Conduction and Radiation

Prof. C. Balaji
Department of Mechanical Engineering
Indian Institute of Technology, Madras

Module No. # 01
Lecture No. # 14
Kirchhoff Law, Absorptivity Contd…

So, in yesterday's class, we started looking at absorptivity. Absorptivity is a key radiative surface property, because in the design of solar collectors and so on, we want high absorptivity in the solar part of the spectrum that is the visible part of the spectrum. If you want a body's temperature to go up, so that you can extract the heat, it should have low emissivity in the infrared part of the spectrum. So, we defined the basic quantity of interest, as far as, absorptive is concerned and that is the spectral directional absorptivity.

Then we looked at the Kirchhoff law, which gave a relationship between the spectral directional absorptivity and the spectral directional emissivity, but I think some people were not very convinced and some lingering doubts were there. What is this Kirchhoff law? Suddenly, I did something and I proved it and it was not clear. So, the first few minutes we will go through the derivation again and it is also eight o'clock in the morning. So, people are fresh, we will understand this and then we go to the other definitions of absorptivity and then to reinforce whatever we have learnt we will work out problems.

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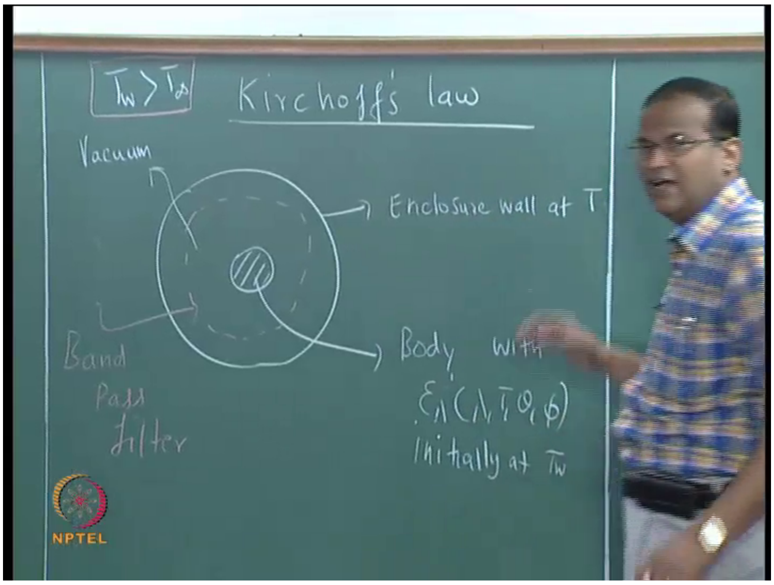

So, the configuration for proving the Kirchhoff law is like this. Before that I want to tell you that the Kirchhoff law is an experiment, is an accepted law of nature, because it can be experimentally verified. Whenever you do experiments, if you have the equipment to measure epsilon lambda dash, and if you have equipment to measure alpha lambda dash, it is found that it is always true. Now, we are trying to do some theoretical explanation, so that our understanding will improve, whether we proved it right or not experimentally. Whenever, you do it, it is always true just like the second law of thermodynamics. If you are not able to prove, then that is inability of intellect, but second law; the second law of thermodynamics is an inherent law of nature. So, it nature does not care whether you know how to prove it or not, it will be here according to its wish.

This we are trying to rationalize and internalize what is going on. Do we have an explanation of Kirchhoff law and that is why we are trying to do this. So, we come up with complicated thought experiment, like somebody kept in a large enclosure that is vacuum and it is initially at different temperature and we do all this, so that with our present level of understanding, are we able to explain? Are we able to understand this better or we able to crystallize our thoughts on this, our understanding on this better? That is the goal now.

Let us take hollow enclosure, the enclosure wall. I will speak little slowly. So, this is key concept. Let us consider a hollow enclosure. The enclosure wall is at T. This can be maintained at T by several ways. I can send cold water at the same temperature. Whatever heat is emitted by anybody in the center, can be taken care, because this body is so big and whatever body we are keeping in the center is very small or I can allow steam to condense, I can have a constant temperature bath and all that. I can maintain the temperature at some T, this T can be 30 degrees or 300 degrees whatever in Celsius.

