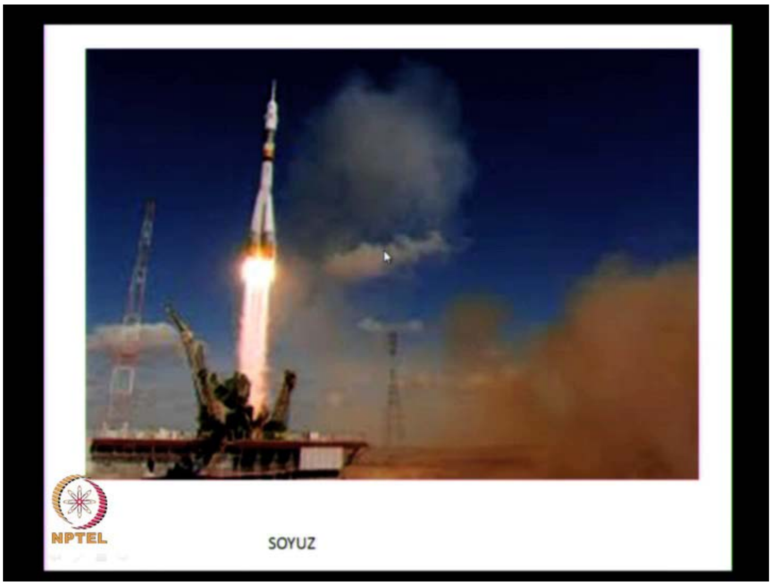

But, I wanted to show "SOYUZ", a Russian vehicle. Wherein you have straps here or a cluster of engines here and the main engine is flying, all the exhausts of this is interacting of something like a ball. And, "SOYUZ" was used for launching our first experimental satellite namely "Aryabhatta". This was in seventy five to seventy six period time frame. Well, again another part of SOYUZ; another slide of SOYUZ.

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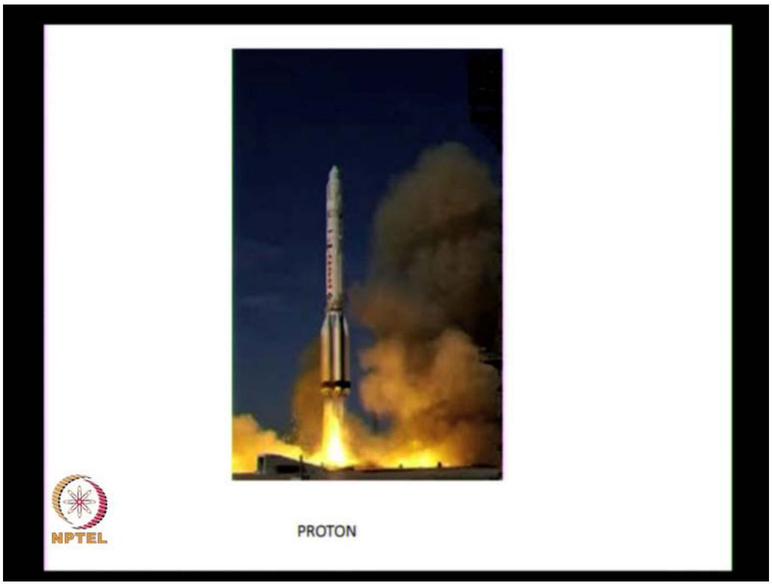

Well, "ATLAS CENTAUR" is something of Russian. "PROTON" is another Russian launch. All what I want to tell you is that, most of the vehicles have a number of rockets which are clustered and a number of stages. This is why I put some of these examples. “PROTON”, you have.

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We told ourselves a launch, a rocket can be launched from the sea, from the submarine it is taking off and you see the water droplets splashing over here. “TITAN” we leave.

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You know this is something about which there, I will give a problem which we will try to solve. This was the SATURN V, which is one of the very powerful launches; which was used to take men to the moon; APOLLO launch vehicle. And, here again and you have a number of stages. I think it is almost like a five stage vehicle; the ground having straps, then one after the other. Then, you have the spaceship module over here which comes back. Maybe we will look when we are solving the problem. Again SOYUZ.

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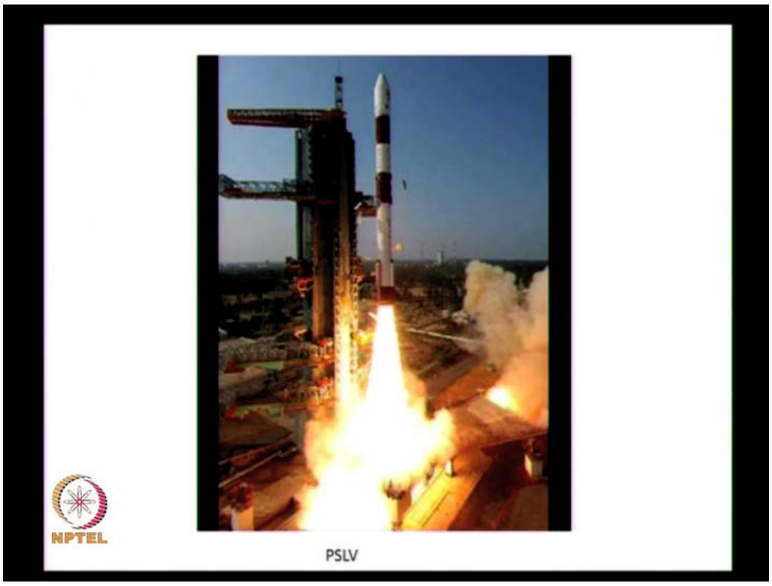

This is our PSLV; Polar Satellite Launch Vehicle of India again. And here, again you find a… see of the first stage you have straps. Six straps or four straps could be put. Then, you have the second stage, third stage over here.

