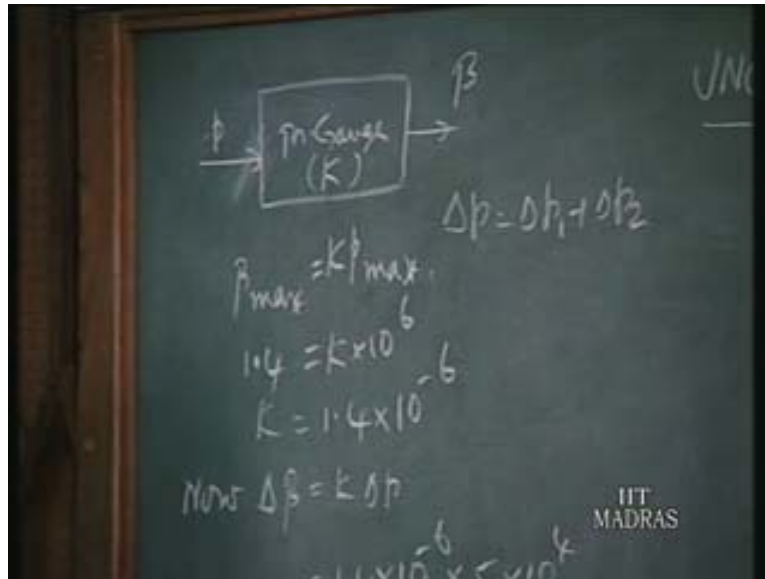


**Principles of Mechanical Measurements**  
**Prof. R.Raman**  
**Department of Mechanical Engineering**  
**Indian Institute of Technology, Madras**  
**Lecture No. # 08**

(Refer Slide Time: 00:01:28 min)



Previously we saw the error due to plain, error due to friction and plain and when you added these two errors that is the error in input signal from the friction, this is the error in input signal due to play in the rack and pinion mechanism. Now since you have found out the total error in the input signal now you have to find out what is the corresponding rotational error in the beta. For that if you consider the whole instrument as one box so what the instrument achieves? Pressure to angular rotation beta. Now this relation if you call it K the gain, now you can write that equation in terms of maximum values of beta and  $\beta_{max}$  and  $P_{max}$ . So beta max 80 degrees equal to 1.4 radians equal to K times, the gain of the whole instrument in to 10 to the power 6, 1 mega Pascal. So from this we find out K as this, this is to change the delta p corresponding to find out from delta p what should be the corresponding delta beta. So delta p is this much, we derived yesterday that is 5 in to 10 to the power of 4 or 0.5 bar. So delta beta corresponding to this now substituting for K you get 0.07 radian. So that is the corresponding angular rotation for an error of 5 in to 10 to power of 4 Pascal. Now for this rotation this is the corresponding rotation for the least count. For the least count we should have 1.5 mm as distance between two graduations.

(Refer Slide Time: 00:03:08 min)

$$L = 1.4 \times 10^{-6}$$
$$\text{Now } \Delta\beta = k \Delta p$$
$$= 1.4 \times 10^{-6} \times 5 \times 10^4$$
$$= 0.07 \text{ rad.}$$

$L$  is the length of the pointer

$$L \cdot \Delta\beta = 1.5$$
$$\text{So } L = \frac{1.5}{0.07} = 21.4 \text{ mm}$$

IIT  
MADRAS

Now if that is 1.5 mm for delta beta of 0.07 radian what should be the pointer length? So if  $L$  is the length of the pointer then  $L$  comes out 21.4 mm, previous class I didn't have time so that is how the length of the pointer is achieved. Now we go to the next topic uncertainty of measurement.

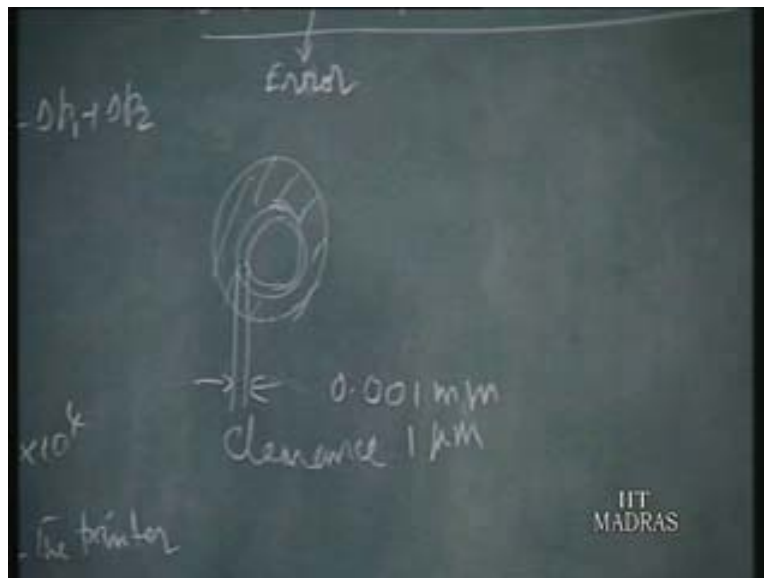
(Refer Slide Time: 00:03:24 min)



Now uncertainty is called error uncertainty error, percentage of error all these things represents only one term uncertainty and error. Now we have learnt previously, how the error is coming in to an instrument through the elements of the instrument from atmospheric condition due to loading effect this we know already.

So that means any instrument has got certain amount of error without error, we cannot measure a parameter. That means the real value, the real value, true value or actual value always remain unknown. There may be an instrument with very less error but still there is some error. So how to select the instrument for a particular measurement. Suppose we want to measure the length of a working table so we use ordinary scale there may be an error of plus or minus one mm or whatever it is. So there we need not have the accuracy in terms of micron plus or minus 2 mm plus or minus 4 mm it is sufficient that is not very controlling dimensions we want to know the approximate dimension of the table or so to say plus or 1 mm it does not matter at all in this measurement.

