Conduction and Radiation
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Lecture No. # 19
Radiation Heat Transfer between Surfaces

So, we will now start off with the very important topic in radiation, the topic all of you have been waiting for, I mean, that is, how do you calculate radiation heat transfer between surfaces. As far as engineers are concerned, there are many surfaces, each is having its own temperature, each is having its own reflectivity, emissivity and so on, absorptivity and when these surfaces are a part of an enclosure otherwise, what is a net radiation heat transfer. This is basically required even if other modes of heat transfer are there, for example, conduction and convection, you have to take care of radiation. So, you may solve a convective heat transfer at every iteration, you may stop, calculate the radiative heat transfer rate, update in a convection solver and proceed, or it may be just a pure radiation problem.

For example, you are interested in the cooling of the, cooling of electronics in a satellite, so you have a lot of equipments, which is generating heat and the temperature of those equipments has to be controlled. How will you do that? You will employ heat exchanger. You will have heat exchanger, it will pick up the heat and then the fluid, which is picked up, the heat, must be cooled again, so that it is in a position to pick up the heat again because these electronics are continuously operating, correct. So, that hot fluid has to become cold fluid somehow, therefore you need an exchanger. Unfortunately, there is no ambient air in the outer space; no convective heat transfer is possible. Therefore, only radiative heat transfer is possible. So, radiative heat transfer is possible, so you design hinges on how you are able to select, select your surfaces, what is the configuration, whether you want to have pins, what type of pins you want to have, how many number of pins, what will be the pin pitch, thickness, what material? And you have to solve by combine, conduction, radiation problem and design this space heat exchanger.

So, you are, so there are so many applications in which this has become important. Satellite temperature control design of combustion chambers, design of furnaces, design of radiant super heater in boilers, even in other problems where your cooling of electronic radiation also has its, has its part to play. I told you in one of the earlier classes, that at, even at lower temperature radiation is significant. It, it is comparable with natural convection so on. Therefore, it is imperative that we have a method to determine to compute the radiative heat transfer between surfaces.

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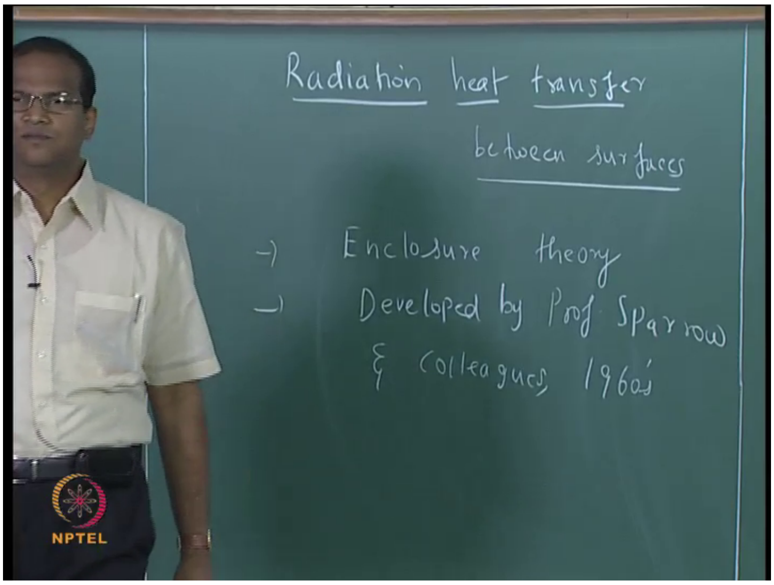

So, let us start off with this topic. So, we learned what is called as enclosure theory. So, enclosure theory was developed by Professor E. M. Sparrow, Professor E. M. Sparrow and his colleagues at the University of Minnesota, Minnesota in the US in the early 60s. So, Sparrow did not do much, he just wrote some 800, 900 papers and get it some 100 PhD students. He is still there, he teaches numerical heat transfer and radiative heat transfer in Minnesota, he is about 82 or 83, he is still alive. If you are going to US, if people are going to US, at least you must go to Minnesota and see him and come, right, when he is alive; so, undergraduate students, 800 papers. So, this enclosure theory is a, has not been challenged, he developed 40, 50 years back, even today it is very popular, we use it, even your fluent and other commercial software use this. So, what is so great? What did, what did Sparrow figure out?

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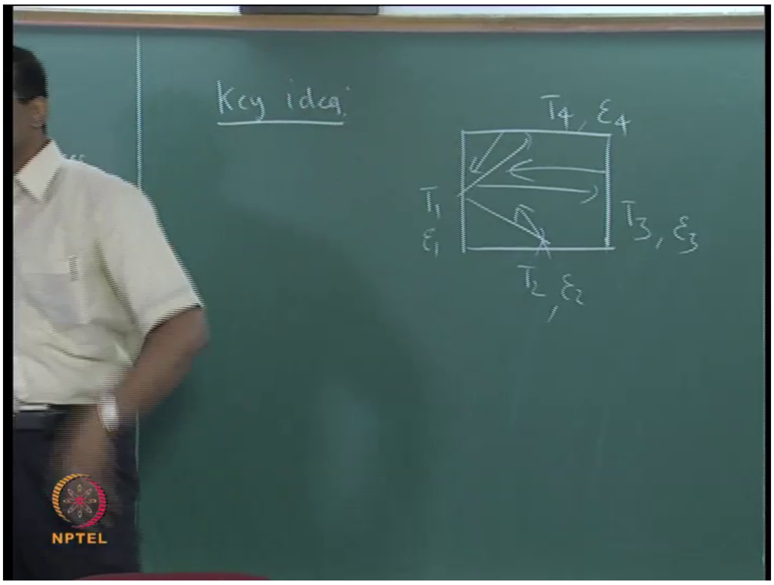

