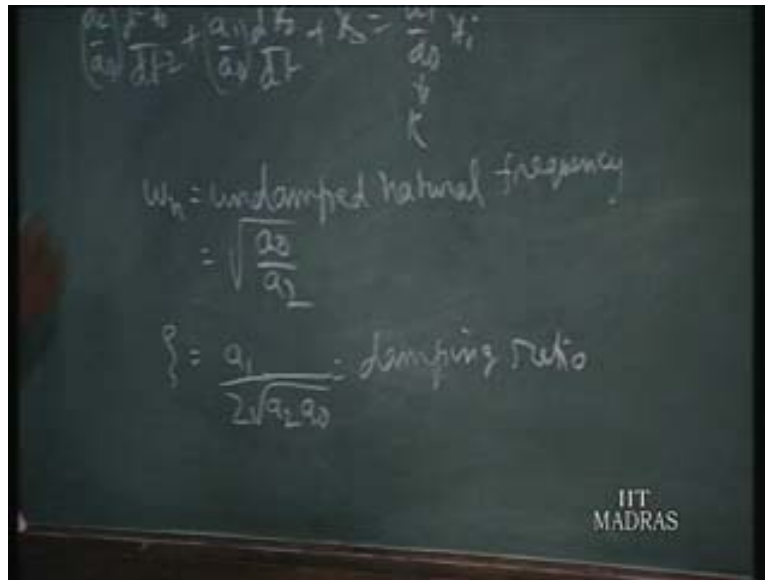


**Principles of Mechanical Measurements**  
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**Department of Mechanical Engineering**  
**Indian Institute of Technology, Madras**  
**Lecture No. #10**

We have seen earlier zero order and first order and now we will see the second order type of instruments. We add one more term to the equation for the first order that is  $a_0 x_0$  equal to  $a_i x_i$  is the zero order instrument.

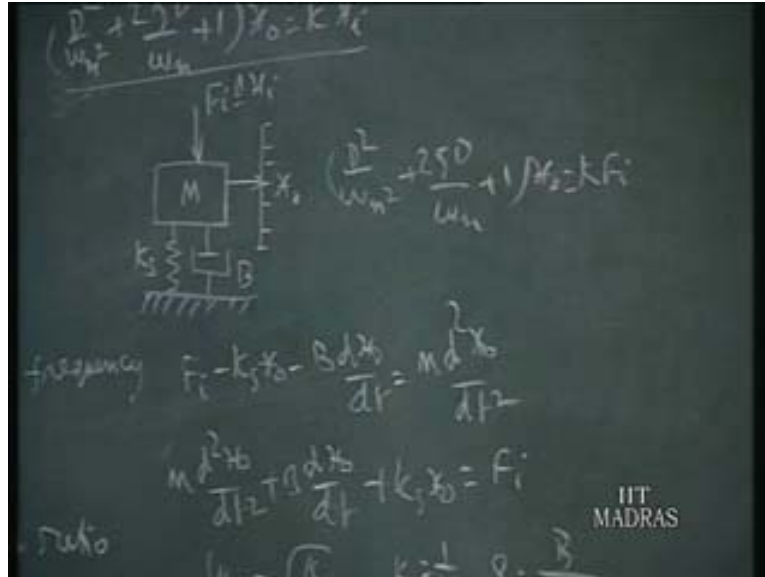
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For first order we added one term  $a_1 dx_0$  by  $dt$  and now second order we add one more term  $a_2 d^2 X_0$  by  $dt^2$ . So this is the equation for the second order instrument general equation. Now we reduce the equation to the standard format, divide everything by  $a_0$  so  $a_2$  by  $a_0 d^2 X_0$  by  $dt^2$  plus  $a_1$  by  $a_0 dx_0$  by  $dt$  plus  $X_0$  is equal to  $a_i$  by  $a_0 X_i$ . Now we know already this is our sensitivity  $K$  and  $a_1$  by  $a_0$  and these are new terms, now we have to define the second order instrument we have got this term, these undamped natural frequency of the system.

Natural frequency is equal to root of  $a_0$  by  $a_2$  that is the definition for undamped natural frequency and next one is damping ratio. Damping ratio represented by  $\psi$  that is  $a_1$  divided by twice root of  $a_2$  into  $a_0$  it's a non-dimensional one so damping ratio. So in terms of these parameters sensitivity and undamped natural frequency and  $\psi$  and  $d$  by  $dt$  we know already so  $d$  by  $dt$  as  $d$  differential operator we rewrite this equation and we get in the following format  $d^2$  over  $\omega_n^2$  plus  $2\psi d$  by  $\omega_n$  +  $1$  into  $X_0$  is equal to  $K$  times  $x_i$ . So this is the resulting equation substituting in terms of  $\omega_n$   $\psi$  and capital  $D$ . So this is the differential equation representing the second order instrument and what is an example in practice. In practice typical example is the force measuring unit.

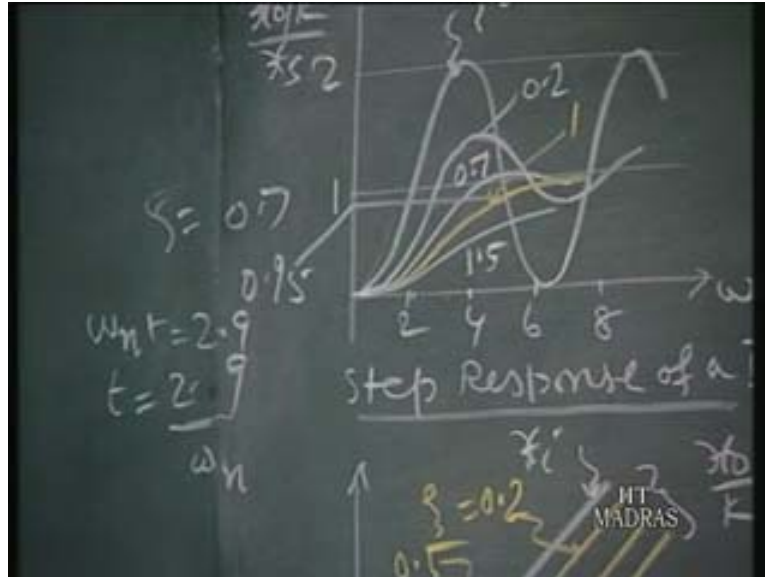
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That is a mass, any force measuring unit or load self to contain a mass supported by a spring and a damper. So this is the frame and now this motion can be measured over a scale say we will say output signal is obtained here, input is  $F_i$  this is your  $X_i$  signal,  $X_o$  is the displacement finally we get so this is a typical second order instrument. Now how to get the equation for this system in the same order. Now consider the force  $F_i$  and part of  $F_i$  is used to overcome the or resistance of the spring or in deflecting the spring and some more part is used to overcome the viscous friction in this damper. The resulting force is used to accelerates the mass. So considering the accelerating the force of mass  $F_i$  minus  $K_s$  into  $X_o$  that is the force used to compress the spring minus if  $B$  is the damping coefficient of the damper so  $B$  into  $dX_o$  by  $dt$  is equal to the force accelerating the mass, so  $m$  into  $d$  squared  $X_o$  by  $dt$  square.

