Conduction and Radiation
Prof. C. Balaji
Department of Mechanical Engineering
Indian Institute of Technology, Madras

Module No. # 01
Lecture No. # 15
Problems on Emissivity, Absorptivity

So, we will continue with our discussion on emissivity and absorptivity before going on to reflectivity and transmissivity I thought it will be appropriate to solve some more problems. So, that we cover wide gamut of problems involving emissivity and absorptivity, so that it reinforces whatever concepts and formulae we have understood and derived in the earlier classes.

Now, I am as you can see if you look at these problems they are graded in complexity and various permutations and combination, I am trying to cover. I will give you I lambda I and then I will give you distribution for alpha lambda and ask you to get the alpha. I may give you the, say that irradiations from sun, and I will give you epsilon lambda. From epsilon lambda you have to get alpha lambda or in this problem one more variant or variation is, I gave rho lambda and say it is opaque then using the relation from rho lambda, one minus rho lambda is alpha lambda you can you can proceed like this.

So, various permutations and combinations, we will do and then the we will work on three problems in today's class that is the program, no lecturing, three problems, but three problems I will also solve along with you. So, this will, because tomorrow you are having the quiz, so this will also be helpful. And the last problem we will have combine radiation and convection problem, some energy balance, how to handle such type of problems. After the quiz, we will start with reflectivity and transmissivity. Once we are done with that, we have to start with the view factors. How the geometric orientations affect the radiation heat transfer between the surfaces? We are not worried so far, because we are always considering one surface.

Now, this one surface is basically to help you understand better, but one surface is such a rarity. It is impossible, virtually impossible to adjust one surface. Sometimes, it is possible, a small surface in large surrounding, but generally there are multiple surfaces. Each of these surfaces, will have its temperature epsilon lambda, alpha lambda, each surface will have its own horoscope, history everything. So, you have to figure out all these properties. Look at the geometric orientation. When everything is acting; radiation is acting at multiple radiations, is being emitted reflected and absorbed at multiple surfaces and we need a methodology to handle this.

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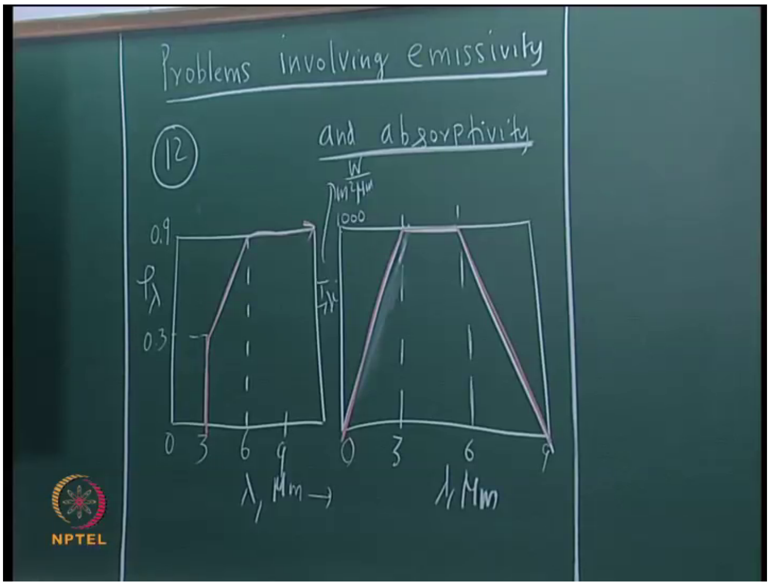

So, now this problem is like this. Problem number 12; takes down this figure, you are able to see. So, rho lambda is given here. What is rho? Lambda means? What about the dash? (( )). What happened we have not knocked out the dash; that means, it is Hemispherical (( ))

So, this is hemispherical spectral reflectivity. So, when I give it in English I will say the hemispherical spectral reflectivity of surface, I will do that. Before that I will just explain this. So, this hemispherical spectral reflectivity, arrow mark is there, do not miss the arrow mark; that means, it is continuing to have the same rho alpha, sorry rho lambda with lambda.