The key idea is, key idea is like this. Suppose, there is a furnace, I am looking on only the sectional view. There is a dimension in the direction perpendicular to the plane of the board. Let us consider only some four surfaces: T 1, T 2, T 3, T 4, temperature of the four surfaces; emissivity is… So, radiation from this surface can go here, can go here, can go here. Radiation from this surface can come, this surface can come and this surface can also. So, the key idea, which professor Sparrow used was, you account for all the radiation is originating from a surface and all the radiation, which is falling on surface and what is going out minus what is coming in, this has to be balanced among all these surfaces. So, by taking care of, by taking into account all the radiation, which is originating from a surface, all the radiation, which is falling on a surface, essentially this you can solve it as a system of simultaneous equation, wherein you can get all the radiative, all the radiative fluxes you desire, this is the idea. Now, you know, what is great, what is so great? Anyway, it is enclosure, radiation will, radiation from one surface will hit the other surface, will get reflected and all that.

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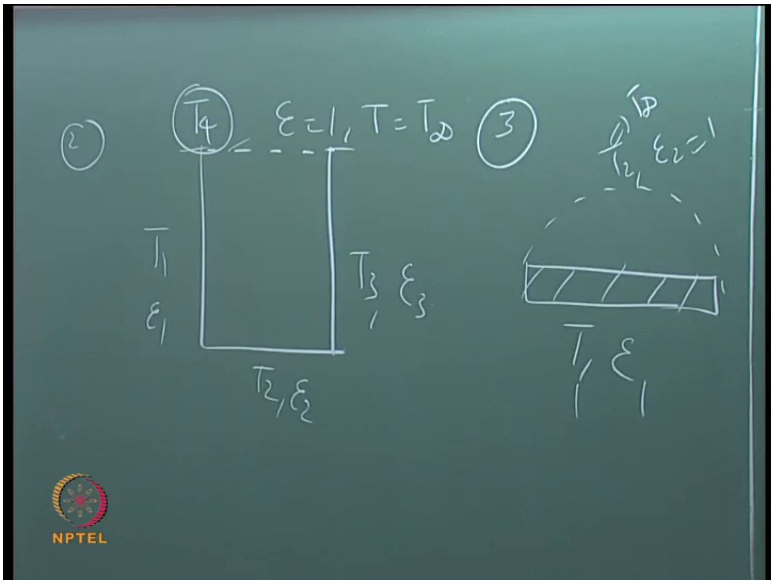

But the key point is, suppose you have a configuration like this. Suppose, you have what is called an open cavity like this. There are three surfaces: T 1, T 2, T 3. The beauty of enclosure theory is, now it is not enclosure, right, it is like open cup, whatever. The beauty of the enclosure theory is you close the top, you close the top by an imaginary surface, that imaginary surface has zero reflectivity, that is, the perfect emitter and has a temperature equal to T infinity, so that is the beauty. So, I consider it as the 4th surface, now I have also treated as enclosure. So, any possible configuration on this planet earth can be treated as enclosure. If, if the surfaces are open, put a dotted line and close the surfaces. For example, even a simple; there is a simple one surface enclosure, so this is, how many surface enclosure? Very good, four surface enclosure. This is a three converted to four; three converted to four.

Now, this also, according to the enclosure theory we can enclose it in hemispherical, enclose hemispherical basket, so make it T 1, T 2, epsilon 2, epsilon 2 is equal to 1, T 2 equal to T… Therefore, any possible configuration, whether the surfaces are plain, convex, concave, whatever it is, if some surfaces open, you put a dotted line and close it and make everything in the world enclosure and then start writing looking at the energy balance, each of this. This is the key idea behind the enclosure theory.

But with the information you already have, that is the last two months, looking at the radiation physics, looking at the Planck's distribution, then the integral version sigma T to the power of 4, then looking at all the emissivities, absorptivities, transmissivities, as a function of angle, as a function of wavelength and all that, that knowledge is not enough because that is all fine if you want to calculate radiation from one surface. But now, you are looking at radiation from enclosure, obviously you can see, that geometry has critical role to play.

The size of the various surfaces in the enclosure and the orientation of one surface with respect to another, with respect to another will eventually decide, what is net radiation heat transfer from each of these surfaces? Therefore, geometry plays a critical part; geometry played a critical part already, in our definition of solid angle. Now, it will keep haunting, it will haunt you again and now more geometry will come because already you can see that you have to calculate the view factors from particular surface 1 to all the other surfaces and vice-versa, 1 to 2, 1 to 3, 1 to 4; similarly, 2 to 1, 2 to 2, 2 to 3, 2 to 4, and so on. We have to, we have to complete the view factors between all the surfaces, therefore the next key idea, which will introduce is the view factor. What is the basic formula for the view factor? How do we go about calculating this view factors if you are solving enclosure problem?

Student: (( ))

No, what is orientation of one surface to another surface, that you do not know, that is one thing. And within a particular surface it can be non-isothermal. That means, you have to subdivide surfaces in to sub surfaces and the other surface may also be non-isothermal.