Now this can be brought  $X_o$  term in one side and the  $F_i$  input can be taken to the other side so  $m$  into  $d$  square  $X_o$  by  $dt$  square plus  $B$   $dx_o$  by  $dt$  plus  $K_s$   $X_o$  is equal to  $F_i$ . Now substitute in terms of the undamped natural frequency and damping ratio that is now  $\omega_{n}$  is equal to root of  $K$  by  $m$  and  $K$  is equal to  $1$  by  $K_s$  and size equal to  $B$  by twice root of  $m$  into  $K_s$ . Substitute in this term then you will get in the same format that is  $d$  squared over  $\omega_{n}$  square plus  $2$   $\zeta$   $d$  by  $\omega_{n}$  plus  $1$  into  $X_o$  is equal to  $K$  times  $X_i$  is  $F_i$ , so in this format we get. So this is a typical example for a second order instrument. Now as we have done earlier we have to find out the first step response and so on. For step response now we substitute as you have done for first order system  $X_s$ .

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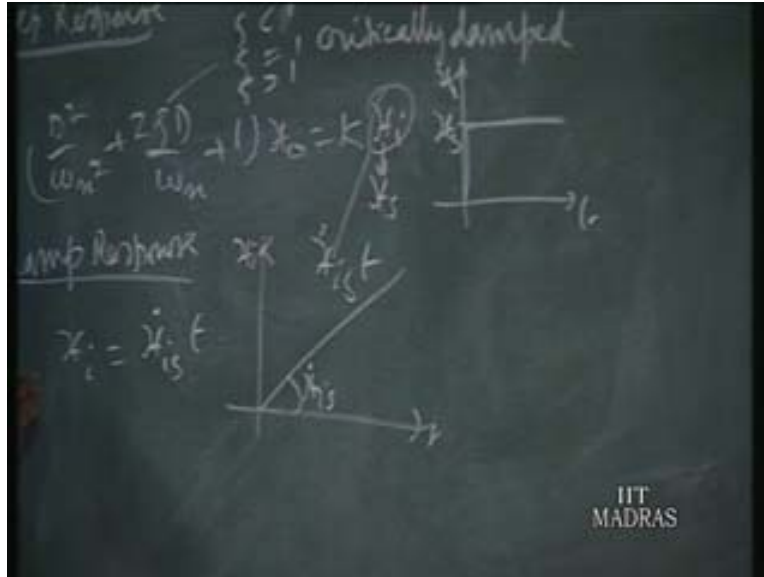


So right hand side will be  $K$  times  $X_s$ . It is a constant so we have solve this differential equation for  $X_0$  and here in this case depending upon the value of  $\psi$ ,  $\psi$  is less than zero,  $\psi$  is equal to one,  $\psi$  is greater than one. Under these three situations we have three equations, there are all very long equations and when they are plotted we get this following curves that is the step response for a second order system. Say  $X_0$  by  $K$  by  $X_s$  that is equal to one this should be the ideal value, one is the ideal value. The output value supposed to correspond to this line 1. That curve  $X_0$   $K$  by  $X_s$  would lie there instead of it how it varies?

Now when  $\psi$  is equal to zero, you get more or less a sinusoidal wave; a size increase you find the oscillation amplitude reduced and when  $\psi$  is equal to one that is critically damped. One is called a critically damped this is critically damped condition, under that condition there is no overshoot it is just goes smoothly tangential to the number one straight line and later on you will find it is 1.5 and 2, 3 and all that is highly damped systems they will behave like a first order systems. For getting the better output normally people go for size equal to point seven because point seven we find the output the output will reaches quickly to the near about one value without much amplitude, without much oscillations that is important thing.

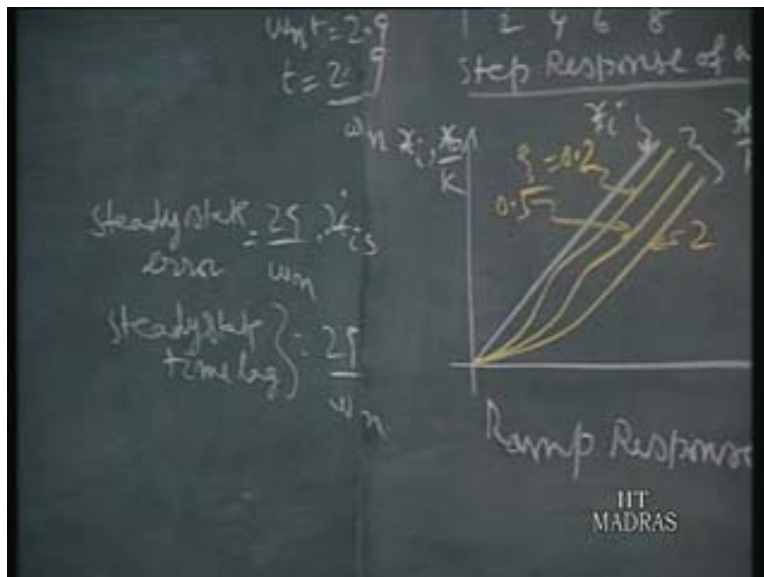
It doesn't oscillate 0.7 very small amplitude is there and later on it will die out and if it is one it takes more time to reach the ideal value so people go for  $\psi$  is equal to 0.7 under that 0.7 and also as in the case of first order system 0.95 reached five, this is call 5% settling time that is 0.95 point it takes say  $\omega_{nt}$  is equal to 2.9; around 2.9 it reaches the 5% settling time. That is time of response equal to 2.9 by  $\omega_n$  that is to reach the 5% settling time, for the second order instrument we call the time of response 2.9 by  $\omega_n$ . That means higher the  $\omega_n$ , time of response or speed of response is quicker or smaller 2.9 by  $\omega_n$ , higher response will give rise to quicker response so that is what do you have to learn here and next we go for the ramp response.

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As for first order system  $X_i$  is equal to  $X_{is}$  dot  $t$  that is velocity at which it varies that is this is the way we have seen this is  $X_i$  versus time  $x_{is}$ . So it is  $x_{is}$  dot so  $X_i$  constantly increases, instead of being  $X_s$  this is  $X_s$  we have suddenly given  $X_i$  increase suddenly to  $X_s$  that is step input this is a ramp input that means in this equation  $X_i$  now we put  $x_{is}$  dot into  $t$  and solve this equation as we have done earlier. Again here also we will have three equations depending upon the  $\psi$  value damping ratio that is under damped, critically damped, over damped these are the three situations and those three long equations when they are plotted you get these curves. Suppose here  $y$  axis represents  $X_i$  and  $X_o$  by  $K$ , the white line represents  $X_i$  where of versus time it constantly increases.