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And, of course this is the GSLV. I think I will show the GSLV. Let us see if I can. Let us… You know, we just watch closely. You know, you first see the configuration. The configuration will consist of, as you see it has four straps or cluster of four engines; one, two, three, four. Then, there is a central engine. First, the four straps fire. It generates thrusts, the vehicle takes off. Then, the core also fires and then you have a huge thrust that keeps going. And, once the four stages at there, it separates and falls down to the ground. Then, the second stage fires this. This is the inter stage. Then, it keeps going further. Then the third stage will fire and it keeps going. The third stage fires and it keeps going and then after its task is over, it again gets separated. And then, only the object or the spacecraft is there. And, once it reaches the particular orbit, it sort of deploys. This is how we get the final vehicle, which… Therefore, staging is the very important exercise in any any of these rockets; staging and clustering.

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And, this is the GSLV. As we just now saw, these were the four straps. This is the first stage, second stage and the third stage. First, these four stages are ignited. Four straps are ignited, gives you the thrust to take off. Immediately after takeoff, the core is also burning. Therefore, you have the huge thrust which pushes within itself. Then the second stage fires, then the third stage fires, then you had sufficient fires. And, this is how a rocket functions.

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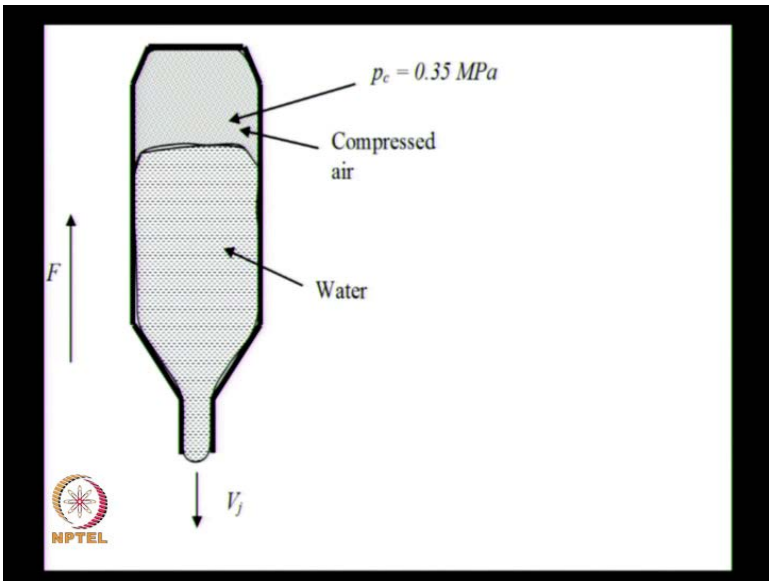

Well, we also talk in terms of a liquid rocket, wherein we said or a water rocket; wherein we could have water and I could pressurize it and push it on. Maybe we will solve this problem in class a little later.

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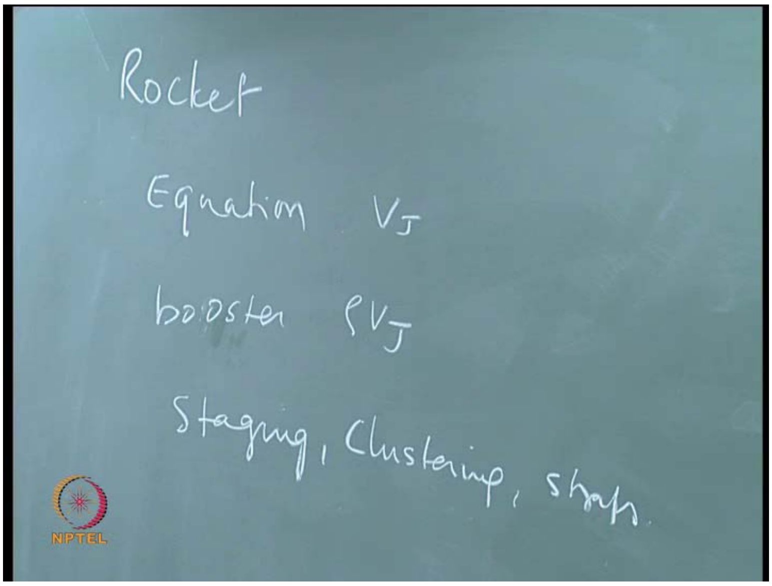

Well, I think by now we should be therefore clear about what is the principle of rocket, what is the rocket equation as it were, what will be the rocket equation modified for a booster, in which case rho V J becomes more important than V J itself. And then, we talked in terms of staging, clustering or and also straps... Is there any questions so far? If it is sure, maybe you should address yourselves. If this if this becomes the essence of what we are going to do. If this is clear, let us go to the next part of the subject. We will ask ourselves, how there are certain things like efficiencies.

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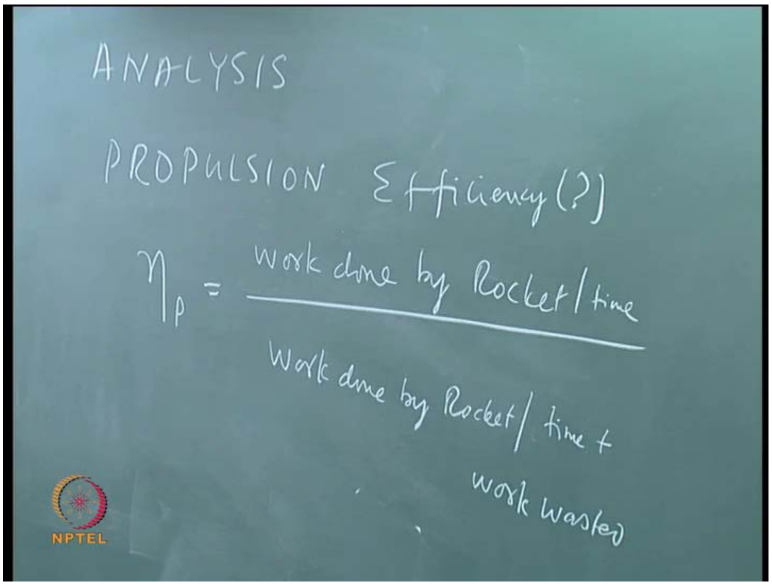

You know we people, you know we people who are used to analysis so much that anything what we want to do we will say… what is the efficiency of a rocket? How do you say efficiency? We are looking at the rocket flying up; therefore I want to find out L. We say rocket propulsion; therefore I say what is propulsive efficiency. How do I define it? It has to do something about the forces, how it is moving up or the power which you are giving, the power it is producing. Therefore, I say, well, propulsion efficiency is something like the power generated by a rocket or let say work done by the rocket per unit time let say, divided by what should it be? What is your guess? Work you do on a rocket to make it go up. And, what should it be? The work done by rocket again let say. Work done by rocket per unit time plus the work wasted by rocket. That is the total work which you are doing on the rocket per unit time. How do how do I put these things …mathematically and tell you what is going to be propulsive efficiency? Any guesses? Anything?