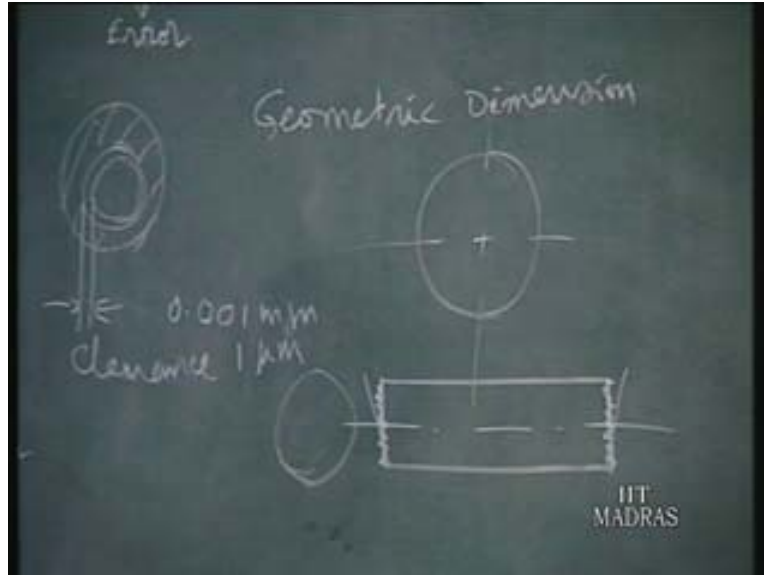
(Refer Slide Time: 00:04:59 min)



Whereas in bearings if you take the bush bearing you have got a bush within this the journal is going to sit and the clearance affects the functioning of the bearing. So this is a functional area, functional dimension and here it should be of the order of 0.001 millimeter this should be the order or one micrometer this should be measured with the accuracy of one micron dimensions. So that means depending upon application we have to select an instrument with its tolerance or error. So whenever I mean we don't go always to an instrument which gives high accuracy because the instrument is costlier when you want to measure with the high accuracy. Only when it is required we go for such an accuracy otherwise we use an even rough errors are accepted in many types of measurements, while making a table the length may be measured within plus or minus 5 mm does not matter if it is its one meter if it is 1.005 meter it does not affect the usage of the table. That is the selection of instruments depends on the required accuracy in an operation.

Next one is there is one more parameter in affecting the measurements. That is geometric dimensions. That is when we say a rod is circular, how good it is or it may have a lobed shaft, lobed surface or it may have the elliptical in shape.

(Refer Slide Time: 00:06:36 min)



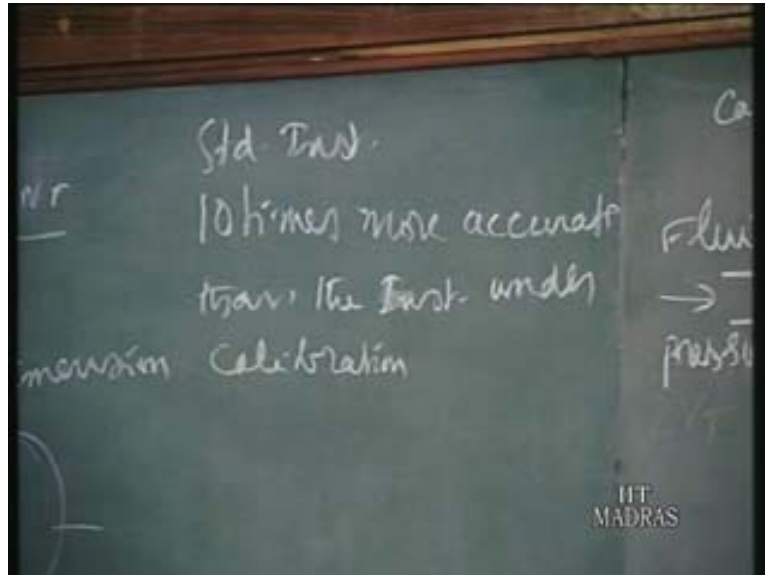
So even though we were told it is a round but realize in practice a round shape is very tuff because they are out of surface and surface irregularities, for example you take a length of rod it's a round rod length of a rod. So when we want to measure the length of the rod because this is abstract geometric shape but when we want to measure we should ensure whether the two surfaces are parallel to each other or the two surfaces the end surfaces circular surfaces is it plain or it has got ups and downs in it's plain configuration or the surface may have the irregularities in terms of microns. So when we measure the length of the rod or we might have measured see if the surfaces are slanting we have one dimensions at the bottom another dimension at the top and if it is not plain if it is having shallow and then the ups and downs in the micro dimension then it depends where we put our measuring instrument.

So even though we say it's simply length but actual length it's difficult to decide. So these are the abstract dimensions which we may not be realized in practice there are many error sources. So that's why we say in a measurement always error exists so theoretical value always remain unknown since any measuring instrument has got certain error. Now another point is after using an instrument for a number of years the calibration of the instrument might have shifted. So after a usage of few years that instrument requires recalibration to update the accuracy or the uncertainty error or inaccuracy.

These are all analogous terms representing the same phenomena. So you have to recalibrate the instrument whether the manufacturer, whatever the error uncertainly he claims still valid today or when we make an instrument for the first time again the scale of the instrument is to be calibrated for that purpose we have we have we have got so called calibration process calibration process. That is in this process we have to realize this atmospheric conditions, same atmospheric conditions in which the instrument in going to be used or whatever we claim the error of the instrument will be within certainly plus or minus 1% under the certain atmospheric conditions those conditions is to be realized for the calibration process.

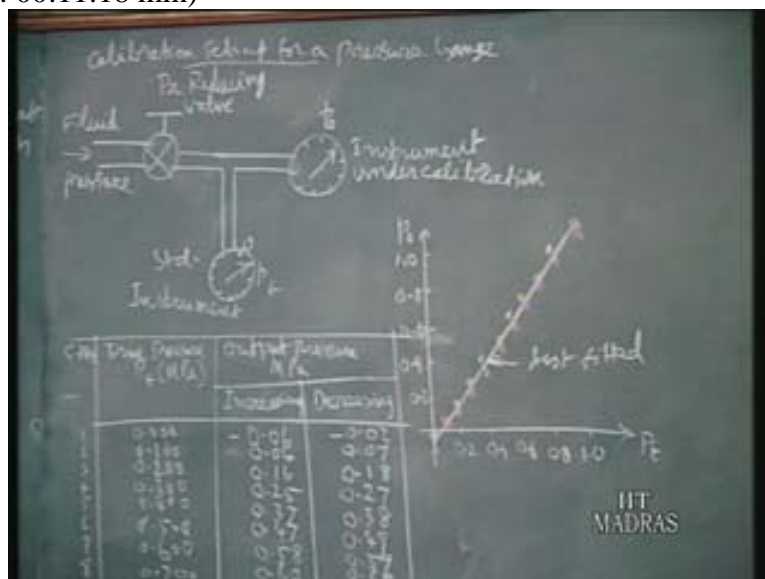
So temperature may be 20 to 40 degrees. Humidity may be varying between say 50% to 100% so like that atmosphere conditions are specified for any instrument in which it's supposed to be used.

