Principles of Mechanical Measurements

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Lecture No. # 17

We have seen the theory of operation of accelerometer that is seismic pickup used as accelerometer. Now in this we know the main construction is mass supported by a spring K_s spring constant and a damper with damping coefficient B. By measuring x_0 that is the relative displacement of the mass with reference to the frame of the instrument. That is $x_{0,}$ we are is measuring by say an LVDT or any other displacement measuring instrument and that reading is calibrated in terms of input acceleration. That is what we have learnt, by theory and all we derived equations but what is the physical explanation for this functioning?

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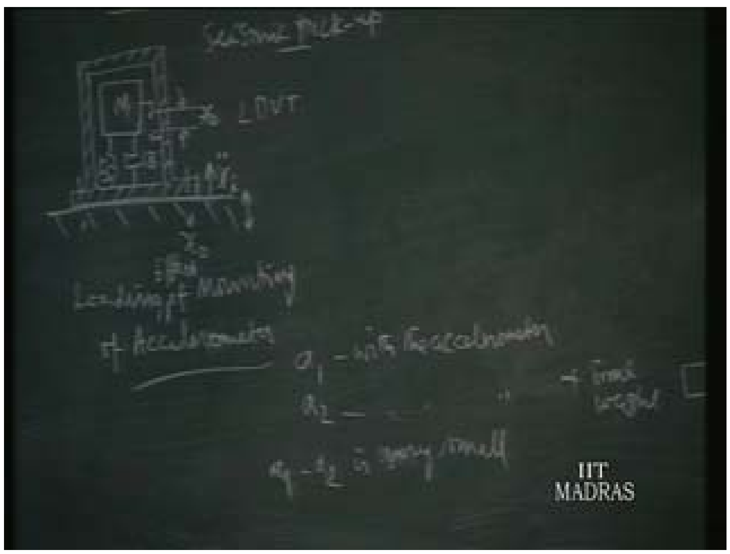

For that you just analyze this case. Suppose the acceleration is just upwards, only one direction a constant acceleration is acting x_i two dot. In that case the mass also getting accelerated to the extent of x_i two dot, to the same extent this is also accelerated. To accelerate the mass M we should have the force, without force no mass can be accelerated. So necessary force to accelerate the mass should come only through either spring or through the damper because mass is suspended and these are the two roots through which the force can reach the mass M.

Since it is a constant acceleration there is no change, no force can be transmitted through the damper because damper output will be proportion to the relative speed of mass and the frame. Since there is no relative speed, it’s a constant acceleration it is moving, so through a damper no force can be transmitted. So force can be transmitted only because x_o two dot proportion to that only force can be transmitted through B. So the force can now be transmitted only through the spring alone.

So the necessary force goes through the spring, when a force goes through a spring this spring gets compressed. That is how the compression of the spring that is nothing but the relative motion of the mass with reference to a frame that is x_o is proportional to the force that is going through the spring. That force is proportional to x_i two dot that is how x_o represents the x_i two dot or x_o is a function of x_i two dot. So by measuring x_o we get an inference, we get a measure of x_i two dot that is acceleration. That is physical explanation how relative measurement, the displace measurement gives rise to acceleration measurement in this instrumentation.

Now we see the loading of mounting of accelerometer. That is what is loading effect? When we mount an instrument over a vibrating body this is what we are going to connect to a vibrating body. This is vibrating body, to this we bolt this accelerometer, whole instrument is fixed on the vibrating body, this is vibrating to up and down. So in a mount such an instrument on the vibrating body whether the amplitude or the magnitude acceleration will get affected because we are bringing extra mass over the vibrating body. So acceleration may get affected that is called loading effect of mounting of this accelerometer.

We have to check by mounting this instrument whether we affected our measured quantity that we have to check. For that you do the following. First mount this instrument to the vibrating body make a measurement say a_1 acceleration one. That is with the accelerometer alone we got a reading a_1 and then next you make another measurement a_2 with the accelerometer plus some weight. You put some more weight say 1 or 1 books also, just put it there of the order of the same weight you put and then make another measurement a_2. Now if the difference between a_2 and a_1 or $a_1$minus a_2 is a very small one. If the difference is very small that means mounting of the accelerometer does not give rise to any loading effect because even after adding some more weight the reading is not changed. So naturally the accelerometer alone has not given rise to any loading effect.

