

Principles of Mechanical Measurements
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Lecture No. #09

We have learnt previously how to find the error of the computed value and if the error of the computed value is known, what should be the error in the individual measurements. So these two things we have learnt yesterday. Now we will work out a problem so that the whole concept is clear. Before working out we have to see what are the rules that we should follow in finding the error in the computed value. So that is what is given here; four steps are given. Four points to be noted when we are computing the error in the computed value.

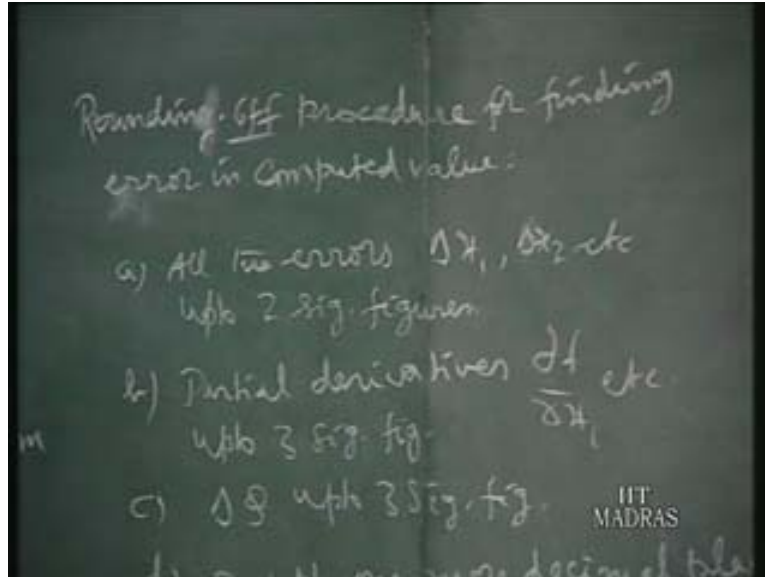
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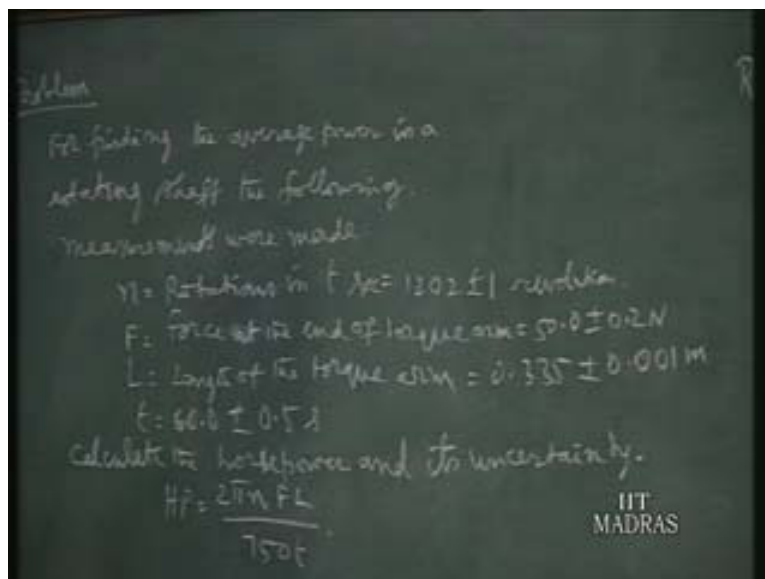
All the errors of say Δx_1 , Δx_2 and so on of individual measurements may be up to two significant figures because it is an error giving more than two significant figures, it is not of much sense. So most of them are given up to one significant figure but we can go up to two significant figures. If it is given more than two significant they are reduced to two significant figures that is the first step. The individual errors of the measurements should be reduced up to two significant figures, it can be one significant figure also. Then while we are computing the propagated error in the computed value, we are finding the derivatives, partial derivatives $\frac{df}{dx_1}$, $\frac{df}{dx_2}$, $\frac{df}{dx_3}$ and so on.

That should be computed up to three significant figures then with these values we are computing ΔQ by using the formula, ΔQ is equal to root of Δx_1 into $\frac{df}{dx_1}$ whole square plus Δx_2 into $\frac{df}{dx_2}$ the whole square and so on we are computing.

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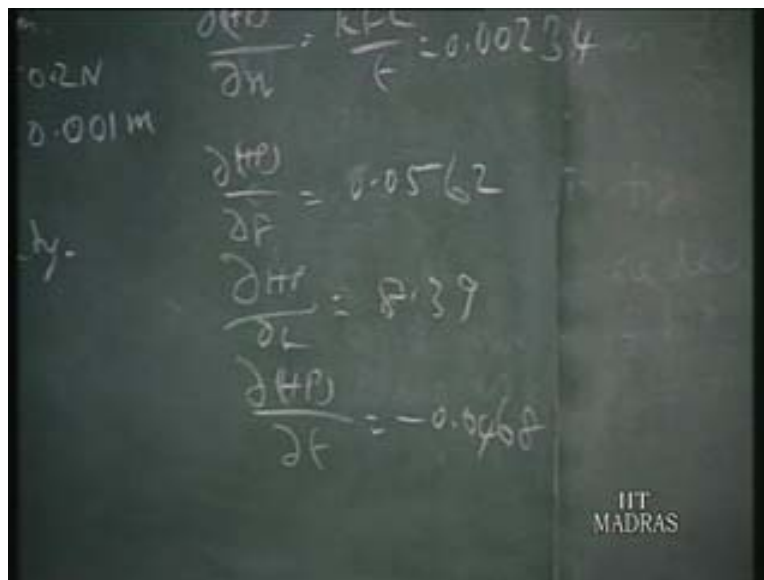
That delta Q, error in the computed value may be computed up to three significant figures that is third and then only you are supposed to compute the Q, nominal computed value. So we don't do it at the first because up to what decimal place Q should be obtained will be known only after computing delta Q. So Q up to one more decimal place we have to find out, one more decimal place than that of delta Q. You have to compute and then round it off to the same decimal place as delta Q. That is why I told Q should be found out, nominal horsepower or nominal computably should be found out after finding the error because up to which decimal place we have to find out that will known only after delta Q is known. So noting down all these steps now we will compute the given problem, they computed error in the computed value. So the problem reads like this.
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For finding the average power in a rotating shaft the following measurements were made. So rotations are measured in t seconds that is t seconds is about 60 seconds, it is measured as 1202 plus or minus one revolution. The error of that instrument is plus or minus one revolution. Similarly force at the end of the torque arm 50.0 Newton because 0.0 we have put first decimal also we can read and that error is plus or minus 0.2 Newton and the length measuring instrument of the torque arm 0.335 meter plus or minus 0.001 meter.

