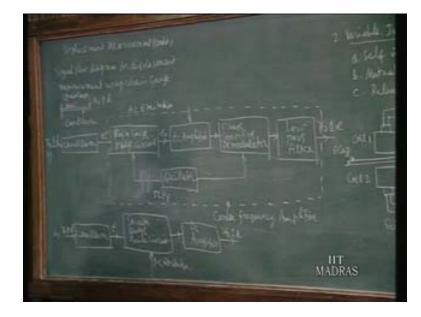
Principles of Mechanical Measurements Prof. R. Raman Department of Mechanical Engineering Indian Institute of Technology, Madras Lecture No. # 14

We have seen how a strain gauge is made up of, how it is fixed in a piece of base metal and how the strain is transduced and how to fix the strain gauges in a bridge network and so on. But now for a displaced measurement how to use a strain gauge that's what I am explaining now.

(Refer Slide Time: 00:01:45 min)

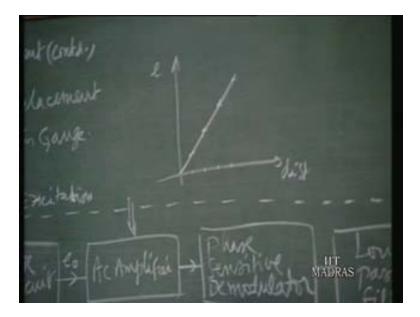


If you want to measure a displacement, give the displacement to a cantilever for example cantilever N its fixed N this is the free end. So give the displacement at the free end so it will deform. So by using this cantilever what you are achieving, the displacement signal is converted into a strain signal. That is what is indicated in the signal flow diagram. This signal flow diagram where the cantilever is shown here first transducer converting the displacement or transducer displacement d into epsilon strain. So afterwards then now the strain is picked up by the strain gauge as resistant change that will form one of the arms of a strain gauge bridge circuit. So by using the strain now that is displacement converted into strain, this strain is converted into a resistant change which forms part of a bridge network.

So now it is the whole thing is our carrier frequency amplifier, we have got oscillator giving a high frequency carrier signal and our strain is our input signal and resistant change correspondingly. You will have the output voltage e_0 from the bridge and this is a modulated signal it's being amplified and amplified one goes to the phase sensitive demodulator and then through low pass filter we get a voltage output. Now this voltage is calibrated, E is calibrated in terms of displacement.

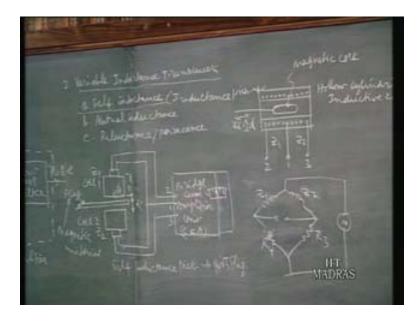
So first what we do it is for AC excitation for the bridge network because oscillator gives near about 5 kilohertz frequency signal. It is used to excite the strain gauge bridge. So that is now here Strain Gauge Bridge is all the four or all the four from few resistances. We can either excite the bridge network by AC or DC. So this is excitation, if you use a DC excitation it is simple circuit, again we use a cantilever to transduce displacement into strain and this strain goes into the strain gauge as resistant change that strain gauge will be forming a part of your bridge network and that bridge network excited with a DC voltage. That output voltage e_0 is amplified in a DC amplifier. Now DC amplifier from operational amplifier we have got many DC amplifiers without any drift problem such a DC amplifier we can use and magnify this voltage and finally you get a voltage output that will be calibrated by giving known displacements that is the calibration. Here also being the known displacement, calibrate the voltage output and later on you may have the curve like this.