Now, the I lambda I, the incident radiation in watts per meter square per micrometer. The scale is 0 to 1000, it is given like this. It has a distribution like this; 0 to 3 to 6 and 6 to 9 and it is 0 in the other parts of the spectrum, because no information is there that mean it is 0 in the other parts of the spectrum. Shall we take down the problem?

An opaque surface has a hemispherical spectral reflectivity, as shown in the figure full stop. it is subjected to a spectral radiation It is subjected to a spectral irradiation, also shown in the figure, full stop, A: sketch the spectral hemispherical absorptivity distribution, B: determine the total irradiation on the surface, C: determine the radiant flux that is absorbed by the surface determine, D: hence determine the total hemispherical absorptivity of the surface. You can write it in telegraphic language, hence determine alpha, and hence determine the total hemispherical absorptivity on the surface.

Please start solving, I will also solve along with you. I lag you by two three minutes. So, that I do not rob you of the expectancy. You just start solving. Shall we draw it first?

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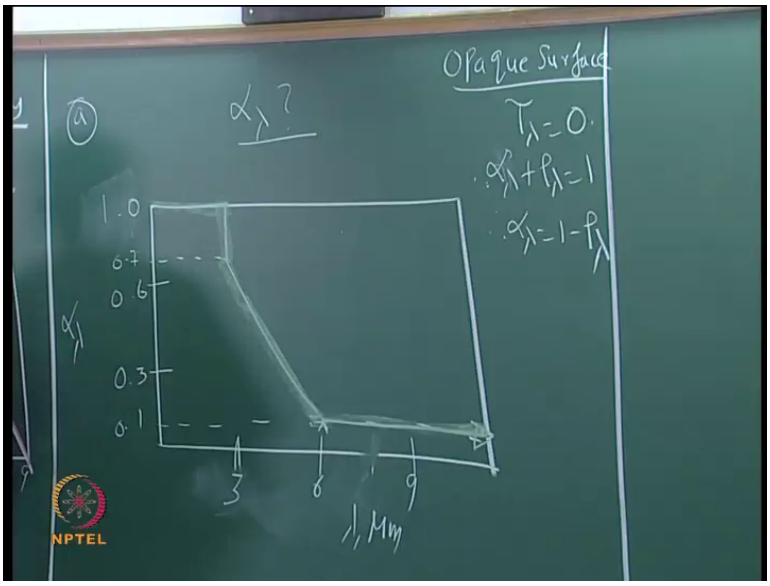

Let us do the part A; alpha lambda dash, correct. So, the solution is like this, opaque surface, opaque surface tau lambda, there is no transmissivity. Nothing comes out of the surface. If there is a mirror image of this, so 0 to 3, correct, no. Use yellow chalk or green, then it drops to 0.7, then at 6, it becomes then straight. Do not get confused, it is not having alpha lambda with a band like this. Just for depiction. That is not the uncertainty associated with that.

There may be very well an uncertainty associated with this, with that and uncertainty can also be estimated using some techniques. Let us not worry about that. So, first part of the story over. So, it is got. So, which means now given epsilon lambda, alpha lambda, rho lambda, any of these three quantities, as long as it is opaque, you can play with that, you can change one to another and then you can start. If adequate data is given you can proceed with the calculations, correct.

