Rocket Propulsion
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Lecture No. # 08
Examples Illustrating Theory of Rocket Propulsion and Introduction to Nozzles

Well. Good morning, in our today's class we will recap what we have done so far first may be in a couple of minutes. And then, we will solve one or two small problems, such that we are fully aware or we will illustrate, how the theory of rocket propulsion works and towards that.

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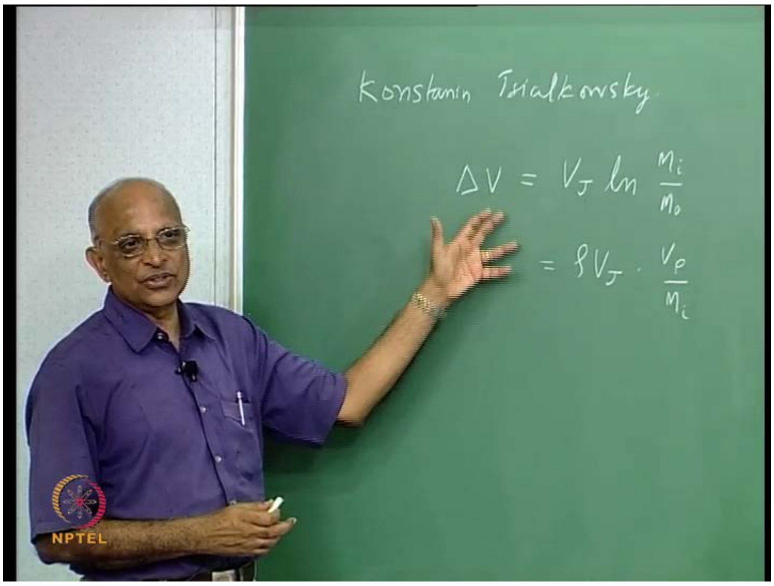

Let, let us quickly a recap, we talked in terms of the Russian school teacher, we said this constant in Konstantin Tsialkowsky. We told ourselves we derived the rocket equation, we derived that in class. We told ourselves that the ideal velocity, which is given by a rocket is given by the, a flux velocity or the jet velocity. That is the velocity, with which the gasses leave the particular rocket and the logarithm of the initial mass of the rocket divided by the final mass of the rocket, and this is what we call as Tsialkowsky's equation or the rocket equation.

We also went one step further, we ask ourselves suppose, the rocket as a small amount of mass so, of propellant and more of inners in it, we found that this equation gets slightly modified as, rho times the jet velocity into we said volume of the propellant divided by the initial mass. Therefore, we will try to do one or two small problems, such that we illustrate how this ideal velocity works, because we have seen what is the payload mass fraction, we saw what is the propellant mass fraction, we saw the structural mass fraction. We wanted to relate these things, which we did do in the earlier class. Let us go ahead and solve one or two small problems.

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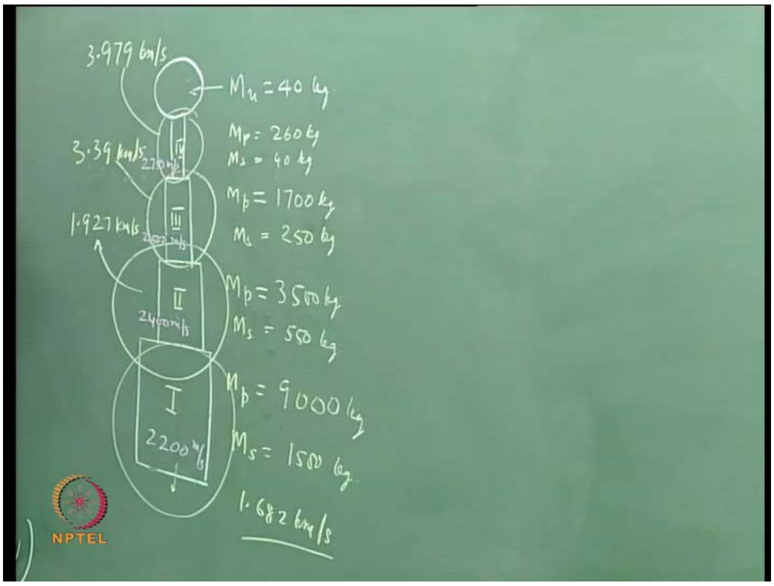

I take a problem of let us say, a 4 stage rocket to begin with. Let us say, I have a booster stage which is the first one, which first takes off and then on top of it I have the second stage, then I have the third stage and then I have the fourth stage and on top of it fix my useful mass which we call as payload. Let me take a typical example, in which the top mass that is the useful mass, which we called as useful mass, is equal to 40 kg. What do you think should be the weight of these stages? Let us take an example, this is one of the large vehicles which we had in India, which I am illustrating may be the numbers are representative, may not be exact.

The first stage has propellant mass of something like, let us put down the propellant mass in yellow color. The propellant mass of the first stage, is equal to 9000 kg and the mass of the structure; that means, the casing other things which are there including in hertz, is equal to 1500 kg. The jet velocity of the first stage is equal to, let me put it inside in pink color and that is equal to 2200 meters per second, is the jet velocity of the first stage. Now, on top of this stage which is; which has a mass of 9000 kg of propellant, the structural mass M s is 1500 kg.

The second stage sets and this stage as a propellant mass fraction, for the propellant mass is equal to 3500 kg and the mass of the structure of this stage is equal to something like 550 kg. On top of the second stage, you have the third stage, in which case the mass of the propellant is equal to 1700 kg, and the mass of the structure here, is equal to 250 kg. Then we said a fourth stage large vehicle, this is the fourth stage over here and the fourth stage as a mass of propellant, which is around 260 kg and the mass of the hardware including the structure is equal to 40 kg.

Just like I said that, the jet velocity of the first stage is 2200, the jet velocity of the second stage is slightly higher a 2400 meter per second. Of the third stage, it is 2500 meter per second and the fourth stage the jet velocity is 2750 meters per second. Therefore, we have this 4 stage rocket, on top of which you have a useful mass or payload mass of 40 kg. It takes off from the ground, and our effort in the to solve this problem is, what is the value of the delta V of this rocket or what is the total value of the ideal velocity, which are rocket gives.