(Refer Slide Time: 00:04:39 min)



This is a displacement versus voltage output at the end of instrument that is carrier frequency amplifier output voltage. So it may be coming somewhat like this, so by giving known displacement you plot it. After you plot it, later on you give unknown displacement to the cantilever N. Previously for calibration you give deform it by using dial gauge so dial gauge gives the known displacement. Plot this curve then at this cantilever end you give the unknown motion and that motion from the voltage you can get the correct displacement that's how the strain gauge is made use of for measurement of displacement. So this is what we learnt about regarding how to use this resistance for the purpose of displaced measurement. Next the passive element is we have already seen that resistance, inductance, capacitance can be used to transduce the displacement signal into electrical voltage.

(Refer Slide Time: 00:05:43 min)



So naturally next one will be variable inductance transducer where inductance is made use of to measure the displacement. Now the inductance is used in three forms that is to change the self-inductance of a coil or mutual inductance between two coils and then reluctance that is resistance for magnetic lines to flow, whatever the resistance that is called reluctance. The reciprocal of the reluctance is called permeance. Sometimes it is also known as permeance pickup, Americans some of them call it as permeance pickup. So these are the three ways in which the inductance is made use of. So first we take the self-inductance. It's also sometimes called simply inductance pickup self-inductance or simply inductance pickup, this is also colloquially used inductance pick it means for example there are two coils in one instrumentation with inductance pickup there are two coils. Each coil is a winding on a magnetic material and so two such coils are arranged, your displacement x_i is given to a flag which is made up of magnetic material.

So when the flag is at the middle of these two coils for example one coil may be 8 mm diameter and say 11 mm length typical inductance coil. So two such coils are arranged with flag in between, when the flag is at the middle position, the inductance of these two coils will be same. So the reluctances are the impedance z_1 and z_2 both will be equal. Now these two coils are connected in the say this is a carrier frequency, bridge cum amplifier unity is nothing but carrier frequency amplifier. So it is connected and you get the excitation. So it's somewhat like this in bridge network z_1 and z_2 will form adjacent terms of an AC bridge and the other two arms can be few resistances. So you can connect it to the carrier frequency amplifier and you can get the reading here that is the output voltage. It is similar to signal flow diagram, instead of cantilever you can say flag is used and then the inductive pickups say displacement we give to a flag in terms of this, this we call it AC bridge. All the other things will remain same as it is in AC carrier frequency amplifier. So this is one form of using the self-inductance pickup that is when the flag is at the middle the inductance of these both the coils will be same. When the flag moves nearer to say a coil one, then inductance of coil one is more and to the same extent the inductance in coil two it reduced. So it's opposite effect, when the flag moves in there, one coil increase and other coil decrease to the same extent.

Hence opposite effects are connected in adjacent terms of a bridge network what you have learnt already under bridge network. So this is a flag type. Another type is we can use a core, the flag itself is made in terms of core and it is positioned at the middle of a hollow cylindrical inductive coil that is it is made up of one coil with a centre taping, to the centre taping you will find then from centre point to one end we have got one coil with impedance Z_1 then from the middle point to the other end of the coil we have got the coil equivalent to coil two and with a impedance of Z_2 . So now when the magnetic core is at the middle, we find magnetic material distributed equally to both the coils so impedance Z_1 and Z_2 will be same and if you move coil by giving x_i displacement, x_i is the input signal same as displacement because it's a transducer for measuring displacement.

So when you give the displacement here suppose we move from left to right to some small extent, of course the displaced limited within this length of the total coil. So if we slightly move what will happen then inductance in coil two will be more than the inductance that the inductance in coil two is increased and to the same extent it is reduced in the coil one. So it is similar effect but here we have a core moving inside a hollow cylindrical construction. So physical construction is different here instead of two separate coils it is made in hollow cylindrical construction and we have a core. So with this again Z_1 and Z_2 opposite effects for a given motion they are connected in adjacent arms and then other two arms can be resistances. So again we connect to AC supply and take an output, this output actually will form part of the carrier frequency amplifier.

The second way of using the inductance pickup is mutual inductance we have seen earlier the self-inductance now mutual inductance and it has got a specific name such transducers, one such famous name is LVDT linear variable differential transformer. So it is very well known in measurements say people simply call LVDT. The construction details are shown in the figure. So this is a magnetic core made up of magnetic material, we will simply call magnetic core.