That is how after making measurements we check whether mounting of accelerometer has given rise to any loading effect. If it is giving weight to the loading effects suppose the difference is much that means the accelerometer you have selected for this measurement is a wrong one. We should go for a lighter version of the accelerometer, mostly you will find piezoelectric accelerometer which we have seen already. Piezoelectric accelerometer is the choice but the problem here is the acceleration should be a varying quantity; omega should be omega minimum of the order say about 10 hertz or 15 hertz depending upon the piezoelectric accelerometer. So it should be a varying quantity in that case we can go for another I mean this to piezoelectric accelerometer or if it is static measurement naturally we go for a lighter version.

Next is inertial navigation. Inertial navigation, that is the usage of accelerometer in inertial navigation. What is inertial navigation? Finding out the location of the ship or aircraft or a rocket after it takes off from a station or from a harbor, after say 1 or 2 days where exactly the ship is located we can find out by using a set of accelerometers. For that purpose what people are using is so called stabilized platform. There will be a stabilized platform in the vehicle whose position we are interested to locate after certain time of starting or after taking off. So the stabilized platform it has got some particular property.

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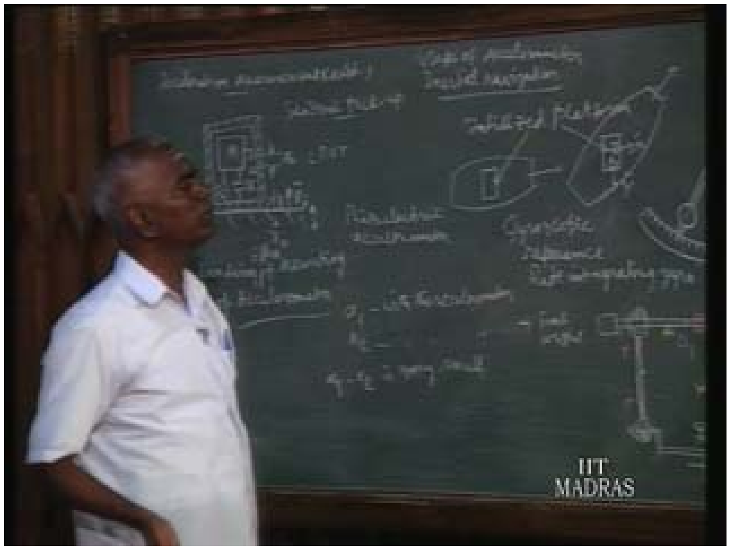

Suppose we have got a ship, suppose this is the ship traveling in this direction and there will be a stabilized platform somewhat like this. Suppose ship turns like this, after sometime you will find ship has turned like this and it is traveling in the direction it has turned. So in that situation the stabilized platform, the orientation it still remains same, this is the stabilized platform. So irrespective of the rotation, irrespective of the movement of the ship the stabilized platform is always maintained in the original orientation, when the ship started that is the stabilized platform. That is the orientation of the platform is always maintained constant irrespective of the movement of the ship.

How this is achieved? Suppose the ship rotates through an angle say theta then stabilized platform is rotated backwards through the same angle theta so that the alignment remains same. So that means the ship motion and rotation about its axis is sensed by so called, it is called gyroscopic reference. By having set of gyroscopes, it is possible to sense the rotation of the ship and one such thing is rate integrating gyro. For each axis suppose for a ship now two directions is sufficient. So for two directions X and Y, you will have one accelerometer here and another accelerometer there, two accelerometers are positioned. So this will measure the motion acceleration in the say Y direction and this will measure the acceleration in the X direction, two accelerometers are put and by having two integrating gyros we can…

Now it is sufficient if we have one integrating gyro, it is just to measure the angle of rotation. About Z only we have to measure the angular rotation because when the ship moves on the surface of the sea water, if you can measure these rotations about the Z axis that is sufficient. So one integrating gyro if you have, the rotation is measured and the rotation is given to a servomotor that servomotor will bring back the stabilized platform back to its original orientation and such a platform is always maintained inside a ship or an aircraft. In aircraft you require 3 axes X and Y and Z, so instead of two you will have another one to measure the perpendicular direction also. So for taking this example of the ship now we have these stabilized platform maintained in its orientation, two accelerometers.