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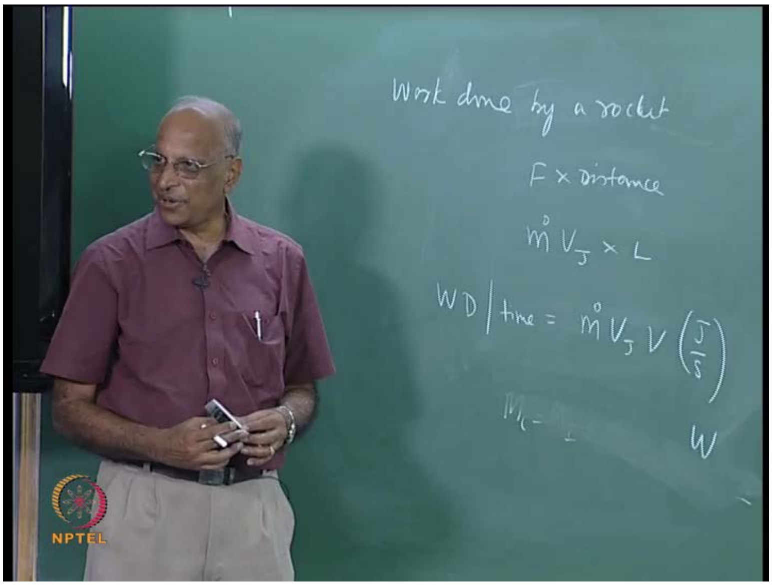

Now, you all should be able to tell me something more. Yes. I will give you an idea. The work done by the rocket is equal to force into distance. Right. The force of the rocket, we told ourselves, it is equal to m dot V J into distance, let say L. Therefore, I say work done by the rocket per unit time is equal to m dot V J into the velocity of the rocket… And, therefore we say so much joules per second or so much watts. Is it all right? Useful work; that is the useful work

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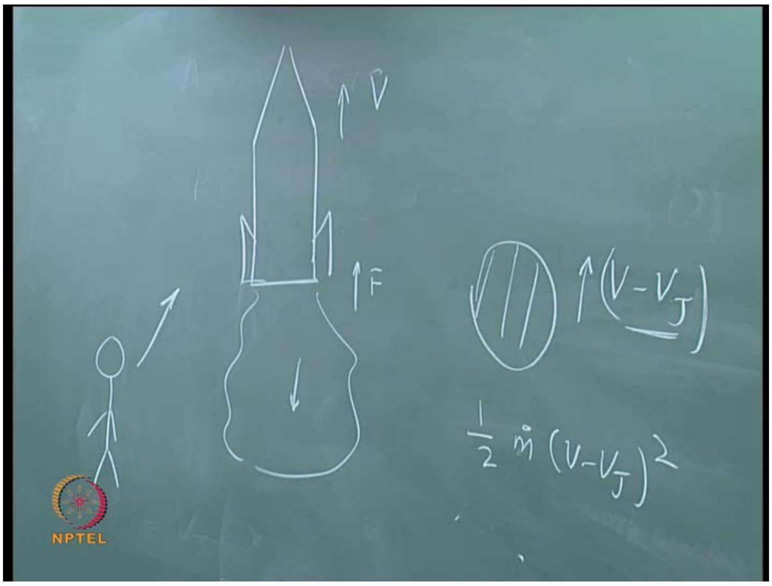

Let us try to sketch this out. All what we are telling is, maybe I have a rocket going up. It goes with the velocity V. It is pushed up with the pressure force. Therefore, the useful work which is done by the rocket is the force into the velocity per unit time. … Does it waste anything? What is the waste? How do I get the waste…? Anything getting wasted? See, as the rocket is getting pushed off the plume is going down. Again, I picture myself. I am in the inertial frame of reference. I am standing here, watching the fun of the rocket going up. What do I see? I see that this plume is now going down with the velocity V J with respect to the rocket or rather if as the velocity of the rocket is going up, the plume is going up with a velocity V minus V J.

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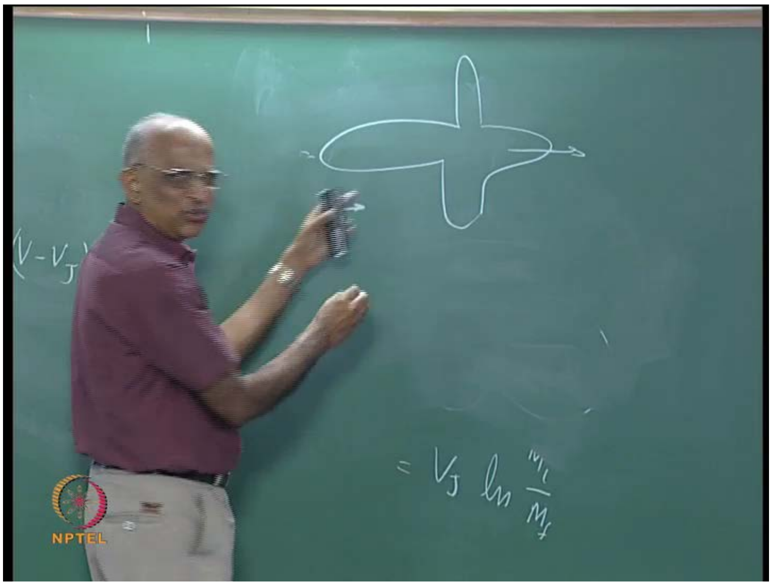

I will give you an example you know. See, on on some of these, you go out of your hostel, maybe at early morning 5: 30 or 6: 00 and watch. May be the, some of these jets go leaving, flying across Chennai, you will see the trail, may be an aircraft is moving. You see the aircraft is going and…

Let us try to picture it out. Let us picture the trail. Aircraft is moving. You see the aircraft going, and then you will find the smoke from this. Trail is also following at a slower speed. Why does it have to happen? Maybe because this fellow is leaving the rocket with the velocity V J, rocket is moving with the velocity this; therefore you see this particular jet or plume as it were following it up with the velocity V minus V J. Therefore, what is being wasted? The energy content of this is getting wasted because it is getting lost. And, what do I see? I, in the inertial frame of reference look at the work done by the rocket per unit time. But, I also see that this is getting wasted. And, what is my waste? That kinetic energy is getting wasted or mass of this into V minus V J squared or what is the rate at which I am seeing is half m dot into V minus V J square. That is the waste. That means the rocket is going up; this fellow is still following it up like this. And therefore, this is waste. It could have done much more job. It did not get wasted.