That is one mm is the error in the measurement of the length, a torque arm and with the time piece we are measuring the time 60 seconds, 60.0 second plus or minus 0.5 seconds is the error of that time piece. So we have measured these quantities, with these you are going to compute the horsepower by using this formula. In this case what is the error of the horsepower because mentioning n , F , L , t we have got different area, different errors and what will the error of the computed value horsepower that is our problem. So we first find the partial derivatives, compute the partial derivatives.

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Any how we can write the following HP is equal to 2π by 750 nFL by t . So one part is constant so I will write the whole thing as k so I write k into nFL by t , this is horsepower. So $\frac{\partial HP}{\partial n}$ is equal to kFL by t which is calculated already 0.00234. Similarly we find the other things $\frac{\partial HP}{\partial F}$ that will come about 0.0562 so $\frac{\partial HP}{\partial L}$ is equal to 8.39.

We are computing up to three significant figures. These are three significant, zeros are not significant and this is not significant we have got three significant 562 and here we have got three significant 839. That is the rule which I narrated earlier and the now $\frac{\partial HP}{\partial t}$ is equal to minus 0.0468. So you found out the partial derivatives now we are going to find out ΔHp .

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The image shows a chalkboard with the following handwritten text:

$$\Delta(\text{HP}) = \sqrt{\left(\Delta n \cdot \frac{\partial \text{HP}}{\partial n}\right)^2 + \dots}$$
$$= 0.028 \text{ HP}$$
$$\text{HP} = 2.814 \text{ HP}$$

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That is the error in the horsepower computation is equal to root of Δx_1 that is Δx_1 is Δn and $\frac{\partial \text{HP}}{\partial n}$ by or $\frac{\partial \text{HP}}{\partial n}$ whole squared for the rpm measurement. Similarly we have got for other measurements we have to write it in the quantities. All the four quantities we have to write and then now if I substitute Δn is given as one and $\frac{\partial \text{HP}}{\partial n}$ we have already calculated and square it and then you find finally the answer is 0.028 horsepower this is the error. So that is error we have found out up to two significant figures and it lies at the third decimal place.

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The image shows a chalkboard with the following handwritten text:

$$= 0.028 \text{ HP}$$
$$\text{HP} = 2.814 \text{ HP}$$
$$\text{Power} = 2.814 \pm 0.028 \text{ HP}$$
$$= 2.814 \pm 1\% \text{ HP}$$

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So now find out the nominal horsepower by using that formula up to I mean it is coming at 2.814 horsepower probably zero might have come. The final value is up to the same decimal place as the error of that measurement. So this is the nominal value so with the uncertainty if you write is equal to 2.814 plus or minus 0.028 horsepower, power developed is equal to 2.814. That is now you will find error is one hundredth of this nominal value so we can write also plus or minus 1% horsepower. That is the error in the computed value of horsepower, making use of this instrument having different errors. Suppose if the error is specified and how to select these instruments that you will see next. We want to find out the horsepower within error of 0.5% what you have found out earlier by using given instruments it gives rise to 1% error.

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Handwritten notes on a chalkboard:

$$\Delta Q = 0.5\% = 0.014$$

$$\Delta x_i = \frac{\Delta Q}{\sqrt{N} \left(\frac{\partial Q}{\partial x_i} \right)}$$

$$\Delta n = \frac{\Delta(HP)}{\sqrt{4} \frac{\partial(HP)}{\partial n}} = \frac{0.014}{\sqrt{4} \times 0.00234} = \pm 3.0 \quad (\pm 1\%)$$

Wanted $\Delta F = \pm 0.1N$ ($\pm 0.2N$)
 $\Delta L = \pm 0.8 \text{ mm}$
 $\Delta t = \pm 0.15 \text{ s}$

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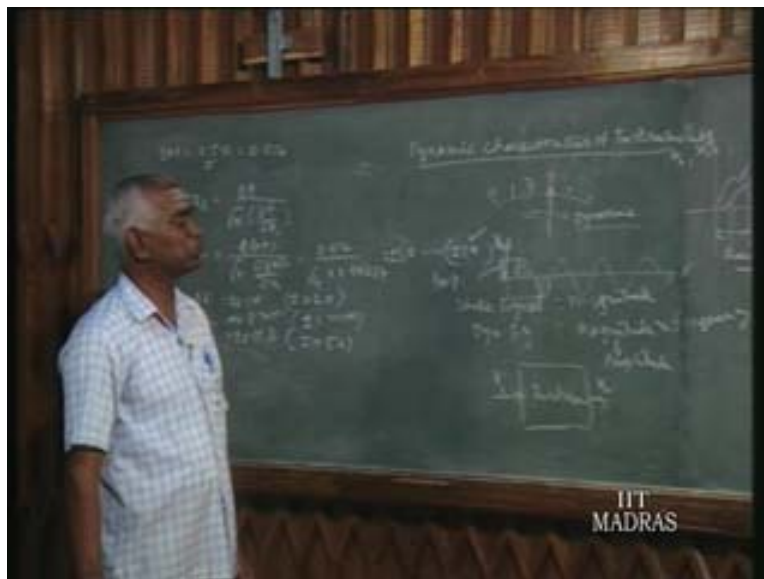
Suppose you are interested only 0.5% error in the computed value of horsepower that is 0.014 horsepower alone should be error then what are the types of instruments you have to select. That is what you are going to find out now. You know the equation that is Δx_i , i being 1 to n number of instruments is equal to ΔQ over root N into $\frac{\partial Q}{\partial x_i}$. This is what you have derived earlier. This equation for the parameter here for example for the rpm rotation measurements Δn that is error in the instrument measuring the rotations if it is Δn then it should be is equal to that is where x_i term is here n rotations. So ΔQ is Δ horsepower for this problem, you rewrite for this problem root N here four measurements are there and then $\frac{\partial Q}{\partial n}$.