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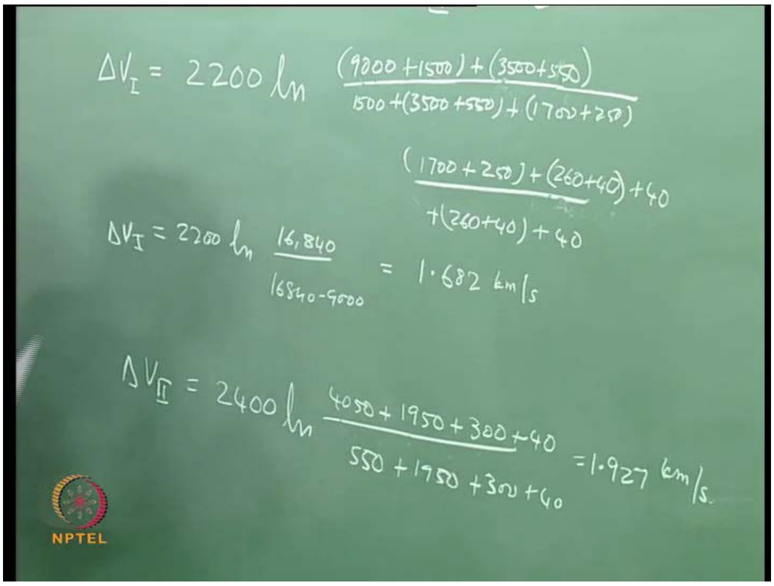

$$\Delta V_I = 2200 \ln \frac{(9000+1500)+(3500+550)+(1700+250)+(260+40)+40}{1500+(3500+550)+(1700+250)+(260+40)+40}$$

$$\Delta V_I = 2200 \ln \frac{16{,}840}{16840-9000} = 1.682 \text{ km/s}$$

$$\Delta V_{II} = 2400 \ln \frac{4050+1950+300+40}{550+1950+300+40} = 1.927 \text{ km/s.}$$

We have already is told ourselves well, this is staging first stage, second stage, third stage, fourth stage. If, the first givers a velocity increment ideal velocity of delta V 1, that is available when the second stage starts of therefore, we can summing of one, two, three and four. I can say, that the velocity, the net ideal velocity of the rocket is equal to what is provided by the first stage plus what is provided by the second stage plus what is provided by the third stage plus what is provided by the fourth stage. Therefore, my main effort has to be, to find out what is the delta V of the first stage, second, third, fourth. I just arithmetically add it up, I will know what will be the velocity with which the payload, the ideal velocity given to the payload by this fourth stage vehicle.

Let us do the calculations. Let us do for one and two and by and similarly, we can say we can do for the second and third stage and the third and fourth stage. The first stage, you find that the exit or the jet velocity is equal to V J, is equal to 2200. Therefore, we have 2200 meters per second into logarithm of the initial mass of the rocket to the final mass of the rocket this is, where we must be very clear. What is the initial mass of the rocket I take off from here? It includes all the mass of propellants, all the mass of structures plus this is over here.

Therefore, if I were to write the expression for the initial mass, which as we saw in this expression, is the numerator, it is equal to for the first stage, the propellant mass is 9000, the structural mass is 1500, now I add the second stage which is equal to 3500 plus the structural mass is 550. I keep adding up let us; let me put the bar over here let me, continue with the numerator here. I find for the third stage, the propellant mass is 1700 plus I have something like 250 is the structure plus the last one I have 260 plus 40 plus, I have the useful payload which is plus 40. Now, what is the final mass of this rocket after the first stage is performed? What must, I remove or what must be done?

First stage propellant gets burnt out and therefore, the total mass will be this gets knocked off, I have 1500 plus I add these two, may be 3500 plus 550 plus I have this one, which is equal to 1700 plus 250 plus I have 260 plus 40 plus I have the value of 40 and this gives me the value of delta V 1. Let us simplify I get therefore, delta V 1 is equal to 2200 log of, I add all the masses 9000 plus this, 3500 plus 550, 1700 plus this, plus this gives me the total initial mass. And when I derive, I will; I get the value that is the total mass is 16840 kg and in that I subtract the value of 9000, that is 16840 minus 9000 and this gives me the velocity of 1.682 kilometers per second or 1682 meters per second, this is in meters therefore, I would have got 1682 meters per second which is 1.682. Therefore, let us put the value here, I get the velocity of the first stage delta V 1 as equal to 1.682 kilometers per second.

Now similarly, I go to the second stage, when I come to the second stage, well, it becomes simple for me to today. Now, I know what the initial mass of the rocket, let say delta V 2 of the second stage, is equal to, now my jet velocity is 2400 meters per second, therefore, I have 2400 meters per second. The logarithm of now let us do it a little faster we find that this has; this is no longer in the picture. I start with the initial mass of the rocket should be the second stage, plus third stage, plus fourth stage, plus the payload weight. And what is the mass of the second stage? 3500 plus 550 that is 4050.

Now, the third stage 7700 plus 250 that is 1950, the stage fourth stage 260 plus 40 which is 300 kg plus useful 40 and what is the mass at when the second stage has stop functioning? Only the inner tray it is left, that is equal to 550 kg, plus the upper stage is are still plus 300 plus 40. And therefore, this will give me the value of 1.927 kilometers per second. Therefore, the second stage gives me a value let us, put this here of 1.927. Now, I repeat the calculations for the third stage and how do I get the calculations for the third stage? The jet velocity is 2500 into logarithm of the mass of this plus, this plus, this which is 1700 plus, 250 plus, 250 plus, 40 plus, 40 which is the initial mass.

At the end I am; I have depleted this amount of propellant, I am left it 250 plus 300 plus 40, which gives me velocity of let us put this over here, this gives me a slightly higher velocity of something like 3.39 kilometers per second.