So you will measure acceleration in two directions and vectorial addition of these two acceleration will give the final or integrate the acceleration in both the directions, twice integrating the acceleration will give the displacement and vectorial addition of the twice integrated signal of this acceleration signal will give rise to actual position of the ship after it starts from the harbor. That is how the accelerometers are used in locating the ship or finding out the total distance after starting from a harbor or for an aircraft after it takes off from the airport, after certain hours how how much distance it is there you can find out by using this stabilized platform plus number of accelerometers. So that completes the portions, I will just workout 1 or 1 problems.

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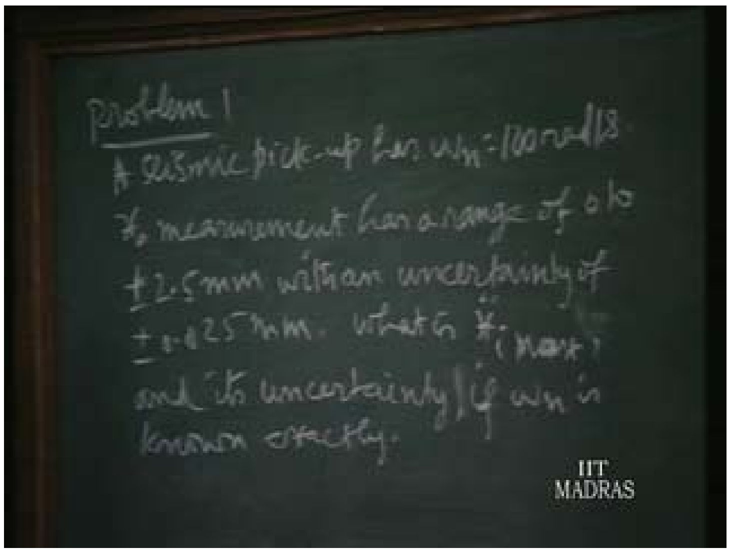

Now the first problem is a seismic pickup as omega$_n$ that natural frequency 100 radian per second and x_o the relative measurement between the mass and the frame is measured with an instrument having a range of plus or minus 2.5 millimeter with an uncertainty of that instrument is 0.025 millimeter that is 25 micron is the uncertainty. What is x_i maximum the instrument can measure, that is acceleration, what is the maximum acceleration and its uncertainty? What it is uncertainty of the acceleration measurement? If omega$_n$ that is 100 radian per second is known exactly that is the case what should be the error?

From error of propagation we know that first we see the equation, x_o by k by x_i two dot is equal to we know 1 over 1 plus d square omega square but assuming a static measurement, we can say is equal to 1. The 1 over d square omega r square that change in acceleration is not there, under static conditions it should be one. That is this equation is obtained from 1 over d square omega$_n$ square plus 2 psi d by omega$_n$ plus 1, that we have neglect it. Assuming a constant acceleration, so this is the equation x_o by k by x_i two dot is equal 1. Now k is equal to 1 by Omega$_n$ Square so we write x_i two dot is equal to x_o by k that is now k is 1 by omega$_n$ square so it will come x_o into omega$_n$ square, substituting k by 1 by omega$_n$ square this is the equation. So this is the relation between under any static conditions of acceleration, x_i two dot is equal to x_o times omega$_n$ square. This is the relation between relative displace measurement and x_i two dot. Now there is no error in omega$_n$, only in x_o there is error.