So this equation is for the rotational measurement, the instrument for rotational measurement. So we substituted here, Δ horsepower is 0.014 divided by root 4 into $\frac{\partial Q}{\partial n}$ we already calculated earlier. That is 0.00234 this is what you have found out in the earlier problem $\frac{\partial Q}{\partial n}$. Now this comes about three rotations. Similarly you find for other parameters, next parameter is Δf that will be coming around in same way we work it out 0.1 Newton and ΔL is equal to 0.8 millimeter and Δt that time measuring instrument should have an error of 0.15 second.

So these are the permitted errors in the measurements, if you want to achieve an accuracy of 0.5% or within an inaccuracy of 0.014 that means what are the instruments we have to change. Here this is plus or minus 3 so plus or minus understood this is the error that we should have but what is existing is instead of 3 we have got already plus or minus one zero, it's already available. So with this instrument we can use it. This is finer instrument than the proposed one so we can use it and for the delta f we have the existing instrument as measuring 0.2. This is plus or minus 0.2 Newton that is existing one but we want 0.1 Newton so we should go for a new instrument. This we can go for new instrument that is how we use this concept here. Then delta L for measuring length what we required is plus or minus 0.8 but what is there already plus or minus 1 mm what you have got already. So with this 0.8, this one mm we can use it, this we are using it this also we can use because not much difference and delta t time existing inaccuracies is plus or minus 0.5 second but what you require is 0.15 second. So we should go for the new one, here also we should have new instrument.

So we have to change the two instruments and the existing two instruments we can use it. Suppose when you select new instrument if you are not able to get 0.1 Newton, only 0.2 is there **0.2 Newton inaccurate instrument that much inaccuracy** finer instrument is not there and here also you don't get, little rougher only is there. You can use those instruments because for the rotation instead of one we have got already one then finer instrument is required. So with the available instrument again compute the computed error and if it is within this we can do it. That is by trial and error we arrive at the type of instruments or the errors in the typical instruments as per the availability and then we check whether it is within the 0.5%. That is how we make use of these equations in deciding the instruments, in selecting instruments for a particular computed error. Next chapter is dynamic characteristics of instruments.

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Why it is necessary? Say most of the times or often we may also encounter a measurement of varying signal. If the signal is constant at least during the measurement we can read the value in the scale where the pointer stops, this is the scale and where the pointer will be standing here 0, 1, 2, 3 so 3 is the reading. That is the pointer has to be stationary to take readings, that is possible only if the parameter doesn't change during the measurement. Suppose it is continuously oscillating and doesn't have any fixed place then you can note down. So such oscillating signals are called dynamic signal. Dynamic means always changing so it is a changing signal, so you cannot make any reading there. In such instances how to study such things, in that case we go for recording. You record these variations, record variation of the signal and later on analyze because in static signal we note down, tabulate it for different conditions what are the output signal. We can tabulate but such tabulation is not possible when the signal is varying or dynamic in character.

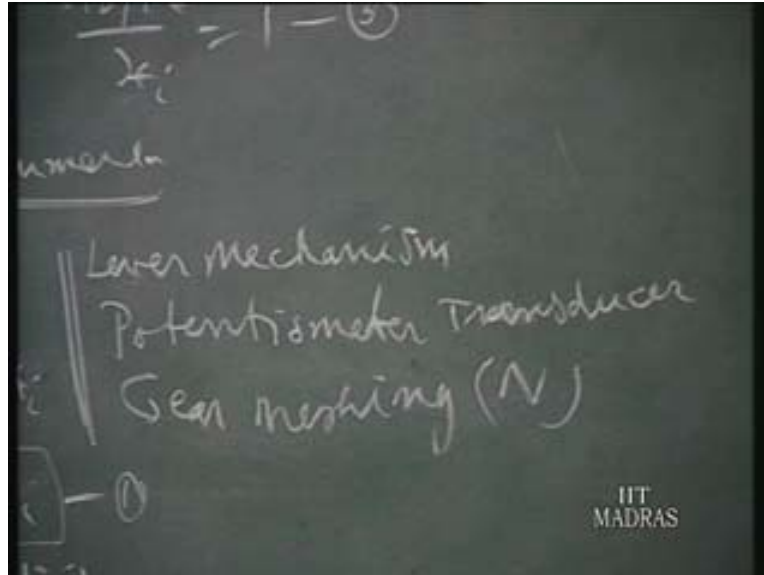
So in the static signal we will measure only the magnitude whereas in dynamic signal we have got 2 quantities magnitude and the frequency. These are the two measurements you have to make because the frequency at which or how often the change is taking place that is given by the frequency magnitude. Suppose this is the time and this is your output signal x_o if it is varying like this, you see magnitude at different instance varies sometimes it is positive magnitude, sometime it is negative magnitude. So we have got this instantaneous varying that we cannot say then we give so called amplitude. That maximum magnitude is called amplitude.

So people give the so called dynamic signal, amplitude. Amplitude is the maximum magnitude during its variation, amplitude and frequency these are the two measurements which will fix the type of variations. Now for a system if suppose we have got an instrument, to that instrument we give the signal x_i which is the varying one. This is dynamic signal, what will be the output? How the output is going to vary? Suppose x_i is a sinusoidal one; t and the x_o this is x_i . So x_i and x_o we are plotting in the same y-axis so the x_o will not be of this same shape and also you may not have the same amplitude, it may be little less depending upon the frequency.