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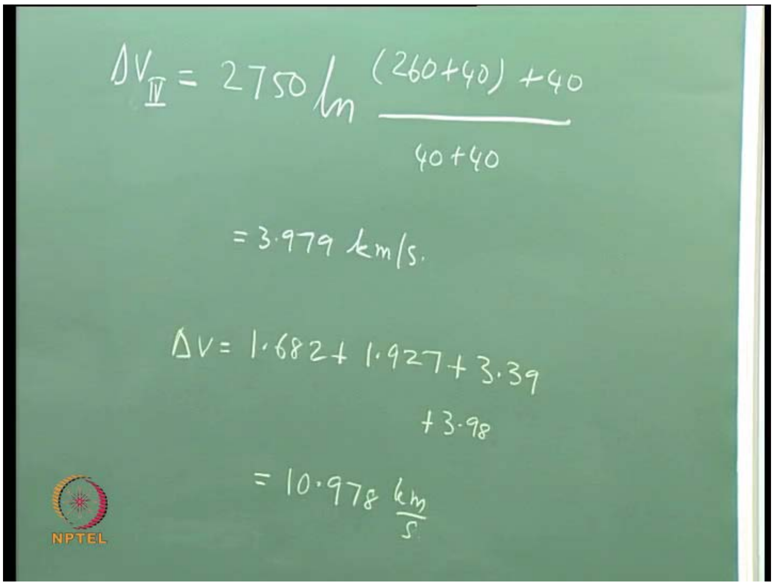

$$\Delta V_{IV} = 2750 \ln \frac{(260+40)+40}{40+40}$$

$$= 3.979 \text{ km/s.}$$

$$\Delta V = 1.682 + 1.927 + 3.39 + 3.98$$

$$= 10.978 \frac{\text{km}}{\text{s}}$$

And let us do the last one and the last one let us say put the delta V 4, such that, we are absolutely clear this is how we do any calculation fourth stage. And we find that the jet velocity of the fourth stage is 2750 meters per second. That is 2.750 kilometers per second into logarithm of the initial mass is, 260 is the mass of the propellant, plus 40 is the structure, plus 40 and what is left after the fourth stage burns? 40 plus 40 is what is left. And this gives me the velocity as equal to something like 3.979 kilometers per second. Let us also put this into this vehicle here, the fourth stage gives me a velocity of 3.979.

Now, let us try to draw some inferences from this, we find that the first stage gave me a velocity increment of something like 1.682 kilometers per second. The second stage gives me a slightly higher value 1.927. Third stage gives me a significantly higher value 3.39, while the fourth stage gives me an; at higher value. That means, the upper stage is contribute to more delta V than the lower stages and the reason being, one is the jet velocity of the lower stages is generally smaller than the upper stages. And you have the benefit of a lighter rocket which can give you a higher delta V. And therefore, what is that total delta V of the rocket we just add the velocities.

That means, the total delta V which is the rocket gives, that is a added together 1.628 plus the second stage which gave us 1.927, the third stage which gave us 3.39 and the last stage which gave us something like 3.98 kilometers per second. And the total velocity what we get is therefore equal to 10.978 kilometers per second. Here, we must remember, that we have neglected the velocity gains when they used or discarded stages are removed. And this is the velocity which is available to you for orbiting plus the potential energy or the velocity to increase the height of the rocket from the ground to the particular orbit. And how do you match it with the orbital velocity and the total velocity?

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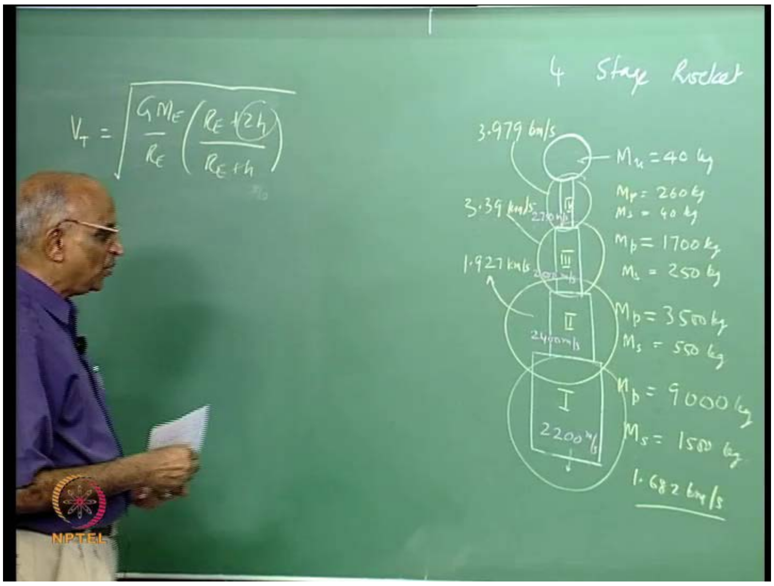

You already derived the expression, that the total velocity provided by the rocket can be written as under root G for the earth mission, M E by R E, into R E plus 2 height, divided by R E plus h and this will tell you at what height this particular rocket can launch at particular satellite. And this is how we do any velocity increment for any multistage rocket. Now, we know to be able to illustrate, do we need a strapons, what will be the initial acceleration. I give you some more data and let us do a small problem as an extension of this, suppose, we tell ourselves this fourth stage rocket, when it takes off the booster stage or the ground stage.

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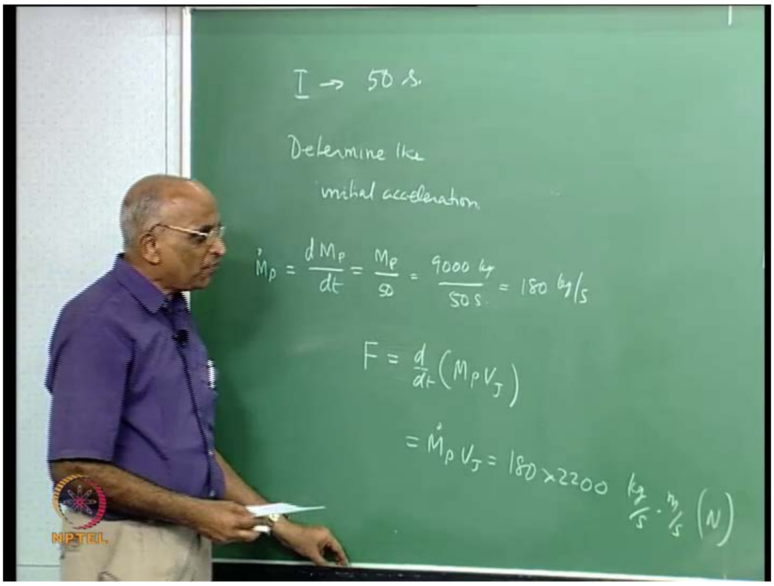

That is the first stage operates for let us say, 50 seconds. And the question post to you, if the first stage operates for 50 seconds determined the acceleration, the initial acceleration of the vehicle of this particular rocket. Now, how do will I do this? That means, each stage operates may be, the first stage operates for 50, the second stage for 35 may be and so on something like this it operates. May be the final stage operates for let say another 80 seconds or something, but the data which is given to you is, this stage which has a propellant mass of 9000 kg, operates for a time of 50 seconds. It is also given to us that the flow rate of the mass leaving the rocket is a constant.

