Advanced Operations Research

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Lecture - 2
Revised Simplex Algorithm

We now explain the details of the revised simplex algorithm using this example.

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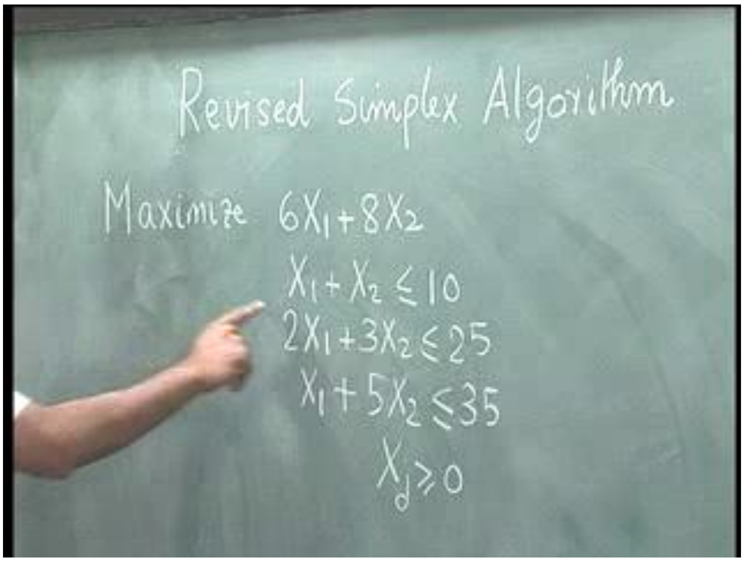

The example problem is to maximize $6X_1$ plus $8X_2$ subject to X_1 plus X_2 less than or equal to 10, $2X_1$ plus $3X_2$ less than or equal to 25 and X_1 plus $5X_2$ less than or equal to 35. We have three inequalities and all three are of the less than or equal to type. So, we first convert these inequalities to equations by adding slack variables, plus X_3 equal to 10; instead of this, plus X_4 equal to 25 and we add a third slack variable plus X_5 is equal to 35.

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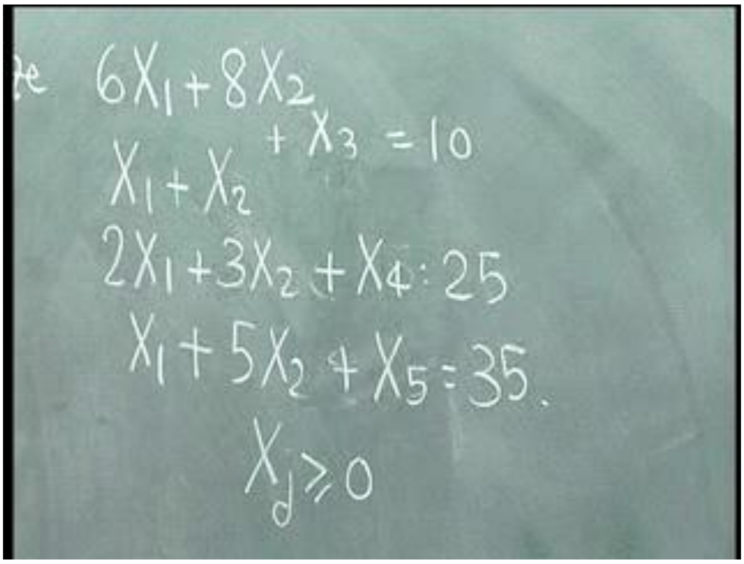

Now, X_3, X_4, X_5 being slack variables with a plus 1 coefficient and as they appear in only one constraint, so they can qualify to be the basis. We start the simplex algorithm with the X_3, X_4, and X_5 as basic variables.

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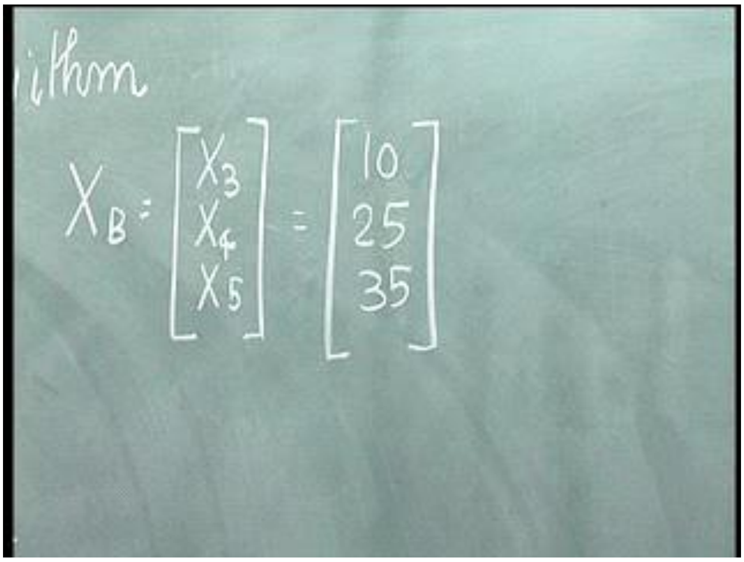

So we say, X_B set of basic variables is the set X_3, X_4, X_5 and they will have values 10, 25, and 35. There are two non-basic variables, which are X_1 and X_2.

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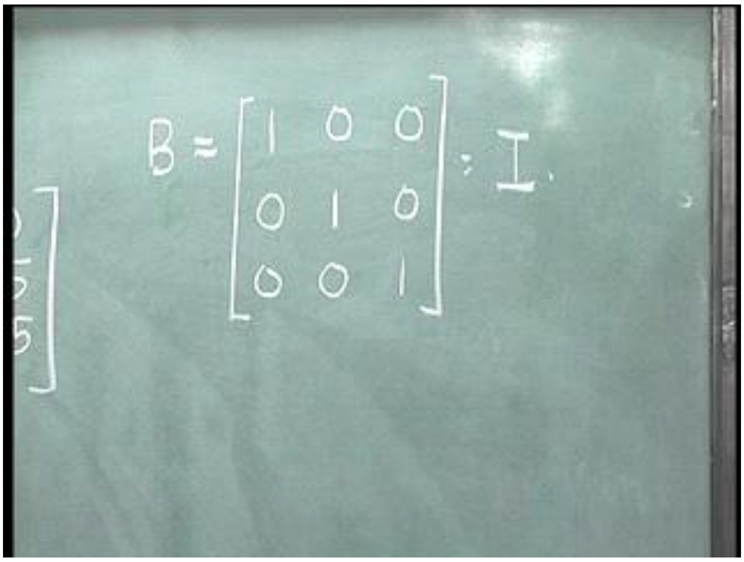

Since X_3, X_4, and X_5 are the basic variables, the basis matrix B is a 3 by 3 matrix, corresponding to three basic variables and the basis matrix B will be the matrix 1 0 0, 0 1 0, 0 0 1 which is I, identity matrix. Since, the basis matrix is an identity matrix, B inverse is also an identity matrix. Now, in order to check whether this present basis is optimum, we need to look at the two non-basic variables, which are X_1 and X_2 and find out if any one of them can enter the basis.

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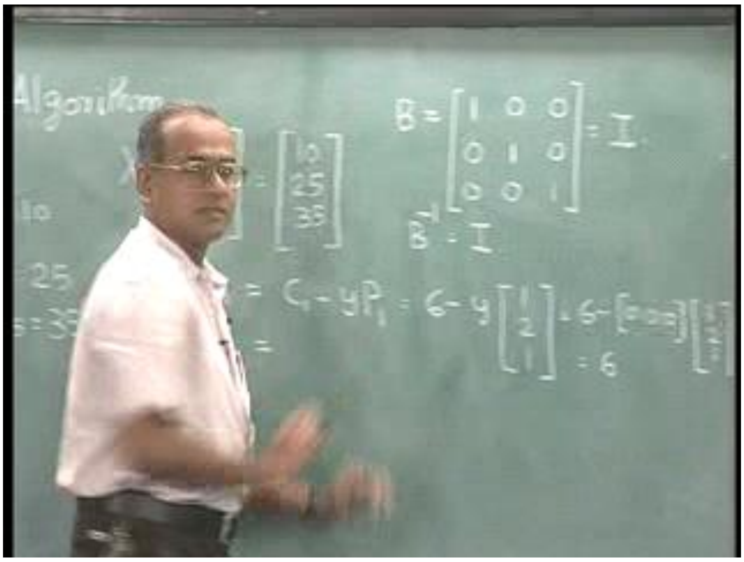

To do that, we have to find out C_1 minus Z_1 for variable X_1 and C_2 minus Z_2 for variable X_2. C_j minus Z_j is equal to C_j minus yP_j. So, C_1 minus Z_1 is equal to C_1 minus yP_1. Y is the value of the dual and P_1 is the vector corresponding to the variable X_1 in the original problem.

Therefore, P_1 will be 1 2, P_2 will be 1 3, C_1 is the objective function coefficient of variable X_1, which is 6 and C_2 is 8, so this will become 6 minus y into 1 2 where y is 1 2.

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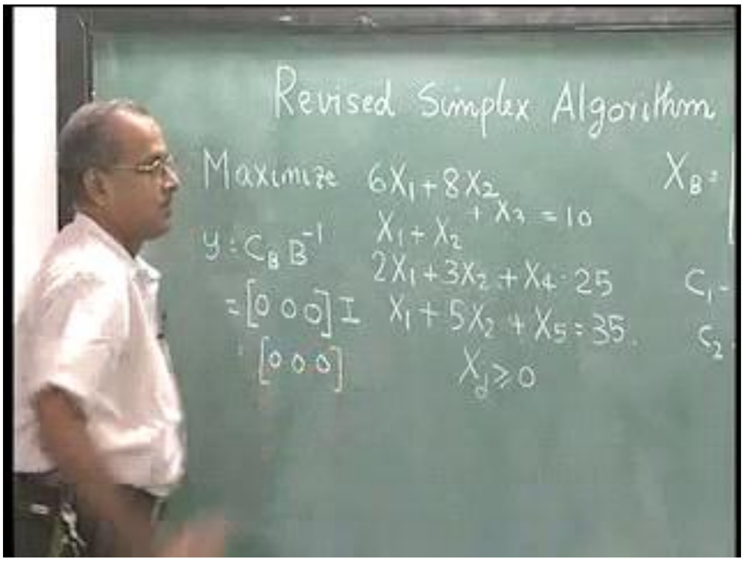

P_1 is the vector corresponding to variable X_1. So therefore, that is the vector 1 2 1; so, y into the vector 1 2 1. y is the value of the dual variables; y is given by C_B B inverse. C_B is the objective function coefficient of the basic variables. The three basic variables X_3, X_4 and X_5, all of them are slack variables. C_B is 0. So 0 0 0 into B inverse is I, which we have shown here. B inverse is equal to I, therefore, y is equal to 0 0 0. Therefore, C_1 will become 6 minus 0 0 0 into 1 2 1 which is 6.