So you have somewhat variations and afterwards it settles. So this way up to here it's a transient; it is a transient region. Later on you find the steady state once it settles then it is steady state. What you find when the frequency increases the amplitude reduces but also you have got some phase lag. This is the phase lag with some difference, it doesn't give immediately the output; it takes some time. So this is the overall behavior of any instrument when we give the varying signal.

Depending upon the signal or response of the instrument we have got three types of instrument. Zero order instrument, first order instrument then second order instrument. So three types of instruments are there depending upon their response for the dynamic signal. So first you will take zero order instrument, it is represented by $a_0 x_0$ is equal to $a_i x_i$ where x_i and x_o are input and output signal and the a_0 and a_i are the physical parameters.

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So it can be rewritten x_o is equal to a_i by a_o into x_i and this a_i by a_o is normally called sensitivity so k into x_i where this is your sensitivity of the instrument. This equation is for the zero order system, what does it say. The moment you give x_i you get x_o according to this equation, there is no time lag and the sensitivity is a constant. The x_o is perfectly proportional to x_i this is the way it is understood. So we say this equation can be written on two more forms x_o by k is equal to x_i . That is what it says, same equation mathematical equation written in different form but here say this equation one same equation written in another form. Here if x_o output signal is divided by sensitivity then you should get x_i , if there is no error for that purpose we can use this equation x_o by k minus x_i is equal to error. So x_o by k should be x_i , in ideal situation since no error x_o by k itself is x_i .

In another way we express this in the form x_o by k divided by x_i is equal to one that is the proportionality equation. x_o by k or we say the x_o is perfectly proportional to x_i if x_o by k by x_i is one, proportionality constant. What are the physical systems which obey this zero order system? So lever mechanism we give a displacement x_i or d_1 and it comes at the output signal as d_2 . So the moment you give d_1 the output signal appears d_2 . Hence we say there is no time lag and the sensitivity is the leverage of the mechanism. So we find that it approximately represents zero order, no time lag immediately without any time lag the output signal appears and the sensitivity or the leverage doesn't change the operation of the mechanism or potentiometer, transducer where displacement it is converted into a voltage. There also we find the moment you will move the output voltage will be appearing or gear up, gear machine with a definite velocity ratio n so the angular rotation of θ appears as n times θ in the output shaft. So these are the three mechanisms where you may have the zero order realization but you find some deformation may be there in the lever mechanism.

Potentiometer transducer assumes pure resistance but there may be some capacitance and inductance to that extent there is some time lag, gear mechanism you may have backlash. So this will affect the functioning as for zero order instruments but otherwise more or less they represent zero order system.

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Now we see the first order instruments, there in zero order two terms $a_0 x_o$ is equal to $a_i x_i$. You have to add one more term that is $a_1 dx_o$ by dt plus same term $a_0 x_o$ zero is equal to $a_i x_i$. This is our in equation for the first order; first order is d by dt once it appears that is one extra term. **Previously from here to here we had for zero order (Refer Slide Time: 23:38).** Now rewriting this you have coefficient x_o as one, divide throughout by a_0 , a_1 by $a_0 dx_o$ by dt plus x_o is equal to a_i by a_0 into x_i . Now you have got one more new parameter that is a_1 by a_0 call it as tau, time constant tau.

So now d by dt we can call it as differential operator D . So tau D plus this we call 1 into x_o , x_o bringing it outside so we can write the left hand side in this fashion where capital D is the D by dt , **x_o is equal to** this we have already seen as sensitivity k say k into x_i . So x_o by x_i is equal to k by 1 plus tau D . So we can write this way also. So this is the typical equation for a first order instrument. Now what are the first order devices we have in practice? Mostly temperature measuring instruments.

Temperature measuring instruments represents first order in nature. For example we can derive an equation for a thermocouple where we will prove that it is of this order. For that consider this thermocouple circuit, galvanometer. So thermocouple A and B is put into a bath. This is thermocouple junction, the other one may be ice cube, it may be at zero degree centigrade and this is at certain temperature T .

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It is a temperature t_1 which is the temperature of the bath. Now this is the thermocouple. Consider the heat transfer from the bath to the thermocouple junction. I will draw the thermocouple junctions alone separately. Suppose this is thermocouple junction and the heat entering through this surface area of the thermocouple junction is equal to the heat gained by this mass of the thermocouple junction. That is principle what we use here to equate that is heat entering through the surface area of thermocouple junction is stored by the thermocouple junction itself.

Suppose u is the heat transfer coefficient through the surface area and a is the area of this surface area of the thermocouple junction and t_i is the temperature at any instant t_i or t_j we call it t_j , t_j temperature of the thermocouple junction at any instant. After inserting the thermocouple junction temperature will increase up to t_1 , at any instant it may be less than this. So t_j is the temperature of the junction at any given instant t_j and m is the mass of the thermocouple junction and c is the specific heat of that material. In this situation we can write this, equating the two heats to a transmitted through surface area is equal to heat stored by the thermocouple junction. We write this equation U into T_1 minus T_j into A for a duration of dt , between the given duration this is the heat that has flown through the surface area; heat transfer coefficients, area and temperature difference.

At any instant T_j , T_1 is the final temperature of the bath. T_j is nearing T_1 , at any instant this is the difference in dt , this is what is heat flown through the surface area and stored by the mass is equal to m into c into dT_j . That is temperature increase in the junction T_{j1} to T_{j2} that we call differential dT_j and also we know the voltage output may be a voltmeter, voltage output is proportional to the T_j . So this is equation one, voltage output e is equal to the sensitivity of this thermocouple k into T_j proportional to the thermocouple junction temperature, we get the output voltage.

Now convert T_j in terms of e the voltage dT_j is equal to de by k substitute for T_j from this equation 2 and rewriting the equation, you can derive the final equation in the following form; mc by UA into de by dt plus e is equal to T_1 times k , k is our input signal that is the temperature of the bath, so this is the form. Now you find this is nothing but our time constant of the thermocouple this we denote D and make e as 1 and bringing it outside is equal to T_1 is that is k is the sensitivity, T_1 is our input signal. So we find x_o this is our x_o , this is our x_i . It is of the form x_o by, so τD plus 1 into x_o is equal to k times x_i . This is the standard form of the or x_o by x_i is equal to k by one plus τD , so this is the standard notation for any first order instrument. That is how the instrument is behaving. This is just to prove the temperature machining instruments are of the first order.