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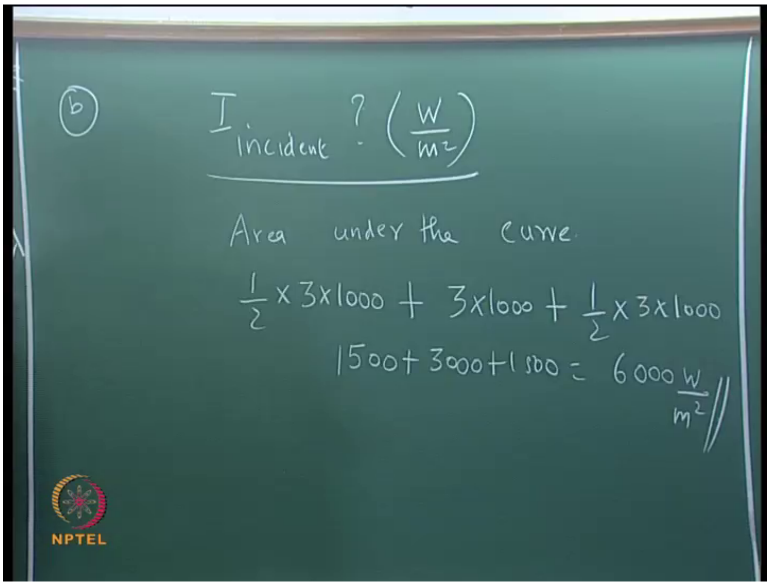

ⓑ $I_{incident}$? $\left(\frac{W}{m^2}\right)$

Area under the curve

$\frac{1}{2}\times 3\times 1000 + 3\times 1000 + \frac{1}{2}\times 3\times 1000$

$1500 + 3000 + 1500 = 6000\ \frac{W}{m^2}$ //

Now, part B so half of plus correct fine.

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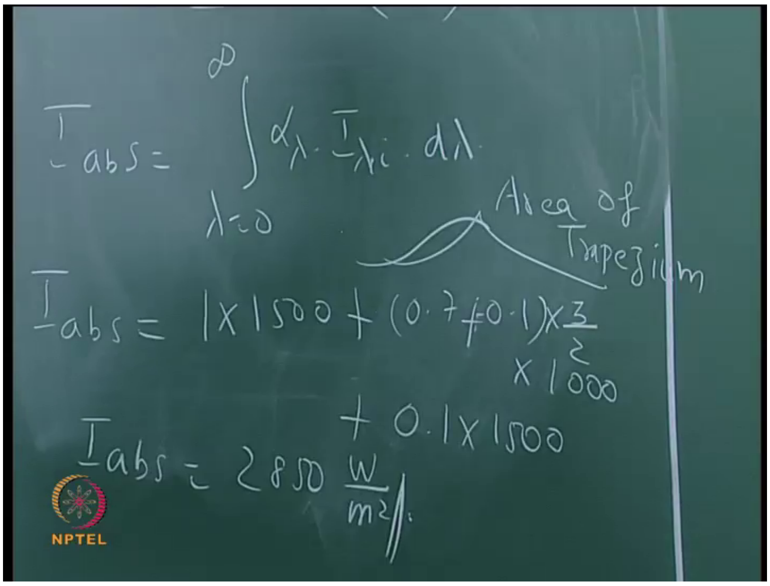

$$I_{abs} = \int_{\lambda=0}^{\infty} \alpha_\lambda . I_{\lambda i} . d\lambda$$

Area of Trapezium

$$I_{abs} = 1\times 1500 + (0.7+0.1)\times\frac{3}{2}\times 1000 + 0.1\times 1500$$

$$I_{abs} = 2850\ \frac{W}{m^2}$$ //

Now, we go to part C. let us keep this. What is part C? I absorb; what is I absorb? Then d, I will just divide I absorb by I incident that is all. Systematically, we have without the individual parts, quickly you can do, but this is in stages, you will do it. So, you can get also get step marks. So, I absorb, please tell me correct (( )) its fine. So, integral alpha lambda, I lambda, d lambda; alpha lambda we got from part A, already, right. So, I absorb will be (Problem discussion: 12:39-13:23). First, is what? 1 into plus 0.7 or 0.7 or what did we get 0.7 (( )) 0.7 into 3000 plus 0.1.

Student: Sir, but lambda varies; alpha lambda varies. (( )) (( )) no 0.7. Sir, 0.1 Area of trapezium what is it 0.7 plus 0.1 into thousand.