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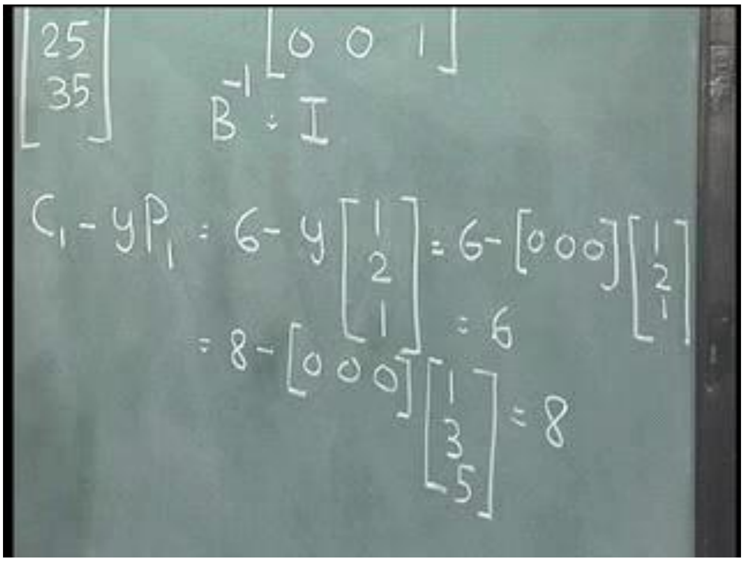

In a similar manner, C_2 minus Z_2 will be 8 minus 0 0 0, which comes from the value of y multiplied by P_2, which is the vector corresponding to variable X_2, which is 1 3 5. So, this will be 8 because y is 0. Now, both C_1 minus Z_1 and C_2 minus Z_2 are positive. Based on the maximum coefficient rule, variable X_2 will enter the basis, because C_2 minus X_2 is 8, which is bigger than C_1 minus Z_1, so variable X_2 will enter the basis.

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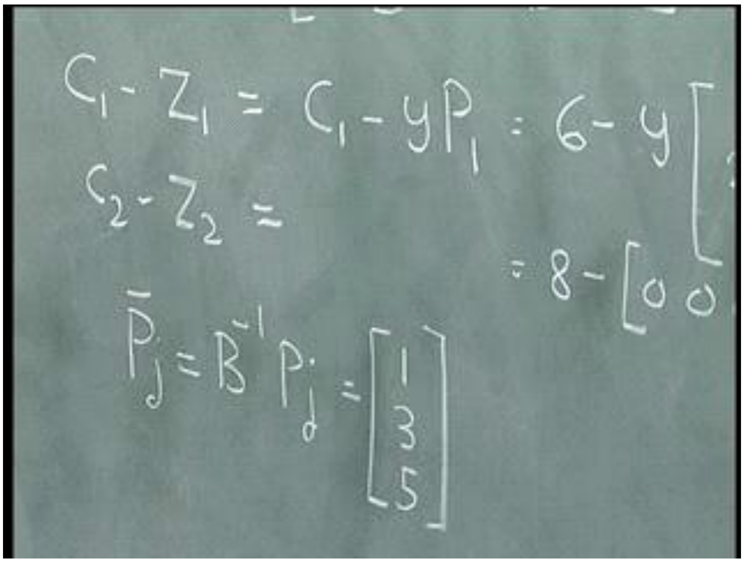

Now, in order to find the leaving variable, we need to find the column corresponding to variable X_2 and that is given by $Pbar_j$ is equal to B inverse P_j and B inverse is I. Therefore $Pbar_j$ will be equal to P_j, which is 1, 3 and 5. P_j corresponds to P_2, which is from here 1 3 5. Now, theta is the ratio between right hand side value and the corresponding $Pbar_j$. So, we should take the minimum of 10 divided by 1, 25 divided by 3, 35 divided by 5.

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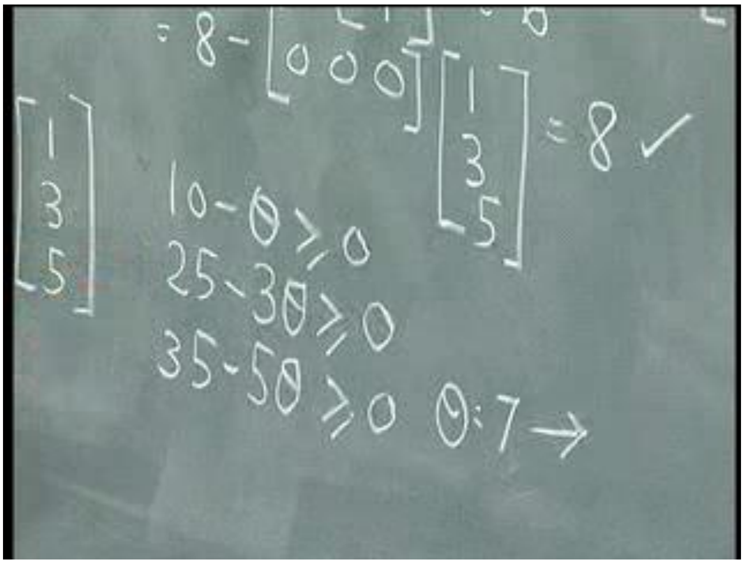

Another way of saying is that, we find out minimum theta, such that 10 minus theta is greater than or equal to 0, 10 minus 1 into theta is greater than or equal to 0, 25 minus 3 theta greater than or equal to 0 and 35 minus 5 theta greater than or equal to 0. This would give us theta equal to 10. This would give us theta equal to 25 by 3. This is 7. 7, is the smallest of the three. Therefore, theta will be equal to 7 and this variable will leave the basis, which means the third variable will leave the basis. So X_5 will leave the basis. At the end of the first iteration, we have understood that we started with X_3, X_4, and X_5 as basic variables and we found out the optimality of this basis and realized that this basis is not optimal, because both of these are positive, which meant that variable X_2 will enter the basis and variable X_5 will leave the basis.

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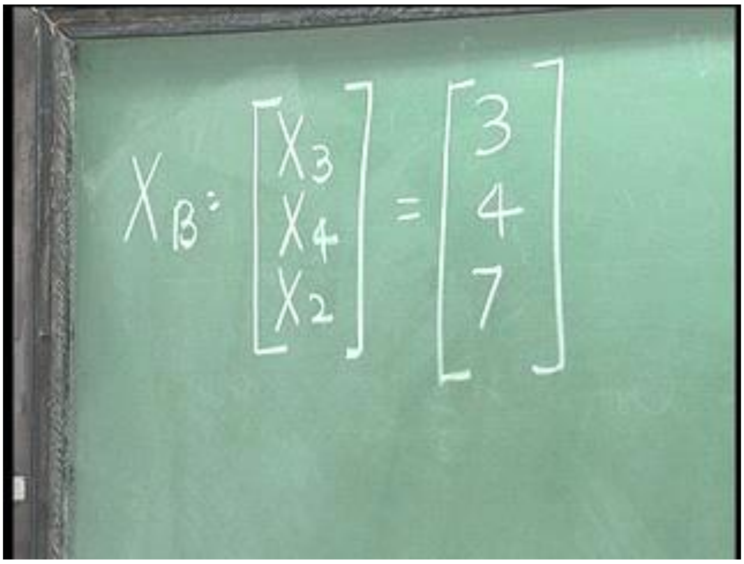

In the next iteration, the set of basic variables X_B is now X_3, X_4 and X_2, because variable X_2 has now replaced variable X_5. Now, the corresponding right hand side value can straight away be given. We have found out from here that theta equal to 7 and the third variable is the one that leaves. So in the next iteration, the variable X_2 will take a value of theta, which is 7, while the rest of the variables will take the corresponding value, when we substitute theta into these two expressions. 10 minus theta is 3, 25 minus 3 theta is 4, so we will have X_3 equal to 3, X_4 equal to 4 and X_2 equal to 7. We can even verify this; X_2 equal to 7, X_1 plus X_2 plus X_3 is equal to 10. So, 7 plus 3 is 10, 3 X_2 plus X_4 is 7 into 3 is 21 plus 4 is 25, $5X_2$ plus X_5, $5X_2$ is 35 plus 0 is 35.

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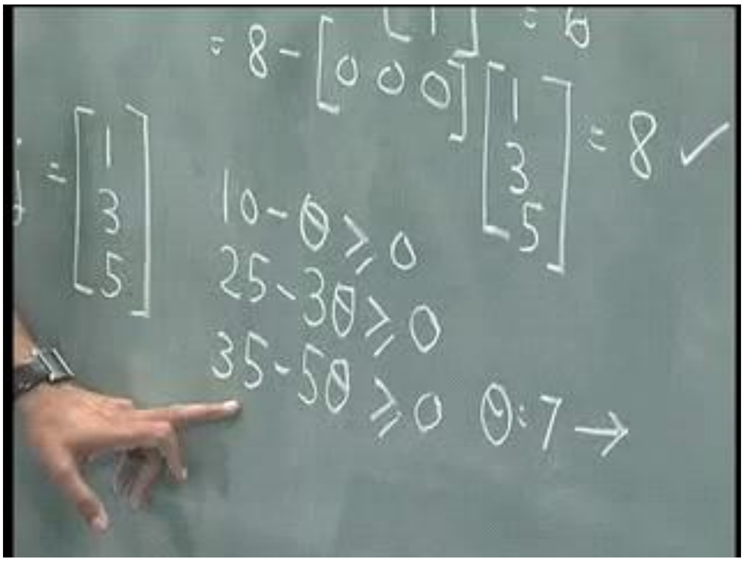

We now have this as the basis and we now have to find out whether this basis is optimal. In order to do that, we need to look at the two non-basic variables, which are X_1 and X_5 and find out their C_j minus Z_j values. Before we find the C_j minus Z_j values, we first have to find out the value of the dual, which is y.

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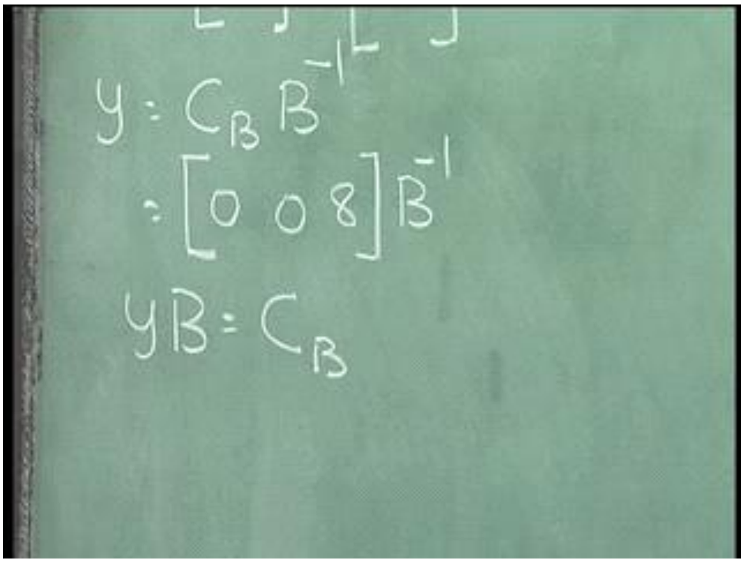

y is given by C_BB inverse. C_B is the objective function coefficient corresponding to the basic variables. This has 0 0 and variable X_2 has 8. So Y will be 0 0 8 into B inverse. Another way of looking at it is instead of saying y is equal to C_B B inverse, you can always say yB equal to C_B. We now know that this basis matrix is a 3 by 3 matrix, corresponding to X_3, X_4, and X_2.