Similarly mercury in glass thermometer can be worked out and you get finally the equation of this form that is why it is a first order instrument. How the first order instrument responds to a dynamic signal, how to find out? In zero order system we didn't have any such problem because there is no time lag and proportionality is maintained always but in this first order and second order instruments, to understand the behavior of this instruments we should give some standard input. The first input is called step input.

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Handwritten mathematical derivations on a chalkboard:

$$x_i = x_s$$

$$(\tau D + 1) x_o = k x_s \quad - (1)$$

$$x_o = k x_s (1 - e^{-t/\tau}) \quad - (2)$$

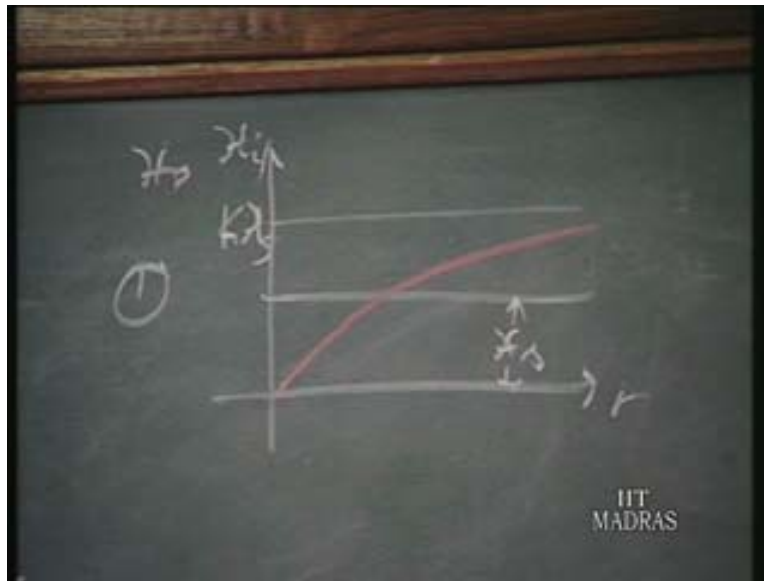
$$\frac{x_o}{k} = x_s (1 - e^{-t/\tau}) \quad - (3)$$

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That is suddenly we give that input all of a sudden and watch how the instrument is behaving then from that we know the behavior of the instrument. Suppose the x_i is equal to x_s , step input x_s is given then we put that x_i as x_s in the character equations τD plus one into x_o is equal to k times x_s . So this is differential equation where you can find out the variation for x_o or you can get an expression for x_o . Say x_o is equal to $k x_s$ into one minus e to the power of minus t over τ , this will be the variation of x_o . That is solving the differential equation you get the complimentary function, particular integral and using that formula method we can get these expression x_o is equal to this much, so to under same this equation we plot it.

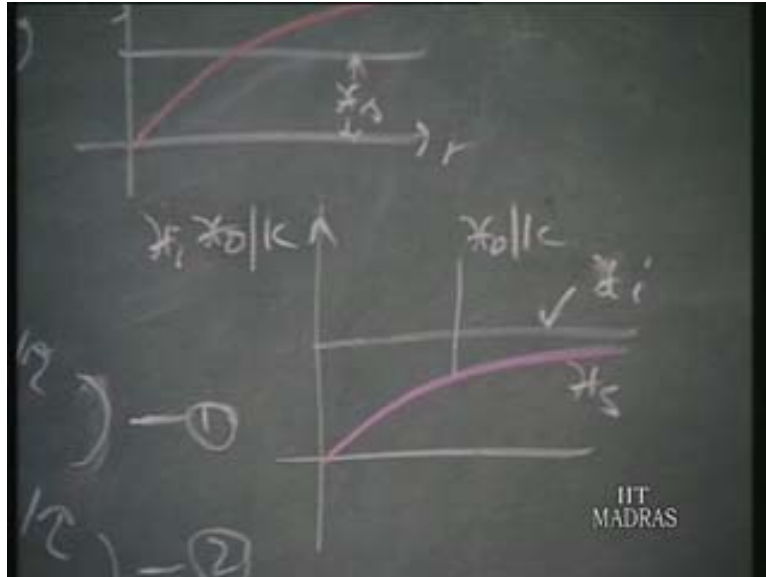
So this can be understood in three ways x_o by k , it can be written in different way. Same equation written in different ways x_s into one minus e to the power of minus t over τ and the another way you can write x_o by k by x_s is equal to one minus e to the power of minus t over τ , it is non-dimensional this is equation three. So these three ways so no bracket is required only two terms, the same equation given in terms of three different forms. Mathematically there is no sense but in measurements it represents something that x_o that is output signal itself is of the form so if you want to plot the first equation, you have to plot like this.

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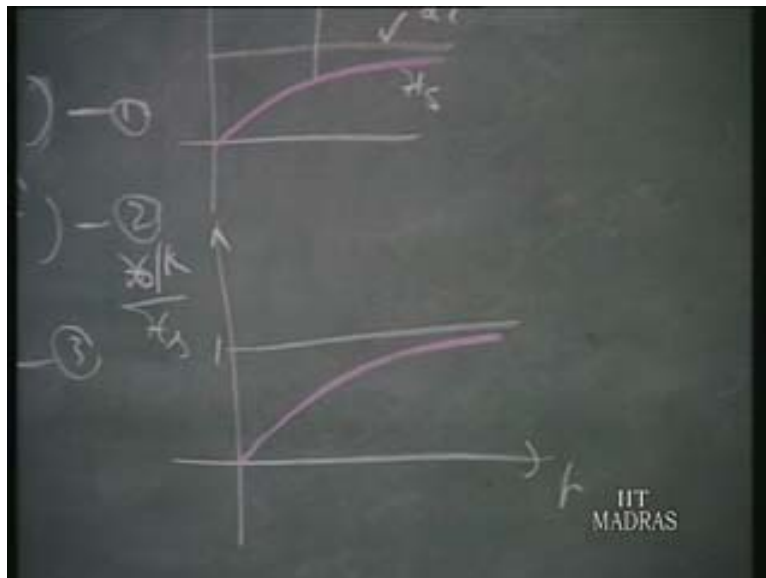
That is here x_i and x_o are plotted. Now this is our x_s constant with a time. The output signal at k times x_s around that it is going to vary that is k times x_s will be there. So the variation will be following like this; this is an exponential variation. At the time t is equal to infinity it will be k times x_s . This is the graph for the equation one and for equation two you write, this is x_o by k and x_i also you can plot. So you find this is x_s and x_o by k will be varying with x_s , when t is equal to infinity you find x_o by k is equal to x_s itself, so this is the plot for the second equations. So you find it varies x_o by k , this is our x_s or x_i .