Now, there is a vacuum inside and there is a small body which is kept inside. So, do not ask me how this small body was kept inside. You have to make hole. How do you do it? You make some hole, you put in, then close it and assume that or if you able to come out with some special machining. I mean, if you start asking questions, then we will sometime it will lead to madness, but assuming that it is possible for us to keep something and still there is a vacuum. Now, this fellow is a body, which is not a blackbody. It is body, which is not a blackbody. It has got an epsilon lambda dash; epsilon lambda dash will be a function of lambda, T, theta, phi, because this is general. This is spectral directional emissivity. So, it is not a blackbody.

Now, initially this fellow was heated T w is greater than T infinity. Now, we allow it to cool. After sufficiently a long interval of time has elapsed, this fellow will also come to T, because of the transfer of heat, this temperature will not change, because this is a small body in a large enclosure. This is capable of which standing any amount of radiation, which is coming on it and that is like our sink; the ambient.

Now, up to this, the story is clear. Now, I want to make some interventions. What I am saying is that I am at liberty to put some filters. These filters could be cellophane sheet, transparent sheets or glass sheets or Perspex sheet, whatever, and each of these sheets allows radiation to pass through only in a wavelength interval d lambda about lambda. Assuming that I have enough knowledge of engineering, it is possible for me to come out with a band pass filter, which can permit radiation in any d lambda about lambda. Are you getting it, regardless, whether it is visible, infrared or ultraviolet.

Now, the beauty of this is, if it is allowing only in a wavelength interval d lambda about lambda, this body is permitted to emit radiation only in this wavelength interval d lambda about lambda, because any other radiation, which is emitted by this body, which is outside this d lambda, will be reflected by this fellow and it will come back again to this fellow. So, radiation only in the wavelength interval d lambda about lambda can escape and hit the surroundings.

And this is inevitable; because the body is at temperature greater than zero; T it is not 0 Kelvin. So, it will continue to emit at the same time, the body is also in equilibrium with the surrounding. Therefore, it must absorb the same amount of radiation upon equilibrium, whatever is emitted must be equal to whatever is absorbed. Therefore, if you apply the first law of thermodynamics, for this body, which is in equilibrium with surroundings the following thing, holds.

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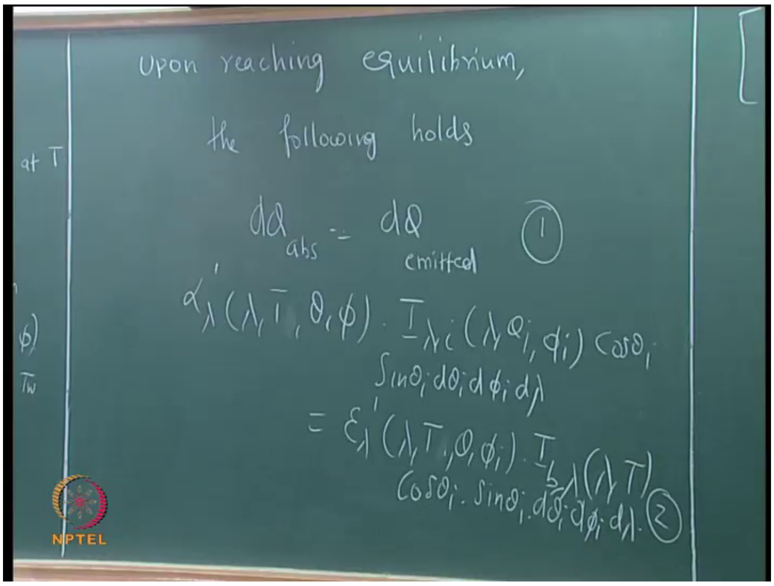

This exactly the same as what we discussed in yesterday’s class. Q net is equal to what is going minus what is coming. So, what is Q net? For this body, what is Q net upon reaching equilibrium? Because, if Q net is nonzero, its temperature has to decrease or increase with time.