Therefore, we can say the rate at which propellant gets depleted that is dM p by dt is equal to it is a at a constant flow rate is leaving therefore, M p divided by 50 which is equal to 9000 kg is the mass of the propellant, it is getting depleted over 50 seconds. Therefore, the rate at which the propellant is leaving the nozzle is equal to something like, 180 kilograms per second. You know, we all should have some fields for this numbers you know, you find the mass flow rate from a nozzle is quite high. We are talking of several hundred kilograms per second and the rocket is huge, like what we said is the moon rocket and all that, it will be even higher. Therefore, the mass propellant mass flow rate or rather dM p by dt, which I can also write as M p dot is equal to this.

Now, I know the value of V J, 2200. I want to find out the initial acceleration of the vehicle, how do; I do it from here. See, to be able to find the acceleration, I need the force and what is the force? The force which this rocket derives is equal to let us, write the expression before I come to this acceleration. Therefore, the force F which the rocket gives is equal to d by d t of change of momentum, which is equal to M p into V J. Which is equal to M p dot into V J, which is equal to now it is 180 kg into the value of V J is 2200 and, what is the unit I get now? Kg per second into meter per second; kg meter per second square this is so much neutron is the force with which the rocket is going on, is it clear? I think we must be able to work out the accelerations for each stage. May be, we will illustrate it a little more.

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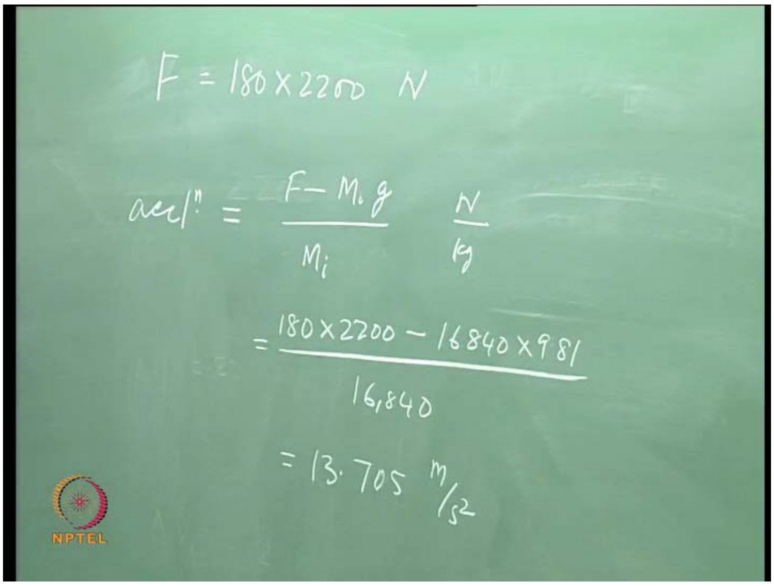

$$F = 180 \times 2200 \ N$$

$$\text{accl}^{n} = \frac{F - M_i g}{M_i} \quad \frac{N}{kg}$$

$$= \frac{180 \times 2200 - 16840 \times 9.81}{16{,}840}$$

$$= 13.705 \ m/s^2$$

Therefore, what is that I have told so far? We told ourselves that, the force which the rocket develops are the thrust is equal to 180 into 2200 neutrons. Therefore, what is the acceleration at take off? We find acceleration of this particular vehicle at take off is equal to the force minus the initial mass, which is subjected to the initial gravitational field of the earth, divided by M i is the acceleration. Mass into acceleration is equal to the net up ward force. And therefore, this is equal to 180 into 2200 minus the initial mass of the vehicle we had calculated, that was equal to how much? 16840 into 9.81 divided by 16840 and this gives me the value of acceleration as equal to 13.705 meter per Second Square. This is neutron divided by kilogram and the unit is meter per Second Square.

Therefore, do you find that, it is living the ground with an acceleration which is something like may be 1.4 to 1.5 G, that is the level with which is leaving and this is how you calculate the initial acceleration. Is there anything else we could do? Supposing, I were to start reflects say that, they are fourth stage just taking off. I want to know the acceleration of fourth stage when it is taking off. Let us say, I know the time over with the fourth stage once I know the value of M p dot, I can calculate the force. I corrected for the attraction of this initial mass by the earth, divided by the total mass that will give me the acceleration over the earth. I can therefore, find out the acceleration at any particular time and this is how we determined the acceleration.

The last thing, which I would also like to know is, we determined a lot of things supposing I were to ask you, because we did exclusively considered, what is the payload mass fraction of this vehicle?

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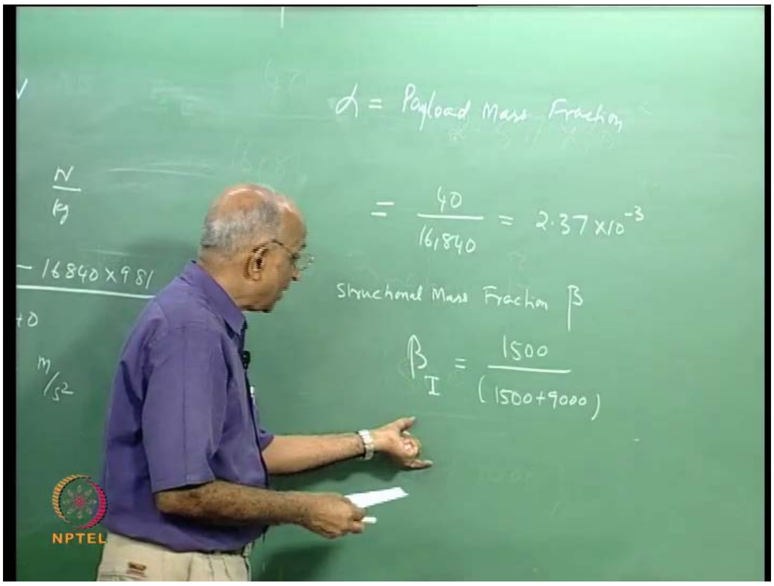

Let us say, what is the value of alpha which we said is the payload mass fraction? Can you tell me what it should be? Useful payload, exactly 40 kg divided by total vehicle weight and that we said is something like 16840, it is number comes out to be a very small number. Let us put down this number, it comes out to be something like 2.37 into 10 to the power minus 3. Or rather the net useful thing what comes out of this rocket is something like 0.02 percent of the total weight that is the only useful thing which is the rocket is doing. Therefore, you find that in rockets view, we waste a lot of things.