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Similarly if we plot third equation it is non-dimensional, y axis is non-dimensional. X_0 by k by x_s then the infinite value will be one, when t is equal to infinity this ratio becomes one so this is non-dimensional one so we have to plot. So you find the same equation written in different way of mathematics, mathematically all are equal but in measurements you will find it represents different graphs. So from here what we learn suppose you take this say second one x_0 by k is equal to this one so x_0 by k should be equal to x_s or input signal.

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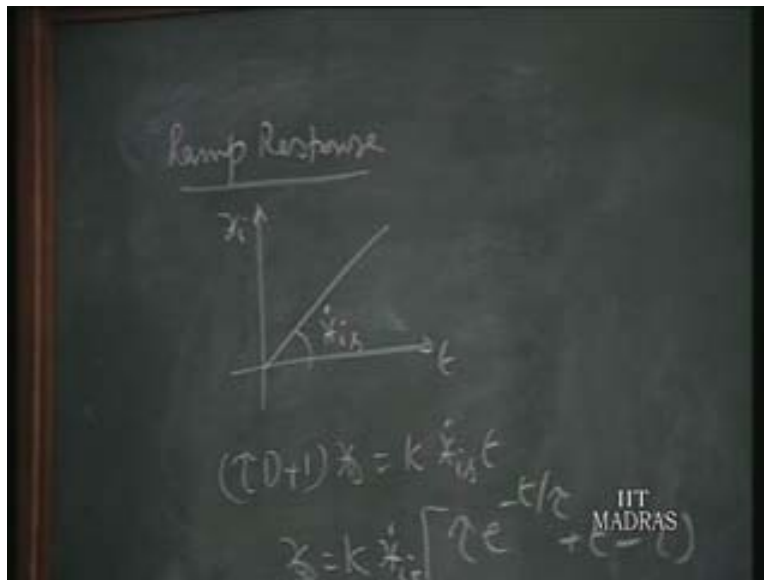


Suppose this is x_0 by k , so 0.95 times x_0 by k , it is called 5% settling or even we can represent in the non-dimensional also. So x_0 by k by x_s if you want, 0.95 times when it comes we say the error zone; this is the error zone for 5% settling time. What is the time taken is called the 5% settling time, within 5% of the final value how much time it takes to come to that value is called 5% settling time. That is called t_1 i.e. 5% settling time for this instrument may be say 10 or 15 seconds. It is equal to normally three times time constants. If you make it 0.95 you will find t over τ will be 3 that is the 5% settling time.

This is one of the specifications for the standard instruments and you will find from here we derive many other characteristics. When t is equal to τ and this x_0 by k divided by x_i will be 0.632. That is if it is 0.632 somewhere here then you will find the time is τ itself. By using the property, for any first order instruments we can find the time constant. Plot this output signal with t as x axis and 0.63 times the final value, if I draw a parallel line and where it meets the time decides the time constant of the instrument. So to measure any unknown time constant of a given instrument we can conduct the experiment and find the time constant.

This is regarding the step input; for step input the instrument will behave like this, at time infinity the output will be vary exponentially and it will reach the final value. So time infinity we cannot wait, where the instrument when it reaches 0.95 times the output value or within 5% of the final value, we say the instrument has given the correct reading. The error will be here with an error of plus or minus 0.5 and plus 8 it doesn't go it is only minus error, this is what is happening.

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Now we see the ramp response, you have seen the step response. Ramp response means at a given velocity the input signal is continuously varying say increasing.

That is we say velocity dot dx_i by dt its velocity, so this is a constant velocity so x_{is} dot is a constant slope. So x_i will be increasing as time increases so that instead of x_i being a constant value x_s here it is increasing from zero onwards. If that is the input it is called ramp input, ramp response. Ramp input, response of the instrument for this ramp input is called ramp response. Now for this we go for the equation for the first order is τD plus one into x_o is equal to k times x_i . Now x_i here is equal to x_{is} dot into t that will decide, when t is equal to zero x_i is zero so proportional to t the x_i is varying so this is the x_i term, x_{is} dot this is step velocity input suddenly at given velocity it is varying into t , time is the second.

Now this again a differential equation, you have to solve it for x_o and this is the equation for the x_o . We rewrite it so that we get the following form X_o by k minus x_i that is x_{is} dot into t is equal to, we know it is x_i that is x_i is equal to x_{is} dot into t , this is our x_i signal. The k we will bring it here then x_{is} dot and t will x_i that is brought this side as x_i and remaining terms are x_{is} dot into τ e to the power of minus t over τ minus τ into x_{is} dot so this is the final form.