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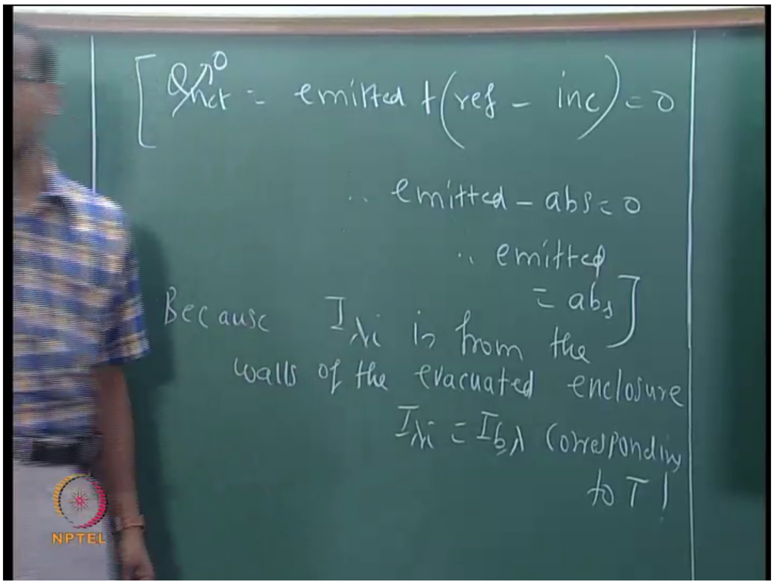

Therefore, emitted plus reflected minus incident equal to what is incident minus reflected, if there is no transmission. This you can put within brackets that is to help you understanding that is all. If some people got a doubt yesterday, when suddenly, I said emitted is equal to absorbed and that is the explanation.

Now, if this equation… are we putting equation one? Is there any number for this? Did we use numbers yesterday? One, now, will have to write expression for absorb and emitted. What about absorbed? Now, apart from deciding that it will be only in a wavelength interval d lambda about lambda, I can also decide about the direction. I will cut out the radiation emitted in other direction; I can allow the band pass filter to allow radiation in one direction only; all that I can control in to, in to, Now, I use only T for this. Now, for epsilon lambda dash, I should not put I lambda I, epsilon lambda dash is multiplied by I b lambda, corresponding to what and which temperature? It is T.

I do not have to put theta and p because I b lambda is independent of theta p, but cos theta and other things will come you can put this is as equation number three and put this as equation number four, if you feel that will make you understand better. For a great diffused surface from the hemi this is not correct. So, there is unit vector. So, there is an elemental surface dA and considering the hemispherical space above the d A. So, radiation from the hemispherical space is coming on to this object and falling on this object. So, it is coming like this.

What is the total radiation, which is coming from the hemispherical space, which is falling on this object, in a particular wavelength interval, divided by… no no that is a denominator. What is the fraction or what is the fraction of total radiation from the hemispherical space above it, in this hemispherical solid angle at a particular wavelength interval, which is absorbed by the body, rather from the hemispherical space what is absorbed divided by whatever is incident. So, that is the goal of defining this, and we always assume that once you go to quantities, which are integrated version of the primary quantity of interest, I assume that alpha lambda dash is, lambda dash is known. So, given the information of alpha lambda dash, how do you calculate alpha lambda alpha dash and finally, alpha. That is the logical way and this way the story progresses.

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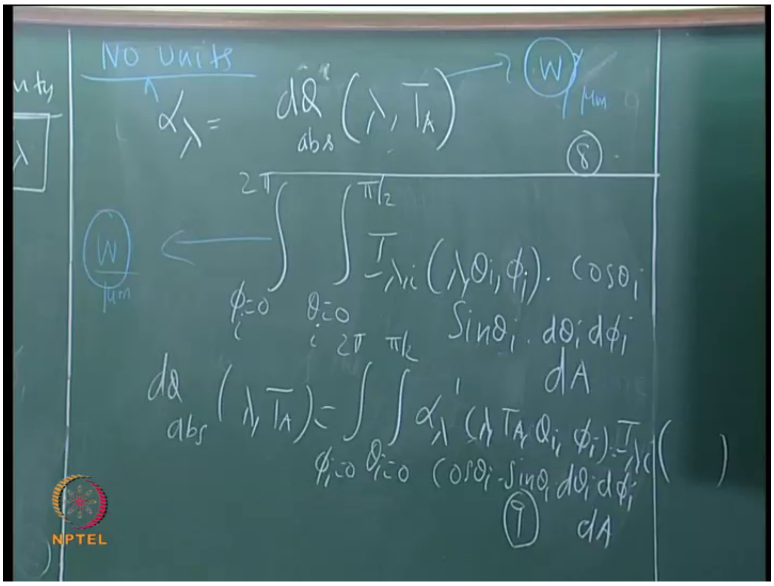

Now, if you want to write mathematical expression for this what should be alpha lambda? The d Q absorbed correct should be a function of lambda and should be a function of the temperature of the absorbing surface; theta, phi i, are removed because we are integrating over the hemisphere, divided by … I can use the notation, phi i 2, i subscript denotes the incident direction. Can you fill up the denominator, I lambda I….