And the aim is now a days, how do I improve this fraction to something like 2 percent, 3 percent, which will be a good one and that is what we will be considering in the subsequent classes. We talked of the payload mass fraction, supposing I ask in terms of the structural mass fraction let say beta, but then, I have four stages, first stage, second stage, third stage, fourth stage. Let me say, I am interested in the structural mass fraction of the first stage, what will be the value? Yes, the structural mass is 1500 kg divided by here we should be a little careful, I am asking for the first stage. The first stage consist of 1500 kg as structural mass and the balance 9000 is the propellant mass therefore, the total mass of the first stage is 1500 plus 9000 and this is the structural mass of the first stage alone.

If I consider the second stage well it is going to be something like 550 divided by 3500 plus 550, I can calculate for this third stage, I can calculate for fourth stage. And this is how we calculate the payload fraction, structural fraction, may be a propellant fraction I can similarly, calculate and we calculate that the total velocity and the acceleration. this is all about the way we go ahead calculate the theory of rocket propulsion namely how much velocity is given, is it alright? Let us do one more problem, you know this was for a rocket. Let us do a very small problem relating to let say a satellite, we told ourselves that, a satellite carries some amount of fuel or propellant and the movement that the propellant is over, we say that the useful life of a satellite is over. Therefore, let us take an example.

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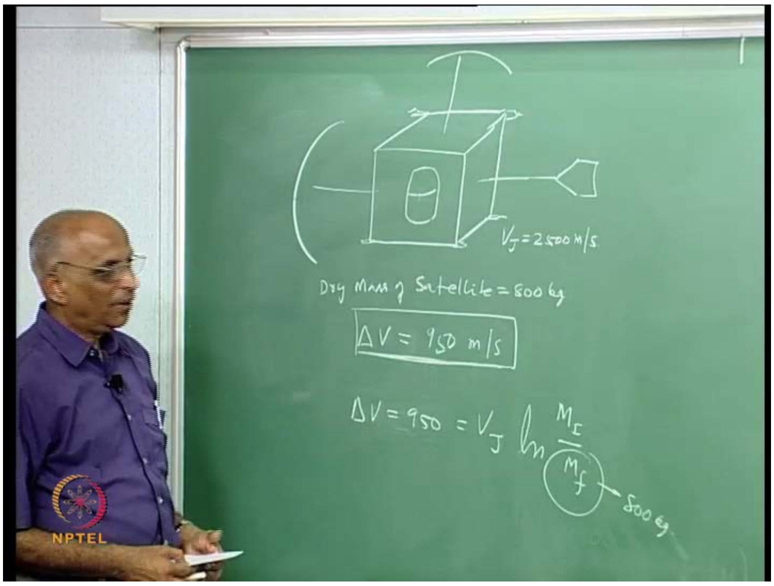

Let me take an example of let say insat. The insat, if you see consist of something like a cube like thing this is the basic structure, inside this compartment, this is are all ribs and all that which are there. I carry something like a propellant tank and in this propellant tank may be I take some propellants may we will get into details of this when we do the liquid propellants. I attach a series of rockets at the edges over here, a lot of them something like 16 of them, such that, I can still can do whatever I want in space. Supposing, I have a problem, the problem given to me is the dry mass of the satellite is given to me as let us say 8000k; 800 kg, what is that dry mass?

Mass of the structure, mass of the tank, maybe I could have something like an antenna over here. May be some antenna over here, maybe I could have something like an solar over here. May I could have some more antennas over here and this is how the satellite is going around round the earth. Now, we find that to be able to maintain this satellite let say for 10 years or 15 years. I need to be able to configure the rockets such that, the total delta V, what is required could be around, let us take an which I worked out this morning, It is equal to let us say 950 meters per second is the total incremental velocity what is require?

I should qualify this, what is required is, whenever you have the satellite going not pointing correctly, I must give a small impulse to the satellite; that means, I have to give some height to the satellite impulse is change of momentum. I know the mass of the satellite therefore, I can find out what is the delta V which I must give to the satellite. I keep on adding the different delta V is for attitude control for station keeping, I find that during the total machine of this satellite for a few years I need to give a delta V of 950 meters per second. Now, I ask myself a question, a designer will ask how much fuel or how much propellant do I carry in this space craft? This is what I want is to solve.

In other words, all what we are saying is, the delta V to be provided by the rockets is equal to 950 meters per second. And then, I need to know, what is the jet velocity of the different rockets and then I say of the initial mass divided by the final mass of the satellite, because initially I have some mass after propellants are exhausted I have a final mass. What is given to me is that dry mass? Dry mass is the final mass when all the propellant is gone, therefore M f is given to you as equal to 800 kg. It is also given to us when we qualify all these rockets, we will find out what is the exhaust velocity with which it moves, the exhaust velocity is given as V J is equal to 2500 meters per second.

In the next class we will find out how we calculate this value we will see how to doing. Therefore, now my question is what must be the mass of propellant which I carry in this particular rocket? How do I do? Yes, please let me know, the initial mass of the satellite will also contain the propellants in it.

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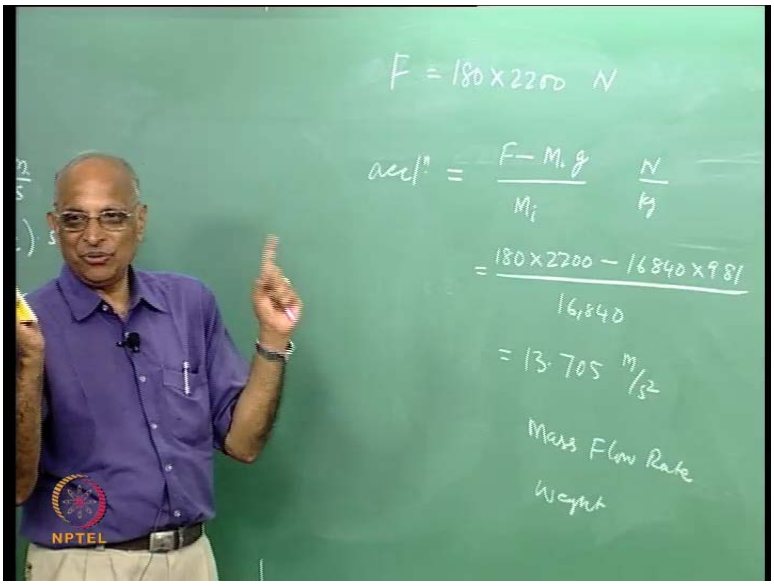

Therefore, the initial mass is equal to m I, 180 is the final mass plus the mass of the propellant. And I find that my delta V what is required is 950 meters per second therefore, I have 950 is the velocity, increment what I require? The jet velocity is 2500 into logarithm of 800 plus the mass of the propellant divided by 800. And we know, what is the mass of propellant to be (( )). Let solve this equation, we get 800 plus M p mass of propellant in kg, divided by 800, is equal to e to the power, I take exponential on both sides, I get 950 divided by 2500 and this gives me a value around 0.34 this is the long value and the other value something like 1.462. Or rather, I get the value of M p that is the propellant mass required is equal to 370 kg and this is how you charge the propellant.