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The image shows a chalkboard with handwritten mathematical derivations. At the top, there is a small sketch of a ramp function. The main equations are:

$$(\tau D + 1)x_o = k(\dot{x}_{is} t)$$

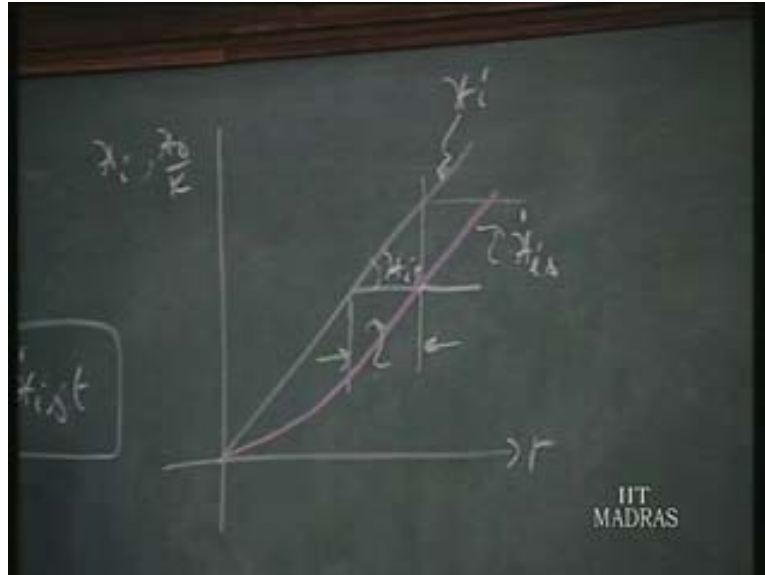
$$x_o = k \dot{x}_{is} \left[\tau e^{-t/\tau} + t - \tau \right]$$

$$\frac{x_o}{k} - x_i = \dot{x}_{is} \tau e^{-t/\tau} - \tau \dot{x}_{is}$$

The bottom equation is circled. In the bottom right corner of the chalkboard, the text "IIT MADRAS" is visible.

Now it has got some interesting shape that is x_o by k should be equal to x_i , if the instrument doesn't give any error. So if there is error x_o by k that is ideal value is subtracted from the actual value x_i then this is actually our error. This is the error at any instant x_o by k minus x_i is error which is equal to x_{is} dot into τ into e to the minus t by τ minus say this is a function depending upon the time at any instant and when t becomes infinity this terms disappears and finally it settles here. So τ into x_{is} dot, x_{is} dot velocity input signal varying by speed into second will give you the input signal dimension. So this will be the steady state error, this is transient error. Now if we plot you will have this form with a time, x_i we plot x_o by k also we plot in this y axis.

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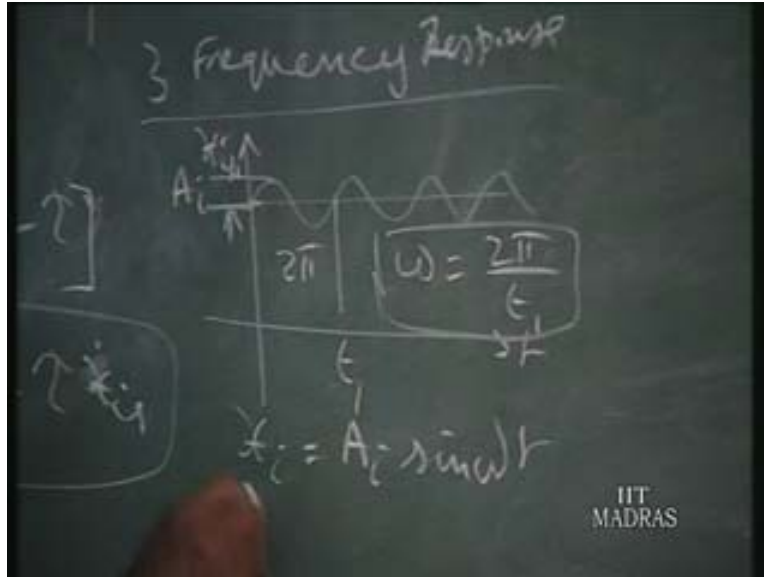


This is our x_i versus t this is our x_i , slope being x_{i_s} dot and if you plot this equation x_o by k now you find it will have this following shape. Here it is a varying one later on it settles as parallel line. So until parallel lines comes it is a transient behavior that is illustrated by this side and once it settles then you will find error is always constant, steady state error. This is the steady state error τ into x_{i_s} dot is our steady state error and the slope is always x_{i_s} and the steady state time constant is equal to τ , steady state time lag is τ that is why you have got this ordinate this side τ into x_{i_s} divided by τ will give you the slope that is how we get it. Steady state time lag is τ in first order instrument and steady state error is τ into x_{i_s} dot this is the inference what we get. That is for the ramp response. Next one is frequency response which is two, first one is step response, third is frequency Response.

That is we are giving an input signal previously we gave step as input signal, second we are giving steadily increasing as input signal, third we give a signal of this order say it may be around a value, may be varying. This may be the variations which you want to steady, suppose this is x_i and this is time, such type of signal if it is given to this instrument how the instrument is going to give the output. For that suppose if it is sinusoidal in nature you can write this signal as x_i is equal to $A_i \sin \omega t$, in this form you can write this variation that is ω is equal to, this is zero to 2π , one full cycle.

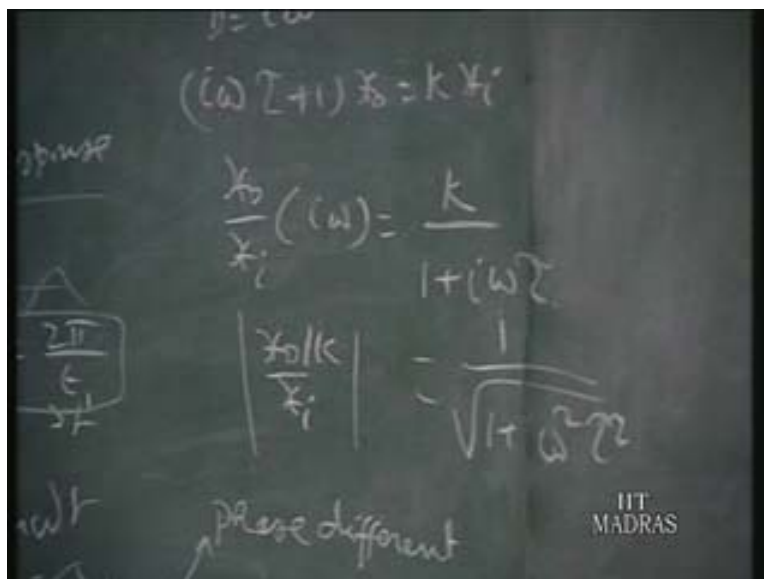
For that one full cycle t then ω in this case equal to 2π by t_1 that is your ω in this variation. So for one full cycle whatever it is taken so 2π is the one full cycle, 2π by t_1 will be the ω value. So any such variations if you can find out plot it in a recorder later on you can write that variation in terms of this input signal this is A_i amplitude of this variations, so ω is this 2π by t_1 , A_i is the amplitude of the variation.