(Refer Slide Time: 00:11:06 min)

So it is positioned at the middle, if the construction is same as more or less similar to our selfinductance with core. Here also you have hollow a cylinder but here you have three coils one primary coil and two secondary coils. In earlier we have only one coil with centre taping that is in inside you have the difference of constructions. So primary coil is excited by an AC voltage which may be varying 5 kilohertz to 20 kilohertz, it is decided by the frequency of the input signal. Now input signal here it is given to the core that is the displacement, input signal is given to the core, this is the extension rod at the rod whatever the displacement we want to measure we give it to the rod, so the core will be moving. Now assuming the core is at the middle, now we find the mutual inductance between primary coil and secondary coils are same that is mutual inductance here inductance between these primary coil and say secondary coil one we call mutual inductance M_1 and similarly the mutual inductance between the primary coil and coil two we call M_2 mutual inductance two.

So proportional to mutual inductance voltage is developed here that voltage suppose we call it C_{o1} is equal to M_1 times dip that is current flow that is ip current flow in primary coil by dt. The rate of current flow into M_1 will give the voltage developed in the secondary coil one. Similarly e_{o2} that is voltage developed in the secondary coil two is equal to mutual inductance M_2 into dip by dt. Now these two coils as we see they are connected in opposite polarity. So we find the net voltage appearing between these two coils is the difference between these two voltages. So e_0 is equal to e_{o1} minus e_{o2} is equal to dip by dt into M_1 minus M_2 this is the voltage developed at the terminal of the secondary coils, to the terminals connected in secondary coils output voltage is proportional to these.

Now when we change the displacement suppose the d is zero that is the core is at the middle then you will find M_1 is equal to M_2 , e_0 is zero. When we give a displacement suppose we move this core towards S_1 , suppose we move in this direction then you will by mutual inductance M_1 is more and M_2 is small so you will find e_{01} is larger than e_{02} that difference between these two things will appear. Suppose e_s is of this nature then e_{01} , so this e_0 when x_i is positive suppose you call this as positive this direction or as per the diagram let us assume here and it does not matter, we can have it in same direction. So if this is positive, if you define towards left is positive then you will find the S_{01} will be somewhat like this, for voltage developed e_0 will be somewhat in phase with the supply voltage.

Suppose we move the other side towards right then if it is a from neutral position it is negative movement considering the left hand motion as positive, this is the negative motion and you will find in this case S_2 mutual inductance between primary coil and S_2 is more. So you will find voltage developed e_{o2} will be larger than e_{o1} then you will have the 180 phase shift that is this, when x_i is negative then you will find the output voltage will be opposite that is 180 degree phase shift would be there that is what is shown. At this point you will have 180 degree phase shift. That is the RMS value of e_o if you consider you will find when x_i increase positive, the output voltage is increasing e_o and with 180 degrees phase shift it also increases the other directions that is when x_i negative and it is moving in this direction. Suppose we withdraw it completely you will find then non linearity starts. So within the dimensions of the pickup only, you are supposed to move the core that is why it is called a linear range for the operation.

So that is how the linear variable differential transformer construction is there and it's functioning but if you see the signal flow diagram, we can see the x_i when that is displacement given to the core and it is given to the core of the LVDT and we have the excitation voltage that is for the primary coil, we have excitation voltage that is taken from the oscillator which may supply any voltage from any frequency between 5 and 20 kilohertz. This rate is depending upon the input signal frequency the carrier frequency, the oscillator frequency should be at least 10 times the maximum frequency of the input signal. The frequency with which we are going to change the input signal. So depending upon that we can select the frequency in the oscillator so that signal is given to primary and the output voltage e_0 will be the frequency of this I mean magnitude of the input signal is superimposed over the carrier signal and we have got so called modulated voltage is obtained as the output from the secondary coils.