(Refer Slide Time: 00:10:12 min)



So for that purpose let us consider a pressure gauge, this is a standard instrument which is going to be recalibrated instrument under test, under calibration. That is after realizing the atmospheric conditions for the calibration process next we have to select the so called the standard instrument. The standard instrument should have an accuracy of 10 times more accurate than, that is standard instrument should be 10 times more accurate than the instrument under calibration. That is if the instrument under calibration has got 1, 2, 3 bar like that as the least count, the standard instrument should have 10 times more accuracy that means we should be able to read 0.1, 0.2, 0.3 bar also. So that's why we select a standard instrument.

(Refer Slide Time: 00:11:18 min)



Now that is a setup shown in figure here that is calibration setup for a pressure gauge. We have the pressure source fluid pressure it may be a gas or a liquid and we have got a valve pressure it is revolved with which we can adjust any pressure downstream and the same pressure is given to this instrument of calibration and also to the standard instrument where we can read where the least count is ten times finer. Now the process is like this before that suppose if the 2 instrument reads up to third decimal of a mega Pascal and if we completely close it is going to read zero but we have to write 0.000 because actually we can simply write zero for this whole reading, one is zero, second is 0.100 or 0.1 but that is not permitted for calibration process.

(Refer Slide Time: 00:12:07 min)

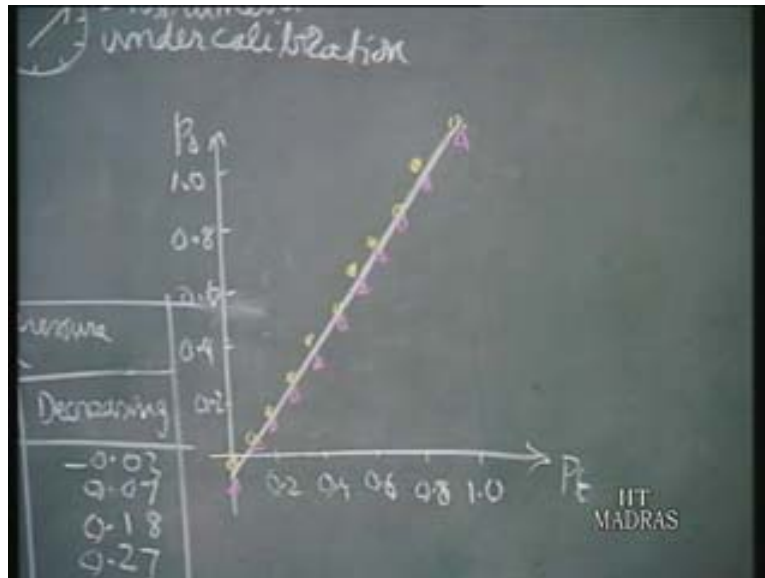
Instrument	True Pressure $P_t$ (MPa)	Gauge Pressure MPa	
		Increasing	Decreasing
0.00	0.000	-0.06	-0.07
0.10	0.100	0.06	0.07
0.20	0.200	0.16	0.18
0.30	0.300	0.26	0.27
0.40	0.400	0.37	0.38
0.50	0.500	0.47	0.49
0.60	0.600	0.58	0.57
0.70	0.700	0.68	0.69
0.80	0.800	0.79	0.80
0.90	0.900	0.89	0.91
1.00	1.000	0.99	1.00

The zero represents, there can be some other number also in this particular instant they are all zeros so you have to simply write all the terms as 0, each term you have write specifically zeros even though if you write 1 0 that maybe sufficient for mathematically. I mean mathematically it is sufficient but not for measurements so measurements you have to write all these zeros. Now at that time you make some measurement, note down in the standard instrument that is for increasing zero onwards we are going to increase. It shows already some reading minus reading it shows so below zero it is there some reading note down. See you find here in the instrument under calibration we can read up to second decimal and that means a standard instrument should have three decimals that is there already. So next we open little more and increase the pressure to point one, we are writing 0.100 because second and third decimal the readings shown by the instrument is zero, so we write it 0.100 at that time it reads 0.06 + 0.06 so plus is not written.

Similarly we increase the pressure in steps of 0.1 mega Pascal and conduct this experiment until the standard instrument reads one mega Pascal. So at that time the instrument shows 0.99. Now you have to take the readings for decreasing the pressure for that don't start from this, you increase the pressure some more standard instrument should show some more reading than one that is 1.2 something like that then reduce the pressure from 1.2 and when it comes one, stop and take reading at that time it is 1.00.

Similarly reduce to 0.9 take the readings and complete it that means you have got 22 readings, I can also write this also below this but this way it is written conventionally it is written one, for increasing, decreasing. That means 22 reading, 11 in to 220 readings we have taken. Having taken these readings, this is the way of static calibration. To do the static calibration you have to take these readings, increase the input in steps and decreasing the input in the same step and note down the readings. Next we plot these values in a graph somewhat like this.

(Refer Slide Time: 00:14:45 min)



So the x axis is theoretical value that is what is shown in the instrument and the y axis what is indicated that is output pressure is from here. This is  $P_t$  readings, this is  $P_o$  readings that is plotted here so this red color, pink color represents the increasing and decreasing readings are shown by yellow color. Now this is called best fitted line based upon the least square principles. That is from the best fitted line, the difference between the best fitted line and the readings that is a difference square of that reading and add for all the readings that will be the minimum of the squared of the differences of the points from the best fitted line will be minimum that is the basis. To obtain such a best fitted line we have got a formula to compute the slope and cut in the y axis. What should be the cut in the y axis? That is the least square principles we have got that equation.