Student: (( ))

Yeah, you have to find out. See from here, totally hundred for example, 100 watts is going out, for argument sake, out of which how much is going to 4? How much is going to 3? How much is going to 2? That depends on the relative size of the surfaces. And the angular, the orientation, is not that we will have to know? So, we have to introduce this concept called view factor. For those people who are very algebraically inclined, view factors, view factors are pure fun puzzles and brain teasers and problems, you will see first, initially I will put some complicated formulae and all that, but hopefully, we do not use that without using the formula, by using algebra we can get. So, will the story will evolve slowly.

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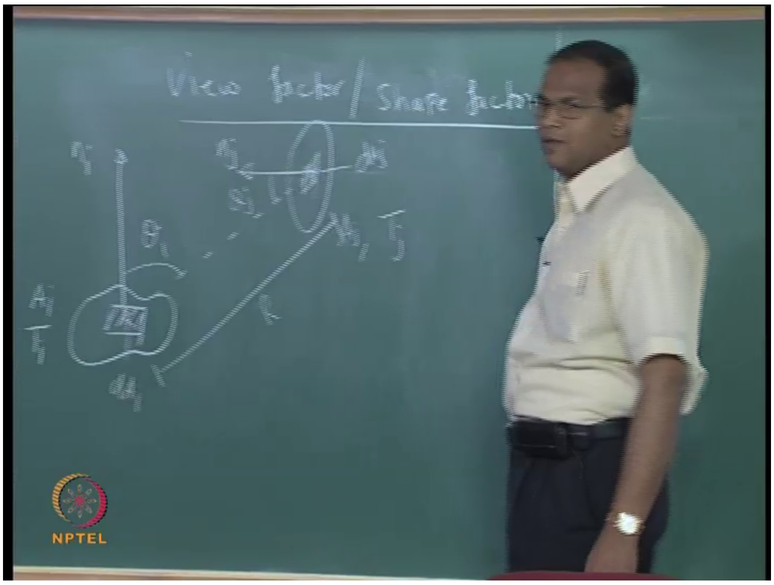

Now, we will have to, this is also called shape factor, angle factor, configuration factor, geometric factor, there are various other names, view factor is most widely used.

Now, there is a surface, there is a surface A i, it has a temperature T i, take an elemental area dA i. Now, the unit vector is n i, there is one more surface A j. This is having emissivity, no, this is having temperature T j, do not worry, let us not worry about emissivity now. Now, I am taking an elemental area d j, dA j, now the unit vector is n j. Now, I connect this, I connect the centroid of this to the centroid of this and this distance, I call it as radius R, R, this angle I call theta i and this angle theta j.

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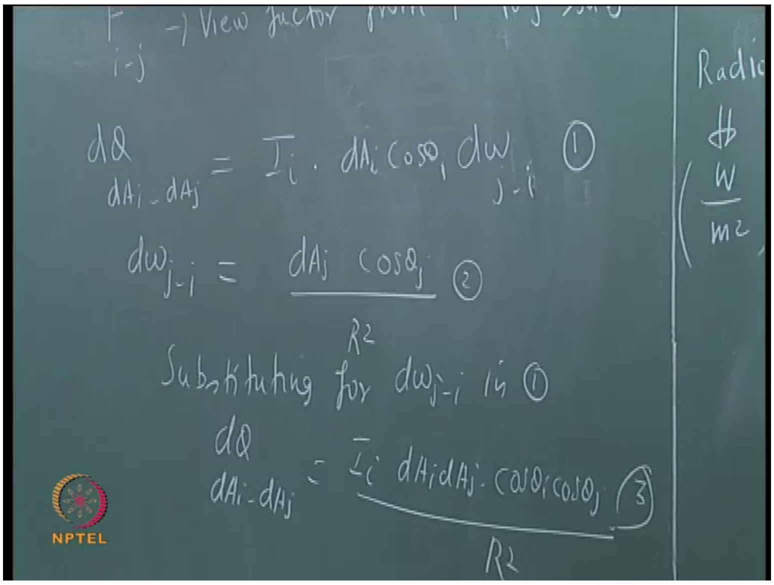

So, F, F i j represents the view factor from the ith surface to the jth surface. So, you can take down this definition. The view factor F i j, the view factor F i j between two finite areas A i and A j, the view factor F i j between two finite areas A i and A j denoted by F i j, so the view factor between two finite areas A i and A j denoted by F i j is the fraction of the radiation, is the fraction of the radiation leaving the surface i, is the fraction of the surface leaving the fraction of the radiation leaving the surface i, that is intercepted by the surface j, is the fraction of the radiation leaving the surface i, that is intercepted by the surface j. So, what are the units of F i j?

No units

No units, dimensionless. What are the limits of F i j, lowest and highest?

Zero to one

Very good, it is an efficiency how much the 2nd fellow absorb with, picks up the radiation the 1st fellow. It, it varies from zero to one, you cannot take negative values, you cannot take values more than one. It does not have units of watts per meter square or steradian to the power of minus 1 or meter to the power minus 1, whatever.

Now, anyway, when we have drawn so much, there is so much buildup logically, mathematically, formula, formulas must flow. So, how do you go about? How to do you get the fraction? How do you get the fraction? So, the F i j will be written by, whatever radiation coming from i received by j divided by the total radiation leaving, leaving i. So, we have to evaluate the numerator and the denominator and make it more general.