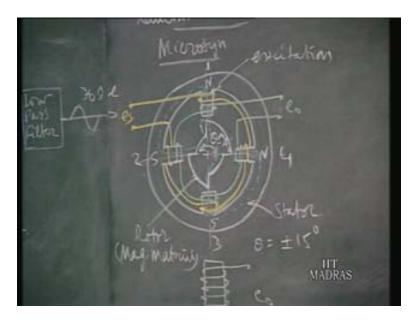
Suppose the motion is only on one side of the neutral positions, suppose the motion is only at the right hand side of this neutral position then we need not have the sophisticated instrumentation that is phase demodulator filter and all is not required because the average value of the modulator signal alone can be read in an AC voltmeter. That will give you the RMS value that is proportional to the displacement what we were given but suppose motion takes place on both side of the neutral position. That is it moves to and fro like this what is given is positive motion, positive side that is this side it no motion and the other side also it goes then you will find if you use only AC voltmeter the direction will not be moved.

In such cases we take the e_o and give it to phase sensitive demodulator where again with a synchronous signal from oscillator it brings these waves to the same side of the signal and after low pass filter you get a voltage output proportional to the displacement what you were given and that can be caliboratored in terms of known distances and unknown distances can be read from the meter. So that is signal flow diagram of a LVDT. We have note here important point is, in instrumentation for LVDT there is no bridge network, only for inductive pickup there is bridge network and LVDT, one more point we do not have u and v amplifier because if it is sufficient magnitude simply without amplifier we can get few volts 2 or 3 volts that can be calibrated in terms of displacements. So there is no amplifier in some instances.

If you want you can add amplifier here also, if this small value you can add but that's not a must and main point is there is no bridge network in the instrumentation for the LVDT. So this is for linear motion along a line x_i is that is displacement along a line, when it is in suppose it is pivoted you can give the rotary motion also. Hence it is called discovered rotary variable differential transformer, we give this angular rotation to again magnetic core. same construction arrived I mean made along an arc, so this primary coil P this is S one, this S two same way it functions but here the angular rotation is given to the LVDT. Hence it is called rotary variable otherwise the functioning is same principle as that one. So that is regarding the mutual inductance.

Now the reluctance variations. That is third way of using variation. Third way of using the inductance is change the resistance to the flow of magnetic lines that is what is achieved in microsyn, this specific name here is microsyn or it is sometimes its called proximity pickup in some other construction that is second construction. So in reluctance variations is achieved like this.

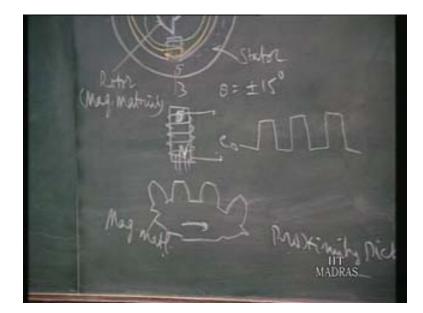
(Refer Slide Time: 00:20:30 min)



So there is a rotator which can make a rotation theta that is our input signal x_i . When it rotates we find the resistance for the magnetic lines of one or two lines are shown here. So the magnetic lines are induced on the poles, 1, 2, 3, 4 poles are there in this stator and there is excitation coil, this is your excitation winding which is shown here in yellow line and each pole has got excitation winding as well as the secondary winding. So the excitation winding is similar to our primary coil in the LVDT and the secondary winding is shown here by say green line. So each pole has got excitation coil and secondary coil because the excitation coil the magnetic lines are induced and that path is shown here. So that is now if suppose the rotor is in the neutral position then you will find rotor material that is shown here. This is rotor material is distributed equally to each among all the 4 poles; half of the each pole is covered by the rotor material. Rotor material is made up of again magnetic material so magnetic material equally distributed among the 4 poles. So number of lines produced because of this we find the reluctance of each against each pole is also same in the neutral position. So the number of lines in each pole is same since North Pole, north pole; south pole, South Pole. So you will find probably two lines here, two lines there and two lines entering here, two lines entering here. So you will find the number of lines entering or coming out of each pole will be same for the neutral position of rotor.