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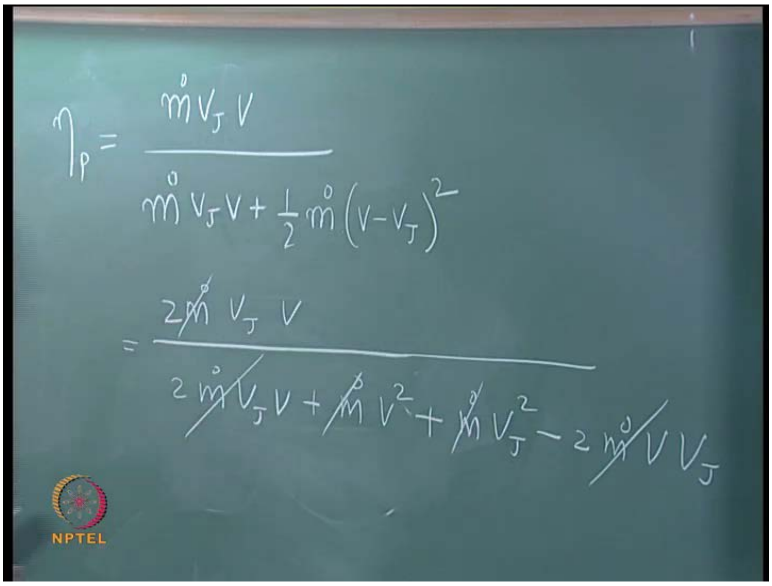

And therefore, how will I put my propulsive efficiency together? I will now write the equation as eta p, propulsive efficiency is equal to the useful work done. It is equal to m dot, V J is the force into V of the rocket divided by useful one m dot V J V plus the kinetic energy by unit time; m dot into V minus V J whole square.

Please let us be very clear. See, this will keep on haunting us. You know, we will have to define the efficiency of the scram jet. We will have to define the propulsive efficiency of an aeroplane. We will find that there are some optimums. And, I find some research work going on. I will refer you to a paper in today's class itself. In that, the way people tend to think, can we improve the rocket by looking the propulsive efficiency? Let us simplify this equation. This is equal to m dot into V J into V divided by, now it finds out to look like this. I bring 2 on top right. Then, I get 2 m; 2 m dot into V J into V plus m dot V square plus m dot V J square minus, sorry, minus 2 into m dot V into V J. Is it all right? V square minus 2 V V J plus V J square V square V J square 2 V V J. You find that this and this gets cancelled; m dot gets cancelled on; the numerator and denominator. And, what is the propulsive efficiency therefore equal to?

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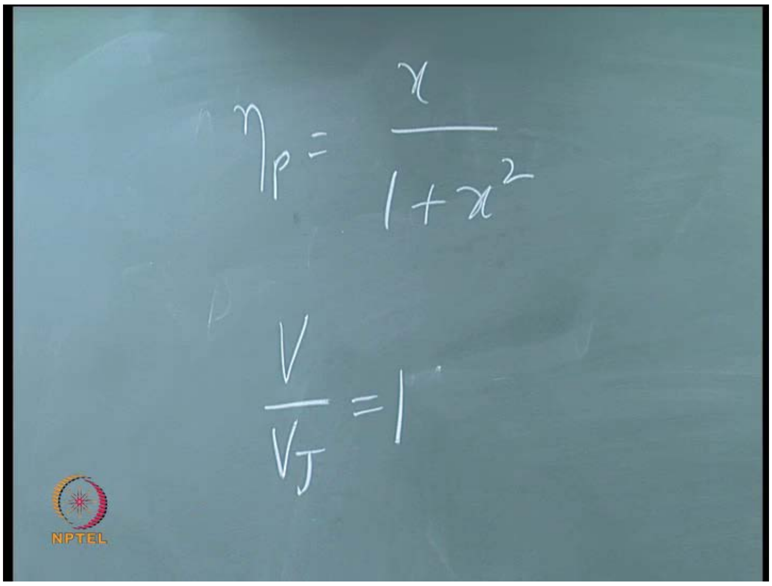

Propulsive efficiency is therefore equal to 2 of V V J by V square plus V J square. Is it alright. V square plus V J square in the denominator. Let us simplify it. Let us divide by V J the numerator and denominator or we get, we divide it by V J square; 2 V by V J divided by 1 plus V by V J whole square is equal to efficiency of propulsion. Is it alright.

Now I want to ask you, when will the propulsive efficiency be a maximum. Just look at this expression. Just be unbiased and tell me whether I can identify a condition for the propulsive efficiency to be a maximum. We will anyway solve. We will find out the maxima and solve it. But, I am looking at this expression. Can you tell me when should the efficiency become maxima? Yes. Anybody? by looking at the expression? All what we are saying is n p square is equal to plus or 2 x is divided by 1 plus x square. What should be the value of x which for which eta p is the maximum? Efficiency cannot be greater than 1. It has to be 1. And, we find that the moment V by V J is 1. It becomes 2 by 1 plus 1; 2. Therefore, by inspection itself I can say when V by V J is equal to 1, then the propulsive efficiency will be a maximum of 1. And, how do we do it? Normally, we go through the Mathematics of it.