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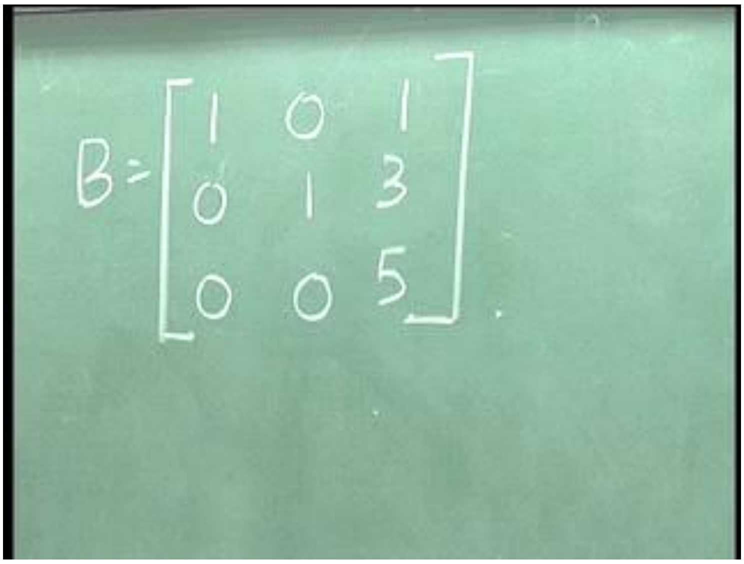

The basis matrix B will be corresponding to X_3, X_4, and X_2; so 1 0 0, 0 1 0 and corresponding to X_2, would be 1 3 5. One way of finding out this y, is by taking this basis matrix B, inverting it in the usual way, either by finding out first the determinant and the cofactor and the adjoint and then getting B inverse or by inverting B in other ways such as Gauss Jordan method or Gaussian elimination or any of these methods.

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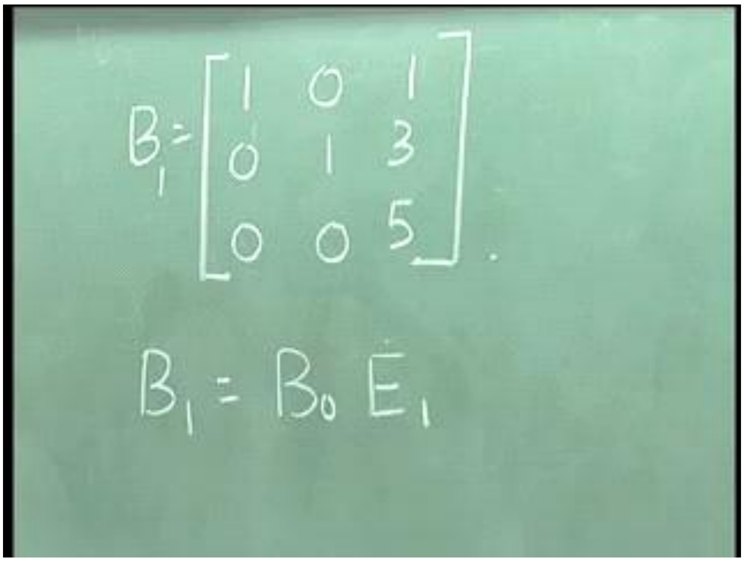

But what we are going to do now is something different. We are not going to directly and explicitly invert B, but we are going to indirectly invert the basis matrix B. What we will do now, is we simply call this B as B_1, which is the basis matrix corresponding to iteration number one. The earlier B, which was the starting iteration, is now called B_0. Now what we are going to do is to say that B_1 is equal to B_0 into E_1 where E_1 is called an Eta matrix. An Eta matrix is also a 3 by 3 matrix. The difference is that, one column of the 3 by 3 matrix is a non-identity column and that non-identity column is exactly the column corresponding to $Pbar_j$.

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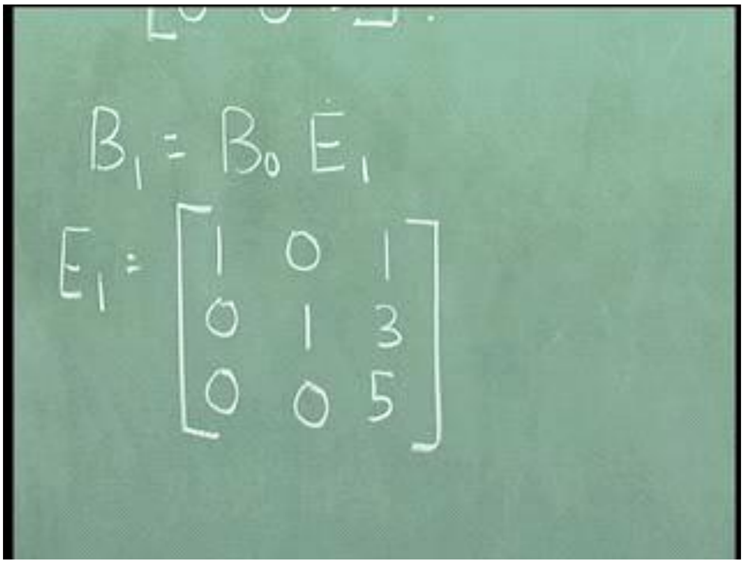

In our case, E_1 is a 3 by 3 matrix. Since, the third variable left the basis, the third column of the Eta matrix E is a non-identity column. The other two columns are 1 0 0, 0 1 0 and the third column is $Pbar_j$, which is 1 3 and 5. It happens that B_1 is equal to $E_1$and in fact, in all the first iteration B_1 will always be equal to E_1. Later we will see that, it changes. Now B_1 is equal to B_0 into E_1. In order to find out y, we should find out yB_1 is equal to C_B.

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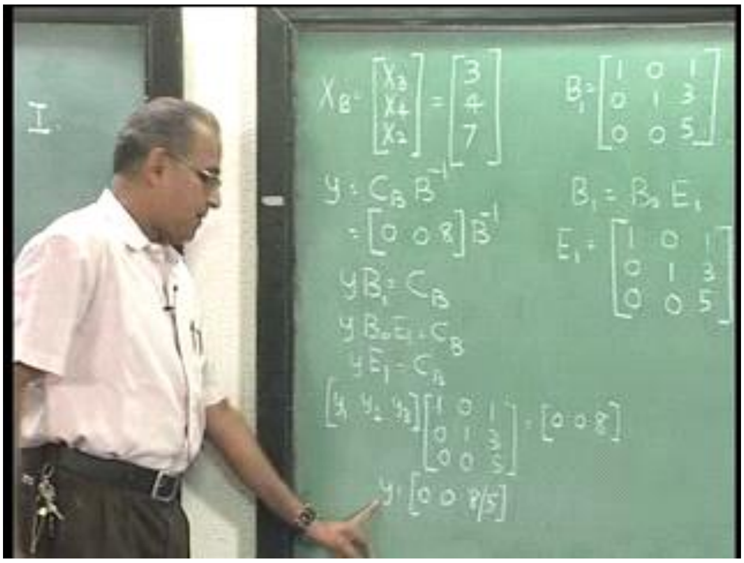

yB_0E_1 is equal to C_B. B_0 is I; so, yE_1 is equal to C_B. Now, y is a vector. So y is now written as y_1 y_2 y_3. Because there are three constraints, there are three basic variables on the primal, there will be three dual variables for y; so y_1, y_2, y_3 into E_1, which is given here 1 0 0, 0 1 0, 1 3 5 is equal to C_B, which is 0 0 8. The nice thing about writing this way is that the computations are much simpler, because the first equation will be 1 into y_1 plus 0 into y_2 plus 0 into y_3 is equal to 0.

This is 1 into 3 this is 3 into 3, so the resultant is 1 into 3. So y_1 plus $0y_2$ plus $0y_3$ is 0, which would give y_1 as 0. Now the second one, $0y_1$ plus $1y_2$ plus $0y_3$ is equal to 0, would give us y_2 equal to 0. y_1 plus $3y_2$ plus $5y_3$ is equal to 8, would give us y_3 is equal to 8 by 5. So y value is 0 0 and 8 by 5. With the value of y that we have obtained, we will now go back and find out the C_j minus Z_j corresponding to the two non-basic variables, which are X_1 and X_5.

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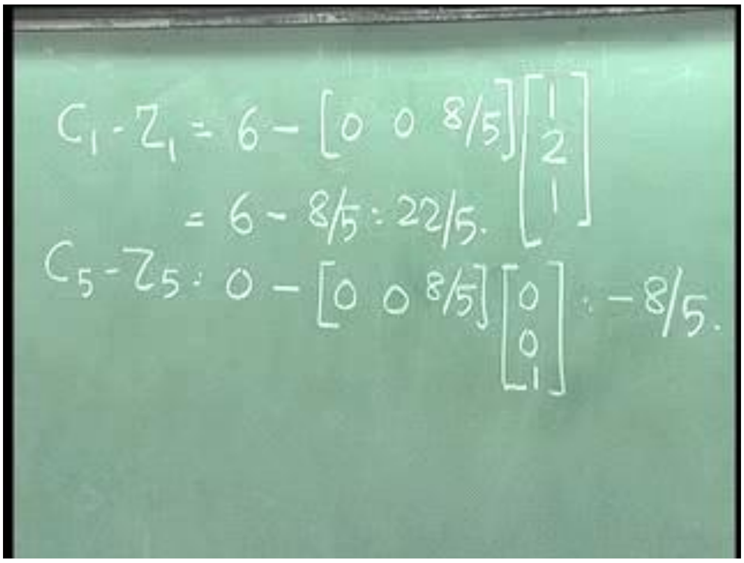

We have to find out C_1 minus Z_1 which is, C_1 is 6 minus yP_1; y is 0 0 8 by 5 into P_1. P_1 comes from the original matrix, which is the column corresponding to variable X_1 in the original matrix which is 1 2 1. So C_1 minus Z_1 is 6 minus this, into 1 2 1, which is 6 minus 0 into 1 plus 0 into 2 plus 8 by 5, 6 minus 8 by 5 which is 22 by 5. Now C_5 minus Z_5; variable X_5 is the slack variable, therefore C_5 is 0 minus y is 0 0 8 by 5 into P_5. P_5 comes from here. P_5 is 0 0 and1, because variable X_5 appears only in this constraint, with the coefficient of 1. It does not appear in these two constraints. So we have 0 0 8 by 5 into 0 0 1, which is minus 8 by 5. So, C_j minus Z_j are the two non-basic variables. Non-basic variable X_1 gives us the C_j minus Z_j positive value. For variable X_5, it is a negative value. There is only one positive C_j minus $Z_,$ therefore, variable X_1 will enter. Now, once variable X_1 will enter, we now have to find out the leaving variable.