Suppose it rotates to a small angle normally here theta is varied plus or minus say 15 degrees, only small angle rotation is possible to convert that into displacement signal into rotary, this rotation of plus or minus 15 degrees can be converted into an equivalent voltage here. So if suppose we rotate towards say anticlockwise through say few degrees say 5 degrees then you will find material comes here more, that is poles 1 and 3 you will have more material and here it will withdraw. So 2 and 4 you will have less material that means more number of lines are there in pole 1 and 3 and less number of line in 2 and 4. Now the secondary coils are bound in such a way, the coils 1 and 3, they add each other and 2 and 4 also connected in series but in combination it will oppose 1 and 3, the second secondary coil will made up of 2 and 4 and the voltage developed in the secondary coils will oppose each other.

So when they are connected in series all of them then you will find two adding and two subtracting, net voltage is appearing here. So you find that is in 1 and 3 voltage is increased and 2 and 3 because when there is change in magnetic lines and they interact with the secondary coil, they induce voltage that's how the voltage induced in the second secondary coils and 1 and 3 they aid each other, 2 and 4 aid each other but in opposite to the polarity of 1 and 3 so net voltage is coming here as it is in LVDT. But what you have made use of is the reluctance, since more magnetic material is brought reluctance reduced in 1 and 3, at the same reluctance is increased in 2 and 4 so it is opposite effect producing in to one pair, increase one pair and in the other pair of poles the reluctance is increased. That is what is made use of here to convert that rotary motion into voltage. So this again you can, as for the signal flow diagram as LVDT you can give. This is a modulated signal we can amplify if you want or you can give it to phase sensitive demodulator and the low pass filter, same instrumentation low pass filter and then finally you have the voltage output. That part of it I have not shown. This is one way of using the reluctance variation, another one is simply called proximity pickup.



(Refer Slide Time: 00:26:06 min)

It's a well known and the construction is we have got a north pole south pole, a permanent magnet on which there is a coil wound over and suppose a gear is just below the or in the vicinity of this device. At one instant say a tooth comes just below this arrangement then you will find this is made up of magnetic material. So you will find since presence of magnetic material induces more magnetic lines in this... (Refer Slide Time: 26:48). So let us have the North Pole, here this South Pole. So now more magnetic lines will be coming out of the North Pole in the presence of a magnetic material and as it rotates, immediately you will find a valley then reluctance is increased then number of lines will decrease. So that means when a tooth and valley passes in front of this permanent magnet, you find number of lines changes. When the number of the line changes then you will find that is cutting the coils will induce a voltage here. So whenever a tooth comes out you find one pulse is produced that's how you get a pulse output from the proximity pickup. Proximity pickup is made up a permanent magnet on which we have got a coil.

So in the coil you have the voltage induced and100 teeth are there, then in one rotation you have the 100 pulses. It can be read in an electronic voltmeter and electronic counter and you can divide by 100 means you have so many rotations. That is how the reluctance is made use of here in measuring the rotary motions of any magnetic material. Now we see the third element, we have seen the resistance, inductance. How they are made use of for displacement transducer.

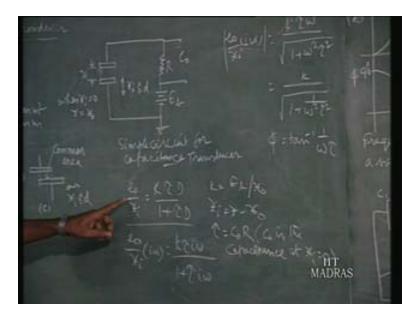
(Refer slide time: 00:28:09 min)

Now we see how the capacitance made use of to transduce a displacement into a voltage. Now we know in a capacitance normally we go for the parallel plate type of capacitors. So having two parallel plates having a distance of d and this is distance in millimeter and area is millimeter square. Then the capacitance assuming dielectric medium say air is equal to 1.006 something like that for air so some dielectric medium is there. So the capacitance of such parallel plate capacitor is given by this expression 0.0086 epsilon dielectric constant, area is the common area but in the two plates and db the distance in millimeter and millimeter square respectively. That means capacitance is depending upon 3 parameters dielectric medium, the plate area and the distance between the plates.