And now, we will arrive at a general formula for F i j. Now, let us start with the general formula of F of dA i to dA j, that is, view factor between two elemental areas, that I will call it as dF because elemental view factor. Then, I will integrate first with the respect to A j, then I will integrate with respect to A, A. Then, two infinity similar small areas, one, infinity means small area to a finite area, then between two finite areas, that is a way our derivation will proceed. So, first what is the, what is the radiation, which is leaving i which is intercepted by, what is dQ, dA i, dA j?

Capital I, intensity of surface I i is intensity of I i j, is intensity of j. It will go back to our definition of solid angle. From there, you have to, same story starts, I i, first we will start with I i.

D a i

Very good, dA i cos theta…

Student: (( ))

Very good, d…

Student: (( ))

d, d omega…

Student: (( ))

d omega j to I, if that is what you are intent implying, it is fine, d omega. For people who have not followed I will come again. Intensity in watts per meter square per, per steradian, watts per meter square per steradian multiplied by dA i. To take care of the projected area, cos theta A is coming because this fellow is asked to go in this direction, right, you want to do this, then d omega j or j i, fine. Now, we can write the expression for d omega, there is elemental solid angle. Next step, equal to, is equals…

A j

Not A j…

D a j

dA j…

Student: (( ))

Very good.

Student: (( ))

Very good, everybody through with this? Now, you can substitute for d omega j i in equation 1, I have not evaluated the fraction, I am just writing dQ first. So, I am getting the numerator in that definition substituting for…

Now, you consider i, now you consider i to be a diffuse emitter and diffuse reflector. That means, it does not have directional preference.

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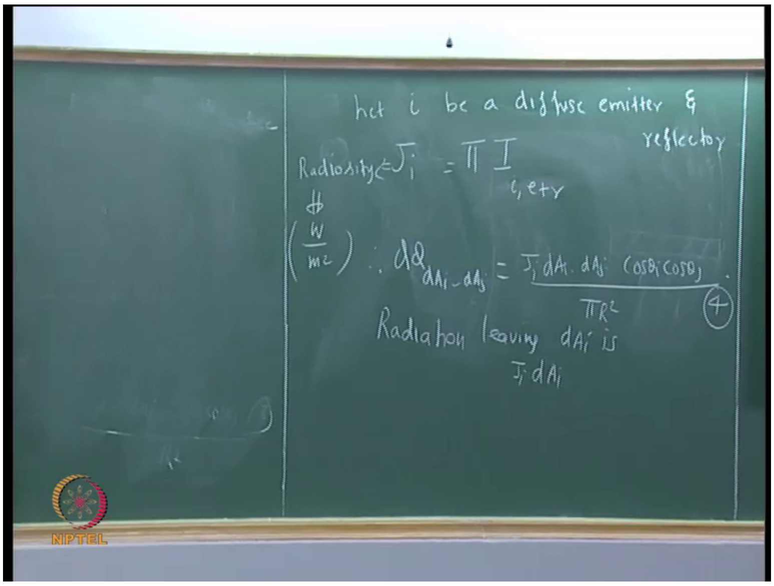

Suppose I, put it as three. So, what will be the total energy, which is leaving or this i consists of what all components? What is leaving a surface consists of how many components? Emitted and reflected; now, I am saying it is diffused, so I will say, I… The intensity, you can have intensity corresponding to emission, intensity corresponding to reflection. Why I am already saying, that it is diffused? So, it does not have directional difference, so I can make it pi. So, to make it clear, I will make it, this I called it as J i.

Did I define this? J is called the radiosity; have we defined this earlier? I am introducing a new term in today's class, J is called the radiosity; J is called the radiosity.

So, radiosity, what are the units of that fellow? So, he is like a flux, that is, the leaving intensity multiplied by pi for diffuse this. Do not ask me, Sir what happens if it is not diffuse and all that, then all this theory will not apply, you have to, it becomes more difficult. This theory is applicable for grade diffuse surfaces. I will tell, towards end we will say, we will list out the assumptions, what are the assumptions under which this theory is valid. Now, can I convert i to J? I think it is fine, right; is it fine? Let us call, let us assign number 4 to this. So, what is the radiation leaving dA i? What is the radiation leaving dA i?

Student: (( ))

The radiation, leaving radiation leaving dA i J i into dA i, radiation leaving dA i, agree, watts per meter square into meter square watt. Then, if you look at the units for equation 1, 2 equation 1, 3 and 4, they are all having units of watt, very good. Now, now, I can get the fraction.