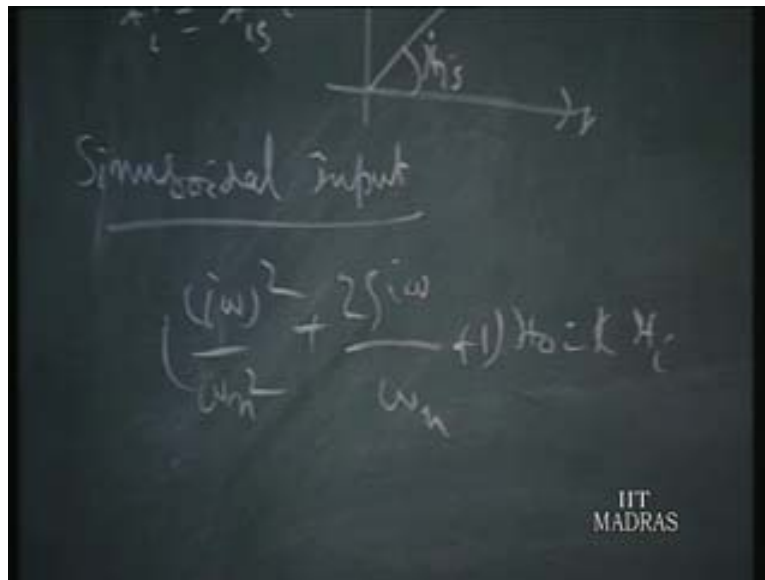
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Now when  $\zeta$  is equal to 0.2 under damped, we find there is some transient behavior after it settles at a certain steady state error. Similarly for other values 0.5  $\zeta$  is equal to 2. One is I mean under damped two curves under damped, one is over damped, for one also it will be something like two between these two values one will be there. Here the steady state error for this ramp input is  $2\zeta / \omega_n$  by  $\dot{x}_{is}$  is analogous to our time constant into  $\dot{x}_{is}$  dot this is our steady state error.

Steady state error for ramp input in a second order system is given by  $2\zeta / \omega_n$  into  $\dot{x}_{is}$  dot and steady state time lag in the first order instrument is  $\tau$  here it is equivalent is  $2\zeta / \omega_n$ , that is steady state time lag. So for any curve we can find, this is our steady state time lag and this will be steady state error, for this curve it is steady state error from the theoretical value. This one is analogous to  $\tau$  that is actually our  $2\zeta / \omega_n$  and this vertical is  $2\zeta / \omega_n$  into  $\dot{x}_{is}$  dot that is for the ramp input.

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Now sinusoidal input we have to substitute in the differential equation  $d$  by  $i\omega$ . So  $i\omega$  square  $d$  squared  $\omega_n$  square plus  $2\zeta i\omega$  by  $\omega_n$  plus  $1$  into  $X_o$  is equal to  $K$  times  $X_i$ . So  $X_o$  by  $X_i$  in terms of  $i\omega$  is equal to  $K$  by  $d$  squared over;  $d$  square that is  $i\omega$  square  $d$  squared by  $\omega_n$  squared plus  $2\zeta i\omega$  by  $\omega_n$  plus  $1$ .

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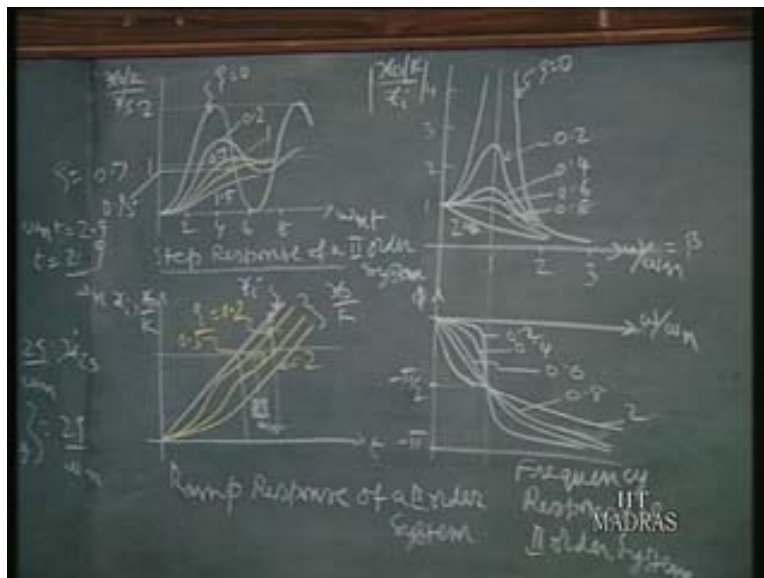
$$\frac{X_o/K}{X_i} = \frac{1}{\omega^2 + 2\psi\omega + 1}$$

$$\left| \frac{X_o/K}{X_i} \right| = \frac{1}{\sqrt{(1-\beta)^2 + 4\psi^2\beta^2}}$$

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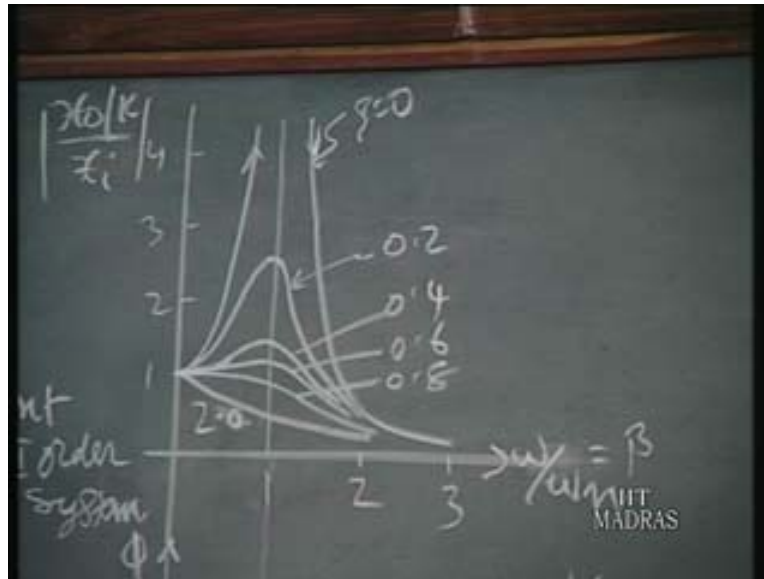
It's a complex number so the magnitude is equal to, I can bring this side  $X_o$  by  $K$  is plotted. So I can bring  $X_o$  by  $K$  this side by  $X_i$  mod that is  $i$  omega mod magnitude is equal to one by root of one minus, will call beta is equal to omega by omega<sub>n</sub>, the frequency ratio. So one minus beta the whole squared plus four psi square beta square plus one and phi is equal to tan minus one of two psi beta by beta squared minus one. That is the axis and now these two equations are plotted here that is a frequency response second order system this is omega by omega<sub>n</sub> is nothing but beta, this is beta value.