All of you got this. That is the area of the trapezium. Then what happen? You did not get. It is 0.7 plus 0.1; not into, sorry. All of you got this. This is because it is varying. Now, I can complicate matters by having a varying alpha and the varying I lambda I then you will have to do the double integration, but I will not do such things. Wherever, this is varying that is not varying, did you see that. So, the problem is carefully chosen. Otherwise, some additional pain is there. It is not that it cannot be done it can be done, but some there is a scope for making mistakes and all. But, the goal in any exam is not to find out what you do not know. If sometimes it becomes a goal, but generally the goal should be to find out what you know. Right, plus the last fellow is 0.1 into 1500, finally, if you do all these integration and you get an answer more than 6000, then we have to stop and go home, because incident itself is 6000. So, then I am happy, 28 very good. So, you are getting some decent value, 0.4, something fine.

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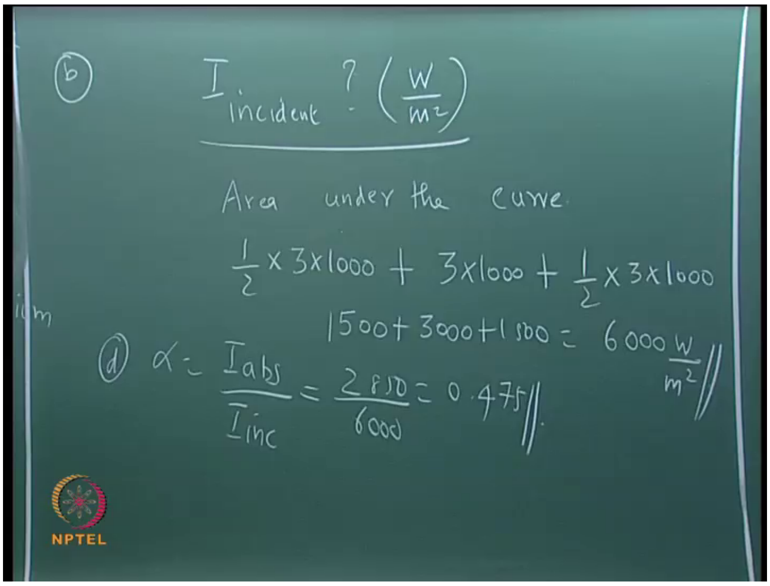

ⓑ $I_{incident}$? $\left(\frac{W}{m^2}\right)$

Area under the curve

$\frac{1}{2} \times 3 \times 1000 + 3 \times 1000 + \frac{1}{2} \times 3 \times 1000$

$1500 + 3000 + 1500 = 6000 \frac{W}{m^2}$ //

ⓓ $\alpha = \frac{I_{abs}}{I_{inc}} = \frac{2850}{6000} = 0.475$ //

You need to specify the temperature of that surface, because emissivity is the function of its temperature. But, if you want to calculate the alpha, one of the two things I have to tell; either I should tell you that it is from a gray body or a blackbody at some temperature or I should give you some story like this.

So, you must understand what is the critical information, which is required to determine the property? For alpha you need either this or some specification that irradiation is from a blackbody at some temperature. Emissivity, even if I give rho lambda, you can do all these conversion. From rho, you know that story, from rho you can get alpha, from alpha you can get epsilon, but you will get stuck up, because you have to use f functions chart. For that you need that lambda into T. What is that T? It is not a temperature of the source, which is giving radiations temperature of the surface itself.

Now, let us do a problem. Yes, (( )) micrometer. I lambda I lambda No, I it was taken care. E lambda not I lambda. Now, if you want you can take it as E lambda, it does not matter. He is saying that whether we should multiply by pi. So, what did I do? Now, I have taken the units of I lambda to be watts per meter square. Otherwise, you can keep it as E lambda, but we generally do not do E lambda. So, he has a good point. Suppose, I may just quietly put some steradian, then you are you are in trouble, but then I should tell you that it is diffuse and all that right. So, otherwise with these units, it is no problem watts per meter square per micrometer into D lambda, the micrometer and micrometer gets canceled. Now, the problem is slightly more complicated than this. The problem number 13 is…