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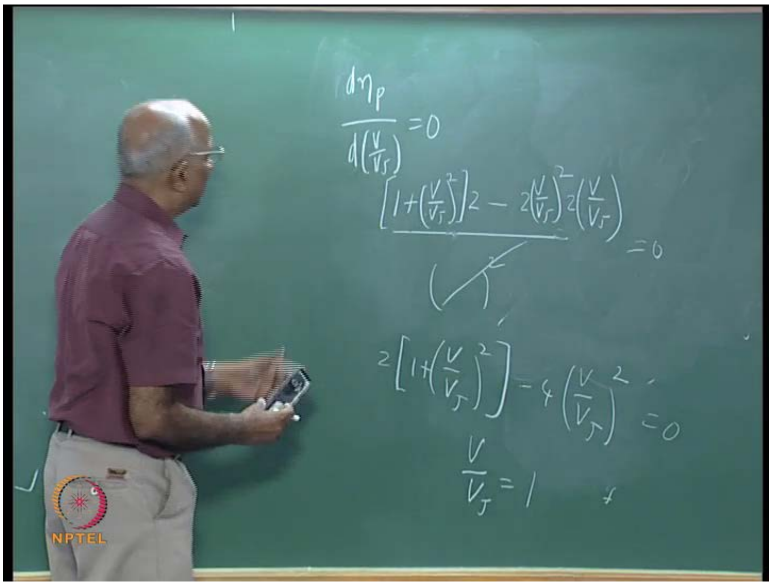

We will tell ourselves, let me differentiate it. Let us lets do that exercise. d eta p by d of V by VJ must be equal to 0 to give me the maxima. And for that, you would say yes denominator square into denominator 1 plus V by V J square into differential of numerator; 2 right, minus what? Numerator into differential of denominator; 2 of V by VJ. And, this must be equal to zero. Therefore, I am not really bothered. I do not want to write 1 plus V by VJ whole square over here. And therefore, what does it give me? It gives me 2 into 1 plus V by VJ whole squared minus four into V by V J square; is equal to 0. What does this give you? 1 plus 2, 1 minus 2 V by V J, 2 minus 2 into V by V j or rather it gives you that V by V J must be equal to 1 to get the maximum… Could you do this? See, but you know we must be able to draw inferences by looking at… some by inspection itself. What, we get this, what comes out of differentiating…

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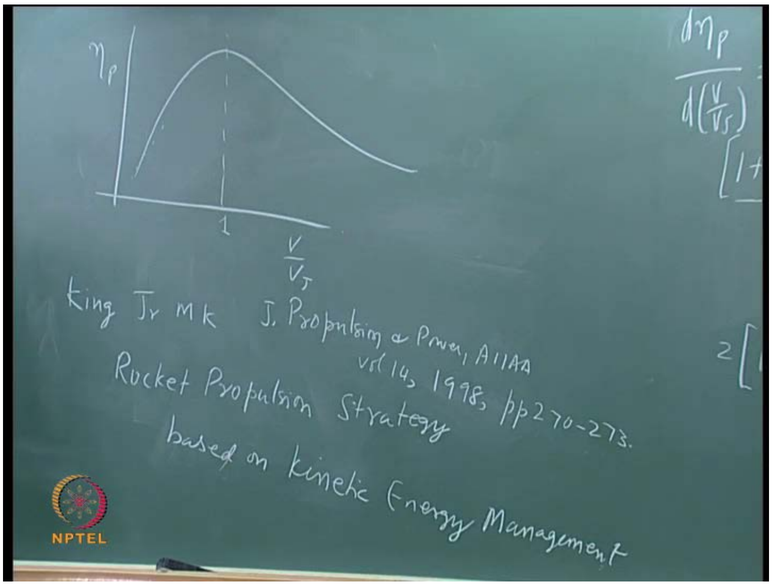

Therefore, all what we are trying to say is, well, if I were to plot the propulsive efficiency of a rocket eta p as a function of its flying velocity divided by the jet velocity, the efficiency becomes a maximum when the velocity is 1. And, there after it begins to go down. Therefore, this gives me some suggestion that if I can have the exhaust velocity equal to the velocity of light, then I may able to get the maxima of the propulsive efficiency. Or rather, as the velocity of the rocket changes, if I can somehow keep on changing my exhaustive velocity, I get the better efficiency.

But then, you know just not possible, a rocket is this, a rocket is a thing which is rapidly accelerating and all that is very difficult to meet this. But, there are some references. And, one paper which deals with this and which is very exciting to read this. I will give you that reference. May be you should take a look at it. It is by “King Junior MK”. The title of the paper is “Rocket Propulsion Strategy based On Kinetic Energy Management”. You know, it appeared in “Journal of propulsion and power”. I just write it down here; “Journal of propulsion and power” of AIIAA. The volume number is fourteen. It is in the year 1998 and the page number is two hundred and seventy to two hundred and seventy three. May be if you all, should go and take a look at it.

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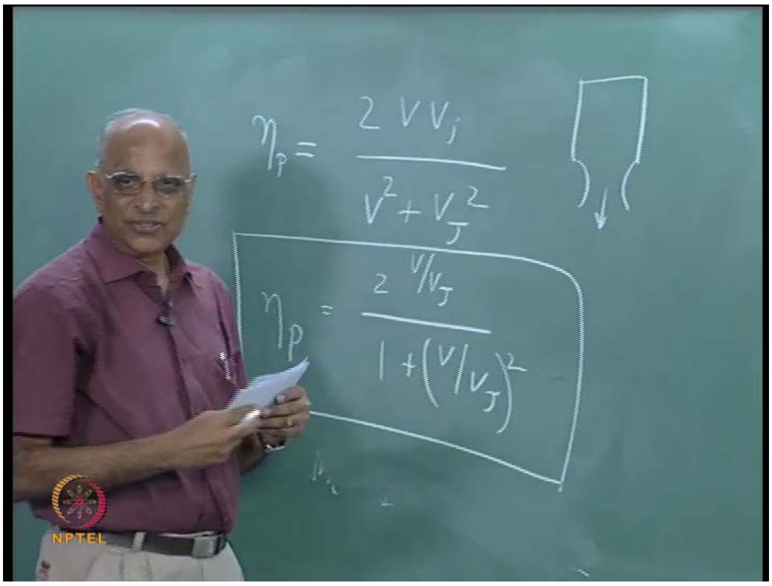

See, we know it is not possible. But still, what does we try to do is, can you somehow get something to do with V J and try to see whether you can get a better propulsive efficiency. Such ideas are useful. You know, because later on we will get into electrical propulsion, nuclear propulsion. We will try to see, whether we can somehow make with a rocket more efficient; because as of today even to go to Jupiter, we saw it takes something like five years. If we have to go to the Kuiper belt it takes something like ten years. Whether be with respect to… going to different galaxies, these different stars, we need to improve the rocket. Therefore, this gives some clue of this. Therefore, this article by King is something which was thought for working at the beginning itself. It says it is a blue eye tutorial for students. But, I think it is something about which we should take a look.