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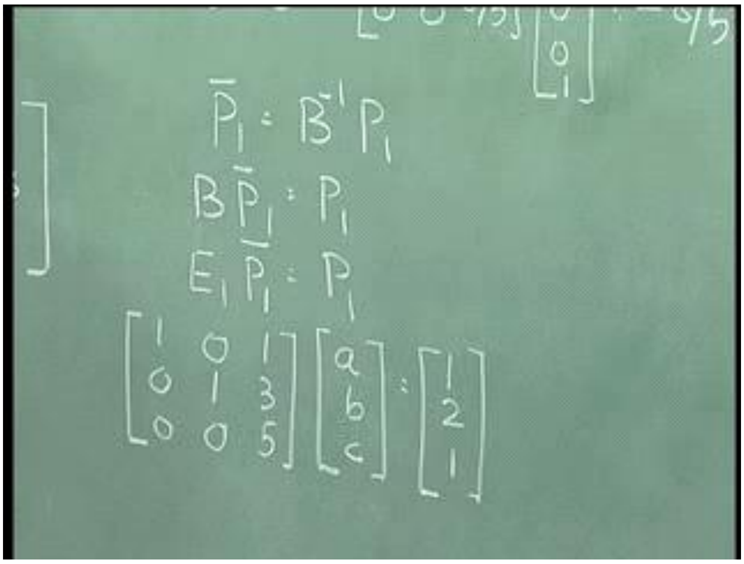

In order to find out the leaving variable, we need to find out $Pbar_1$ which is equal to B inverse P_1. Once again, we do not have B inverse explicitly computed, so we pre-multiply by B to get B times $Pbar_1$ is equal to P_1, because B into B inverse will give I, so I into P_1. Now this B is nothing but B_0 into E_1.We will have E_1P_1bar is equal to P_1. B_0 is also an identity matrix, so this will reduce to E_1 into $Pbar_1$ is equal to P_1. Now E_1 is known, E_1 is here. E_1 is 1 0 0, 0 1 0, 1 3 5 into P_1bar is what we want to find out. We simply call it as some a b c that we want to find out. P_1 comes from the original matrix and P_1 is here. P_1 is 1 2 1. Once again, because of the very nature of Eta matrix, it is easy to compute this, a b and c.

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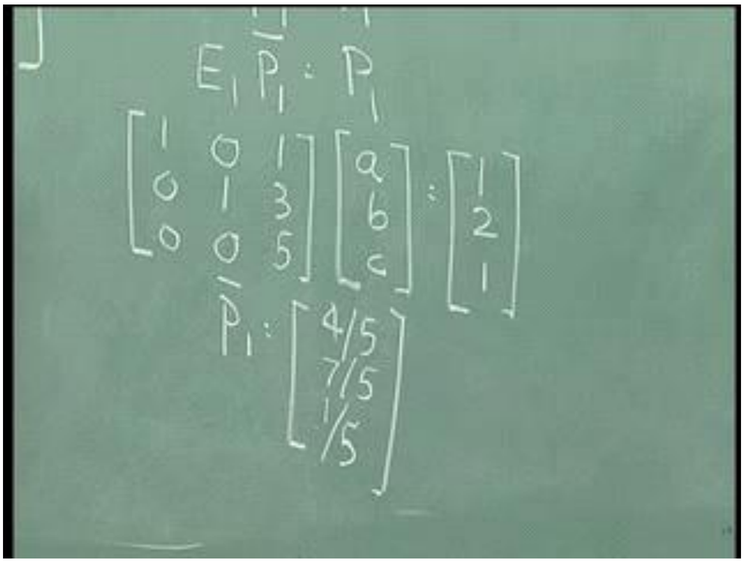

What we will do now is we will take $Pbar_1$ equal to 0 into a plus 0 into b plus 5 into c, which is the third equation. 0 into a plus 0 into b plus 5 into c is equal to 1. So, c is 1 by 5. From the first equation, 1 into a plus 0 into b plus 1 into c is equal to 1; so, a plus c is equal to 1. C is 1 by 5, so a is 4 by 5. From the second equation, 0 into a plus 1 into b plus 3 into c is equal to 2. So, b plus 3c is equal to 2; b plus 3 by 5 is equal to 2. So, b is equal to 2 minus 3 by 5, which is 7 by 5.

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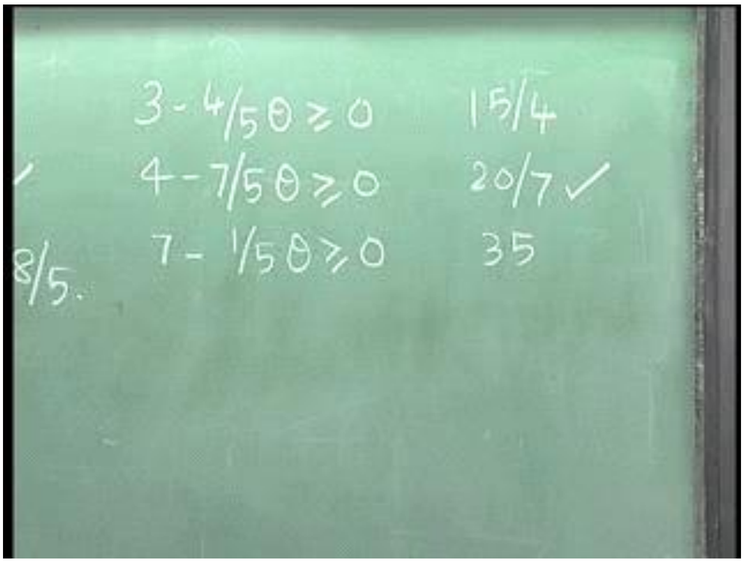

We have now computed $Pbar_1$ and having computed $Pbar_1$, we now have to find out theta, the right hand side values are here. We have to find out the minimum theta, such that 3 minus 4 by 5 theta is greater than or equal to 0; 3 comes from here, 4 by 5 comes from here. So, 3 minus 4 by 5 theta is greater than or equal to 0, 4 minus 7 by 5 theta is greater than or equal to 0 and 7 minus 1 by 5 theta is greater than or equal to 0. This would give us theta is equal to 15 by 4. This would give us theta is equal to 20 by 7 and this would give us theta is equal to 35. Here, theta value is 5 into 3, 15 by 4. So this is 15 by 4. This is 20 by 7 and this is 35. Now, 20 by 7 is the minimum value of theta. So this variable, which is the second variable, will leave the basis. So, this variable is the leaving variable.

We found out that this basis is not optimal, because C_1 minus Z_1 is positive. Variable X_1 enters and when variable X_1 enters, we found out that variable X_4 leaves. We go to the next iteration where the basis changes. From this basis, we know that variable X_4 is leaving and variable X_1 is entering. So, we will have X_3, X_1 and X_2 as the set of basic variables.

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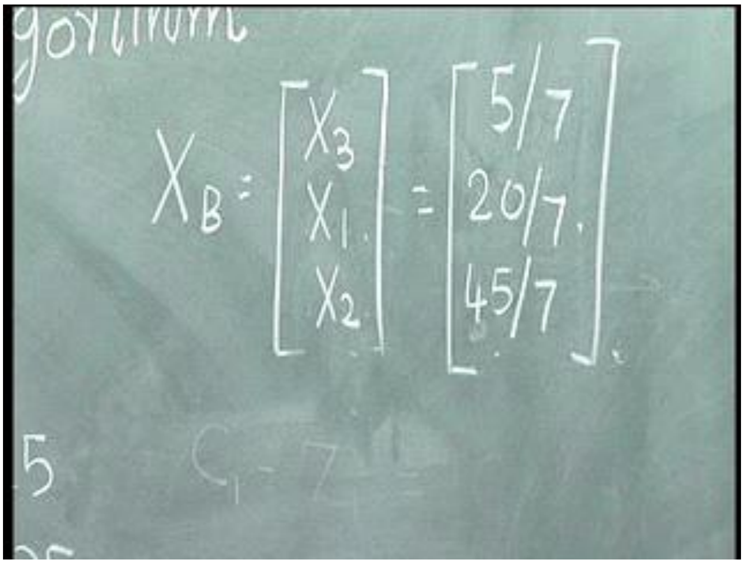

The values of these three are given by: the second one was the one which left the basis with theta equal to 20 by 7, so, this variable X_1 will take 20 by 7.

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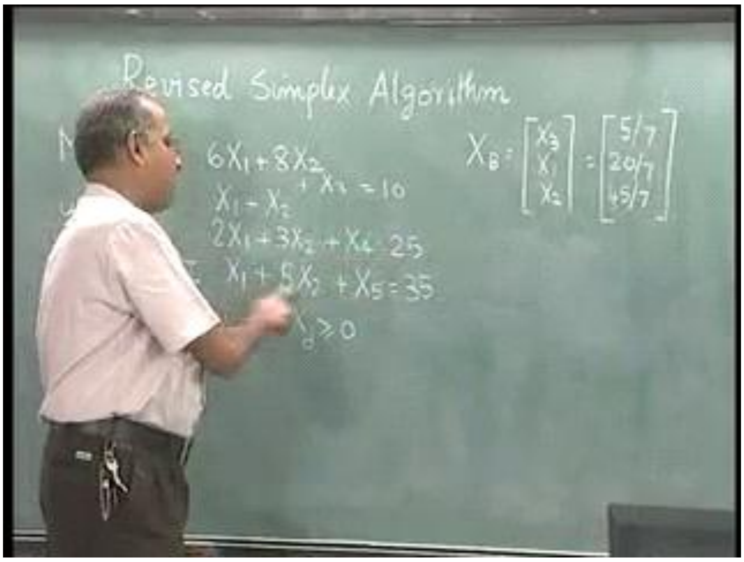

The other two will take the corresponding values. So this will become 3 minus 4 by 5 into theta; so, 3 minus 4 by 5 into 20 by 7 is 16. This would give us 21 by 7 minus 16 by 7 which is 5 by 7 and the third one will be 7 minus 20 by 7 into 1 by 5 which is 7 minus 4 by 7, which is 45 by 7. A quick check again would give us X_1 plus X_2 plus X_3 is equal to 10, 45 plus 20 is 65 plus 5 is 70; 70 divided by 7 is 10. $2X_1$ plus $3X_2$ plus X_4 is 0. So this is 25. $2X_1$ is 40; $3X_2$ is 135, 175 by 7 is 25 and X_1 plus $5X_2$ plus X_5. $5X_2$ is 225 plus 20 is 245 by 7, which is 35. So this has been checked.