(Refer Slide Time: 00:15:52 min)

$$p_0 = m p_t + b$$

$$m = \frac{N \sum p_t p_0 - (\sum p_t)(\sum p_0)}{N \sum p_t^2 - (\sum p_t)^2}$$

$$b = \frac{\sum p_0 - m \sum p_t}{N}$$

Suppose the equation is assumed at  $p_0$  is equal to  $m p_t + b$  this is the equation of the line,  $p_0$  is equal to  $m p_t$  that is  $m$  is the slope of the line and  $b$  is the cut where the line cut the  $y$  axis. So if this is the equation of the best fitted line then  $m$  is equal to in terms of readings,  $m$  is equal to  $N$  times,  $N$  is the number of readings here number of readings equal to 22,  $N$  is equal to 22 readings,  $N$  in to sigma of  $p_t$  in to  $p_0$  minus sigma  $p_t$  multiplied by sigma  $p_0$  divided by  $N$  sigma  $p_t$  squared minus sigma  $p_t$  the whole squared this is  $m$ .  $p_0$  you can write in one line and multiply each individual one, call the  $p_t$  in to  $p_0$  term add all those things at the end that is the sigma of total of that and then again open  $p_t$  square one column, square all these things write  $p_t$  square at the end,  $p_t$  square you find for each one total will be the sigma  $p_t$ , the total will be taken that will be squared here.

Sigma  $p_t$  square you have write it here and sigma  $p_t$  already we know total of it, that is square of it. That is  $p_t$  square you will have a separate column,  $N$  you find out summation that summation you put it and find out the value of  $m$ . Similarly  $b$  is equal to sigma of  $p$  minus  $m$  sigma of  $p_t$  divided by  $N$  that means from this set of readings we can find out the slope and the cut of the line at the  $y$  axis as  $b$ .



(Refer Slide Time: 00:17:52 min)

$$N = 216 - 11216$$
$$p_0 = 1.06 p_t - 0.05$$
$$p_0 = 0.25 \text{ MPa}$$
$$p_c = p_t = \frac{0.25 + 0.05}{1.06} = 0.28 \text{ MPa}$$

Now the equation of the best fitted line is  $p_0$  is equal to for the given set of readings  $p_0$  is equal to  $1.06 p_t$  plus this minus value  $b$  is equal to,  $b$  you have got a minus so minus  $0.05$ . So this is the equation of the best fitted line that means whatever the reading you take in the instrument corresponding value will be a value to be obtained like this,  $p_0$  is equal to this is the output reading. So  $p_t$  you will get from this,  $p_0$  plus  $0.05$  divided by  $1.06$  will be the corrected value that's the corrected value. For example if  $p_0$  is equal  $0.25$  mega Pascal that is the instrument when we are using the instrument to measure unknown pressure that instrument gives a reading of  $0.25$  mega Pascal then corresponding value of  $p_t$  is equal to  $0.25$  that is  $p_0$  plus  $0.05$  divided by  $1.06$ . So that comes about  $0.28$  mega Pascal.

So a  $0.28$  mega Pascal is shown as  $0.25$  in the instrument. So by using this equation we calculate  $p_t$  that is we say we will again name it  $p_c$  that corrected value, for  $p_0$  the corrected value will be  $0.28$  mega Pascal. Now next step is you have to find out what is the limit error limit for this reading. Having found out the corresponding corrected value we have to find out what is the error zone within which the instrument gives a reading.

(Refer Slide Time: 00:19:42 min)

$$\sigma^2 = \frac{1}{N} \sum (A - p_c)^2$$
$$= \frac{1}{N} \sum \left( \frac{p_0 - b}{m} - p_t \right)^2$$
$$\sigma = 0.02 \text{ MPa}$$
$$\pm 3\sigma \text{ (99.7\%)} \\ p_t = p_c \pm 3\sigma \\ = 0.28 \pm 0.06 \text{ MPa}$$

0.34 to 0.22

IIT  
MADRAS

For that you have to use the phenomena the standard deviation that is sigma. This is what we have to find out for a given set of readings that sigma is equal to, if sigma is standard deviation then sigma square is equal to one by N sigma of  $p_c$  minus  $p_t$  the whole squared where  $p_c$  is the corrected value. We have corrected value for 0.25 as 0.28 mega Pascal that is corrected value.  $p_t$  is the corresponding true value for 0.25 corresponding true value is 0.300 that is our true value. The difference between the true two values square and divided by 1 by N that is the standard deviation. So for this set of readings now  $p_c$  you can substitute sigma squared is equal to one by N sigma of  $p_0$  minus  $b$  divided by  $m$  that is our  $p_c$  minus  $p_t$  the whole squared. So we open another column with readings where  $p_0$  minus  $b$  by  $m$  also is also calculated and tabulated for each reading and  $p_t$  is there already so from this we can find out the value of sigma. So sigma here it is obtained as 0.02 mega Pascal.

By Gaussian distribution if you take a limit as plus or minus 3 sigma then 99.7% of the reading the error will be within plus or minus 3 sigma limit. So this is normally adopted, the error we will put as plus or minus 3 sigma. So now the  $p_t$  is equal to  $p_c$  plus or minus 3 sigma that is theoretical value lies between the limits of plus minus 3 sigma from the corrected value, corrected value as you have found out earlier this is the corrected value  $p_0$  minus  $b$  by  $m$ . So here in this case it comes to be  $p_t$  is equal to 0.28 we have already calculated for one reading, 0.28 plus or minus 3 sigma comes about plus or minus 0.06 mega Pascal.

Now you find this is only 0.2 reading is 0.3 so that is within this limit. That is it's varying between point plus means 0.34 to 0.32. That is 0.3 is lying within this that is true value lies within this limit of 0.28 plus or minus 0.06. That is how we arrive at the error of the instrument that is after recalibrating the pressure gauge now we can specify the new accuracy of the instrument like this. The corrected value should be found out by using this equation that corrected value with the limit 3sigma then it gives the true value. True value it has a range within the 3 sigma limit so that is how a recalibration. Recalibration for a given reading what should be the correct reading and its uncertainty that is what we have found out now.

Next one what we are going to learn is the error in computed value. We also say error propagation, when we have this problem, suppose in an engine power is to be computed power output or power input, the efficiency of the engine then you have to measure the flow rate of the fuel then you have to measure the power output in the shaft in terms of torque developed in the shaft on that rotating speeds. So 3, 4 measurements you have to make in order to find out the efficiency of an engine. So from the measurements we have made 3, 4 measurements we are going to compute a value and we also know any measurement has got its own error zone. Now what is the error of the computed value obtained from the different measurements having different inaccuracies.