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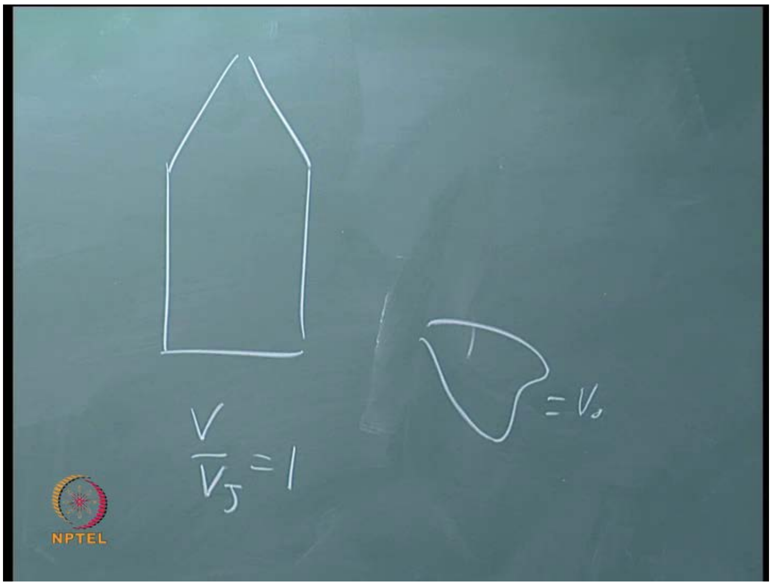

But having said that, I want to ask you one question; under what condition is propulsive efficiency a maximum? See, we told ourselves the rocket is flying up and now when this is flying up, you see the trail following it. What is the condition of the trail or the plume when V by V J is 1? That means the kinetic energy of the plume is what? 0. What happens to the trail? I see a rocket going up; the plume is also going up, what will happen to that plume when propulsive efficiency is one or maximum? That means V minus V J is 0; that means it will be kept static; that means it has no energy at all.

In other words all what we are saying is, the value of this plume becomes V is equal to 0. It does not follow the rocket. It just stays, put at a particular place. And in other words, we have made use of the total kinetic energy for pushing the rocket up. This is the influence of the propulsive efficiency. I do not think there is anything else to do. I think I shall go through two or three case studies. I will start with the SLV three-problem or how we calculate the masses, payloads and all that, the first part. And then, do a simple problem using this water rocket. But, since this water rocket is here on this on this particular slide, maybe I get started with this.

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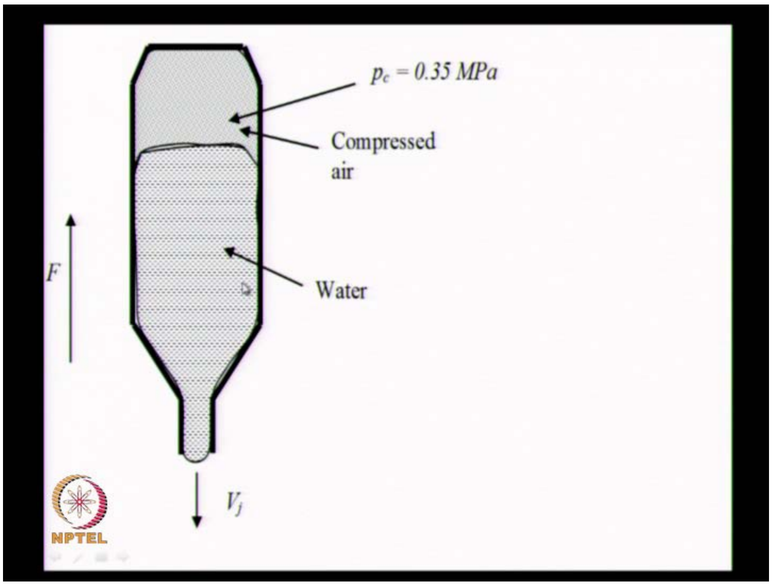

Let us try to solve this problem. See, the problem which I force to you is the following. May be I will put it upon the… See, we have the bottle which contains water; we have high pressure gases above it. I want to push out the… I push the liquid water or the water out through this using compressed air. The compressed air pressure is told to be something like 0.35 Mega Pascal. That is, 3.5 atmospheres. I want to find out the size of the hole such that the rocket leaves the ground at a given acceleration. This is all about… how do I size this rocket? You know this volume is given, this volume is given, but I need to know the size of the vent of the hole such that the rocket can leave with the given acceleration. Let us do this problem.

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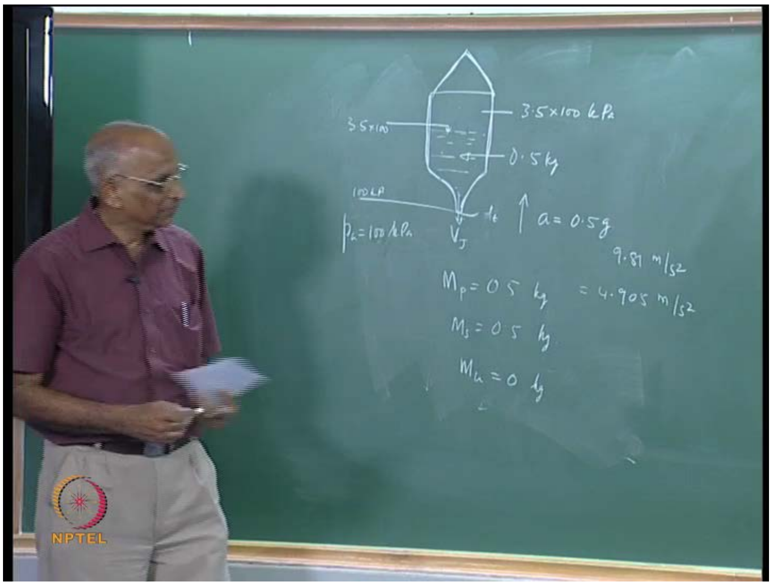