So if we can give the displacement to vary any of one these parameters then capacitance will vary. That is the principle what is made use of in these three types of capacitors because same displacement we can give it to in three ways. This is the option available for the designer when he wants to design the capacitance transducer, to convert the distance into a voltage you can have any one of these three designs. For example here the distance d is varied by having one plate fixed and giving the displacement to the other plate x_i input signal as displacement then d is varied when d is varied and inversely you have got the variation in the capacitance and the capacitance will be made into some circuit which will see little later. The other way of using the d is you have some dielectric medium, some mica or some material I mean insulating material, they are good dielectric materials. So some mica is there and it is put in some initial position like. We can move either this way or the other way then you will find actually in this position half of the volume is made up of air other volume is made up of mica.

So when we move it, when it moves towards a left then the air medium is reduced and mica is more and you will find the epsilon is increased. Similarly you will find when you give different displacement, epsilon is varied when epsilon is varied proportionately capacitance is varied. So similarly here we find the common area. suppose this is the normal middle position that is this point is just middle of this other plate then you can move, the one way you can call it positive, the other direction it's negative. So it will produce the opposite effects. Suppose if you move here A will be increased, the capacitance is increased. You move the other side then capacitance reduced since A is reduced. So from middle position we can increase or decrease and accordingly the distance from zero either positive side, negative side you can make use of. So these are three ways either varying the distance between the plates or varying the dielectric constants or varying the common area, we can convert the distance into capacitance. Now how capacitance is made use of in a circuit to get a voltage that is explained in this simple circuit.

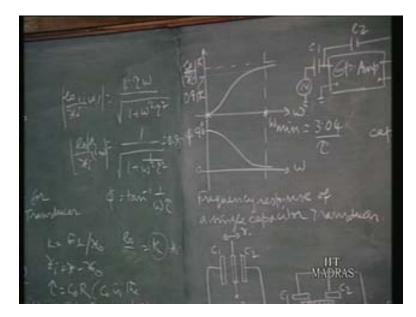
(Refer slide time: 00:31:35 min)



The parallel plate capacitor is forming a one of the elements in the circuit with resistance. So this is your supply voltage E_b . Now for deriving an expression it is assumed like this, at any instant let the distance between the two plates be x and we are giving the distance input signal to bottom plate. Now when x_i is zero let us call x as x_0 . So at any instance x equal to x_i plus x_0 , at any instant x is x_0 is the, when we don't give any input signal x_i that is x_0 . So x_0 plus x_i will be any time x or you can write x is equal to x minus x_0 and we can derive an expression, if e_0 is the voltage drop across the resistance or then voltage drop across R is our output voltage e_0 and e_0 and x_i , the plate displacement x_i input signal under these definitions we can derive this equation in electrical technology where K tau D by the time constant tau, 1 plus tau D where K is equal to E_b minus that is excitation voltage that circuit supply voltage divided by the x_0 , when x_i is zero whatever the distance is x_0 that is volt pet millimeter you will have. Then the x_i already we have seen, tau is equal to time constant C_0 , that is this will have its capacitance when x is here x_0 or x_i is equal to zero, C_0 into R will you give the time constant for the circuit. So in that terms you have got this relation and when you want to find out the frequency response of this equation normally what we do is substitute the differential operator D by i omega.