You will have to do a machine supposing the satellite has cooperate for 20 years, may be I require more of fuel because more corrections are require. If I want it for one year I can have this and that is why you know whenever we read the news paper somebody is says the initial orbit has a significant error and therefore, the life time of this satellite comes down. Why does it come down? Initially, itself I take some of this fuel tube give me the orbital velocity or the corrections and therefore, my station keeping of this satellite and the attitude orbital corrections are decreased, because I have less amount of fuel. I think these two examples illustrate the total and may be for assignments, I will give you some more problems and we all should be able to do any problem relating to theory of rocket propulsion. I think this is how we worried about it. Are there any specific questions? Ya.

(( )) for 10 years.

Ya, how does we; your question is, how did I get 950 meters per second over a given period? See you know that, the satellite is slightly getting drifted, because I have the gravity of the moon, may be gravity of some other planet which is there or else some aerodynamics itself. Therefore, I know the mass of the satellite, I know what is the force which is required to give the orientation. Therefore, I can calculate what is the value of I for particular correction, once I know the correction at that movement of time, I know what must be the initial mass. Therefore, I know what is the value of correction required may be for the first correction like that you know daily or once in two days I require corrections. Therefore, I keep on adding all these things and the sum of all these things is what gives me 950 meters per second.

We will do one or two small problems as we go a long, we will look at some of these things and do some more, but this is how the velocity, that is the total velocity what is required for corrections is also expressed in meters per seconds such that, I can do the problem. We do not really go and do a force balance at each stage, we just specify the ideal velocity required for correction. Further any other questions, I think we must be fairly clear about the theory of propulsion and once we are clear about it, it is just a cake (( )) One you know as we will see in the next part of this class, we will try to see how you will get a high jet velocity?

In fact, you seen the jet velocity is somewhat small, the thing is that, when you carry rockets in space, I cannot afford to have a large expansion like what we will be talking in the next class, we will come back to this. Are there any other questions? If not, let me go to the next part, but before going to this, let summarize once again. In order to appreciate the points made so far, I show a scale model of the G S L V launch vehicle of the Indian space research organization. Here we see that, we have a core stage at the bottom followed by the second stage over here, followed by the third stage over here. The core stage is surrounded by 1 2 3 and 4 straps; that means, you have a core stage followed by 1 2 3 4 rockets, these 4 rockets are a cluster and are known as straps.

At the beginning of the machine, the core or rather the straps either fire together or the straps fire before and immediately the core fires so that, you get a huge thrust, which can carry the entire mass of the launch vehicle. Once, the firing of the straps are over, they are discarded and there after the core fires for a small additional time and then this is also removed there after the vehicle goes up. The second stage fires and once the second stage operation is over, it is also removed and it is thrown out and then the third stage takes over. The third stage fires and takes this space capsule which is sitting on the top of the third stage and puts it into orbit, but before putting into orbit, the third stage is also removed.

Therefore, we see in this particular rocket you have something like a core rocket, you have force straps, you have the second stage rocket, you have the third stage rocket on which you have these space capsule of the satellite and it is this satellite which is put into orbit. The satellite, which is put in orbit, is the insat satellite, I show a scale model of it, the satellite consists of a box like structure, which is shown in brown over here. You have the solar over here, which takes the energy from the sun converts it to electricity. You have a balance for the mass, you have a balance over here and you have these antennas.

But what I really wanted to show was, you have in red 1 2 3 4 similarly, you have 1 2 3 4 something like 16 to 18 thrusters which are rockets which are mounted at the edges of the satellite, which will correct for the position of the satellite, may be the orbit and also the position of the satellite. Therefore, even a satellite in orbit as rockets attached to it which give it some impulse.

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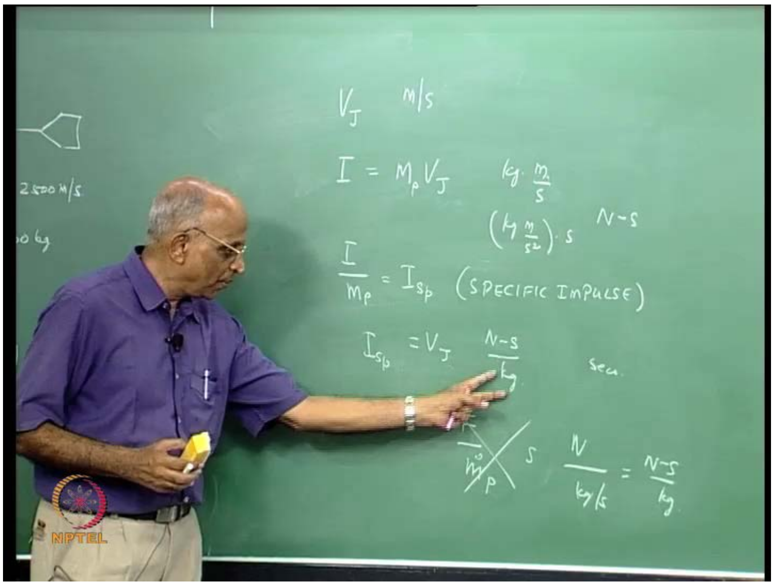

What are the things we have learnt so far? What did we learn? We told ourselves well, V J is a very important quantity that is the flux velocity, unit is meter per second. We talked in terms of Tsialkowskys equation or Rocket equation which told delta V is equal to V J of initial mass by final mass. We also told ourselves that, if I have the impulse of a rocket, what is impulse? The change of movement, among that should that has been your question again, is equal to mass of the propellant into V J is the impulse, what is the unit here? We are talking of kilogram meter per second this could also be written as kilogram meter per Second Square into second, which is same as Newton second that is the unit.