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When beta is equal to one it is a resonant conditions that is the excited frequency is same as the natural frequency of the system then we say it is a resonant condition. Under resonant condition we find under resonant conditions the amplitude ratio goes to infinity that is what is indicated here, it goes to infinity and comes back.

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When psi is increased from zero then you will find the maximum amplitude occurs when the omega is equal to  $\omega_{a_n}$  that is what is depicted here but we find between 0.6 and 0.8 say again psi is equal to 0.7, as you have learnt in the for step response 0.7 is very ideal value. Similarly here also you will find around 0.7 you have the maximum that is ideal value should be around one, this should be the ideal condition. So  $X_o$  by  $K$  by  $X_i$  should be equal to one that is perfectly proportional conditions without any error due to the dynamic conditions.

So that is achieved when psi is equal to around here also 0.7, in between 0.6 and 0.8, you will have maximum range some somewhat here it will come in between. So this may correspond and may be around 0.8 up to beta is equal to 0.8 you may have the usable ranges. Hence for the second order instrument this is called the bandwidth of the instrument. That is a region in which the instrument is supposed to be used that is in this region it is equal to one. Later it deviates that's how the frequency response is made use of in fixing the bandwidth of the instrument. Also when psi is equal to 0.7 it is noted 0.6 and this is 0.8 that is the phase difference also so more or less its linear that is acceptable in a dynamic response higher frequency this is  $\omega_{a_n}$  that is beta.

So at higher frequencies the phase shift will be proportional to the frequency. So phase shift will be the proportional to frequency that is also acceptable, more or less in this measurement region it is more or less straight line. So these two things are attained by the second order equation. Now we'll work out one or two problems in this chapter dynamic response, first problem.

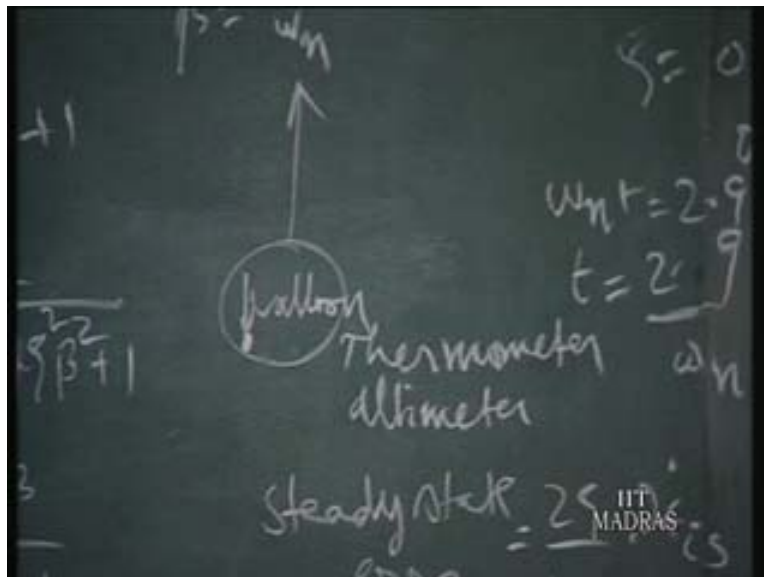
Problem one there is a balloon rising in atmospheric conditions it's just rises at a rate of 7 meter per second.

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That is our speed at which, speed of rising of the balloon this is balloon rises at this speed and it has got a thermometer inside and also an altimeter, thermometer and altimeter.

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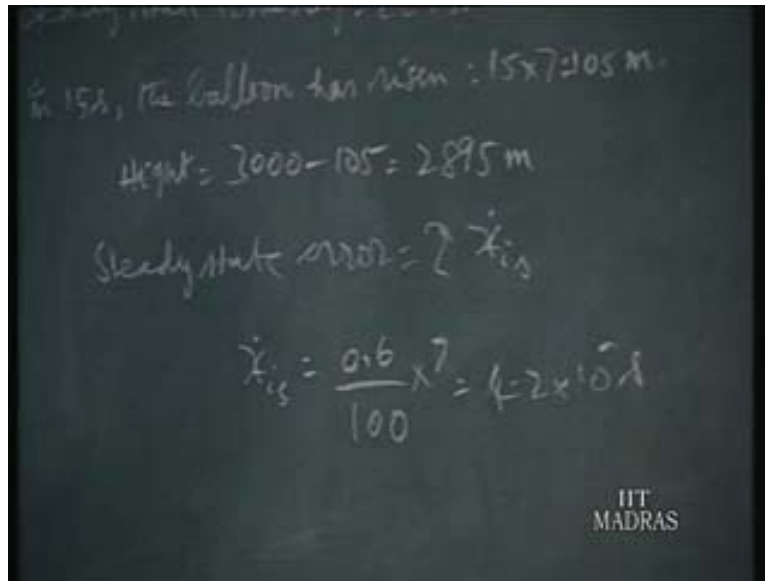


It measures the height of the balloon at any instant, the thermometer has got a time constant of 15 seconds.



The temperature varies at the rate 0.6 degree centigrade per 100 meter that is atmospheric conditions, the temperature reduces every 100 meter by 0.6 degree centigrade and the balloon is sending the signals for temperature as well as for the altitude. When altitude is 3000 meter there is a signal corresponding to minus one degree centigrade but 3000 meter height, it sends a signal of minus one degree centigrade. What is the actual temperature T at 3000 meter and also this temperature is indicated in heights. This is what is done. It is a typical example of a ramp input, it's a ramp response so we will see how to work out this problem.

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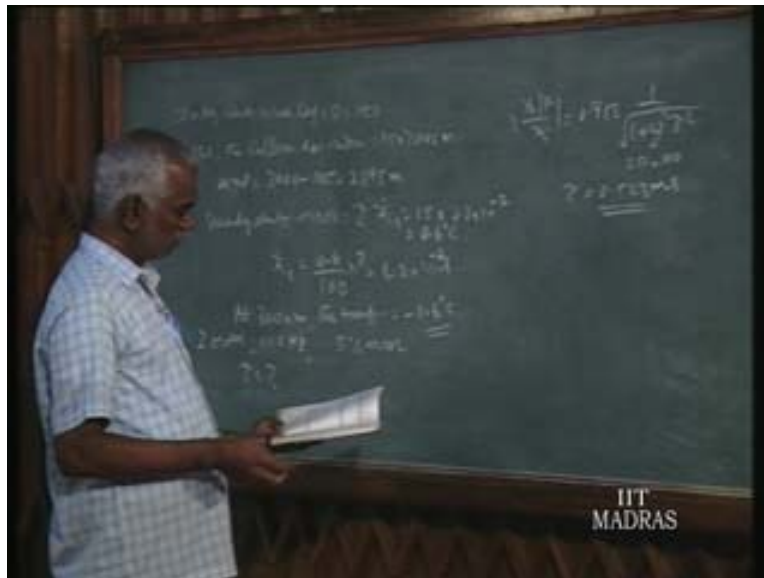


Now we know that steady state time lag is equal to tau say whatever reading we get that was taken on one time constant earlier. So in the time constant that is in 15 seconds earlier where it was the balloon that is the temperature. So where it was previously? It is rising at 7 meter per second so in 15 second 105 meter, so 105 meter below when it was there, this temperature was minus one degree centigrade was shown. So height is equal to 3000 minus this earlier this, so it is equal to 2895. When it was at this height the temperature was shown as minus one centigrade but it was shown 15 seconds later as per our theory we have found earlier.