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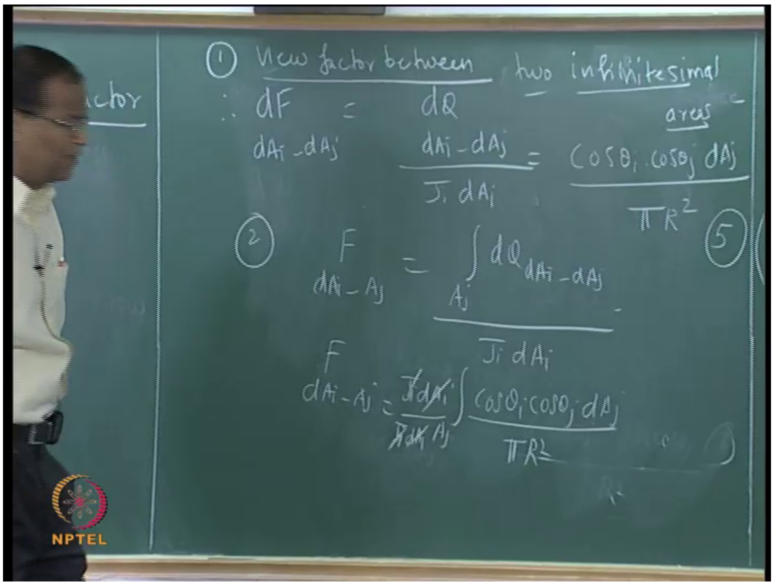

Therefore, equal to cos theta i, cos theta i cos theta j…

F is a fraction, f is dimensionless; right side, is it consistent? So, if, if it is not consistent, we made a mistake, dA has got meter square, R square has got meter square, meter square meter square gets cancelled. Suppose, you do not have, this is a fundamental formula, which can be used. For example, you are, you are computationally very rich, right. You have lot of resources with you, then each of the surfaces you can divide into thousands of surfaces and treat each of, each of the surfaces dA i; to divide that wall into another thousand surface, each of this dA j, you name, it is one to thousand, that is one thousand, one to two thousand. So, one to one thousand, one; one to one thousand, two; one to two thousand, three, simply computer will calculate. Locate the center, locate the center there, root of x 2 minus x j minus x j whole square y j minus y j whole square root of, root of all sigma of all the three, root of that will give R, calculate the cos theta, then we can do this.

Now, you can see, that view factor is not easy. So, if you want to subdivide and do, then it involves lot of computational power, but one good thing is if you have a, if you have a geometry fixe,d before you started heat transfer problem, you just, if the geometry is fixed, your view factor calculation is done. But you see, it is not always like that, the stupid thing is every time the velocity will get updated, it is non-linear problem here, it is like, it is like a mess. People are using fluent; you know it, gambling, you do it once and for all, that is it. Unless your geometry is changing, so view factor, the geometry is fixed. You can determine the view factor once and for all.

Now, here, we are not interested in this view factor between infinitesimal areas. Now, you want to look at view factor between first one infinitesimal to finite area, then between two finite areas. So, this is one in (( )), now, now I want to calculate the view factor between dA i to A j. Now, what is the numerator?

Student: (( ))

Over, over

Student: (( ))

Be confident, over…

Student: A j

Very good, divided by…

j I d a i

J i

D a i

J I dA i, that fellow stays because it is from dA i. Now, that dA I, can we pull out J i dA J R will be function of this. I cannot take R out, R will vary between two infinitesimal areas. Pi, if you want you can put it inside or I hope I have not made any mistake, correct; any problem? Fine.

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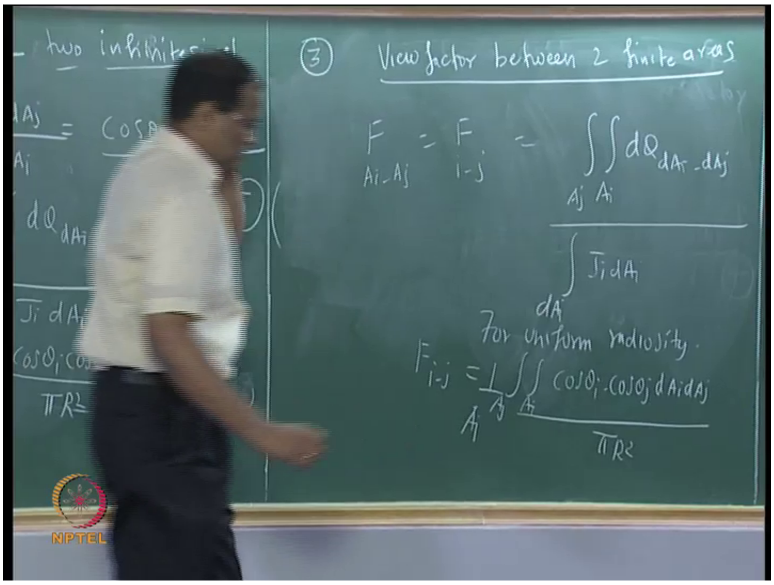

Instead of saying each time, F of A i to A j, I just say F of i to j, what is that? Divided by

Student: (( ))

Very good, no integral.

If I assume, that it is a uniform radiosity, then can I get rid of that for uniform radiosity equal to, is that correct? There is a J i, J i A i in the denominator, there is a J i in the numerator, so the J i J i gets cancelled. Any problem?

Student: (( ))

What is that?

Student: (( ))

There is no integration.

Student: (( ))

I did not get you.

Student: (( ))

Now, if you want to get view factor between the surfaces, now very good, this room is very ideal for explaining view factor. One brown color box is there, surface, one brown color box, another brown color box, that is, i to j, each of i to j, what you can do? You can subdivide this boxes, each hole is there, know, consider around each hole, four holes as dA i there, then four holes as dA j, calculate the view factor like that. It will, it will vary, right, the R will vary from this last hole from there. If you look at the south-east corner and the north-west corner, there, there will be changed in R and please do not underestimate the importance of this, do not underestimate that, difficulty associated with this integral A i.