The incident radiation could be a function of lambda and it could be a function of direction. I do not have to write the temperature if I know what is I lambda I is. Only in the case of incident radiation from sun, I need to write the temperature, because if I know that temperature of the sun, I can assume it to be a blackbody. Otherwise, I know that somebody will give me I lambda I, somebody will give me how I lambda I varies. Is that clear? Now, multiplied by cos theta i, multiplied by d A, because I want the numerator to be in watts.

Lambda (( )) No, that is correct not correct Yeah, phi I and then is it in watts? (( )) So, it should be watts per micrometer then. What is the denominator? Is it watts per meter square per micrometer steradian? No, I can multiply by or I can multiply by whatever. So, I do not want to confuse you for that. Now, what are the units you tell me? Yeah, you write and it is also no problem.

Even if you multiplied by d lambda in the numerator and denominator, I have no problem, because d lambda can we pull out of the integral, and d lambda, d lambda can get cancel. We should not get in to such type of these difficulties. All right, that d is also outside integral. So, it is depends on my liberty to define d Q, I can call it d Q or d Q dash d whatever.

Now, I should write an expression for… now, what is this? Is it three, four, no yesterday we did lot of things know. 4, 5, 6, 7, have we written? This was seven then we will keep it as seven. So, we will call this as eight. Now, tell me d Q absorbed, alpha lambda dash in to I lambda I; and put that bracket. What you want to call this? Equation nine; so, what is exactly the departure from what I taught you yesterday? Yesterday, I gave you the final answer and now I am writing the intermediate step. What is the basic definition of alpha lambda numerator by denominator? Denominator, everybody knows. For, numerator, I wrote it as equation nine and I am substituting for the numerator in the equation eight.

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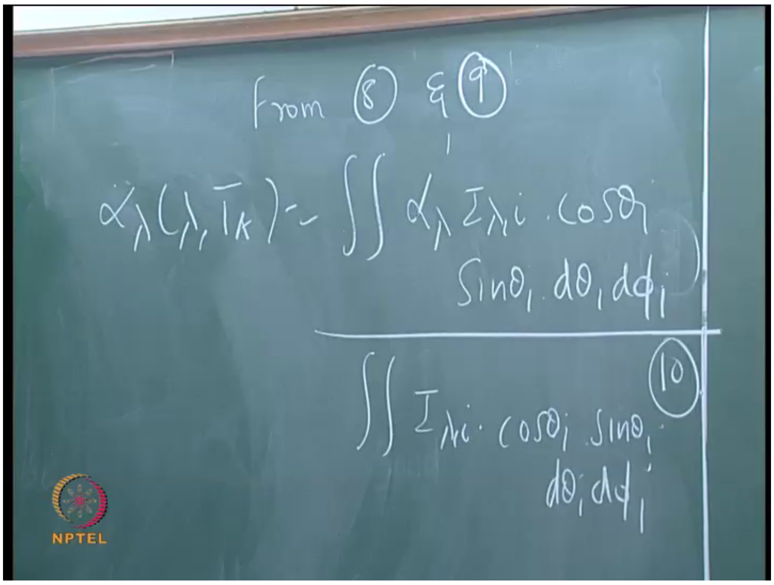

So, from eight and nine, once I show it for hemispherical spectral, the other two quantities, I will directly give you. I will not do this detail derivation. Is it ok? So, alpha lambda absorptivity total, but directional total, correct.

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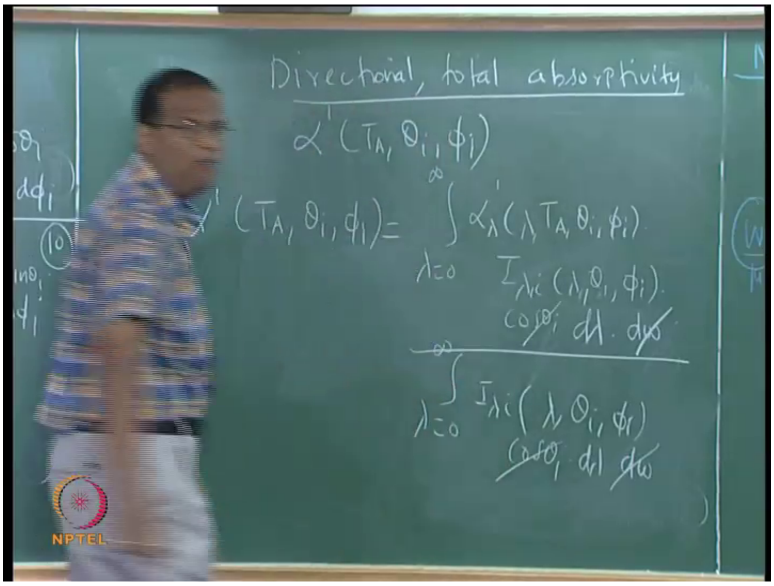