That is why we say the computed value, how the error propagates in to the computed value that is error propagation in to the computed value, that is what we want to find out. The computed value will have some error based upon the error of the individual measurements and first we derive the mathematical equation for this phenomena.

(Refer Slide Time: 00:25:00 min)

$$Q = f(x_1, x_2, \dots, x_N)$$

$$Q \pm \Delta Q = f[(x_1 \pm \Delta x_1), (x_2 \pm \Delta x_2), \dots, (x_N \pm \Delta x_N)]$$

$$Q \pm \Delta Q = f[x_1 \pm \Delta x_1, x_2 \pm \Delta x_2, \dots]$$

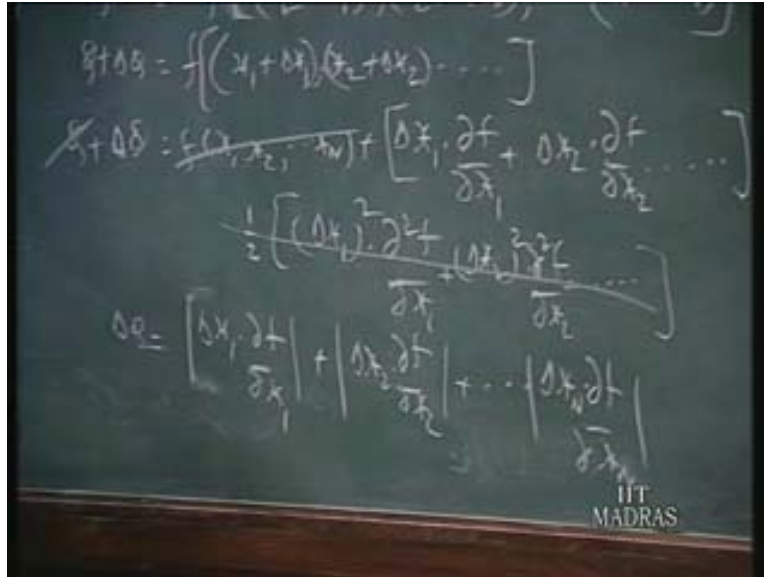
$$Q \pm \Delta Q = f(x_1, x_2, \dots, x_N) + \left[ \Delta x_1 \frac{\partial f}{\partial x_1} + \Delta x_2 \frac{\partial f}{\partial x_2} + \dots \right] + \frac{1}{2} \left[ (\Delta x_1)^2 \frac{\partial^2 f}{\partial x_1^2} + (\Delta x_2)^2 \frac{\partial^2 f}{\partial x_2^2} + \dots \right]$$

IIT  
MADRAS

Suppose  $Q$  is a computed value obtained from measurements of  $x_1$   $x_2$  and so on  $x_N$ ,  $N$  measurements are made and using a function  $Q$  is equal to function of this we can compute the value of  $Q$ . Each measurement has got an error of  $x_1$  plus or minus  $\Delta x_1$  and  $x_2$  will have  $x_2$  plus or minus  $\Delta x_2$  and so on. So the computed value will have error zone of  $\Delta Q$  is equal to function of  $x_1$  plus  $\Delta x_1$ , I will write the square bracket and  $x_2$  plus or minus  $\Delta x_2$  and so on, until  $x_N$  plus or minus  $\Delta x_N$  for the class  $N^{\text{th}}$  instrument. So that is the error in the computed value and its error. Now we want to find out the magnitude of  $\Delta Q$  so we simply omit one of the signs. So we can also write  $Q$  plus  $\Delta Q$  equal to function of  $x_1$  plus  $\Delta x_1$  and  $x_2$  plus  $\Delta x_2$ , so omitting one of the signs we can write it like this. Now we expand that right hand side by using Taylor series, we can write is equal to function of  $x_1$   $x_2$  up to  $x_N$  plus  $\Delta x_1$  in to dow  $f$  by dow  $x_1$ .

This is  $\Delta x_1$  term plus  $\Delta x_2$  in to  $\Delta f$  by  $\Delta x_2$  like that plus of  $\Delta x_1$  square in to  $\Delta f$  by  $\Delta x_1$  and  $\Delta x_2$  square in to  $\Delta f$  by  $\Delta x_2$  and so on it goes for the second derivative and then plus one third of the  $\Delta x_1$  cube and so on, the Taylor series will be expanding like this.

(Refer Slide Time: 00:27:33 min)



Now if you reduce now you find here you have got  $Q$  plus  $\Delta Q$  and we know  $Q$  is equal to function of  $x_1, x_2$  so both side we have got, this is equal to this we can cancel. So  $\Delta Q$  is equal to this plus this and we know the error is going to be a small quantity, when it is small  $\Delta x_1$  square this square term you can neglect it, so this all will go away (Refer Slide Time: 28:48). Finally  $\Delta Q$  is equal to considering only the magnitude of each term,  $\Delta Q$  is equal to  $\Delta x_1$  in to  $\Delta f$  by  $\Delta x_1$ . Consider only magnitude a mod, total error we will find out in to  $\Delta x_2$  in to  $\Delta f$  by  $\Delta x_2$  mod and plus like that  $\Delta x_N$  so  $N$  instruments  $\Delta f$  by  $\Delta x_N$ . So this is the total error, that is the error in the computed value obtained from the individual measurements  $N$  measurements. Now this equation assumes that all instruments are working with the maximum error. Then what is the maximum?

Later on we are going to put  $Q$  is equal to  $Q$  plus or minus  $\Delta Q$ . So that is plus, so this we have got sum value that plus or minus you are going to put always. That plus value when you take, that is individual instrument itself as that is so we will say  $Q$  maximum is equal to  $Q$  plus  $\Delta Q$ . So  $Q$  minimum is  $Q$  minus  $\Delta Q$  minus  $\Delta Q$  that is the these limits are at, suppose we take the plus limit maximum value that is each instrument is assumed to work simultaneously at the maximum limit and then you will get the  $\Delta Q$  maximum limit for the  $Q$ .

Similarly the minus sign you know that the negative side all the instruments are supposed to work in negative maximum and you will get it but such an operation when one instrument measures with a maximum error another instrument need not measure at maximum error, so this equation gives very wide limit for the computed value.