Therefore, let me write this problem on the board. The Mass of water is given to be 0.5 kilogram. You have in a bottle, 0.5 kilogram of water. The air is relatively massless which is above… and the pressure of air is 3.5 3.5 into 100 kilo Pascal. That is the 0.35 mega Pascal. You know, it is also told that the mass of the bottle that is the structure of this, all these things put together is again, that is the mass of the structure is 0.5 kilogram. And, all what we are interested to know is, we would like this rocket to leave with acceleration.

The level of acceleration is to be 0.5 g. That is g is the… acceleration due to gravity or a gravitational acceleration is 9.81 meter per second square. With respect to that, it is half the value of the gravitational field. That is the value with which it must get pushed up. Now the question asked is, what must be the size of the diameter of the vent or the hole by which the water… how should we doing this problem? See, everything is available to you. You have the mass of water 0.5 kilogram. Let us say the mass of water is mass of propellant which is used for pushing add up 0.5. The mass of the structure is equal to 0.5 kilogram. What is told to you; what is not I have not told is, there is no payload in this. Just the bottle is moving up. Therefore, the useful payload M u is equal to 0 kilogram. Now, you are asked to find out what must be the rate at which water is getting pushed out; because if you know rate at which water is getting pushed out, I can find out what must be the diameter of this hole.

Therefore, how should I do this problem? What are the things that I should do? Let us say I first want to find out what is the velocity with which the water will leave this particular hole. In other words, I am interested in finding out the jet velocity V J. Yes. How do I get V J? We said V J, I must find out. How do I get the value? The gas pressure is 3.5; the ambient pressure is 100 kilo Pascal. That is, the air pressure p a is equal to 100 kilo Pascal. Water is incompressible, how will I find out? Let us neglect the height of water. May be from Bernoulli's equation, we can say p for the compressed air which is over here or compressed gas let us say. p by rho plus what? How will you write the equation? How do I calculate the value of V J is the question. I am pushing out with pressure over here. What is the flow rate or what is the velocity with which water level…?

Let us say that you have on this surface of water, the pressure is 3.5 into 100 kilo Pascal. On the surface of this, the pressure is 100 because this is how it is seeing the ambient. Therefore, for the water column I am writing the Bernoulli's equation. Therefore, I have pressure on top, which is equal to 3.5 into 10 to the power 3 plus 2 5 in 3.5 yes into 100 into 1000 Pascal divided by the density of the water plus, this is being the large diameter that the velocity is 0. Therefore, velocity square by the 2 is equal to 0 is equal to the one which is at the bottom. 100 Pascal divided by the density of water plus the velocity with which it is exhausted out, divided by 2. Is it alright. Or rather, you know we could just told ourselves, well, V J square is equal to under root of 2 of delta p divided by rho. No, no, we are talking in terms of V square is equal to delta p divided by rho. And, that would have given us the same thing what we are solving for; namely is equal to under root 2 into 3.5 minus 1 is it? 2.5 into 10 to the power 5 divided by the density of water; 1000 kilogram per meter cube. Is it alright. Then, what is the value coming out to be? If you calculate, you get it to be equal to 22.36 meters per second.

You know, we should be able to do any problem. You know, that is why I use this particular water problem. You are pushing off water, pushing off some pressure. You know the velocity with which it is coming out. And that, the velocity with which coming out is 22.36 meter per second. What about the… See you know, I say let us neglect the height because the height is small; because you have the pressure which is so high, then the height will not really matter. But in a real problem, yes, I would like the height of water… the weight of water due to the height also to be mentioned. Now, we just wrote the Bernoulli equation. Now, I want to make use of this jet velocity and find out what must be the diameter of the hole. But, what is given to me? Something important is given to me. It is told that the rocket should leave with an acceleration of 0.5 g or rather with an acceleration of 4.905 meter per second square. How do I get this? Somehow I have to get this. That means I must be able to calculate the force. And, that force I have to convert it to acceleration and make sure I get this acceleration. How do I do this problem? Let us revise what we have just now done and in that in that process, do this problem.

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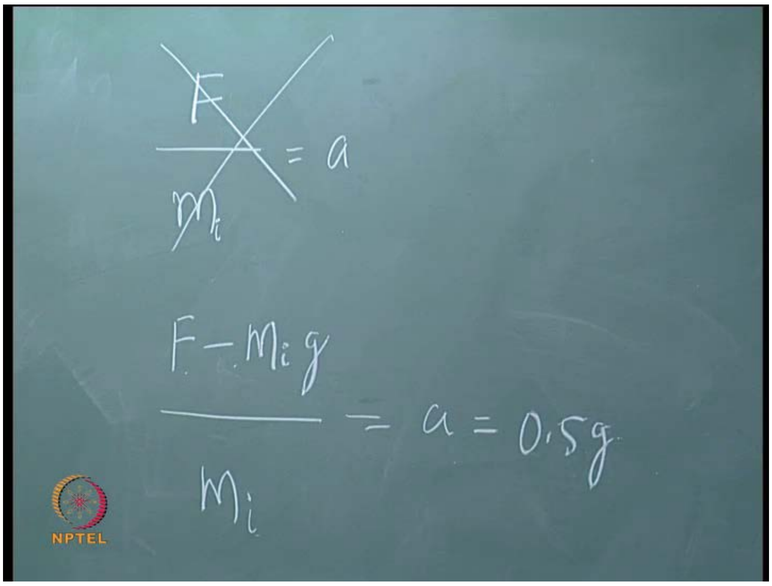

I would like to find out what is the force which is generated by the particular rocket divided by the initial mass of the rocket, which is the acceleration of the rocket. And, the rocket is going up. Therefore, what is the acceleration with which it is going up? It is equal to F minus the gravitational acceleration of the mass divided by m i; is the true acceleration with which it is going up. See, this differentiation is important. I say force; force is what it is pushing it up. As it is pushing up, it is it is the acceleration; the gravitational field is also exerting a force is equal to m i g. Therefore, F minus m i g divided by m i must be a and not this. This is what when we have no gravitational field. Therefore, now how do I solve? This a is given to be as 0.5 g. Let me erase this portion by just writing that V J is equal to 22.36 meter per second. Yes.