So this I lambda I watts per meter square micrometer. I lambda I is so high. 40000, I will make take it as 8000, 6000, but 40000 may be too much. I have no problem. Somebody wants 40000; we will take 40000. Let see what happens. So, if you do not get the right answer then we would catch hold of Vikram. He suggested 40000. Now, take down the problem.

The spectral hemispherical absorptivity of an opaque surface and the spectral distribution of incident radiation are given in the figure. (Refer Slide Time: 38:31)

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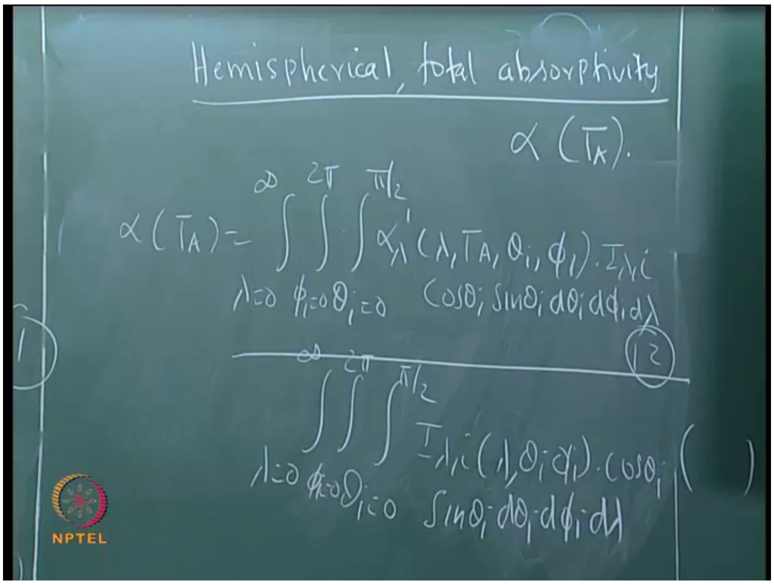

A: What is a hemispherical total absorptivity of this surface? Two, if this surface is diffused, I gave A or one, A, sorry, B. If this surface is diffused and is at 1000 Kelvin. What is the question? What is the amount of radiation emitted? Find the emitted radiation. No, before that, emissivity. What its total hemispherical emissivity? If the surface is diffused and it is at 1000 Kelvin, what is it hemispherical total emissivity? This is part B.

Part C; will please tell me what should be part c? What is the net radiation heat transfer from the surface? That is outgoing minus incoming; Part C is what a net radiation heat transfer from the surface is? First thing, I will calculate the denominator. Why? So, alpha but still one or two people always worrying. Sir, what happen to the sin theta coos theta d 'theta. So, for those doubting Thomas's, I will, we can write the numerator denominator and cut it out if you want be clinically correct. Now, denominator is very easy for us to calculate.

Vikram, what is the answer? (( ))

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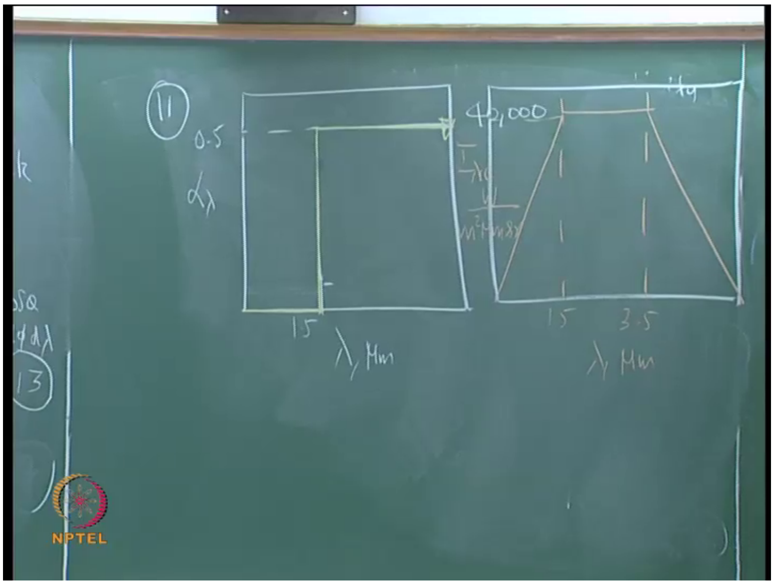