Now, for this set of basic variables, we now need to find out whether this is optimal. To find out whether this is optimal, we have to find out that the C_j minus Z_j corresponding to the two non-basic variables, which are X_4 and X_5.

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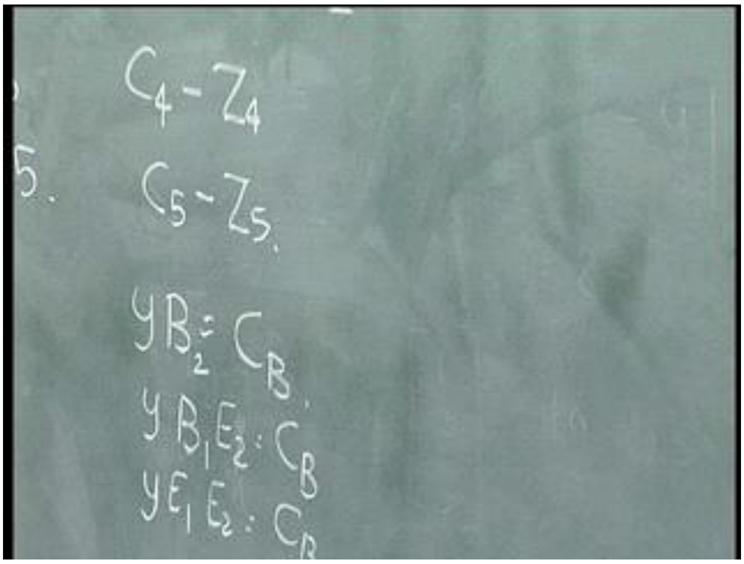

We have to find out C_4 minus Z_4 and we have to find out C_5 minus Z_5. Before we do this, we need to find out the value of the dual, which is y. Dual y is given by yB is equal to C_B. We now call the basis corresponding to this as B_2; so yB_2 is equal to C_B. Now, B_2 will be written as B_1 into E_1, so yB_1E_2 is equal to C_B where E_2 is the suitably defined Eta matrix.

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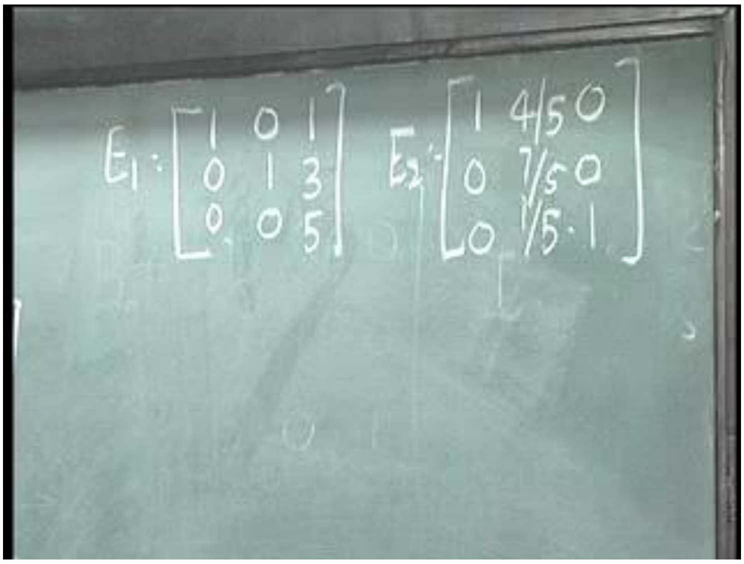

B_1 is already known to be B_0 into E_1, so we will have yE_1E_2 is equal to C_B. We already have E_1 written here. E_1 is this, E_1 we just copy down for our reference. So, E_1 is 1 0 0, 0 1 0 and 1 3 5. E_2 is another Eta matrix with one column, non-identity column. From here, we understand that the second variable is the leaving variable. So the second column in E_2 will be the non-identity column. E_2 will be 1 0 0, 0 0 1, which is the third column and the second column is the non-identity column and that second column is given by $Pbar_1$ which is computed here as 4 by 5, 7 by 5 and 1 by 5.

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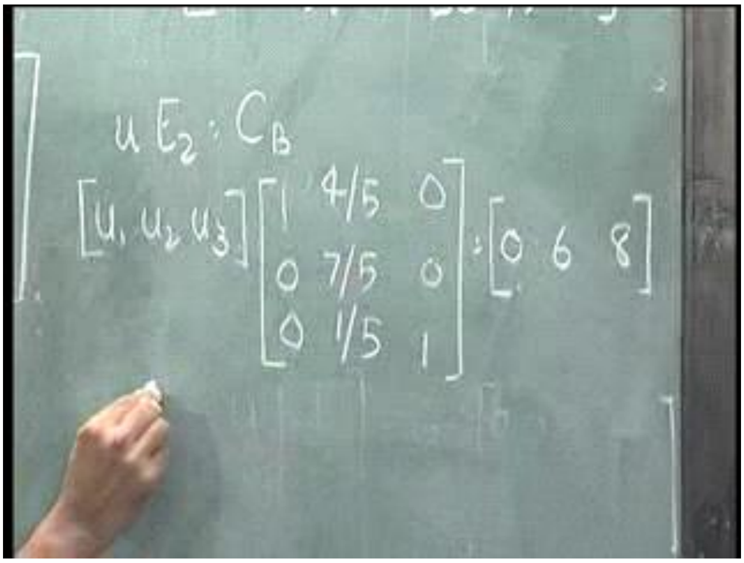

Now, we know that yE_1E_2 is equal to C_B. First what we do is we call this y into E_1 as some u, so we write u into E_2 is equal to C_B. u is written as u_1 u_2 u_3 into E_2 is 1 0 0, 4 by 5 7 by 5 1 by 5, 0 0 1 is equal to C_B. C_B is the objective function coefficients of the basic variables, X_3, X_1, X_2; so, 0 6 and 8. So, this will be 0 6 8.

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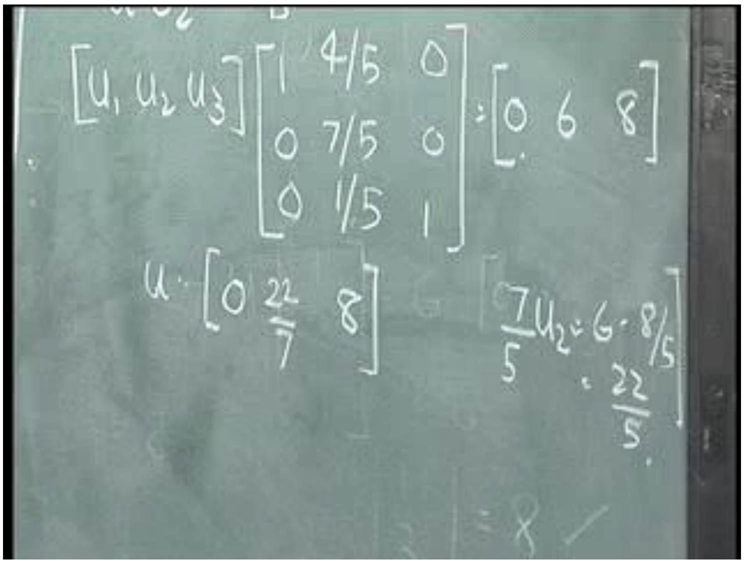

From this we can find out that u_1 plus $0u_2$ plus $0u_3$ is equal to 0. So u_1 is 0, 1 into u_1 plus 0 into u_2 plus 0 into u_3 is 0. From the third, 0 into u_1 plus 0 into u_2 1 into u_3 is 8. So u_3 is 8. From the second one, 4 by $5u_1$, 7 by $5u_2$ plus 1 by $5u_3$ is equal to 6. So, 7 by $5u_2$ plus 8 by 5 is equal to 6. 7 by $5u_2$ is equal to 6 minus 8 by 5; so this is 22 by 5, from which u_2 is 22 by 7. Now, we know this u. Now, we come back to this and say y into E_1 is equal to u.

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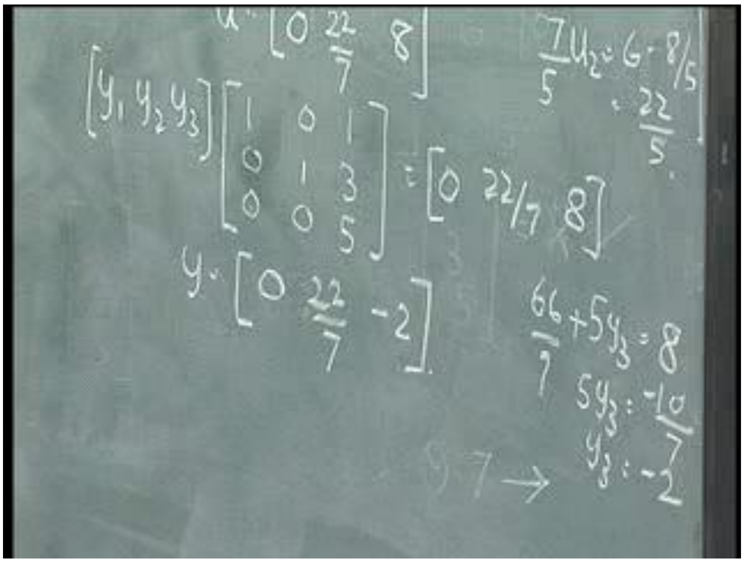

So, you will have y_1 y_2 y_3, which is y into E_1, which is 1 0 0, 0 1 0, 1 3 5 which is equal to yE_1. yE_1 is equal to u, so this is equal to 0 22 by 7 and 8. From this, y is equal to 1 into y_1 plus 0 into y_2 plus 0 into y_3 is 0, so y_1 will be 0. 0 into y_1 plus 1 into y_2 plus 0 into y_3 is 22 by 7, so 22/7. 1 into y_1 plus $3y_2$ plus $5y_3$ is equal to 8, so $3y_2$ is 66 by 7 plus $5y_3$ is equal to 8, so

66 by 7 plus $5y_3$ is equal to 8. So $5y_3$ is equal to minus 10 by 7, y_3 is minus 2. So, you have 0, 22 by 7 and minus 2 coming in as the values of the three variables. Having found out y is 0 22 by 7 and minus 2, with this value of y, we need to find out the C_j minus Z_j of the two non-basic variables X_4 and X_5.