So by substituting i omega you have got this equation which gives rise to magnitude ratio and the phase angle like this. This we can reduce this to this form and that is when omega is equal to zero, this is zero e_0 by e_i is k. That is when omega is equal to zero then you will find e_0 by x_i magnitude that is i omega will be k that is magnitude ratio of e_0 to x_i will be k, that is zero. When omega is zero this becomes infinity. Then it becomes infinity then you will find K by infinity will be zero that is why you have got zero voltage.

(Refer Slide Time: 00:33:46 min)

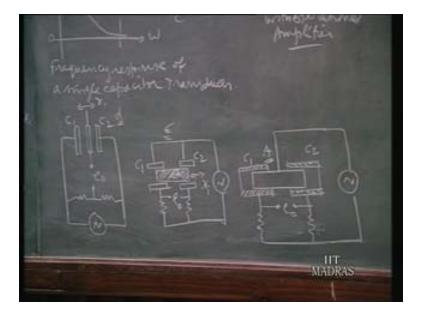


Suppose when omega is infinity then this term will become zero, then you will find e_0 by x_i will be same as K that is how when omega becomes infinity the curve approaches the value of k. That is how the magnitude e_0 varies with reference to omega. Similarly you will find when omega is zero, phi will be that is this infinity, tan 90 is infinity. So you have got 90 degrees tan phi by 2 is infinity. So when omega is equal to infinity that should be zero then time zero, it becomes zero so that is how the angle is varying. Now that means when omega is very large then only that e_0 is, K means e_0 is proportional to K.

Suppose e_0 by x_i is equal to K that means it's a constant and you will find k into x_i , taking it to that side so output voltage is proportional to K, is proportional to x_i only when omega is larger than say omega minimum assuming a 5% error. So this if you say 0.95 times K, 5% error in this instrumentation then you will find assuming making this side will become 0.95, 5% error this side will become 0.95. Then you will find omega minimum that is we can also write e_0 by K that is making it into one. We can also write this as one bringing e_0 by K by x_i i omega and making it for 5% error the ratio will be 0.95 then you have got omega minimum. Omega minimum is equal to 3.04 by tau. So if you substitute in this equation then omega minimum you get as 3.04 over tau. So for higher value of omega being more than omega minimum then you will find the error is always less than 5% or it will be more than 0.95 the magnitude ratio. In order to have omega minimum as small as possible what is the requirement, tau should be larger. To make omega minimum smaller so that even we can use this instrumentation for smaller frequency of... (Refer Slide Time: 36:48).

What is omega? Omega is the frequency at which we vary the input signal that is the input signal variation, frequency is omega. So if you have lesser omega minimum then are even at lower frequency we can measure it. Also we have to note when omega is zero that is static measurement when we just move it and keep it there that is no omega, omega is zero, no frequency that is under static measurement we see output voltage is zero. So the simple circuit with capacitor you cannot use it for static measurements. So only for dynamic measurements and dynamic frequency should be more than so 3.04 over tau for 5% error. If we vary the percentage error then accordingly you can find out. Suppose 2% error means you put 0.98. If you allow only 1%, 0.99 for that what is the omega minimum you can find out but 5% error is normally allowed so having 5% error omega minimum so much, so in order to have as small value for omega minimum as possible then you should have as large tau as possible.

Now what is tau? Tau is equal to C_0 times R that a capacitance at x_i zero and other one is R. So normally having larger R we can reduce omega minimum. So R is of the order of some mega ohm 1 or 2 mega ohm. The problem here is we have reduced omega minimum but the voltmeter which we bring to read this voltage will have 10 times this, so it should be 10 mega ohm should be the resistance of the voltmeter. So that is how it is a high impedance circuit, to convert it we go for the impedance transform amplifier. So by that we convert this high impedance circuit in the low impedance circuit and later on you can connect a voltmeter. That is the usage of the impedance and semi circuit, in the transducers using capacitance and usually transducers we make use of the impedance transform amplifier like that. So the simple circuit with one capacitor has got this drawback it cannot be used for static measurement.