The instruments may not be working simultaneously at the maximum level or minimal level of the error. So to get a realistic view we have got so called by using the principle root sum square by using the root sum square we get this equation delta Q is equal to root of delta x<sub>1</sub> in to dow f by dow x<sub>1</sub> whole square plus that is individual instrument error component is squared and then find the square root.

(Refer Slide Time: 00:30:03 min)

Root-sum Square

$$\Delta Q = \sqrt{(\Delta x_1 \frac{\partial f}{\partial x_1})^2 + (\Delta x_2 \frac{\partial f}{\partial x_2})^2 + \dots + (\Delta x_N \frac{\partial f}{\partial x_N})^2}$$

±3σ

1), (x<sub>2</sub> ± Δx<sub>2</sub>), ... (x<sub>N</sub> ± Δx<sub>N</sub>)

IIT  
MADRAS

So delta x<sub>2</sub> in to dow f by dow x<sub>2</sub> whole square and so on up to N term all these things. So this is the realistic limit that is if you have found out the delta x from x<sub>2</sub> as plus or minus 3 sigma principle which you have learnt under the calibration process then this delta Q also correspond to three sigma limit. That is more than sufficient, 99.7% of the readings will be within this limit so that is the understanding. So we will use this equation always to find out the error propagation of the computed parameter and suppose we have a measurement to be made and the error of the computed only if it is given, suppose delta Q should be within certain limit then what are the type of instruments that error limit we have to select to obtain a particular error in the computed value.

If that is given that means we have to select based upon the error in the computed value we have to select individual instruments and for that there is infinite number of procedures or methods to obtain it. But we will follow one method that is we will say the component of error contributed by each measurement is equal that is this one, from the first instrument this is the part contributed by the first instrument, error contributed by the first instrument towards delta Q. We assume the component of each instrument is same that means when measurement is there then N in to root of this will be giving rise to delta Q. Under that assumptions that is the equal error portion is given by each measurement then we can easily find out what should be the error of individual measurements.

(Refer Slide Time: 00:32:45 min)

$$\Delta Q = \sqrt{N \cdot \left(\frac{\partial f}{\partial x_i}\right)^2} \quad i=1 \text{ to } n$$

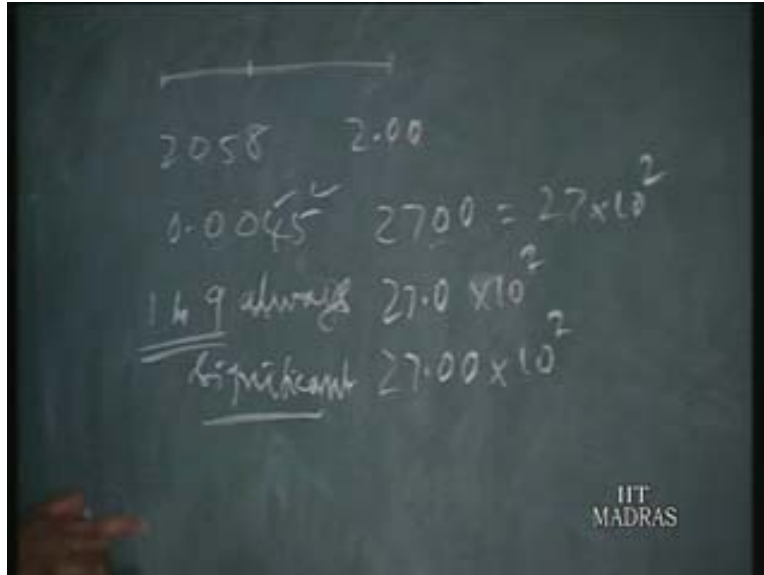
$$\left| \Delta x_1 \cdot \frac{\partial f}{\partial x_1} \right| = \left| \Delta x_2 \cdot \frac{\partial f}{\partial x_2} \right|$$

$$\Delta x_i = \frac{\Delta Q}{\sqrt{N} \left(\frac{\partial f}{\partial x_i}\right)}$$

So now we get that is delta Q is equal to root of N times dow f dow x<sub>1</sub> for example dow f by dow x<sub>1</sub> whole squared that is what we have got and assuming delta x<sub>1</sub> dow f dow x<sub>1</sub> whole squared is equal to this. So N instruments are there so like this and then this gives rise to that is under the assumption that delta x<sub>1</sub> in to dow f by dow x<sub>1</sub> is equal to delta x<sub>2</sub> in to dow f by dow x<sub>2</sub> mod. They are all equal under that condition we write the above equation. Now we can obtain suppose instead of one, i will vary to 1 to n, i is equal to one to n then we can find out what should be the delta x<sub>i</sub>. So delta x<sub>i</sub> is equal to from this we can easily obtain, delta x<sub>i</sub> is equal to delta Q divided by root N, this root N comes this square will be under the square root will be same value dow f by dow x<sub>i</sub>. That is this gives the equation for selecting instruments for obtaining a particular error in the computed value.

So now we can work out some example also how this equations are made use of before we learn how to use those equations, it is important we know something about significant figures and rounding off. This is very important in the sense, suppose we use one time a precision instruments another time a very rough instrument and if you add suppose two stretches.

(Refer Slide Time: 00:34:44 min)



Suppose we want to find out the length of the line, for up to this we have used only instrument and from here another instrument and if they have different inaccuracies then you cannot simply add those two numbers because the rough instrument what you have made use of it has eaten away the accuracy of the good instrument. So final measurement will be accurate only to the accuracy of the rough instrument that is the basis for this. So how to do scientifically that is what you have to learn? For that the signal figures are rounding off is to be understood. Suppose you take a number, suppose some number 2058 some number, it's a whole number so here we say each number here is significant. That is zero which is within a given number is significant but zero will not be significant in some other instance suppose we write 0.0045 like that then 4 5 are significant and 0 0 are used only for fixing the decimal place of the significant numbers. In that case the 0 0 is not significant.