Area under the curve is 1.5 in to 40000 plus 80000, plus, I did not give you this value. What you take it as? You just took it five and that is good, it is zero. So, this is again same; 30000. What is this? 140000; it is correct. Vikram, I do not even know, we are given 1000 Kelvin. I do not know how it is going to come up? Let see now, alpha is equal to, what about the numerator?

So, this is no, yes, it is correct, 0.5 to 0.5 and then no no it is correct, 3.5 to 5 what is the… again 0.5, you are not getting more than one. Right, we should not get dout, 0.5 in to what is this guy? 80000, good plus 0.5 to 30000, just 30000, is it too bad or 0.5 this 0.5. (( ))

No, this 30000 is correct

Student: Into 0.4

Into 0.4 (( ))

Student: Into 0.5, into 0 .5.

Vikram, is it ok? no you are not convinced, but, 30000 is the area under the curve, but, you should multiplied by the absorptivity 0.5. Now, you got 55000 divided by 140, what is the answer?

Student: (( ))

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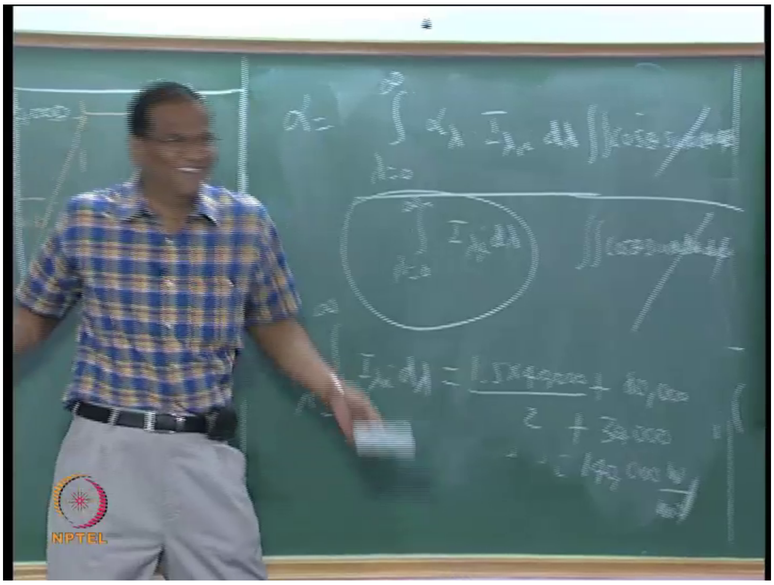

Vikram, suppose we did not had have, or had we not had this; is it correct English? I do not know. Suppose, we did not to have this, then after all these I will get 40000 plus 30000 due to 140000 just 0.5. What a silly answer is because this started with 0.5; alpha equal to 0.39. What should be the reflectivity of this surface? Now, what should be the reflectivity of this surface? 0.61, the hemispherical total reflectivity of this surface will be 0.61. Everybody is clear with this. Now, we have to go, so part A is over. A part is over. Now, B part, just 45 seconds.

You have to use a f function chart. I think I will stop here. Please, use the f function chart and what is the emissivity we are getting? 0.49, so it is got emissivity about 0.5, absorptivity is point 0.39 and then you have the net radiation flux. What is it? Plus or minus, it is losing heat. It is emitting faster than. So, we will work out all these numbers in the next class. So, now, we have a good idea. If there is single surface how to handle like that, there may be many surfaces; each may be having its own crazy behavior. Then you have to integrate and with the respect to angle and lambda and then put all these fellows together. Eventually, the radiation problem will become a problem of solution of linear equations, where there, may be many surfaces, you have to invert the matrix or gauss-sidle method or something, but what is the engine, which is driving all this, sigma T to the power of four, spectral emissivity, spectral absorptivity and all. This is the basis for the whole thing. Eventually, what you get as equation they have to flow the information has to flow from this. So, we will stop here.