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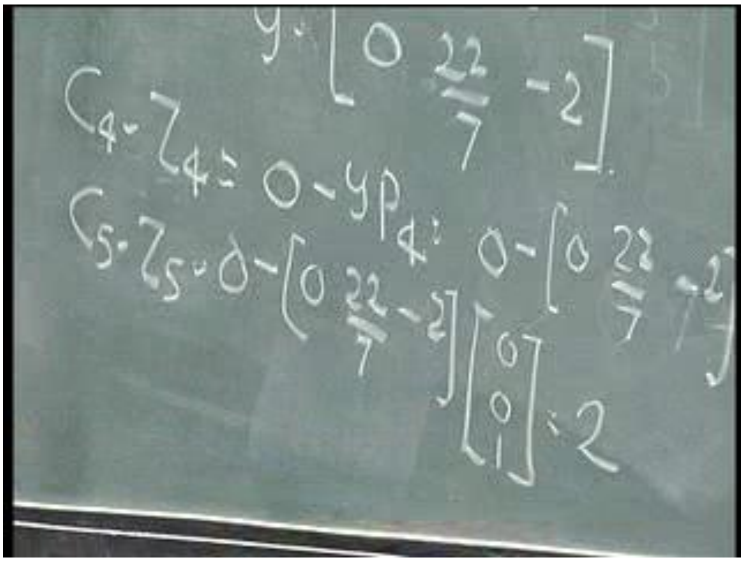

So, C_4 minus Z_4 will be equal to C_4 is 0 minus yP_4, which is 0 minus 0 22 by 7 minus 2 into P_4, is from here, which is 0 1 0. This will be minus 22 by 7 and C_5 minus Z_5 is equal to 0 minus 0 22 by 7 minus 2 into 0 0 1 which is plus 2. Now, C_5 minus Z_5 is positive, therefore variable X_5 will enter the basis. This basis is not optimal and variable X_5 will now enter this basis. In order that variable X_5 enters the basis, we need to find out the leaving variable from this.

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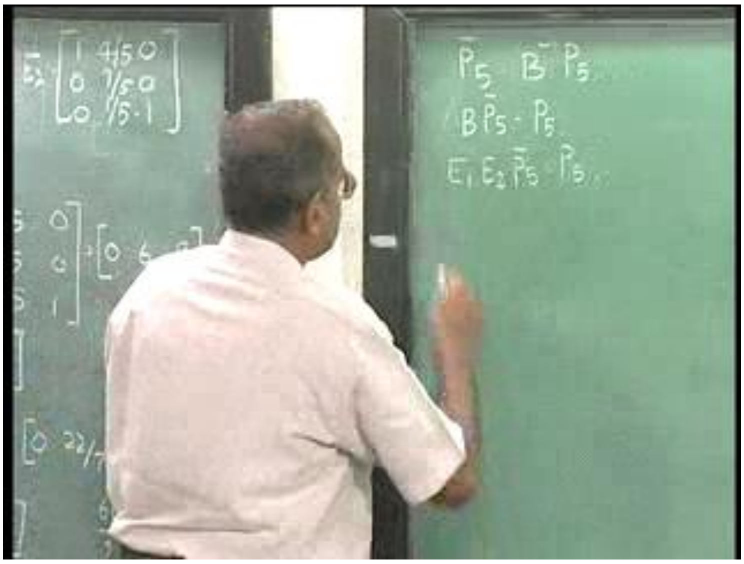

In order to find out the leaving variable, we need to find out Pbar$_5$ which is equal to B inverse P$_5$. Pre-multiplying by B, we get B into Pbar$_5$ is equal to P$_5$. This B is B$_2$, so B$_2$ is equal to B$_1$ into E$_2$. B$_1$ is B$_0$ into E$_1$, so we will have E$_1$E$_2$Pbar$_5$ is equal to P$_5$. We already know the values of E$_1$ and E$_2$. E$_1$ and E$_2$ are matrices, which are here.

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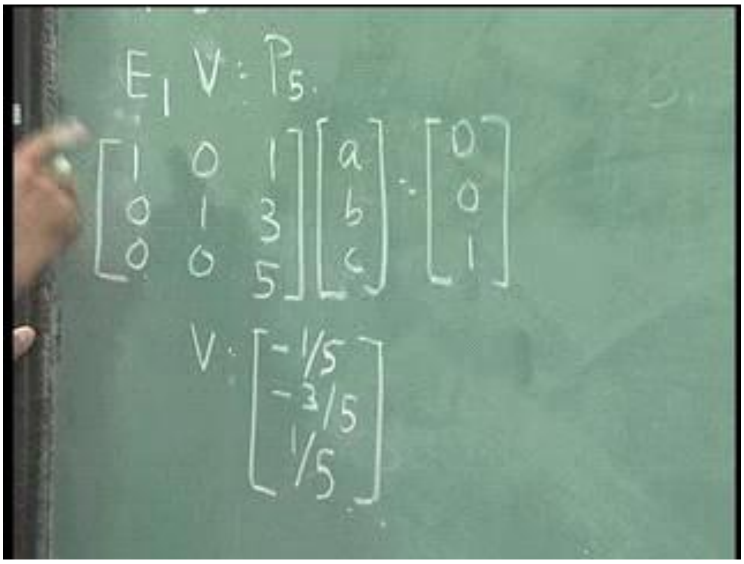

What we do now is, we simply call this E$_2$ into Pbar$_5$ as some V is equal to P$_5$, where this V is a vector, the some V equal to P$_5$. From this we get E$_1$ is 1 0 0, 0 1 0, 1 3 5 and we could simply call this V right now as some a b c will be equal to P$_5$, which is 0 0 1. P$_5$ is 0 0, 1 because in the original problem variable X$_5$ comes only in constraint number three. Because of the nature of the Eta matrix, it becomes easy to compute a b and c. From this, we can easily get V is equal to the third one is 0 into a plus 0 into b plus 5 into c is equal to 1; so c is 1 by 5. Now 1a plus 0b plus 1c equal to 0 would give us, a equal to minus 1 by 5. 0a plus 1b plus 3c equal to 0, would give us b plus 3c equal to 0, b plus 3 by 5 is equal to 0. So b is equal to minus 3 by 5. Once, we know V is equal to this, we now go back here. We have already written E_2 into P_5bar as V. E_2 is here; so 1 into 1 0 0, 4 by 5 7 by 5 1 by 5, 0 0 1 into Pbar$_5$ is now written as some a b c is equal to minus 1 by 5 minus 3 by 5 plus 1 by 5.

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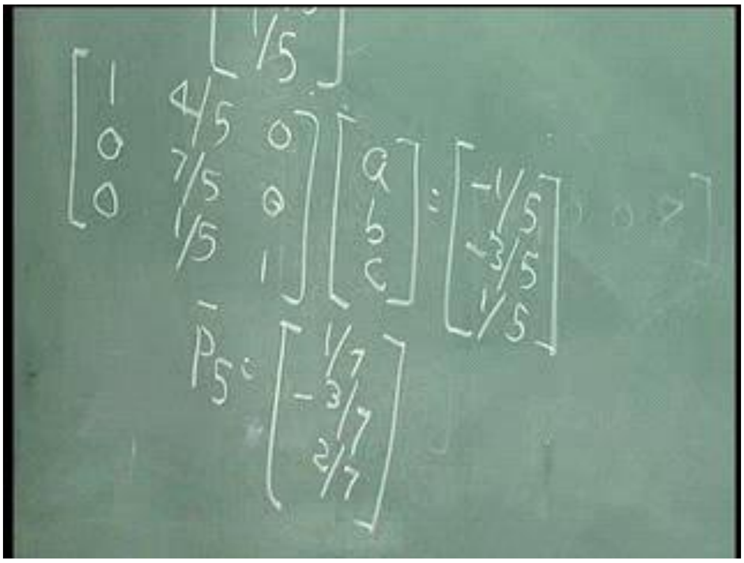

Once again by the very nature of the Eta matrix, we can get Pbar$_5$ by simple substitution process as 0 into a plus 7 by 5 into b plus 0 into c, is minus 3 by 5 from which b is equal to minus 3 by 7. This gives us 7 by 5b is equal to minus 3 by 5, so b is equal to minus 3 by 7. From this, a plus 4b is equal to minus 1 by 5. a minus 12 by 35 is equal to minus 1 by 5. So, a minus 12 by 35 is equal to minus 7 by 35, which is minus 1 by 5; therefore take it to the other side, a will be equal to plus 5 by 35, which is plus 1 by 7.

To find out c, 1 by 5b plus c is equal to 1 by 5, so 1 by 5b is minus 3 by 35 plus c, is equal to 1 by 5 minus 3 by 35 plus c is equal to 7 by 35. Taking it to the other side, we get 10 by 35 which is 2 by 7, so this is 2 by 7. So, now we have found out Pbar$_5$ and once we know Pbar$_5$, we can find out theta. The corresponding values are 5 by 7, 20 by 7 and 45 by 7.

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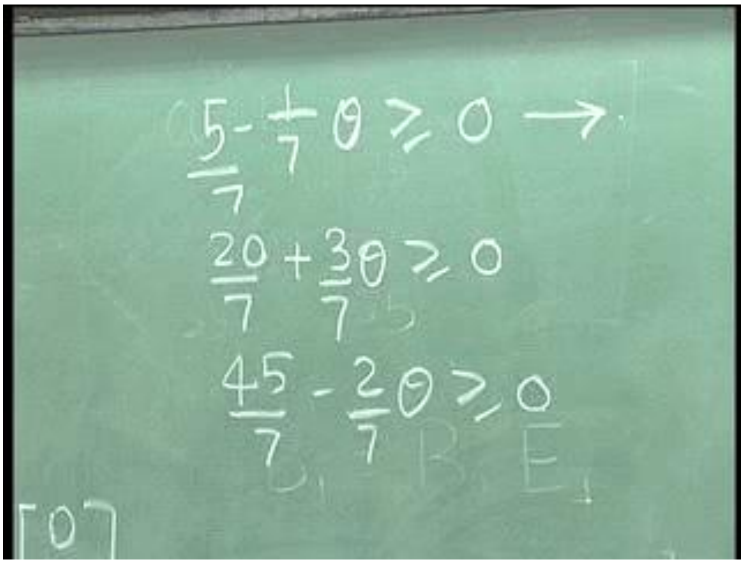

The corresponding value of theta will be such that, 5 by 7 minus 1 by 7 theta is greater than or equal to 0; from the first one, 5 by 7 minus 1 by 7. 20 by 7 plus 3 by 7 theta greater than or equal to 0, the plus comes because of the minus sign here and 45 by 7 minus 2 by 7 theta greater than or equal to 0. This would give us theta equal to 5. This would give us theta equal to any value, because we have a positive coefficient here and this would give us theta equal to 45 by 2; therefore, this is the variable that leaves. Variable X_3 is the variable that leaves and the basis now changes.