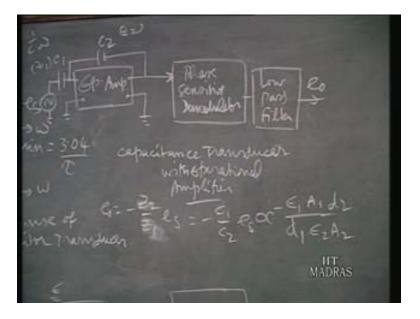
(Refer Slide Time: 00:39:03 min)

If you want to use the capacitor for the static measurement then we go for the bridge circuits with capacitors, AC bridge circuits with capacitors such three constructions are shown here in this figure. That is a capacitance is varied, by varying the distance between the two plates of the parallel plate capacitor or by changing the epsilon that is the dielectric constants or by varying the area. This is area variation.

Here it is an epsilon variation, here it is d variation, distance variations but they are connected in adjacent terms of the Wheatstone bridge. This is the AC Bridge; it is diamond construction so it is drawn in a different way. Across two corners you find the output voltage, the other two corners you give excitation, same circuit in different way it is drawn. Suppose we give the displacement here x_i then we will find the distance here is reduced, the distance other capacitor C_1 is increased accordingly the capacitance is varied and for the changed capacitance you will have the one given output voltage.

Similarly here we find the dielectric medium say mica is moved from this middle position this way or that way, accordingly the epsilon is varied correspondingly they are connected in adjacent arm. For any given variation the variation in $C_1 C_2$ will be opposite in nature. Similarly by having a cylindrical construction for the plate this is cylinder 1, cylinder 2 and this is a rod having a common area equal in both now to start with, later on if you move right hand side C_2 will increase the area, C_1 will reduce. So they are connected again as adjacent terms of an AC bridge with excitation and so forth. So the output voltage we know it will be a modulated one, we can amplify it and then phase sense demodulator, low pass filter all these things are understood these things I am not doing it. It's for using carry frequency amplifier so you can read the output voltage corresponding to this. So this is one construction. Another construction where static signal can be studied is with operation amplifier.

(Refer Slide Time: 00:41:05 min)



We have got two capacitance one is in the forward line other is feedback path C_2 capacitor and now suppose e_s is the supply voltage then e_0 is equal to minus C_1 by $C_2 e_s$. That is e_0 the output voltage of this setup that is operation amplifier with a feedback capacitor and then it is going to phase sensitive demodulator, low pass filter finally you get the output voltage. The output voltage is varied depending upon the variation in C_1 and C_2 . You can select the variations either you can put the variation in C_1 or C_2 because you find this Z_2 by actually e_0 is equal to z_2 by z_1 . This is giving raise to impedance of z_2 this impedance of z_1 .

Now we know impedance is equal to one over C omega this is our impedance of capacitor C is equal to z by one by C omega. So now since it is inversely proportionate you will find a minus C_1 by C_2 e_s because Z_1 is inversely proportional to C_1 so it goes up. Now we know each one is made up of these three parameters now C_1 is equal to epsilon is proportionate to epsilon₁ A_1 by d_1 and similarly C_2 you have got epsilon₂ A_2 by d_2 because capacitance is epsilon proportional to a area into dielectric constant divided by the distances. So converting in terms of parameters. Now if you have the area change, you give because we want e₀ proportionate to proportional quantity then we give area as well as the dielectric constant variation in capacitor one, this is capacitor one. If we want to vary area to change the capacitance then give it to capacitance one and when we want to give the distances input signal to vary the capacitance then that distance give it to capacitance two that d_2 comes up at the top. So you will find e_0 is proportional to d_2 . So we can give the signal in form displacement between the plates or the common area between the plates or the dielectric constant for that we can select anyone of these two capacitors. That is for area and dielectric constant capacitor one, distance we give to capacitance two then we will have e₀ always proportionate to our displacement signal. So that is what is demonstrated by this equation. That means you have seen all three passive elements resistance, inductance and capacitance, how they are made use of to transduce a displacement into corresponding voltage.