If it is two-dimensional surface integral, A i will represent dx dy, integral A j will represent dx dy, A i will be dx i dy i, A, dx i dy i dx to the J d. So, Sparrow and his colleagues, then after doing all this, they developed technical contour integration, contour integration is special craft known only to radiation heat transfer because to breakdown, to breakdown this four contour pole integral, there are, it is not in, we are more than triple integral. So, fortunately, for us these four integrals have been solved and it is available in the form of charts, simple cylindrical disk to cylindrical disk to disk, parallelogram, parallelogram safe surface rectangular surfaces, cuboidal surfaces, perpendicular surfaces and so on. Now, this is the view factor for uniform radiosity.

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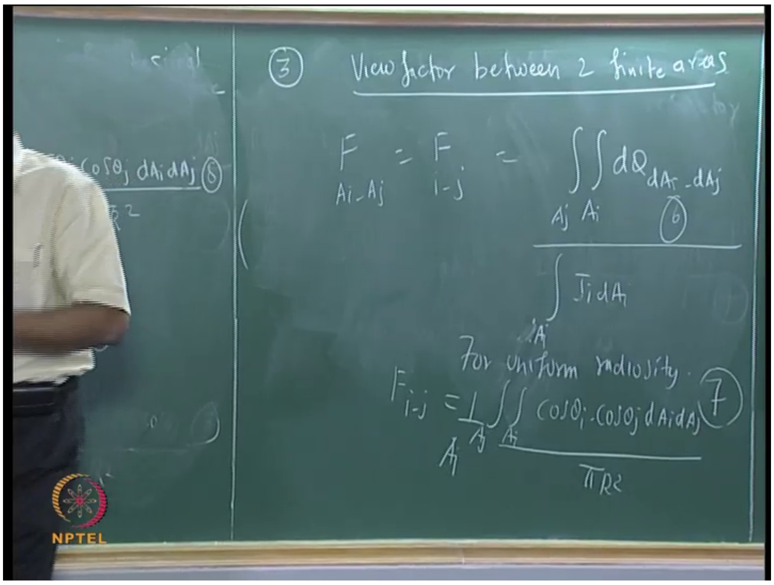

By intuition can you tell me what will be, what F J i will be?

Student: (( ))

1 by…

Student: (( ))

I think we need to put some numbers, what is this number?

Student: (( ))

6, 7, by looking at 7 and 8, what you can say?

Student: (( ))

A i F ij equal to A j F ji, this is what is called the reciprocal rule or reciprocity relation. This is called the… Now, I want to solve four zone enclosures if you have used it double integral. Now, this is going to take a lot of time. Now, I want to see if there are some clever ways of getting the view factors without going through these painful integrals. So, this whole subfield where we try to manipulate algebraically and get view factors with minimum recourse to the original formula, which involves integration is, is a subfield called as view factor algebra.

So, we will now get into view factor algebra because fine, this is, if you can write your program and you are, you are computationally very rich, use this get view factors between whatever surface you have and continue your problem, but now, since this could be very messy, this could involve lot of computational time and effort can be simplified for simple surfaces, simple enclosures. Can we get better than this without taking recourse to this? Can we get view factors? Yes, there are ways of getting this. So, that is a very interesting field called view factor algebra.

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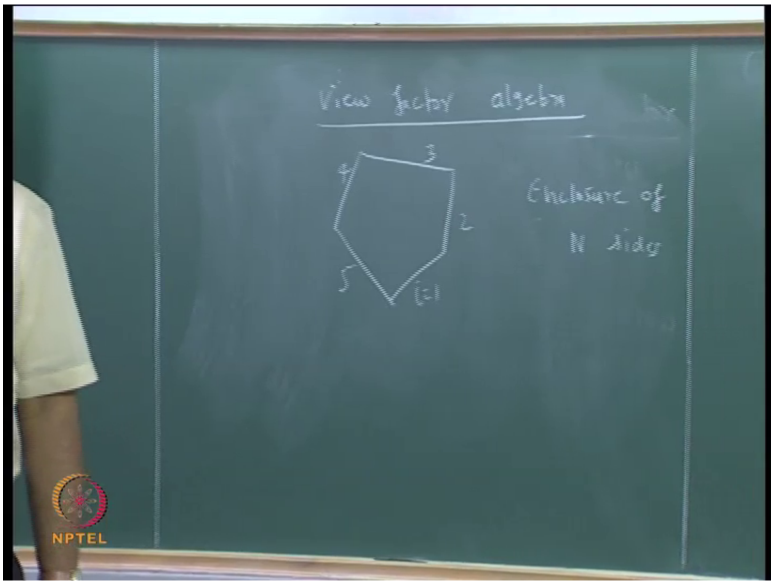

Let us take an enclosure; consider an enclosure of N size.

Soumya, what is your problem, difficult, difficulty staying awake, not well?

Taken a medicine

Taken a medicine, cetrizine? So, there are two medicines, which are working, I hope everybody got it? Now, for a, you got it Deepak? He is feeling sleepy because he has taken medicine. I am saying that there are two medicines, which are working; apart from that medicine, there is one more.

Now, if there is a N surface enclosure… N square view factors, why so much difficult? N square view factors will be there. What are these view factors?