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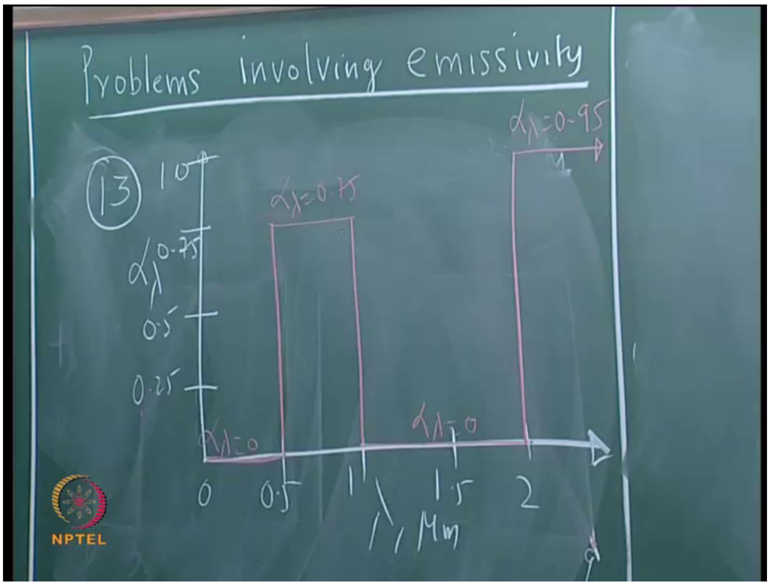

So, now, I straightaway give you alpha lambda for a surface; alpha lambda versus lambda, 0.5, 1, 1.5, 2. Lambda is in micrometer, 0.25, 0.5, 0.75, and 1. Zero Watch very carefully, do not start integrating and wait; is 1200 watts per meter square, you cannot equally distribute it around lambda. So, do not think that it is a very simple and straightforward. 1200 integral alpha lambda; it would not work. I have given you a solar. So, calculate the solar absorptive of the surface. Assume this outer surface of the sun to be a blackbody at 5800, use the f function chart, convert alpha lambda to epsilon lambda. As though, you determining epsilon lambda, epsilon for this surface, first calculate the solar absorptivity.

Once, you calculate the solar absorptivity that is alpha s, alpha s into 1200 watts per meter square is how much it is absorbing from the sun. Now, you have to again use the f function chart with 120 degree centigrade and calculate the epsilon, and then epsilon sigma three to the power of four, will be the emissive power. Then the third part will be this. So, it is not easy. When I give solar absorptivity, 15 minutes will take for you to solve the problem. Are you able to follow? (( )) At 1200 watts per square meter is integrated, it does not help me to get the alpha s, because if you integrate that solar I have to get 1365, because of the clouds and the other things, I am able to get only one 1200. So, the golden rule is once I say sun or solar radiation somewhere, immediately use the f function chart, assume 5800 and get the alpha.

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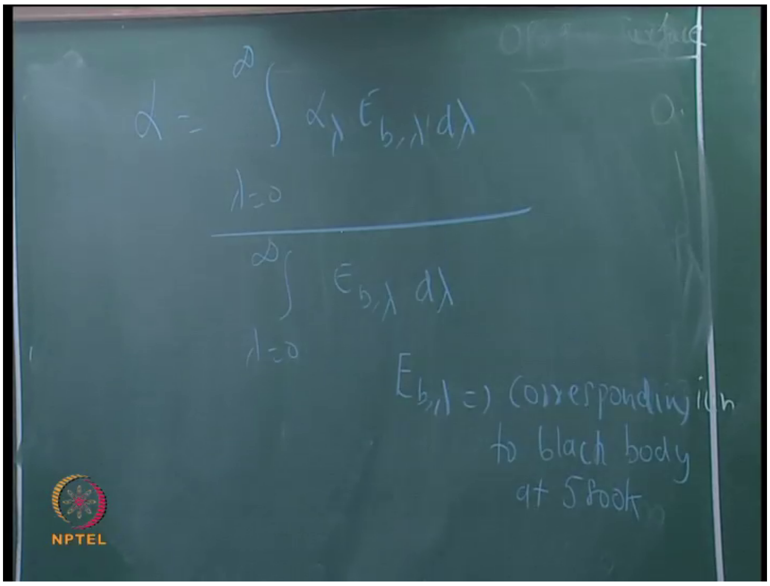

$$\alpha = \frac{\int_{\lambda=0}^{\infty} \alpha_\lambda E_{b,\lambda}\, d\lambda}{\int_{\lambda=0}^{\infty} E_{b,\lambda}\, d\lambda}$$