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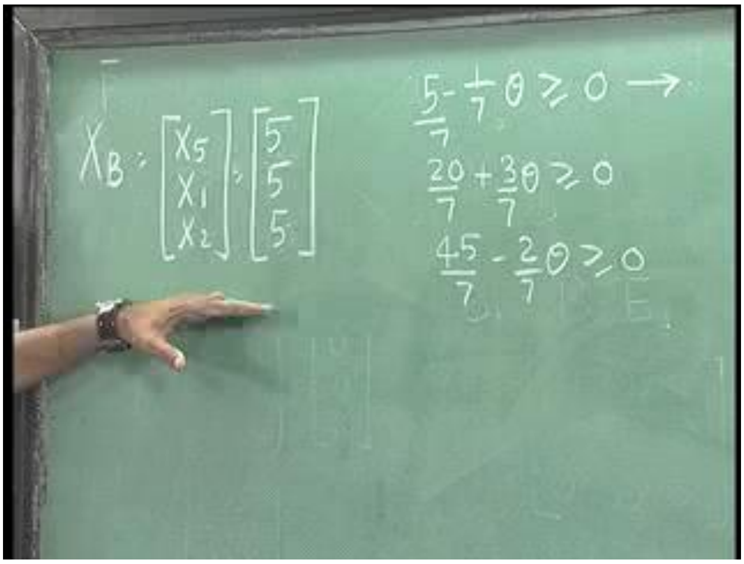

The basis now changes to X_B equal to, we have found out now that the variable X_5 enters the basis and variable X_3 leaves the basis. We have X_5, X_1, and X_2 as the basis and the corresponding values will be, here this was the variable that left, so this will have theta value which is 5. At theta 5, 20 by 7 plus 15 by 7 is 35 by 7, which is 5 and at theta equal to 5 this will become 45 by 7 minus 10 by 7, which is also 5.

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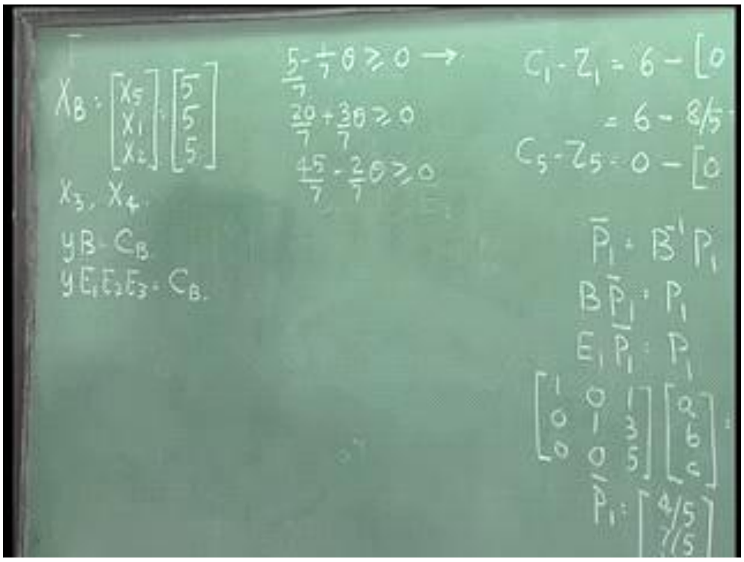

We now have another basic feasible solution with X_1 equal to 5, X_2 equal to 5 and X_5 equal to 5. Once again we need to check whether this basis is optimal and in order to do that, we have to find out C_j minus Z_j corresponding to the two non-basic variables which are X_3 and X_4 and before we find out C_3 minus Z_3 or C_4 minus Z_4, we need to find out y, which is the value of the dual, so yB is equal to C_B. Now, this B is called B_3. So B_3 will be B_2 into E_3. B_2 is B_1 into E_2. B_1 is B_0 into E_1 and B_0 is I. So putting all of them together, we will get y into $E_1E_2E_3$ is equal to C_B. We right now have values of E_1 and E_2, which are written here. So, we need to write the value of E_3. E_3 is another Eta matrix, which is an identity matrix, with one column not being an identity column. From this we found out that the first variable actually left the basis, therefore the first column is a non-identity column.

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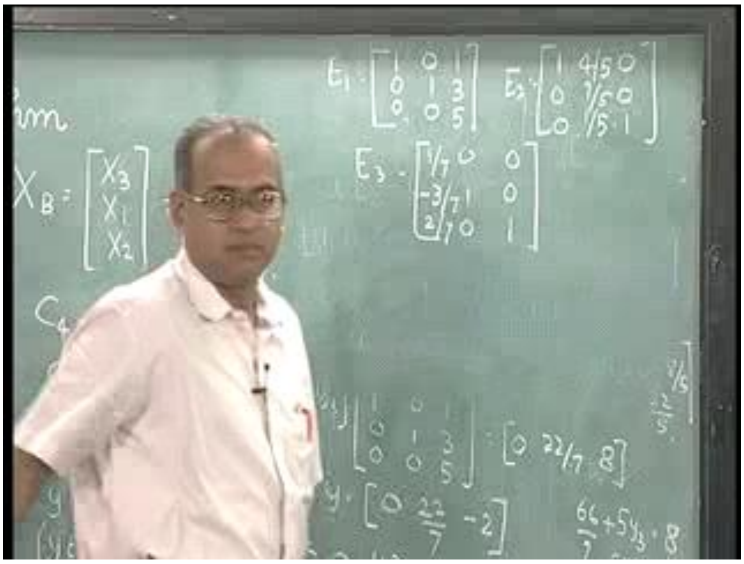

The other two columns will be 0 1 0, 0 0 1 and the first column will be corresponding to Pbar$_5$, which we found out as 1 by 7, minus 3 by 7, and 2 by 7. Because we wrote this out of Pbar$_5$, so if Pbar$_5$ is 1 by 7, minus 3 by 7, 2 by 7, we would have got 5 minus 1 by 7 theta, 20 by 7 plus 3 by 7 theta 2 by 7 theta. So this is our E_3.

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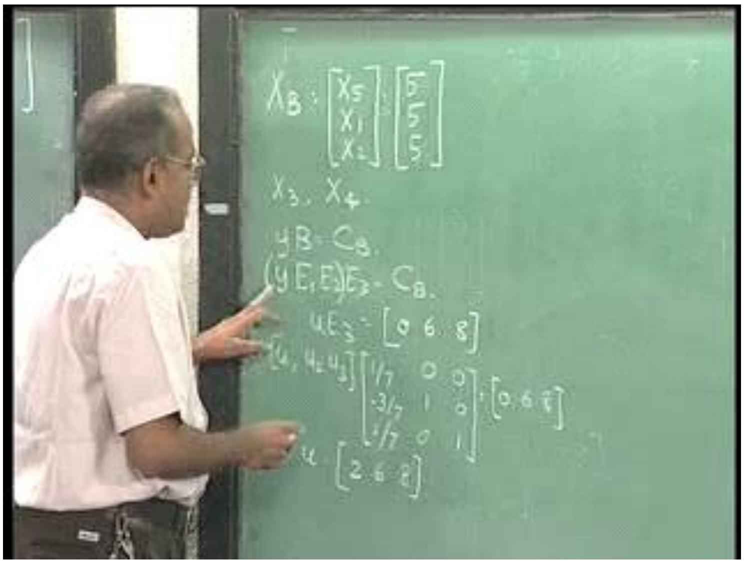

Now using this E_1, E_2, E_3 and C_B, we should now find out y. What we first do is we call this whole thing as u and say u into E_3 is C_B. C_B is the objective function coefficient of the basic variables, which is 0 6 8, so this is 0 6 8. We now substitute E_3. So, u_1 u_2 u_3 into 1 by 7 minus 3 by 7 2 by 7, 0 1 0, 0 0 1 is equal to 0 6 8. Now from the third one 0 into u_1 plus 0 into u_2 plus 1 into u_3 is 8. So u_3 is 8. 0 into u_1 plus 1 into u_2 plus 0 into u_3 is equal to 6, so u_2 is 6. 1

by $7u_1$ minus 3 by $7u_2$ plus 2 by $7u_3$ is equal to 0. So, 1 by $7u_1$ plus, minus 18 by 7 plus 16 by 7 is equal to 0. So, 1 by $7u_1$ minus 2 by 7 is equal to 0. 1 by $7u_1$, this is minus 18 by 7 plus 16 by 7, so minus 2 by 7. 1 by $7u_1$ minus 2 by 7 is equal to 0, from which u_1 is equal to 2. Once we know u_1 is equal to 2, we now go back and find out the other one.

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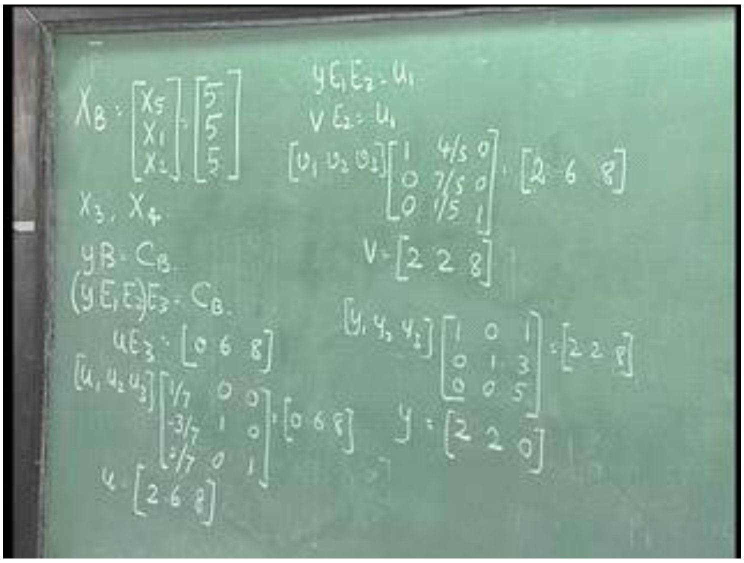

This is called u_1. So yE_1E_2 is equal to u_1, so we now call this as VE_2 is equal to u_1. So, V will be v_1, v_2, v_3 into E_2 is 1 0 0, 4 by 5 7 by 5 1 by 5, 0 0 1 is equal to 2 6 8, because we found out that u is 2 6 8. Once again, by the very nature of Eta matrix we can get V is equal to v_1 plus $0v_2$ plus $0v_3$ is equal to 2, from which v_1 is 2. Again $0v_1$ plus $0v_2$ plus $1v_3$ is 8, so we get 8.

Second one is, 4 by $5v_1$, which is 8 by 5 plus 7 by $5v_2$ plus another 8 by 5. 1 by 5 into 8 is 8 by 5 which is equal to 6; so 8 by 5 plus 8 by 5 is 16 by 5. So 6 minus 16 by 5 is 14 by 5. So 7 by $5v_2$ is equal to 14 by 5 therefore, v_2 is 2.

Now we have found V. We again go back, we have written y into v_1 as V. So, we write y_1, y_2, y_3 into E_1, which is from here 1 0 0, 0 1 0, 1 3 5 is equal to 2 2 8 from which y is equal to, once again by the very nature of Eta matrix, y_1 plus $0y_2$ plus $0y_3$ is 2; so y_1 is 2. $0y_1$ plus $1y_2$ plus $0y_3$ is 2, so y_2 is 2. This alone is y_1 plus $3y_2$ plus $5y_3$ is 8. y_1 is 2, $3y_2$ is 6, 6 plus 2 is 8, y_3 therefore is 0. 2 2 and 0 is the value of y and we can actually even verify the value of y from here. We also know that at any point the value of the objective function of the primal and the dual are the same. So primal has X_1 equal to 5, X_2 equal to 5; substituting, this is 30 plus 40 is 70 and the other dual is 2 2 and 0; 2 2 and 0 would give us the same 70. This is another verification that the dual that we computed is correct. Once we know that this is the value of the dual, we now go back and there are two non-basic variables X_3 and X_4.