So this is the actual height but at 3000 meter what should be the temperature? That is steady state error which we know already. Steady state error is equal to tau into  $\dot{x}_{is}$  dot, this is steady state error we know tau already 15 second given  $\dot{x}_{is}$  dot we have to find out. What is this rate at which temperature is varying? That is we know it is rising at a speed of 7 meter per second and the temperature is for 0.6 degree centigrade for 100 meter. So for 7 meters whatever be the degree centigrade that is for 7 meter it rises in a second. So this is equal to point, that is 4.2 into 10 to the power of minus two second. That is for one degree varies degree per second, it varies 0.042 degrees it varies per second minus 2 minus, 4.2 into 10 to the power minus 2 second.

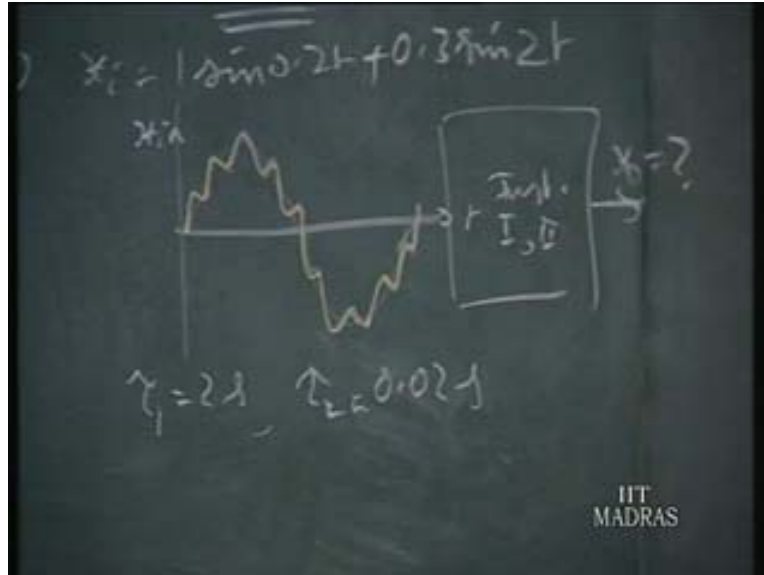
So the steady state error is equal to  $15 \times 4.2 \times 10^{-2}$  this gives rise to 0.6 degree centigrade. So as the balloon rises we have the lower temperature so at 3000 meter the temperature is lower by this much. So is equal to minus 1.6 degree centigrade this is the temperature at 3000 meter but it will be shown after 15 second this is the way the ramp system functions, so this is problem one and problem two. First order problem two, first order instrument measure signals with the frequency up to 100 hertz with an amplitude inaccuracy of 5% error and this is the first order instruments measures up to signal 100 hertz with 5% error then calculate the maximum allowed time constant. What should be the time constant of the instrument, if it has to measure 5% error and up to 100 hertz signal has to be handled and it is a first order instrument what should be the time constant of the instrument? For this you have to use this magnitude ratio equation.

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That is  $X_o$  by  $K$  by  $X_i$  is equal to now for 5% error it should be per magnetic ratio it should be 0.95 that is equal to one by root of one plus omega square tau square. The sampled ratio should be 0.95 for 5% error magnitude ratio, remaining magnitude should give rise to this much. Omega is known as now two phi into the 100, 100 hertz signals is the maximum then from this tau only is unknown and we can find out tau as 0.523 millisecond. So that's how for measuring with any given inaccuracy what can be the time constant of this system can be found out by using the amplitude ratio equation. Another typical example is there, two third problem three.

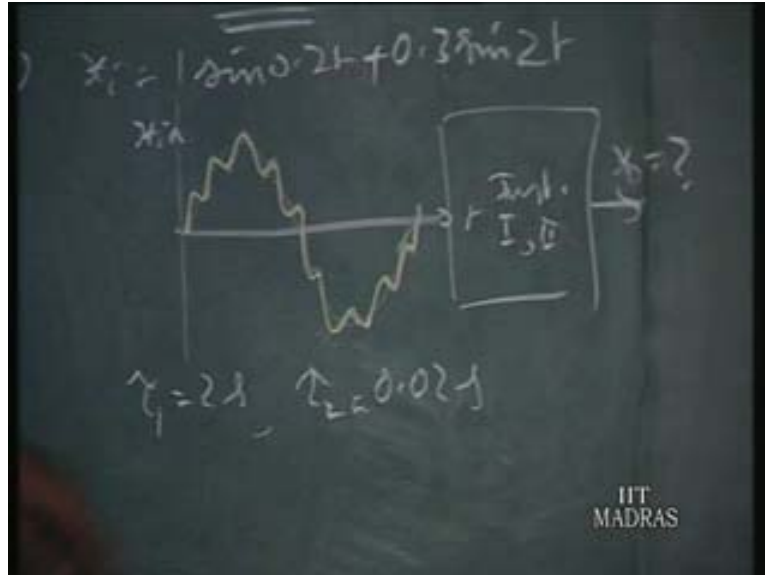
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We have a signal  $X_i$  is equal to, one is the magnitude and  $\sin 0.2 t$  plus  $0.3 \sin 2 t$ . That is it's a signal having two components. This is  $X_i$  versus  $t$ , this is one component and here we have the variation superimposed goes like this, so it has two components, this is the final signal (Refer Slide Time: 25:45). So one is the smaller,  $\omega$  is equal to  $0.2$  radian per second other one is two radian per second that is higher frequency. Higher frequency signal superimposed over a lower frequency and a low frequency signal has got amplitude of one and a high frequency has got  $0.3$  with which the other signal is varying.

Suppose this signal is given to two instruments having a time constants, two first order instruments time constant of  $\tau_1$  is equal to two second and another instrument has got a time constant of  $0.02$  second. These are the two instruments to which this instrument is given, this is the instrument this is input what will be the output signal? Instrument one and instrument 2, two instruments are there. First give it to first instrument having two seconds later by point naught two seconds find out the response of this instrument. We have to find out the amplitude ratio and the phase lag so first it take the instrument one.

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That is time constants equal to two seconds. So for this an amplitude ratio  $X_o$  by  $X_i$  i omega mod is equal to  $K$  over root of one plus omega square tau square. So here we have the frequency is given 0.2 and the other one two frequencies are there we can take one frequency that is omega is equal to 0.2 radian per second and then we call calculate  $A_o$  by  $A_i$ , this is equal to  $K$  by; that is omega is equal to 0.2 input and tau is equal to 2 seconds, omega is equal to 0.2. Tau is equal to two seconds we will have the ratio 0.93 or this is equal to 0.93k and phi is equal to tan minus one of minus omega tau that is equal to minus 21.8 degrees.