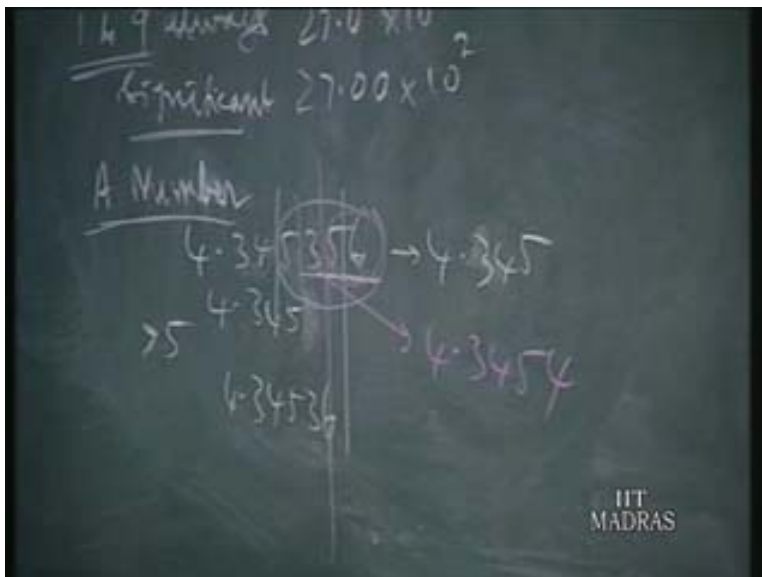
Suppose if you write 2.00 so just like some reading in our calibration measurement, the calibration process we have written there 0 0 to indicate there are supposed to be readings in the particular measurements they are zero. It indicate that you have to write 0 0 so that we know the instrument is measuring up to second decimal. So here 0 0 even the value zero but it represents something so they are significant. So here it is significant, here also it is significant in this place zero is not significant and in another place zero it suppose 2700. So this if you write 27 in to 10 to the power of 2 then in this number we are interested only how many hundreds are there, that means only 2700, 27 only important the 0 0 are not important.

Suppose we write 27 same number we can write, 27.0 in to 10 to the power of 2 this number can be written like this also. So in this case 1 0 is here important, up to one decimal we are finding out so out of this one zero is significant, second is not significant but if you write zero in to 10 to power of two here all the zeros are significant that means zeros can be sometime significant, sometime need not be significant but here if you write 27.00 in to 10 to the power of 2, both of them are significant. If you write 27.0 in to 10 to the power of 2, only zero is significant. If you write 27 in to 10 to the power of 2 both the zeros are not significant.



That means there are two instances where zeros are significant, two other instances where zeros are not significant but all the numbers one to nine are always significant this is the concept of significant numbers, 1 to 9 are always significant, 0 is sometime significant sometimes it's not significant that you have to understand under different context. If it is entrapped within numbers then it is significant and if it is used only for fixed decimal place it's not significant and a number like this when it is written in a particular fashion sometimes significant, sometimes it's not significant so that concept of significance.

(Refer Slide Time: 00:38:12 min)

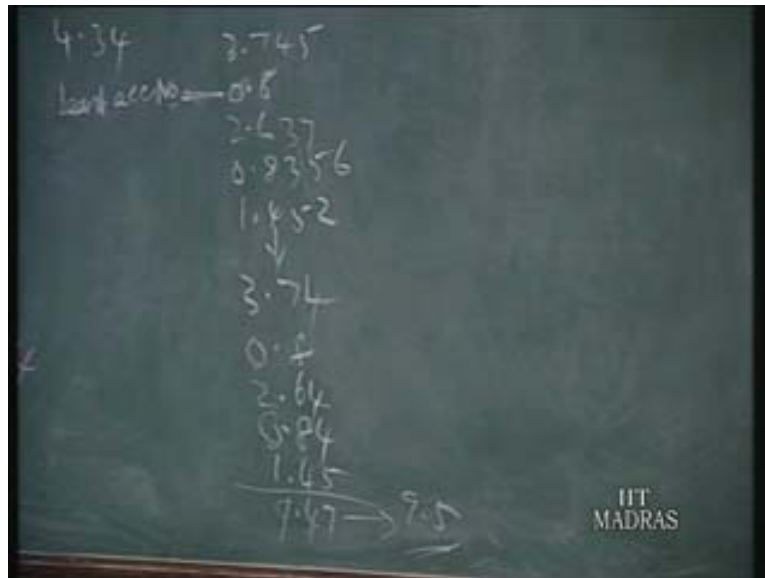


Now rounding off of a given number. Suppose a number is there, how the rounding off is done? Suppose we have a number so 4.345356 suppose now we have got a number like this and we want to round it off to how many decimal places? 6 decimal places are there in this number and suppose we round it off to a particular decimal place instead of say 6 it is sufficient if I have only third decimal place. Then how to do the rounding off? There is a particular rule. Look at the number that is being discarded now 356 is discarded, if it starts with a number greater than 5, add one to the desired decimal place. So in this case this rate comes to 4.345 but because this number is less than 5 and I leave the number as it is, so we have left the number as it is. If it is greater than five suppose we want to have 5 decimal place now it is 4.3453.

Now it is greater than 5, let's discard a number, starts with greater than 5 add one to the desired decimal place. Now it is 3 6 that is for the fourth decimal place. If it is less than 5 we leave as it is that is 5 alone. Suppose we want only 3 decimal places, neglect a number starts with 3 so leave the decimal place untouched. If it is greater 5 add one, if it less than 5 leave it as it is suppose it is exactly 5. Now it is up to second decimal place we want to fix and we find the discarded number starts with 5 and how to round off this number? Now look at the desired decimal place what is the number 4? If it is even number leave it as it is, so 4.34. If it is an odd number then add one extra that will be the case.

Suppose I want to leave it with the four decimal places, round it off to 4 decimal places then we find the neglected number, discarded number starts with 5 and desired decimal place has got an odd number. So for this the rounded off number will be 4.3454 because odd is number is increased to next higher even number. So it is as a rule there are 3 points, the discarded number if it starts with a number higher than 5 always add one to the desired decimal place. If it is less than 5 leave the desired decimal place as it is. If it is exactly 5 discarded number starts with the 5 then add one if it is an odd number, leave the number as it is if it is an even number, desired place decimal place if it is even number leave it as it is. That is how the rule is adapted for rounding off a number.