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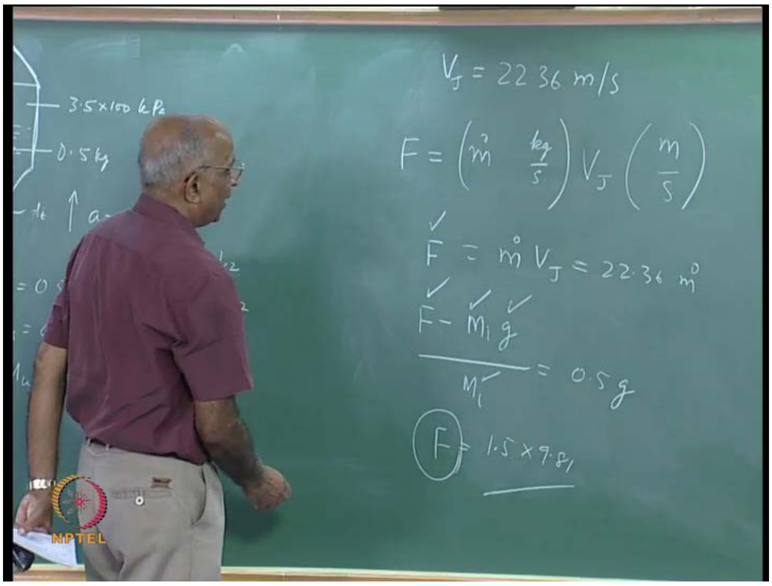

What is the value of F? Let us assume, let water flow at the rate of m dot kilogram per second; because if I know the water flow rate, I can calculate this diameter. Therefore, F is equal to m dot kilogram per second into what? How do you calculate the force? We have done it. Change of momentum, m V J is momentum; force is equal to d by d t of m V J which is equal to m dot V J. Therefore, what is missing here? What should I write here? V J, m dot V J is force; so much meter per second. That means the thrust is equal to m dot V J. And, V J we have already calculated. It is equal 22.36 into m dot is equal to the force.

Now, what is the force? I go back to the equation what I wrote here. I get F minus m i into g divided by m i is equal to 0.5 g. Here g is 9.81. What is the value of initial mass of the rocket? 0.5 plus the structural mass is 0.5, which is one. Therefore, I know the value of m i, I know the value of g, I can calculate the value of F. Then, let us calculate it out. Therefore, F is equal to… I take it on this side. We said m i is 0.5 plus 0.5 is 1. It becomes 1.5 g; that is 9.81 and m i was 1. Therefore, force is equal to 1.5 into 9.81 into… Is there anything else missing? What is the value of m i we have got? See, we said m i is equal to totally 0.5 plus 0.5. right. And therefore, I can calculate the value of F and once I calculate the value of F which is 1.5 into 9.81, I put it over here.

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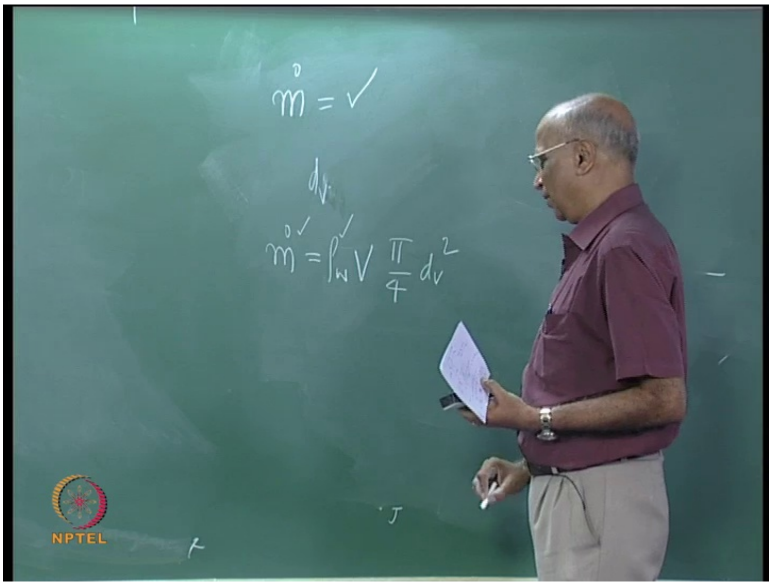

And, what I get is the value of m dot. And, if I know m dot, how do I calculate the hole diameter or diameter of the vent…or the diameter of the hole? m dot is equal to; we get the value, the density of water into the velocity of water into the area. Area is equal to pi by 4 into the whole square. You have already calculated the mass flow rate; density of water is 1000 kilogram per meter cube. You have calculated the V J. This is the velocity with which the liquid is leaving; V J square pi by 4. Therefore, the only unknown is d V and this is how you design the vent diameter.

I think I leave it as carryover homework for you all. All what I want to tell you is, it is possible to calculate the thrust. You need the value of V J; V J we find through simple calculation. It is possible. And, once you know this, I can always relate it to the acceleration. I will not complete it. You know I will just be spending too much time on it.

I rather go to next problem. But can we all, can we compare the notes on let us say in the next class. Just, let sure what is the number of the diameter of what we get. Let us say is it going to be a few millimeter? Is it going to be a centimeter? Is it going to be meter? May be, I would like to take a look… You know by infusion, you expect it to be about the centimeter…. Right. Let us do it.

I think I think with this, maybe I think I will stop the class. I want to do a problem on the masses; structural masses structural mass, may be the propellant mass and the acceleration for some multi stage rocket vehicle. May be in the next class, I will just go through one or two small examples on it and then we go to the next topic, which is on nozzles. That also we will cover it in next class. Well, thank you.

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