Let us be very clear about units, let me derive this again, we have kilogram meter per second which is impulse. We can also write it as kilogram meter per Second Square into second which is nothing but Newton second. Therefore, impulse could expressed in kilogram meter per second or Newton second. This is the total impulse which is given by the M p into V J, because this is what is moving out. Now, I ask myself a question, can I say impulse per unit mass of propellant, which now I call as specific, instead of calling as impulse call it as specific impulse that is impulse per unit mass and that gives me the value as V J and what is the unit I get? Newton second by kg and therefore, you find that the jet velocity and specific impulse are the same.

We will make some corrections for a after we consider the mechanism of flow taking place in a racket. We will find that it may not be identical, but this is the way to go about it. You know, in many text books, they express the value of specific impulse in seconds which is really not right, we are talking of force divided by mass flow rate, because I could have also written I s p from this, I could also write it as equal to force divided by M dot p. Why do I write it? Force into time is impulse and therefore, this is same thing as impulse over total propellant mass.

Therefore, I can also write specific impulse as equal to, instead I write it as equal to force divided by M dot p and this gives me, if I express the force also in kilograms, the M p also kilograms per second, I am left with a unit like seconds which is really not right because the unit of force should have been neutron and neutron divided by kilogram per second, which gives me neutron second by kilogram. In fact, this units are important therefore, please be careful when you read a book, this says specific impulse seconds may be is talking in terms of force, in terms of kilograms. Therefore, is necessity for us to correct it multiply by 9.81 and then have this unit come in with really so on.

May I will take an example as I go long. You know, see there is always a problem you know, people talk in terms of mass flow rate, they talk in terms of wait flow rate as you are mentioning, but when we saw wait flow rate? It is actually mass flow rate. You cannot have wait flow rate, you cannot have force which is flowing out. We must distinguish between mass and wait right that is whenever I measure a mass by a spring balance, it is the attraction and therefore, we measure a force that is a weight, where as when we consider quantity of matter it is mass of matter it is mass.

Let us keep our definition clear for as masses always quantity of matter which is Kg; impulse is Newton second, impulse per unit mass of propellant is Newton second by kilogram or force by mass flow rate over here. I think these definitions are important. I think (( )) at the end of the flow through (( )) for which we get the high jet velocity which I will do now, and then maybe we will again go through this unit second, because units tends to be important.

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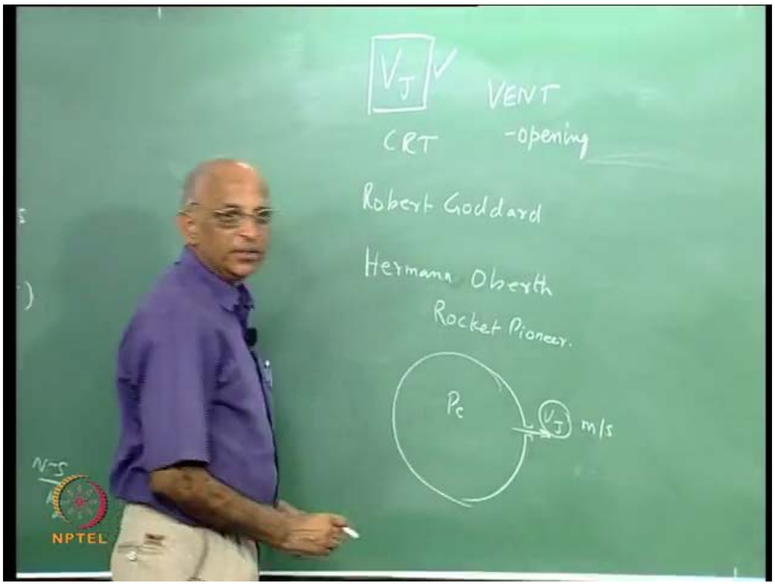

Let us now come back to the next part, namely we ask ourselves how do I derive a high value of v j. We said that it extremely essential to get a high jet velocity. Can this is what people have been after. We told, in fact when (( )) is looked at the rocket equations in those days, what was coming up was this cathode ratives. And what is cathode ratives, we have these electrons moving at high velocity, therefore his idea of rocket was why not we use electrons going at high velocity and you have a high jet velocity that is what you was thinking. In fact around that time you have Robert Goddard in US, he was also thinking in terms of maybe this think like a let us say electrons which can move a high velocity some can we get some high jet velocity from (( )), electrons terms to be light mass and therefore force what you can get is small.

In fact, you also had that the third person by name Herman, Herman Oberth. He was in Austria. In fact in around nineteen twenty, (( )) beautiful book of rocket propulsion and you will be surprise many of the things we do in rocket today's remains exactly similar to what he did at that point in time. And in fact, what he said was you put one stage after other and I can get a high velocity like what we did over here. And he is also a Greek rocket pioneer. He will keep coming to this people again and again, but what I want to do in the in this part of the left over period is, how do I get the high jet velocity.

In other words, supposing I consider a chamber in which I have a high-pressure gas filled with pressure of P c. ok I make a hole over here, I know the gas will (( )). I want to find out the jet velocity. Let me take you through a small example, I will just got a balloon, because this tends to be somewhat fascinating. Therefore, what I am trying to consider is I have a balloon filled with air, at a pressure P c. I I open, I make a small opening here, and I want to find out the velocity with which the gases are going out. And this is exactly what is the theory of rocket propulsion. In other words, I allow it to go and it goes up. It just pushes it up and this is what a rocket propulsion. Therefore let just take a, I need another one wherein I have something like a control opening. In that case the opening was not control. I just put a small opening here, which I will now call as let say a vent – nothing but an opening.

And therefore, let us again fill this with air I close this vent. I now open this vent, (( )) I find the air going out at a certain velocity. I want to calculate the jet velocity of this particular air, which is moving. Let us do this problem, I may be increase the pressure, I increase the flow rate and the velocity. My aim is through this small opening what I have here, what is the value of V j we want to calculate in meter per second. Well let us do the simple problem, here you have done it in your thermodynamics course in the first year engineering, but let us repeat it because it tends to be very illustrated.