(Refer Slide Time: 00:41:50 min)

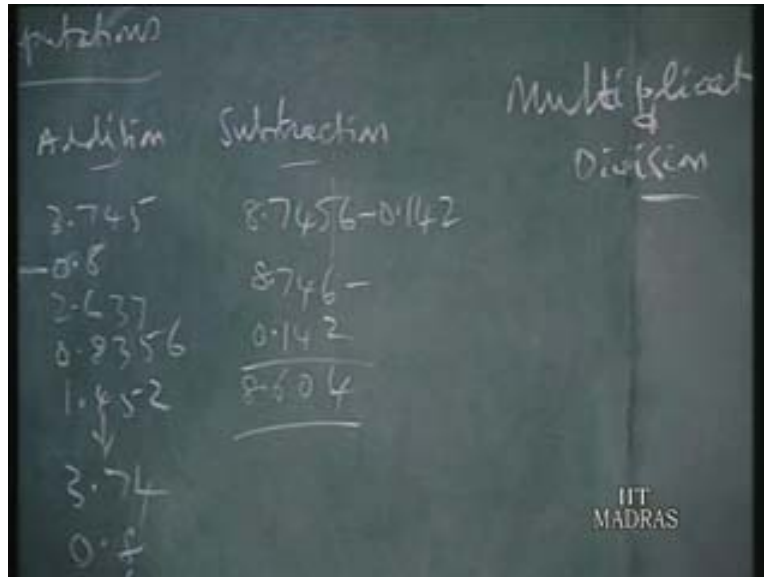


Now what is the rounding off procedure for computations? How many computations we have? Addition and subtraction and multiplication and division, multiplication and division because there are only one rule for both of them and division these are three basic mathematical functions we have and what are the additions, what are the rule for rounding off for computations. So first we will see for the additions. Suppose we have some 5 numbers, so 3.745, 0.8 and then 2.637 and then 0.8356 and then 1.452. So suppose these are the numbers to be added, what is the rounding off procedure?

First point is note down what is the least accurate number that is the one which has got least number of decimal places. This is the least accurate number, note down the number of decimal places and round off all the other numbers to the next decimal place. So from here what we get next decimal to second decimal place we have to round it off. So when you round it off to second decimal place it comes like this 74, because 5 is discarded number and desired place is an even number so leave it as it is. Then 0.8 as it is you write then 2.63 we have to write up to 3 but the discarded number its more than 5 so 2.64, it's a rounded off number. Now up to 83 we have to have, so discard number starts with 5 and it's an odd number 3 so make it next higher number 0.84 and then here the discarded number is less than 5 write it as it is up to that place 4 5.

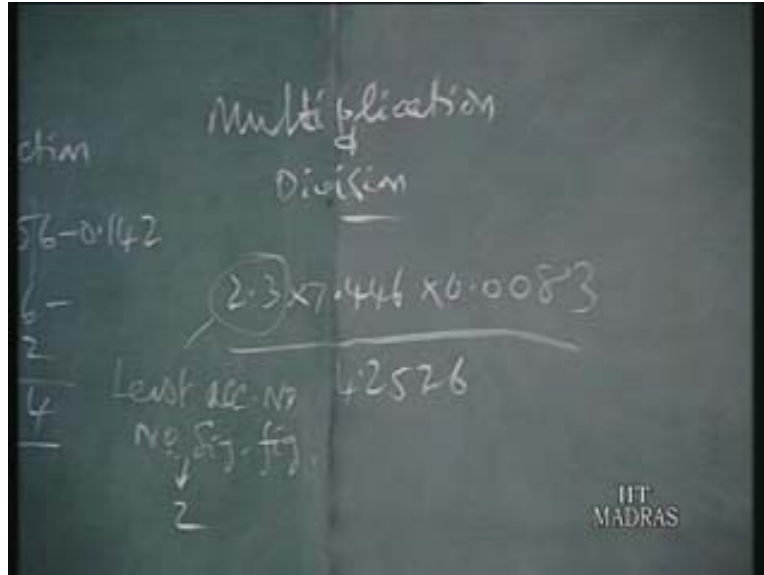
Now add this total so 9.47 and round it off to the same decimal place as least accurate number and from here we have to reduce to the same decimal place as the least accurate number so 9.5 because discarded number is greater than 5. So add one to the desired decimal place so it comes 9.5. So this is the rule you have adopted for the addition.

(Refer Slide Time: 00:44:32 min)



For subtraction suppose you have got two numbers  $8.7456 - 0.142$ , subtraction involves only two numbers addition can be any amount of numbers but here only two number, rule is simple. If they have got different number of decimal places identify the less accurate number having less number of decimal places, round off the other number to the same decimal place and then subtract it and leave it as it is. So here we have got second number is less accurate so round off the first number to the same decimal place up to here third decimal. So it means 8.746 because 6 is higher than 5 and add one always so minus 0.142. So the answer will be also same number of decimal places 068 so this is the answer for the subtraction, rounding off procedure adopted for subtraction.

(Refer Slide Time: 00:45:42 min)



Suppose multiplication division we have got this, that is 2.3 in to 7.446 in to 0.0083 divided by 4.2526. Suppose this is the computation we have to do and how to round off the final answer. Here you use the concept of significant figure also, up to this that the addition subtraction we are not using the concept of significant figures but here you have to use the significant figure, find out what is the least accurate number in this multiplication and division? Find out the number of decimal places? Find out number of the significant figures in the least accurate number? Now 2.3 is one which is having the least number of decimal places so it happens to be the least accurate number. Next is number of significant figures. Here it is two significant figures that is here it is two and round off all the other numbers to one more significant figure and write the computation again that is one more significant figure means up to 3 significant figures.

So now I will write it 2.3 here we have got 4 significant figures all numbers are significant so we have to have only 3 so it has to be left somewhere here, 6 should be left. So it is giving rise to 7.4 because 6 is higher than 5 add one number 5, in to here only two significant figures you write it as it is 83 divided by here 3 significant figures so 26 we have to leave and is smaller than 5 write it 4.25 without any modifications. I will compute the number to three significant figures that is now obtained as 0.0335 that is we have got three significant figures and then reduce this answer to the same number of significant figures as the least accurate number that is now here were three significant figures in the computation reduce to have the same significant as the least accurate number so only two.

(Refer Slide Time: 00:47:48 min)

The chalkboard shows the following work:

Least acc. no. 42526  
No. sig. fig. 2

$$2.5 \times 7.45 \times 0.002$$
$$= 0.0335$$
$$= 0.034$$

ITT  
MADRAS

This gives rise to 0.003 this, we are going to neglect it is exactly 5 this is odd number make it a next higher even number so this is the answer. That is how we have to use the concept of significant figures and rounding off for individual numbers as well as for the computation of all these things.