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Then the output signal as we have seen earlier the output signal of a present transient will settle at a particular value. So that is equal to we can write in terms of these sin omega t plus pi so output will be of this nature where the A_o may not be same as A_i , it will be different from A_i . Mostly it will be different from A_i , this is a phase difference that is the output signal doesn't appear immediately. It is lagging or leading so whatever it is, some phase difference will be there for the output signal. In the output we have got two parameters A_o and phi, they have to be found out. In that case we can write that equation for the output signal. How to find this A_o and phi? For that we go back to our first order instrument equation. That is $\tau D + 1$ into x_o is equal to k times x_i .

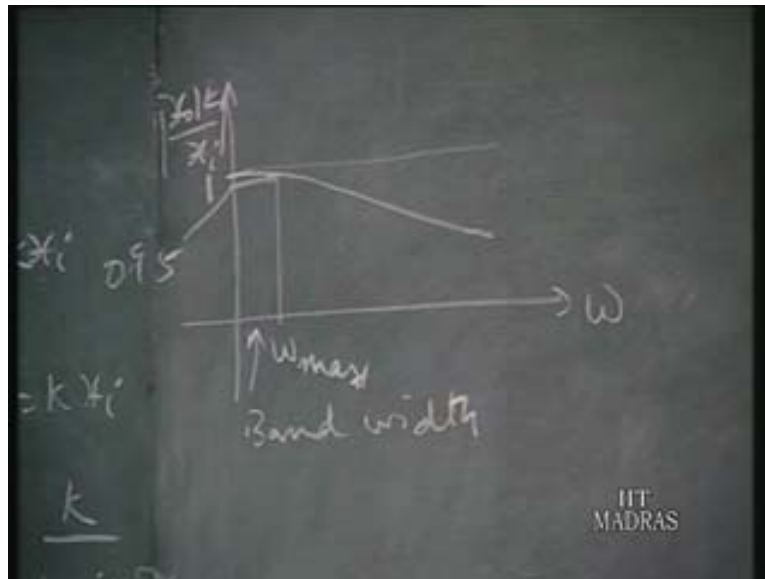
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One standard procedure is write D is equal to $i\omega$ then $i\omega\tau + 1$ into x_o is equal to k times x_i and then writing x_o by x_i in terms of $i\omega$ is equal to k by $1 + i\omega\tau$. This is a complex number, its magnitude ratio is obtained x_o by x_i we can bring k this side by k , the magnitude ratio is equal to one over where k is brought there, square root of this one plus $\omega^2\tau^2$ square. That is A_o is represented by x_o , amplitude of x_o is A_o , amplitude of x_i is A_i but here it is written in terms of x_o and x_i . So from here we write the magnitude ratio from which we get A_o or x_o by k or x_o and then phase difference equal to $\tan^{-1} \omega\tau$.

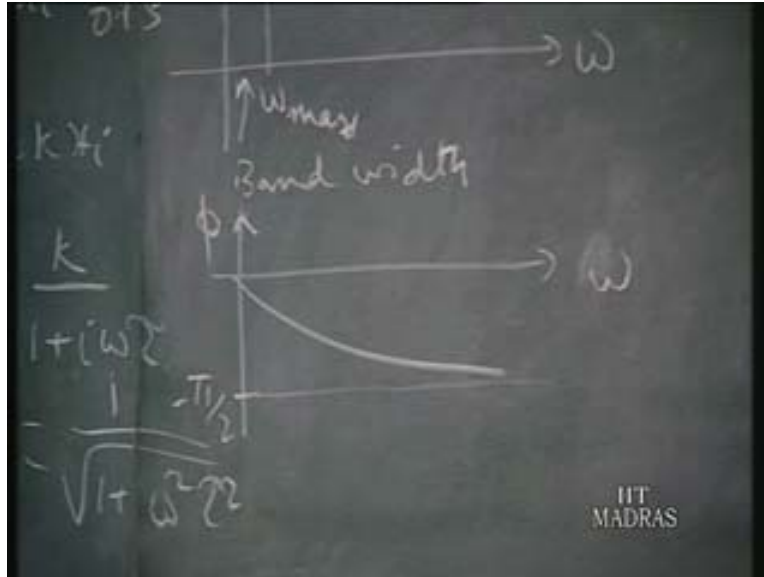
In this case it is in the denominator so ϕ is this. So we find A_o can be obtained or A_o in terms of x_o can be obtained, ϕ also is obtained from this equation. Now if we plot we have got this equation, even if you plot the frequency response that is ω frequency x axis and x_o by k by x_i mod we plot it and that is supposed to be one always but as ω increases it is looping down. This is one ideal value.

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So if error can be 0.5% error so this may be 0.95, so what is the ω_{max} . So that is how bandwidth is fixed for instrument, this is the bandwidth of the instrument. If 5% error is permitted due to dynamic signal we find there is always error, this is another error source. We have learnt many error sources, from inside instrument, outside instrument and so on but for dynamic signal that was a static signal; for random signal one more source is as frequency increases again error is increased and for 5% error ω_{max} is obtained from here.

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This is 0.95 just come here, that will be omega maximum. The phi will be, suppose phi frequency and phi, this is minus value so suppose it is minus phi by 2 so it will be varying like this. That is a phase difference will be proposed to omega, such thing is permitted. So that is the frequency response of the first order instruments.