$E_{b,\lambda} \Rightarrow$ corresponding to black body at 5800k

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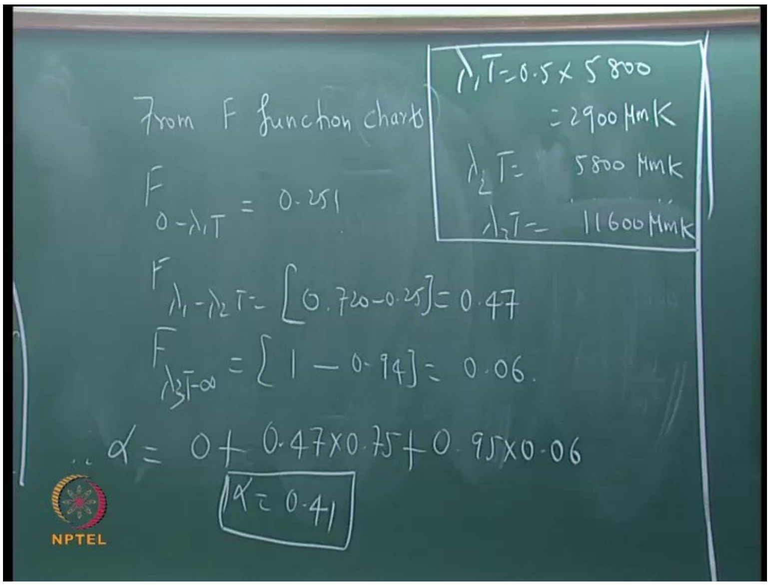

From F function charts

$F_{0-\lambda_1 T} = 0.251$

$F_{\lambda_1-\lambda_2 T} = [0.720 - 0.25] = 0.47$

$F_{\lambda_3 T-\infty} = [1 - 0.94] = 0.06$

$\therefore \alpha = 0 + 0.47 \times 0.75 + 0.95 \times 0.06$

$\boxed{\alpha = 0.41}$

$\lambda_1 T = 0.5 \times 5800$
$= 2900\ \mu mK$
$\lambda_2 T = 5800\ \mu mK$
$\lambda_3 T = 11600\ \mu mK$

So, alpha is, Since, nothing is specified about the variation of alpha lambda or epsilon lambda with temperature, I am assuming that the same characteristic hold good for all the temperature. That assumption you can write. In tomorrow's paper also you can write, I mean make suitable assumptions, wherever required. Justification, in the absent of any other data, I got only alpha lambda data, I will use it as epsilon lambda data and I am assuming that it is valid for all the temperature you can write that. Now, can you do this? So, alpha, this is very important. It is very important point (Refer Slide Time: 27:18). Now, I have to get lambda two. Now, please tell me f of 0 to lambda, you just keep in the margin first, through this.

One is also there, (( )) correct, F of 0 to lambda one is 0.25, f of lambda one to lambda two is something minus 0.25. What is it? No, you have already done minus; tell me before subtracting, minus 0.25, 0.720. How much is this? 0.47, f of lambda 2 to infinity, lambda one lambda three to infinity, one minus, it is not 0.72. It is 0.9394. Therefore, f of 0 to lambda one is 0 plus and how much is it. (( )) 0.41.

300 and 400 and very good. Unfortunately, the sun distribution is not like a rectangular trapezium or triangle that follows which law? That follows Planck's law. So, we cannot integrate. The previous problem we just put area of the trapezium area, you cannot integrate, because people could not integrate, what they figured out? F function chart; that is why that integral between lambda one to lambda two is given as f function chart. So, that connection, whenever you study of that connection; it should not for some problem use f function chart some problem I do integrate f function chart is also integration, you cannot do this simple integration.