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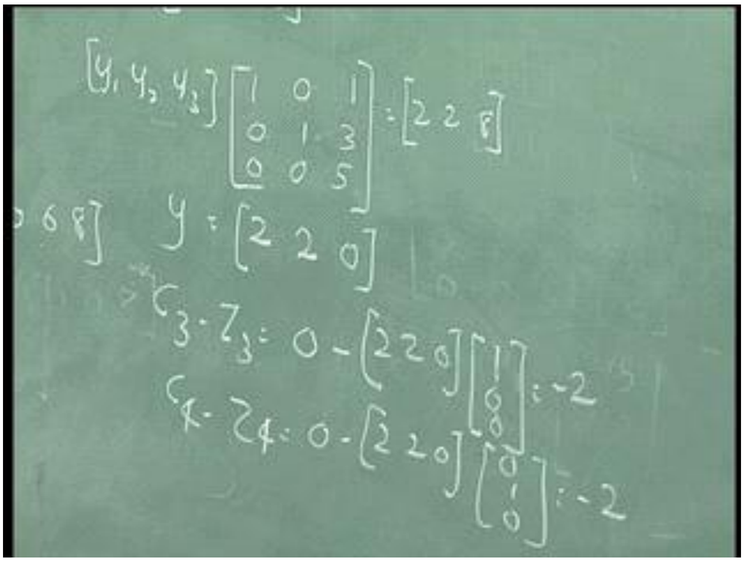

We need to find out C_3 minus Z_3, which is 0 minus y which is 2 2 0 into P_3 which is 1 0 0, and that is minus 2. Now, y is 2 2 0, y into P_3. P_3 comes from here, corresponding to variable X_3. So here the coefficient is 1 0 and 0. Similarly P_4 will be 0 1 and 0, because this does not appear in the first equation. So C_4 minus Z_4 is equal to 0 minus 2 2 0 into 0 1 0 which is minus 2. So, C_j minus Z_j corresponding to both the non-basic variables are negative. There is no entering variable and the algorithm terminates. The algorithm terminates now with the objective function value of 70 with X_1 equal to 5, X_2 equal to 5, Z is equal to 70 and from the dual y_1 is equal to 2, y_2 is equal to 2 and y_3 equal to 0; so, 20 plus 50 is also 70. The algorithm terminates.

This is how the revised simplex algorithm works. On the face of it, it looks much more computationally intensive or laborious compared to the tabular form. Tabular form is much more comfortable, largely because we are used to a tabular form. Now, the only change that actually happened between the tabular form and this way of doing it, is the fact that B inverse was not explicitly computed either using the determinant cofactor adjoint method or by using Gauss Jordan, which we did in the tabular form.

If we look at this very carefully, in this iteration to find out y we had something like y into E_1 into E_2 into E_3 equal to C_B, which means, we have to do this three times to get y. In case this was not optimal, then to find out the entering variable Pbar we need to again do it three times.

In the third iteration it will be three multiplication to get y and perhaps another three multiplication to get the Pbar, which means, in the hundredth iteration, we will be multiplying it hundred times in the forward direction to get y and another hundred times in the reverse direction to get a Pbar. Therefore, the question naturally arises is, is this a very efficient method?

Thus, apart from this, everything else was the same and more importantly we also need to store the E_1, E_2 and E_3. We do not need to store any other thing; we will be storing this. But in addition to that, we only need to store E_1, E_2 and E_3. From a storage point of view, it is said that we normally want to store it as 3 into 3 matrix. E_1 will be stored only as this vector 1 3 5 and a simple identification that this vector is in the third column. This one will be stored as 4 by 5 7 by 5 1 by 5 and 2, which means, this first three are in second column and so on. You do not end up storing a large matrix, instead you store only a vector; you need to store only the vector for every E.

The other question is this laborious process of forward multiplications and reverse multiplications efficient? Incidentally, the answer is yes. For the normally small sized problems that we worked out, we know that the tabular form is much better than this. But as the problem size increases, say we have linear programming problems with 100 constraints or more then, we realize that actually this way of calculating y and Pbar in forward and backward directions is actually much faster. For the simple problems like 2 by 2, 3 by 3 that we normally encounter in a classroom, the tabular method is still very efficient. The moment we get into solving large sized linear programming problems typically more than 100 constraints, this method is found to work much better than the tabular.

In terms of speed, this method is much better and one of the things with which we started this course is to find out how do we make the simplex work a little faster. So we zeroed in on the fact that the most important thing in the simplex is actually the matrix inversion. If we do the matrix inversion, well, by a better method and do it faster automatically, time taken per iteration will come down. The number of iteration, we have already said, we could do based on the largest coefficient rule or any suitable rule. The revised simplex method helps in actually bringing down the computation of this simplex algorithm, because the time taken per iteration comes down, when the problem size is large. But the method that we have seen particularly to invert is called the Eta factorisation of the basis, which we have seen to convert the basis as a product of several Eta matrices and we also use this method called product form of the inverse where to find out the inverse, we try and multiply these Eta matrices once in the forward direction and once in the reverse direction; forward direction to get y and in the reverse direction to get Eta. This is the brief description of the revised simplex algorithm through a numerical illustration.

The next thing that we like to see in linear programming is another important aspect of linear programming. Now, in the next important aspect of linear programming, we also wish to see how well we can actually make the linear programming problem work, if constraints in linear programming are of a certain type. We take a simple example. To start the example what we will do is maximize $8X_1$ plus $3X_2$, subject to $3X_1$ plus X_2 less than or equal to 7, $2X_1$ minus X_2 less than or equal to 8, $2X_1$ plus $3X_2$ less than or equal to 20; X_1 less than or equal to 2, X_2 less than or equal to 6, X_j greater than or equal to 0.

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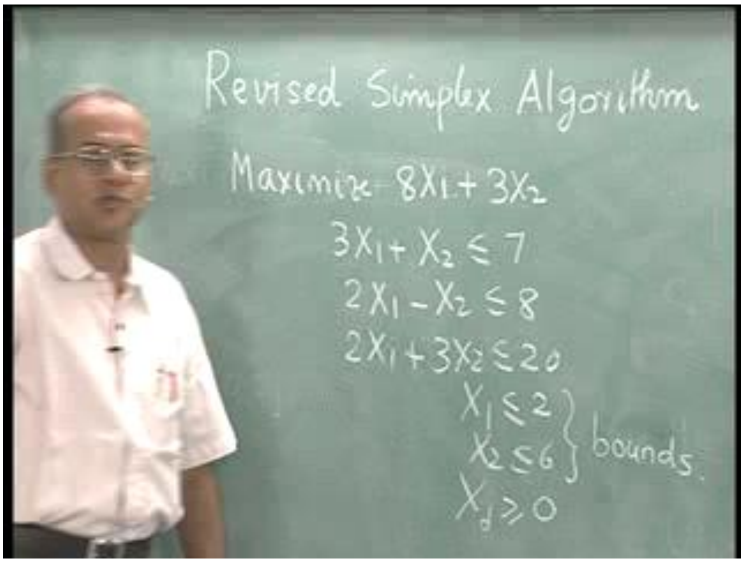

So, let us consider a linear programming problem of this type. Right now, this linear programming problem has two variables X_1 and X_2 and it appears that this linear programming problem has five constraints, three constraints involving both X_1 and X_2, $3X_1$ plus X_2 less than or equal to 7 and so on. The last two are constraints of the type X_1 less than or equal to 2 and X_2 less than or equal to 6. Now, this problem can be solved using the revised simplex that we just now saw in this lecture. The thing is if we use the revise simplex or even the tabular form of the simplex, we will have five constraints and in every stage or every iteration, we will be inverting a 5 by 5 matrix.

A much closer look at these two constraints would tell us that these constraints involve only one variable and therefore these constraints are not treated as constraints, they are called bounds on the variable. Now, the question is do we have to look at this as 5 by 5 problem and solve or can we simply take these bounds outside, look at it only as a three constraint problem and then at every stage suitably incorporate or bring these bounds back into the solution. If we end up doing that, then we have reduced the number of constraints. We already saw in this lecture that the biggest aspect of an iteration of simplex is the matrix inversion.

If we do not treat this as a bound and treat it separately, we end up having five constraints and we end up inverting a 5 into 5 matrix in every iteration. Whereas, if we pull these two things out and solve the resultant three constraint problem also making sure that, these are satisfied, but solving it as a 3 constraint problem, then we end up having a 3 by 3 matrix to invert in every iteration. So, can simplex be modified in such a manner that if there are bounds, these bounds are not treated as constraints, these bounds are taken separately and only the resultant constraints are solved; which also means that a constraint is a constraint, when it involves more than one variable. If it involves only one variable, then the constraint becomes a bound. So can simplex be adapted to problems, which has bounded variables.

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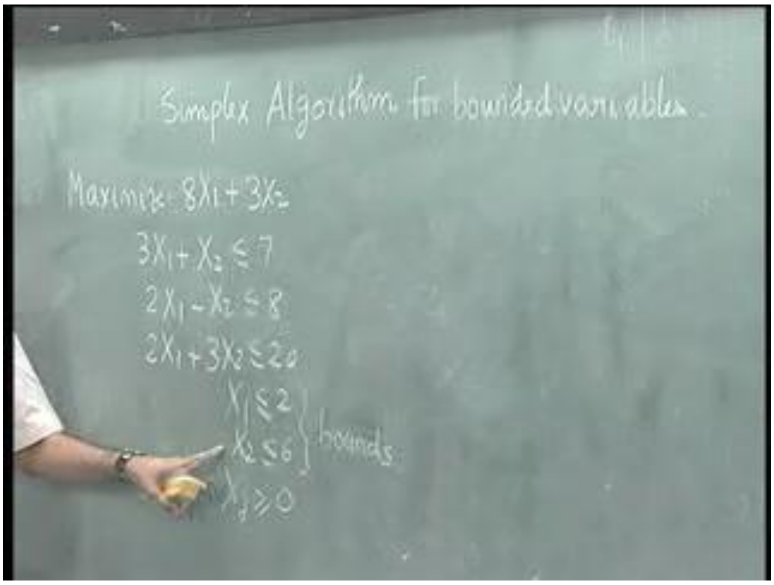

We will now address what to do with simplex, particularly when simplex has bounded variables and therefore the algorithm that we will see is called simplex algorithm for bounded variables. As I said, these two are the bounded variables. We will simply take these variables outside, treat them as bounds, write them separately as X_1 less than or equal to 2 and X_2 less than or equal to 6. Now, this bounded method can be solved using the reverse simplex algorithm. We can also solve it by the normal algebraic method, with the motivation to simplex; not the typical algebraic method, where we evaluate all the basic solutions, but we can solve it by the algebraic method. How we do the simplex method for bounded variables in using the algebraic method, we will see in the next class.