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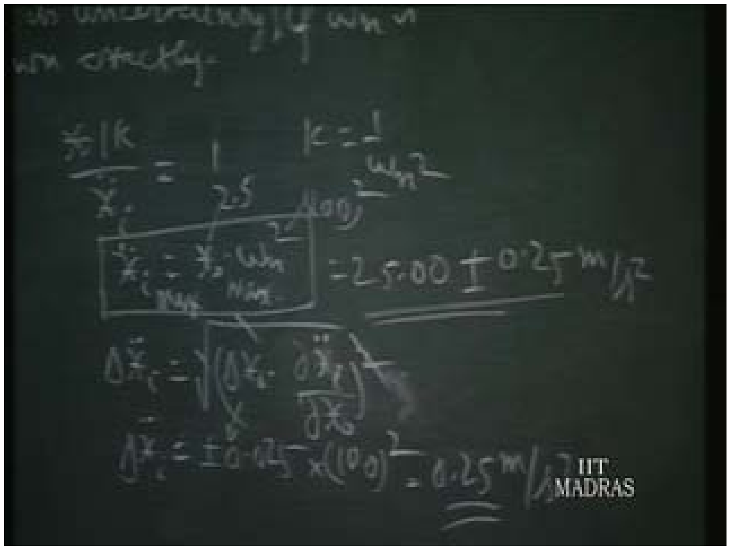

So we find delta x_o that is error in acceleration measurement, delta x_i two dot is equal to root of delta x_o that is error in displacement measurement into dow x_o by dow x_i two dot that is the function with reference to dow x_o into whole squared plus the other parameter, other parameter is known exactly so delta omega$_n$ zero, so totally it will be zero. This is the equation, in a sense square root of square so you can forget this square also and square root also you can forget, so delta x_i is this much. Now delta x_i is given as plus or minus 0.025 and now dow x_i two dot by dow x_o is equal to, from this omega$_n$ square. So into dow x_i two dot divided by dow x_o is equal to, this is constant so into omega$_n$ square. Omega$_n$ is given as 100 radian per second, so 100 square. So this is our delta x_i two dot and this you find it is coming about 0.25 meter per second square. It was millimeter and convert into meter, finally you will have this error. So this error is up to two significant figures 2.25 and they lie up to second decimal place.

Now you find out the maximum value, you add here x_i two dot maximum is equal to x_o maximum into omega$_n$ square. So now x_o maximum is given as 2.5 mm and omega$_n$ square is 100 square is equal to x_i two dot maximum and up to second decimal place then if you calculate it will come as 25.00. So with error plus or minus 0.25 meter per second square. This is the answer for this problem. So what is the x_i maximum? That is 25.00 meter per second squares and its uncertainty is plus or minus 0.25 meter per Second Square, so this is answer for this. Second problem the uncertainty is 1%. So now we have this equation x_o by k divided by x_i two dot is equal to 1 over root of 1 minus beta square whole square plus 4 psi square beta square. So this is the magnitude equation beta 1 minus beta square whole square.

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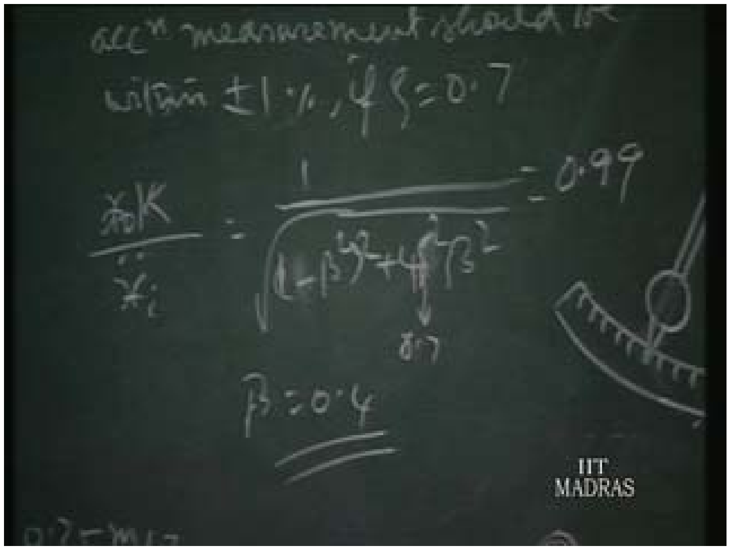

Now this is equal to 1% error so the magnitude ratio will be 0.99. So now substitute, size given as 0.7 and beta can be found out from this equation, except beta all the other are known so beta comes about beta is equal to 0.4, it’s a straight forward problem. So with this we close the chapter on acceleration measurement.