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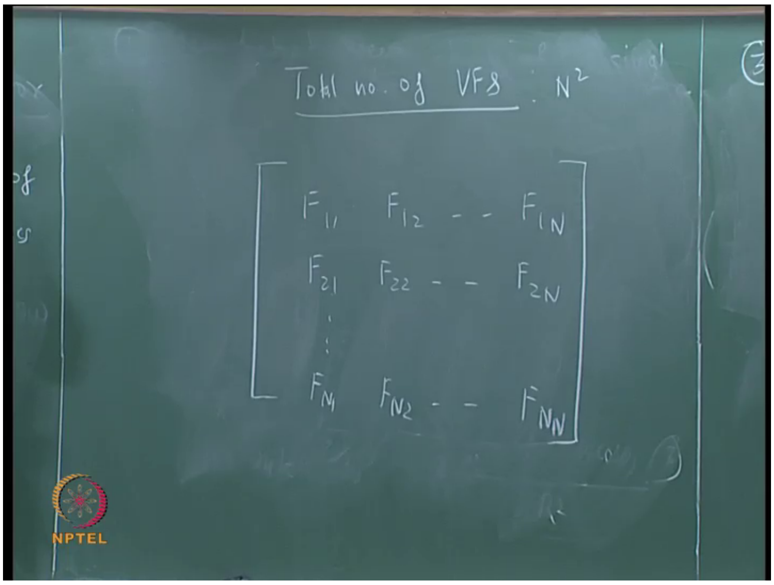

So, I will say, total number of VFs, VFs is view factor N square, I can write it like this. F 11, F 12, so on up to F 1N; F 21, F 22, F 2N; F N1, F N2… Do you think it is very pragmatic for us for five surface enclosure to get all the 25 view factors by using that stupid integral? So, there should be some other way out. Already, already we got some idea, A 1 F 12 equal to A 2 F 21, A 1 F 13 is equal to A 3. So, some fellows I am knocking off because of the reciprocal rules. Let us see if additional rules are available. All this constitute the algebra.

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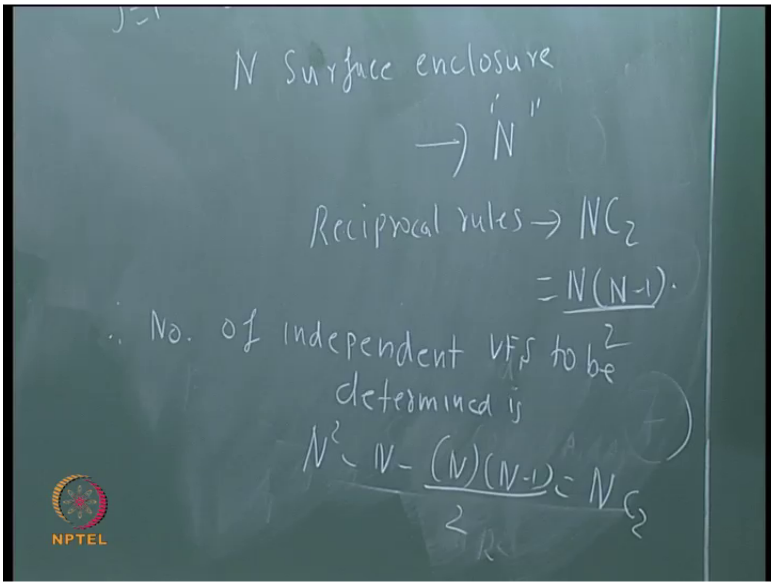

Fine, now please tell me, yeah please look at the please look at the board and tell me what is this

Student: (( ))

Very good, it has to be 1 because it is, this is energy balance. From, from surface 1 if there is a 5 surface enclosure, F 1 to 1 plus F 1 to 2 plus F 1 to 3 plus F 1 to 4 plus F 1 to 5 must be 1, where else will the energy go? So, this is, now for N surface enclosure how many such rules are available?

Student: (( ))

N, so by the way, this is called the summation rule or the sum rule. For N surface enclosure, how many reciprocal rules are there? You take…

Student: (( ))

NC 2, very good NC 2 relations, two combinations take two surfaces at a time, N is equal to N into N minus 1 by 2. So, how many independent view factors have to be evaluated? N square minus N minus N into N minus N by 2, correct, because you can, you can exploit the summation rule as well as reciprocal rules. Therefore, the number of independent view factors to be determined, what is this? 2 N square minus N minus N square plus N, do all that, this is again NC 2.

Student: (( ))

What, what, what?

Student: (( ))

No, that I, I am coming; they are already jumping the gun, NC 2.

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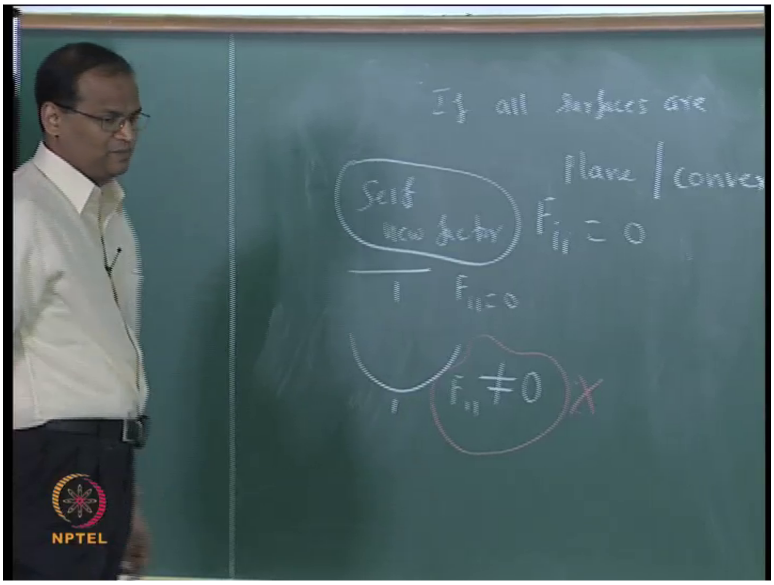