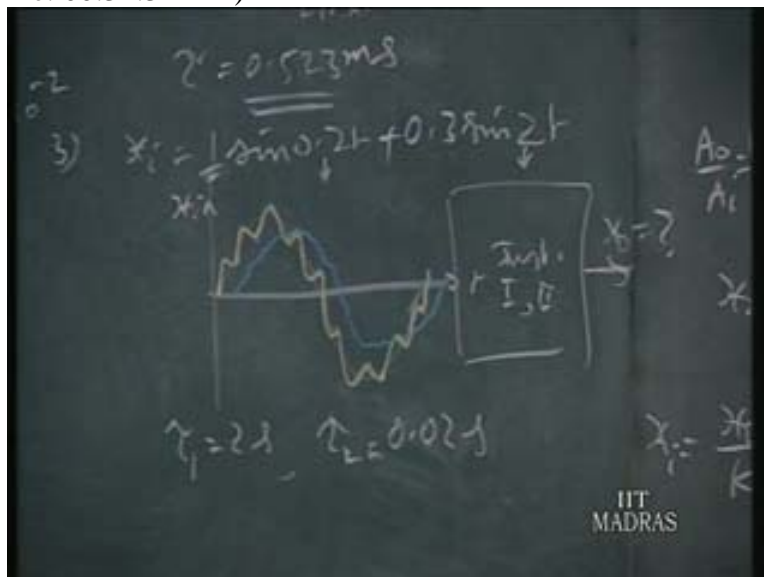
Similarly when omega is equal to two and again to tau is equal to 2 then you will have the same ratio  $X_o$  by  $X_i$  i omega that is magnitude ratio is equal to 0.24 K so to say or we can also write this is  $A_o$  by  $A_i$  that is amplitude of the output signal and amplitude of the input signal ratios. Here the phi is equal to tan minus one of this, it gives rise to minus 26 degree. So we have found out both the low frequency, high frequency component the amplitude ratio and the phi and we write this. now the output signal  $X_o$  is equal to 0.93 K into  $A_i$ ,  $X_o$   $A_o$  is equal to this into  $A_i$ ,  $A_i$  is one for this signal into one that is sin omega t, omega t is a here 0.2 t and the phi is minus 21.8 degree.

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$\omega = 2 \quad T = 2$   
 $\frac{A_o}{A_i} = \left| \frac{x_o(\omega)}{x_i} \right| = 0.24K$   
 $\phi = -76^\circ$   
 $x_o = 0.93K \sin(0.2t - 21.8^\circ) + 0.24K \cdot 0.3 \sin(2t - 76^\circ)$   
 $\frac{x_o}{K} = 0.93 \sin(0.2t - 21.8^\circ) + 0.072 \sin(2t - 76^\circ)$

So this is the first term, the second term is now 0.24 K into the  $A_i$  that is 0.3 into 0.3 sin 2 t minus 76. This is the final equation of the output signal. So now we divide this  $x_o$  by K, K we will bring this side is equal to 0.93 sin 0.2 t minus 21.8 plus 0.24 into 0.3, 0.072 that is plus 0.072. That is what is remaining without a sin 2 t minus 76. So this is the equation  $x_o$  by K now this would be equal to  $x_i$  if it is not  $x_i$  there is error. What is the error now? Previously it was amplitude one and here it is reduced to 0.93 and the lag is 21.8 degrees. So slowly varying signal, the amplitude is not reduced much but what happened to high frequency signal there it was 0.3 amplitude but it has come to 0.072. It has almost disappeared that means the output signal will be of this nature, so it is following that is the high frequency common and is completely eaten away.

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This is 21.8 degrees, later it comes and this one it has reduced to 0.93. That is the first instrument responds only for the low frequency signal and high frequency signal it only produce only small ripples, it is completely eaten away. So if you work out now for the second instrument has got a time constant of 0.02 seconds that is 100 times smaller time constant. If you work out the same thing for this time constant of 0.02 same steps then you will get finally the equation  $x_o$  by  $K$  is equal to 1.00 up to the second decimal  $\sin 2 t$  minus 0.23 degree plus 0.30  $\sin 2 t$  minus 2.3 degree.

So now you find the original signal is almost there up to second decimal, here is very small phase lag and here also amplitude remain same and the phase lag is also is very small. So such a signal where there is a high frequency component we should always go for an instrument having very low time constant. Then only the input signal will be truthfully reproduce in the output side otherwise the higher time constant instruments will eat away the higher frequency component so that is being well demonstrated by this example. Here with this we will close this chapter and we go to the next chapter.

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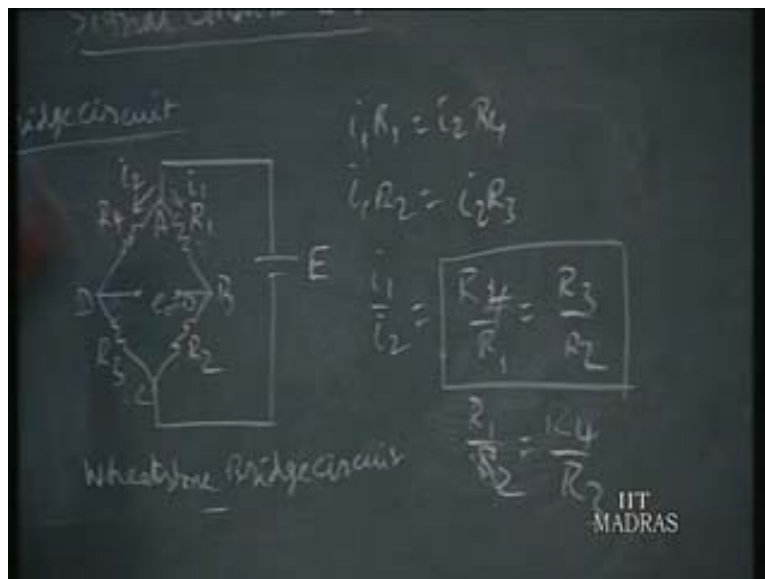


We have seen in one of the earlier lectures an instrument inside can be divided into three sub divisions, first is signal input unit then signal conditioners and last is signal output unit. Now you are going to learn here is signal conditioners that comes in middle of the instrument that is it takes output of the signal input unit and the processes the signal so that it can be given output unit. For example if signal output is a digital device then suppose signal cover from the input unit as analog necessarily the condition should contain an analog to digital conditioner or suppose the signal input is resistance change for example strain gauge we are using and then we have to convert that resistance change into a voltage change that is done normally by a bridge circuit. So bridge circuit becomes part of the signal conditioners. Now the bridge output may be a very small value of voltage and it cannot be given directly to output so it has to amplified.