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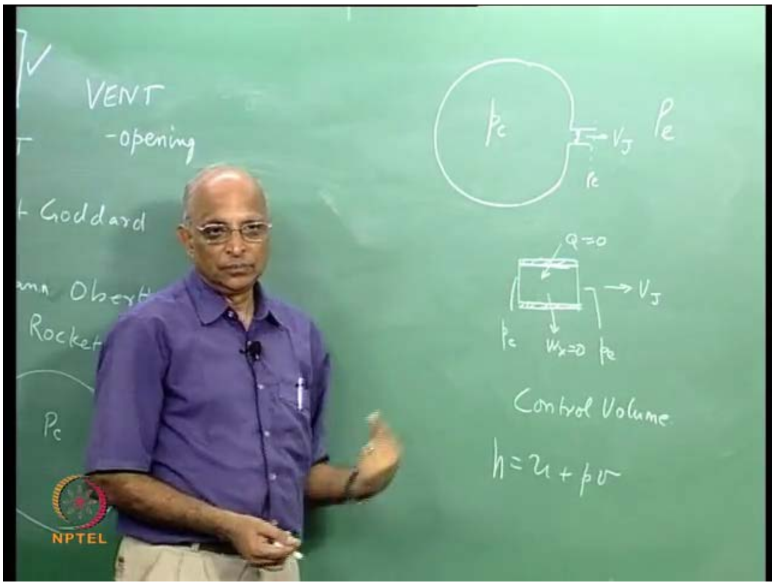

Therefore I again draw a huge reservoir here, which I call as a chamber. The pressure is P c; I give a vent over here. I allow to this particular area of opening here. Calculate V j, I want to calculate the value of V j, when the ambient pressure is let us say the exit pressure of the nozzle; that is the pressure at the exit is p e. What I have to solve for, I know the pressure here, I know the pressure at the exit, I want to know the jet velocity.

Therefore, I deal with this problem. I say yes I am interested in this particular vent, which has the shape. I do not know the shape, what I just now showed you something like this. Air enters at the pressure P c; air leaves at a pressure P e I am interested to calculate in the value of V j.

Therefore, this is nothing but a control volume, why do I say a control volume; I have a fixed volume in space. And what is happening to this volume air is entering at a low pressure at a high pressure leaving at a low pressure. And therefore, I solve this equation for a control volume. And to be able to solve this problem, I have to make some assumptions. What are the assumptions I could probably made? Let us say this is my vent, what are the assumptions you normally make? Let us assume it is the adiabatic, that means the vent is such that there is no heating of the vent or no heat comes from outside into the nozzle. In other words, I say Q, which is entering this particular area is zero. Let us for the present also assume that this vent is rigid, if it is rigid, it cannot expand; it cannot do any work. Therefore, the work done by this vent, that is W x is equal to zero.

See, I could have something like a flexible vent, which is moving and it can do some work. But I assume that it is rigid, when it is rigid I have the work done is zero. Therefore the assumptions are make sure for this high-pressure gas enters a vent, leaves the vent at high jet velocity. The vent is such that it is adiabatic, that is the heat transfer is zero and the work done by the vent is zero. Mind you the might be the work which taking place here, there is work interaction taking place here, but across the surface what we get the work done is zero. Now I have to write the equation, how do derive this equation, let us go back on that (( )) and put the equation together.

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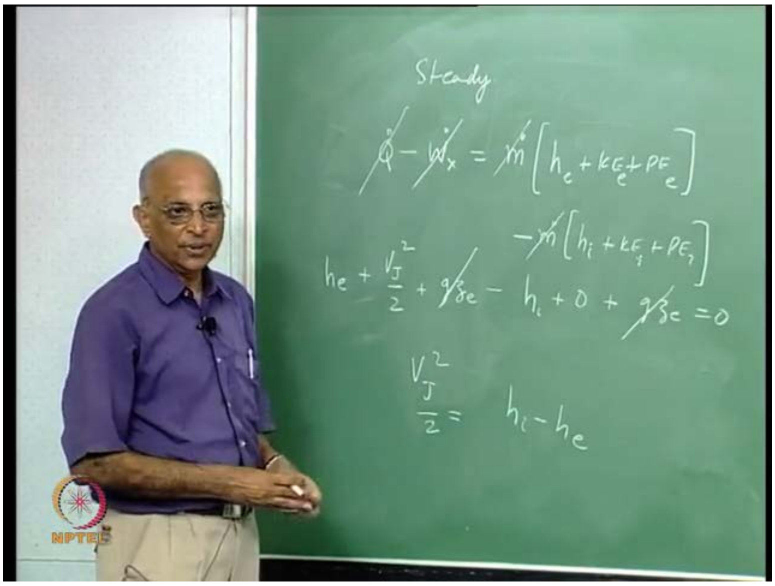

Let us also assume, that the flow is steady. What do you mean by that the flow is steady? The mass, which is entering by vent, the mass leaving the vent is the same, steady flow energy equations. If I have Q the rate at which heat is entering the vent minus W x rate at which work is done is equal to… What happens? The mass, which is entering the vent, let say and leaving the vent is m dot. You have enthalpy, which is entering which is leaving, because you have some heat which is coming on, the work is done. Something is entering h at the exit plus I have a kinetic energy at the exit plus I have a potential energy at the exit minus the same mass which is entering will have enthalpy h i plus kinetic energy at entry, the exit, exit, may be at the inlet plus potential energy at the inlet.

Why do we write enthalpy here, because you have something like gas has some internal energy and it also has a specific volume that is some flow work? Internal energy we say enthalpy is equal to internal energy plus p into specific volume right. This how we define enthalpy. Therefore, what we have been done, we just show the energy balance Q minus W is equal to what is the enhancement it has got the enthalpy, kinetic energy and potential energy over work, it had over here. Let simplify, let see what could be done, we told ourselves Q dot is zero W dot x is zero. We also find this left hand side is zero; therefore, m also drops out, m dot that is rate of flow drop.

So if I have h e plus what is the kinetic energy per unit mass, it is equal to V j square divided by two that is the exit velocity. I have potential energy, potential energy is equal to g into z or the height above the datum at the exit and this minus I have h i plus velocity with which the gases leave from the chamber. See, the chamber is huge, this is small; therefore the velocity what I get here is almost zero, that means I can I can neglect the velocity with which is something over here. And I get this value as equal to zero plus I have g z e is equal to zero.

Therefore this will tell me, well you know the nozzle or the vent is so small, but the change in height is very small, I can neglect this. And now, I write V j square divided by two is equal to I take it on the other side. I get the value as equal to still I having the square term over here, therefore I get h i enthalpy at the entry and enthalpy at the exit. Therefore what we will do in the next classes, we will start with this equation and try to see under what conditions can I get a high jet velocity. Well, in the class therefore what we have done this morning is, we looked at two problems illustrative of rocket propulsion. And then we went ahead and try to find out what is the jet velocity with which a pressurized gas will (( )) right. Thank you.