Therefore, somebody did it and put it in the form of a table. Now, the next part of the story is emissivity. Part a is over, part b, fortunately for us, this is, but lambda one is changing, lambda T and all will change; T is now. I will put T s, which is temperature of the sun. Now, this is T. What I did say? 120 degree centigrade is 393 Kelvin. If somebody does not like T, you can put T w temperature of wall or temperature of surface T s, whichever way. I will just use T. Lambda one T, I think all are 0; right all are 0. Last one is having something. Now, f of 0 to lambda one T is 0; f of lambda one to lambda two is 10 to minus (( )) Emissivity of the surface is absolute 0. What is the emissivity of the surface finally?

(( )) , 0.94 , (( )) 1 minus , Oh 1 minus. So the emissivity of the surface is 0.95. That is what it should be, because the surface temperature is 393. If you do the Wien displacement law; lambda max T is 2898 micrometer Kelvin; 2898 micrometer Kelvin divided by 393 or 400 will give you a value of seven micrometer. So, it has a peak around seven micrometers. Now, I am saying after two there is no change. It will be 0.95. Therefore, even without using f functions chart from experience I can say, all this is irrelevant. So, it will have value, which is close to this. The only effect of this, all this is to create the stupid 1.64 into ten to the power of minus five. Now, I think we are getting more comfortable to evaluate this; in evaluating the properties, fine.

Now, we know the emissivity also. So, what is the net radiation? I asked emitted power? So, E tell me; you want to put it in watts per square meter or watt. I gave you area also, anyway it does not matter.

(( )) Put a (( )) Then I will put (( )) 91.0 I should put E or Q, Ok E, So, 2891.0 2.8 kilowatts 2.89 kilowatts. Some people who are not been following or you busy with doing your own stuff. I gave you the area in this problem, which is 1.5 into1.5. So, you can put the land and convert the watts per meter square to watts and keep it like that.

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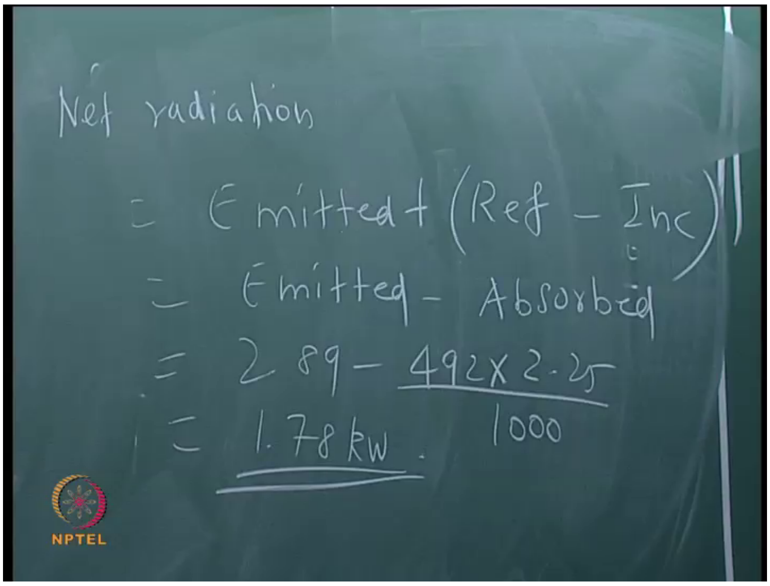

Net radiation

$= \text{Emitted} + (\text{Ref} - \text{Inc})$

$= \text{Emitted} - \text{Absorbed}$

$= 2.89 - \dfrac{492 \times 2.25}{1000}$

$= \underline{1.78 \text{ kW}}.$

Now, part C, the net emitted plus reflected minus incident; why? Emitted plus reflected is what is going out minus what is coming in, but what is reflected minus incident, rather than incident minus reflected. Therefore, this is also emitted minus absorbed. Is it correct? Now, tell me, emitted is 2.89 minus I did so many things here like 4 into watts per meter square multiplied by the area 1.5 to 1.5 divided by 1000, because this fellow is in kilowatt. The answer is 1.78. Determine the steady state temperature of the plate.