So next comes amplifier so like this bridge circuit, amplifier, analog to digital converters or digital to analog converters all these things form part of the signal conditioners. So first we will see the bridge circuit. Bridge circuit is often used for converting resistance change into voltage change. We have got both ac and dc bridge and dc bridges we have only resistances, in ac bridge resistance, capacitor, inductance these three passive elements can be combined in any way for each arm that will be ac bridge that we'll see little later. Now what is the condition of balancing for a Wheatstone bridge? For a Wheatstone bridge, under balanced condition it is understood  $e_o$  will be zero. If it is balanced then we find the voltage drop across this will be called A, B, C, D this are the corners we name it.

So the voltage drop across resistance  $R_1$  should be equal to voltage drop across  $R_4$  that is voltage drop across A B or B A should be same as B A. That means if  $i_1$  is the current flow through  $R_1$  and  $i_2$  is current flow through  $R_4$  then voltage drop here will be  $i_1$  into  $R_1$  that should be equal to  $i_2$  into  $R_4$  under the balanced condition then only here we will have  $e_o$  zero. Similarly since here it is voltage zero B and D in the same potential, no current flow can take place in this path that means this current  $i_1$  should go through  $R_2$  whatever it has gone through  $R_1$  should go through  $R_2$  also. Similarly  $i_2$  which has gone through  $R_4$  should go through  $R_3$  that means here also  $i_1$  into  $R_2$  is equal to  $i_2$  into  $R_3$ , here also voltage drop should be equal then only you will have the  $e_o$  as zero. So under the situation now you find  $i_1$  by  $i_2$  is equal to  $R_2$  by  $R_1$  from the first set of equation, from second set  $i_1$  by  $i_2$  will be equal to  $R_3$  by  $R_4$ . So this is called the ratio of arms, ratio of the Wheatstone bridge for balanced condition.

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By using the law of ratios you can write this ratio in different way,  $R_1$  by  $R_2$  is equal to  $R_4$ . This is  $i_1$  into  $R_2$  is equal to  $i_2$  into  $R_3$ .  $R_2$  by  $R_1$  is equal to  $R_3$  by  $R_4$ , it will be  $R_2$  no this is  $R_2$   $i_1$  by  $i_2$  is equal to  $R_3$  by  $R_4$  (Refer Slide Time: 37:28).

This is  $R_4$  so  $R_1$  by  $R_4$  we can write  $R_1$  by  $R_2$  is equal to  $R_4$  by  $R_3$  this is one way of writing. So  $R_1$  by  $R_2$  is equal to  $R_4$  by  $R_3$  that is the ratio of adjacent terms  $R_1$  by  $R_2$  is equal to  $R_4$  by  $R_3$  so now again go to the same side write this equation or you can write  $R_1$  by  $R_4$  is equal to  $R_2$  by  $R_3$  or  $R_4$  by  $R_3$  three is equal to  $R_1$  by  $R_2$ .  $R_1$  by  $R_4$  is equal to  $R_2$  by  $R_3$  any way you can write but what you have to remember is the side from where you start for the other ratio you go to the same side. That we start with  $R_1$ ,  $R_1$  by  $R_2$  then write  $R_4$  never write  $R_3$  by  $R_4$  that is the only way to remember it. So this arms ratio should be satisfied for the balancing conditions. Now what is the voltage output that is unbalanced voltage  $e_o$ , what is the voltage  $e_o$ ? If there is unbalance in this bridge then the difference between voltage drops between A B and A D will be the unbalanced voltage. So that is  $e_o$  is equal to  $E_{BA}$  minus  $E_{DA}$ .

Now what is voltage drop across this? The total voltage is across the two terms  $R_1$  and  $R_2$ . Similarly  $R_3$  and  $R_4$  the full voltage. So if we consider only  $R_1$  and  $R_2$ , the  $R_1$  by  $R_1$  plus  $R_2$  will be the portion across  $R_1$  similarly  $E_{DA}$  will be  $R_4$  by  $R_3 + R_4$  portion of  $E$  that will be coming there so you find this is equal to  $E$  into  $R_1$  by  $R_1$  plus  $R_2$  minus so  $E$  we take it outside so  $R_4$  by  $R_3 + R_4$ . So reduce this form, reducing this we have got this equation  $e_o$  is equal to capital  $E$   $R_1 R_3$  divided by minus  $R_2 R_4$  divided by  $R_1$  plus  $R_2$  into  $R_3$  plus  $R_4$ . Now the resistances are changed from  $R_1$  to  $\Delta R_1$  when the resistance are picking up signals there is some changes. So  $\Delta R_2$  and  $R_3$  changes to  $\Delta R_3$  and  $R_4$  changes to  $\Delta R_4$ ,  $R_4$  plus  $\Delta R_4$ .

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The image shows a chalkboard with the following handwritten equations:

$$= E \left[ \frac{(R_1 + \Delta R_1)(R_3 + \Delta R_3) - (R_2 + \Delta R_2)(R_4 + \Delta R_4)}{(R_1 + \Delta R_1 + R_2 + \Delta R_2)(R_3 + \Delta R_3 + R_4 + \Delta R_4)} \right]$$

$$\frac{R_1}{R_2} = \frac{R_4}{R_3} = m$$

$$e_o = \frac{m}{(1+m)^2} E \left[ \frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} + \frac{\Delta R_3}{R_3} - \frac{\Delta R_4}{R_4} \right]$$

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In such situations  $E_o$  will be equal to just  $E$  into  $R_1$  plus  $\Delta R_1$  into  $R_3$  plus  $\Delta R_3$  minus  $R_2$  plus  $\Delta R_2$  into  $R_4$  plus  $\Delta R_4$  divided by so  $R_1$  plus  $\Delta R_1$  plus  $R_2$  plus  $\Delta R_2$  into  $R_3$  plus  $\Delta R_3$  plus  $R_4$  plus  $\Delta R_4$ . Now it can be reduced because  $\Delta R_1$  is a small quantity,  $\Delta R_1$  into  $\Delta R_3$  will be still smaller we can reduce, we neglect them to be zero.

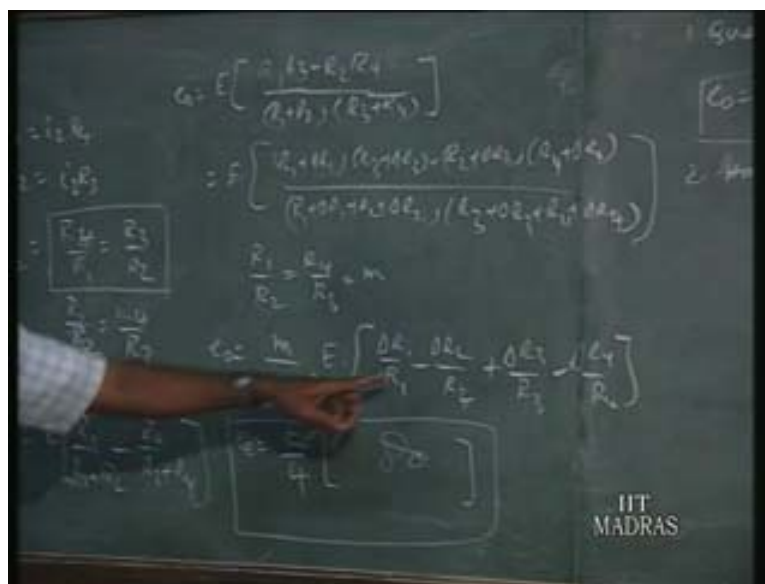


So higher orders of delta terms and also routing down for balance condition  $R_1$  by  $R_2$  is equal to  $R_4$  by  $R_3$  calling it as arms ratio  $m$ , it can be reduced or neglect higher order terms we can reduce this equation into  $e_o$  is equal to  $m$  by one plus  $m$  square into  $E$  into  $\Delta R_1$  by  $R_1$  higher order terms are neglected minus  $\Delta R_2$  by  $R_2$  plus  $\Delta R_3$  by  $R_3$  minus  $\Delta R_4$  by  $R_4$ , to this form the above equation can be brought down, so this is the final equation. Suppose  $m$  is equal to one that is the case in mechanical measurements, we are using strain gauge of 120 ohms each or equal arm bridge so  $m$ 's equal to one in that case you will find  $e_o$  is equal to  $E$  by 4,  $m$ 's equal to one means 1 by 2 square that is  $E$  by 4 into the same term you write it whatever inside the bracket. So this will be the imbalance voltage of a bridge.

We will use this equation to find out the voltage output in different configurations of the bridge. First configuration is Quarter Bridge that is the bridge is not always used with all the four terms active. Sometimes one of the four arm will be active sometimes two of the four arms are active sometimes all the four arms are active. When we give signal to only one of the arms the other three arms are kept constant that is  $R_1$  alone is varied and the other three will be kept constant it is called quarter bridge only one of the four arms is active others are constant. If it is half bridge two of the arms are active other two will be constant if it four terms means all the four will change that is whenever you give a signal all the four will be changing.

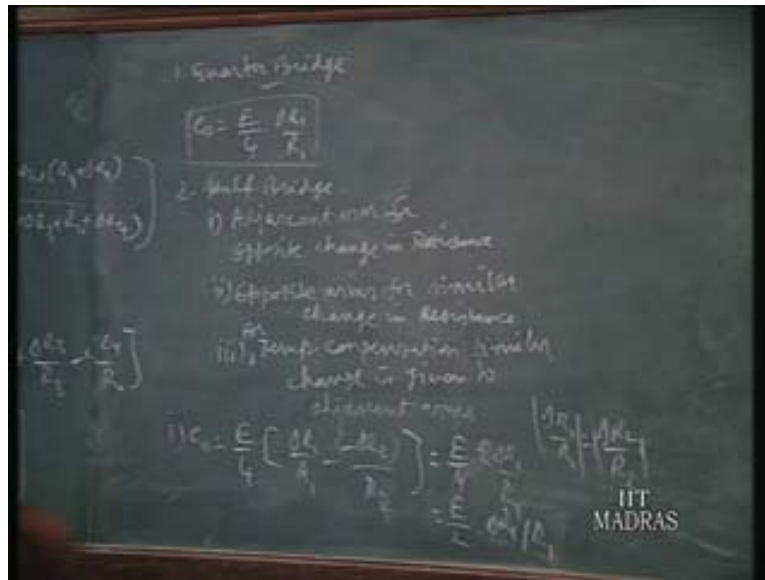
So this are the three configurations quarter bridge, half bridge and full bridge these are the three configurations and the quarter bridge what is the voltage output. So that is  $e_o$  is equal to  $E$  by 4 where delta are alone exists all the other three are zeros because it's a quarter bridge so into  $\Delta R_1$  by  $R_1$  this is simple. In quarter bridge voltage outputs is equal to  $E$  by 4  $\Delta R_1$  by  $R_1$  relative change of a resistance. Now in the half bridge that is half bridge what is the voltage output?

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Now half bridge's we have got two configurations, you can have  $R_1$  and  $R_2$  as active arms or  $R_1$  and  $R_4$  or the adjacent arms may be made active or opposite arms may be made active this are the four situations. Take the adjacent arms, you know adjacent arm adjacent arms is active average now if you want to make an adjacent arms as active and if you want to get any voltage output then adjacent  $R_1$  to  $R_2$  they are adjacent arms now in that case the polarity for delta  $R_2$  should be negative. If it is negative only minus and minus will becomes positive you will get some voltage if this one also positive then it gets nullified and voltage output will be zero. That means for adjacent arms for opposite change in resistance.

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When the resistance changes opposite way in both the adjacent arms then you can use this bridge. Similarly half bridge you can use for adjacent arms for opposite change in resistance and opposite arms that is one I will say 1, 2 and opposite arms are for similar change or similar change say similar change in resistance. So this is to be observed, if same type of strain then delta  $R_1$  will change and delta  $R_3$  will change both get added up  $R_2$  and  $R_4$  being zero because half bridge only two of them are changing other two are zeros. So following this condition only we have to connect the resistances. If you give similar change to adjacent arms that the adjacent arms in  $R_1$  to adjacent is  $R_2$  or  $R_4$  any one.

For example we take  $R_2$  if you give similar strain then you will find similar strain mean they get cancelled out we don't have any output but that is what is made use of in temperature compensation. That is three for temperature compensation similar change is given to adjacent arms. Otherwise for active arms we should follow these rules. So if you follow this rules now what is the output  $E_o$  is equal to  $E$  by 4 that is case one adjacent arms this is delta  $R_1$  by  $R_1$  we are giving the opposite change so that is minus of minus delta  $R_2$  by  $R_2$  and other thing are zeros so this equal to  $E$  by 4.

Now if  $\frac{\Delta R_1}{R_1}$  is equal to magnitude wise  $\frac{\Delta R_2}{R_2}$  magnitude wise then we find  $\Delta$  twice it becomes twice  $\frac{\Delta R_1}{R_1}$ . So it is equal to  $E$  by two into  $\frac{\Delta R_1}{R_1}$ . That is now it is giving twice the voltage output as Quarter Bridge that is advantage. Quarter bridge gives  $E$  by 4  $\frac{\Delta R_1}{R_1}$ , half bridge will give you output of  $E$  by 2  $\frac{\Delta R_1}{R_1}$ . That is twice the voltage output of the quarter bridge. Similarly we find in the second case that is the opposite arms for similar change,  $R_1$  and  $R_3$  alone we give so these things are zero but also we find  $\frac{\Delta R_1}{R_1}$  magnitude wise if it is equal in that case also you will have  $E_s$  equal to  $E$  by 2 into  $\frac{\Delta R_1}{R_1}$  that is the  $\frac{\Delta R_2}{R_2}$   $\frac{\Delta R_4}{R_4}$  zeros now temperature compensations you have to look into.