Back sided insulated. Please, look at the board; back sided is insulated. Here, there is air, which is flowing over it. T infinity is 300 Kelvin. So, let the temperature be T w. So, it is emitted plus reflected. In this case, since T infinity is also there, you can assume that T infinity to be a blackbody. Therefore, net radiation or you can say that what will be the energy equation for this. What is going out?

Student: (( ))

What is going out minus what is coming in must be equal to 0; net emission plus convection equal to incident. So, net emission, please note that net emission is not just epsilon sigma T to the power of 4. It is epsilon sigma T to the power of four minus T infinity to the power of 4. Vikram had a point that how I put epsilon into the T infinity. As I told you, please accept this as a correct formula. We will derive this formula in later part of the course. Net emission will be epsilon sigma T to the power of four minus T infinity. Is that correct? (( )) which one? (( ))

No, (( )) net emission. What is going out? No, plus reflection. Correct, is it correct? Do not need the reflection; plus rho times I incident plus h to T minus T infinity is equal to I incident. Please, look at this, right side, you got incident radiation, minus rho incident, then one minus rho will become alpha. So, right side will be net absorb that is also another formula.

So, you can say that net emission plus convection equal to net absorb that is more intuitive. Now, epsilon I gave you. What are the values? It is 0.1, solar absorptivity is 0.9. So, right side is 0.9 into 950. So, how much is that? (( )) It is a non-linear equation, which has to be solved only by iteration. It is called transcendental equation because there is a T to the power of four. You never thought that it will lead iteration; you thought very well now. So, it has to be solved by Newton Raphson method or method of successive substitution.

Let us assume that we are going to solve it using the method of successive substitution. Therefore, the algorithm is written like this. Please, look at the board. I will say T of I plus one equal to all this right side is T I. So, I will assume that since outside is 300, I will start with the temperature of 320. I will put 300 to the power of four minus three hundred to the power of four, and get T I plus one. If T I is plus one is also 320, I can go home, but it is very unlikely that it will be 320, it will keep on and the non-linear correction will take place, till you hit the right answer. Why do not we you start with 320? Let us do two iteration and then I will leave it. Are you getting the point? People, who have taken a optimization course, say will know that two information through diagrams are possible. From T to the power of four, you can get T or you can either use the T to the power of four minus T infinity to the power of four on the left hand side. Or T minus T infinity on the left hand side of the equation. One of these things will cup, because the T to the power of four is so non-linear that we will drive it crazy. So, I am using this because I think this algorithm will work right. So, here also I can put 300 to the power of four. I hope all of you are through with the idea. This problem needs iteration. So, it is called a transcendental equation, with from T 2 get T 3, from T 3 get T4, from T4 get T5, till T5, T n or T I n T I plus one are sufficiently close to each other.

Student: (( ))

385, very good. So, how much iteration? After three iteration, it is believable if you have flat plate collector you can get around 110 or 120 degree centigrade, but we did some simplifying assumption on the backside it is insulated all that. Then you have to put cork, wool, mineral wool and all that. So, some laws will take place on underside also and then this 900 watts per meter square, it is not available for 24 hours a day; it is available only between morning six o'clock and evening six o'clock and this 950 may be available only at twelve o'clock and morning eight o'clock, then I have to integrate it.

It may vary from 0 to 900 and then you have to integrate. All this may give a realistic temperature of 80 degrees in Rajasthan. On hot summer after noon, you may get 90 degrees. So, people want to work more, they are concentrated either in a tube or this thing concentrating solar collector or they put everything on a tower and then concentrate all the mirrors on to it, increase the temperature then send the fluid that will pick up the heat then make that fluid run the fluid in heat exchanger with water that will become a steam and run a power plant.