Fine, now let us look at the, if all the surfaces are plane or convex, what will, what will this F ii be? Zero. For a surface like this… Of this surface…

Student: (( ))

Please note, concave surface will give some [FL], so you have to handle concave surfaces with care, convex surface there is no problem because if it is like hemispherical cup, it will look at itself, so that is called F ii. It is called, give some name, self view factor.

Now, if you, if all the surfaces are plane or convex, total number of independent view factors…

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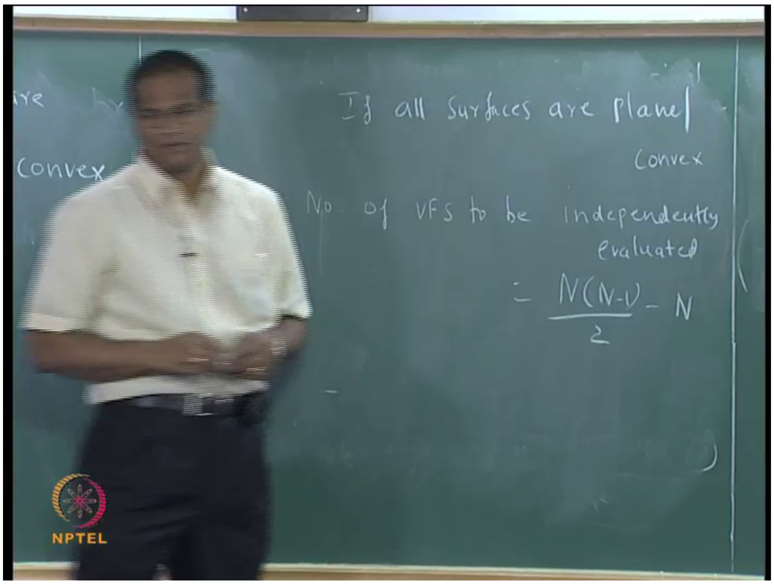

How many self view factors are there?

Student: (( ))

Very good, yeah, so the goal, the goal of view factor algebra is to ultimately determine the number of independent view factors, which have to be evaluated necessarily by adopting the cos theta A cos theta J…

Now, let us look, let us look at a very elementary problem, simple problem. Let us take a three surface enclosure, that is, triangular duct. Some gases are going inside and all that convection radiation is taking place. Do not worry about convection triangular surface, N equal to 3. You apply this formula, how many independent view factors have to be determined? Zero, that means, without using that integral you can get all the view factors. Can you get those view factors?

Now, shall we look at general triangular enclosure? Let us do that problem and close the discussion for the day, which means, you will write all algebraic relationships, manipulate the algebraic relationships and get all the view factors.

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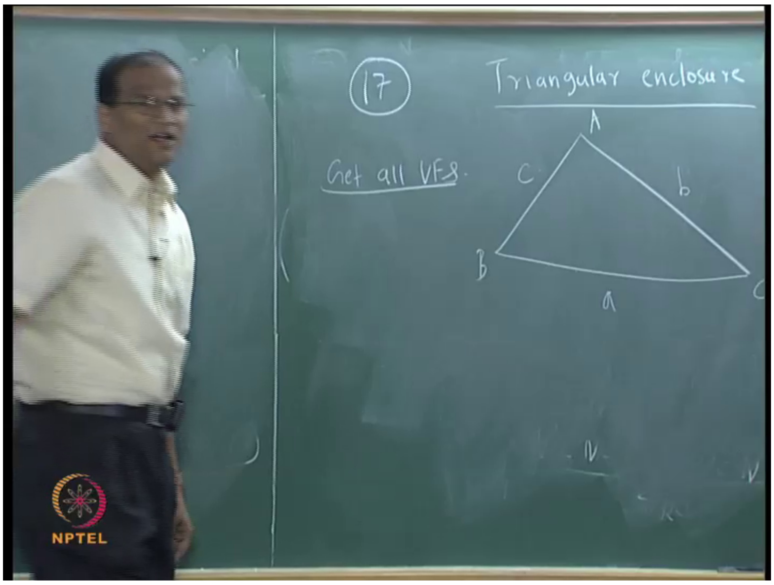

A, length is A; yeah, what is this problem number, problem number?

Student: (( ))

Seventeen, you can, what is the problem appropriately? Consider a triangular enclosure, consider a triangular enclosure, three surfaces of length A, B, C. Determine all the view factors. Consider a triangular enclosure, how many of you got it already?

Student: (( ))

Yeah, what is it you get? I want a very nice answer.

Student: (( ))

Is it what? I will, I will demonstrate, please look at the board. It is unit depth in the other direction; no problem.

Student: (( ))

What area is length (( )) into 1?

Student: (( ))

I will show you quickly, I will show you quickly a very cheeky way of doing it. Agreed, is everybody through with this? Pawan, Amrita, you have written all this?

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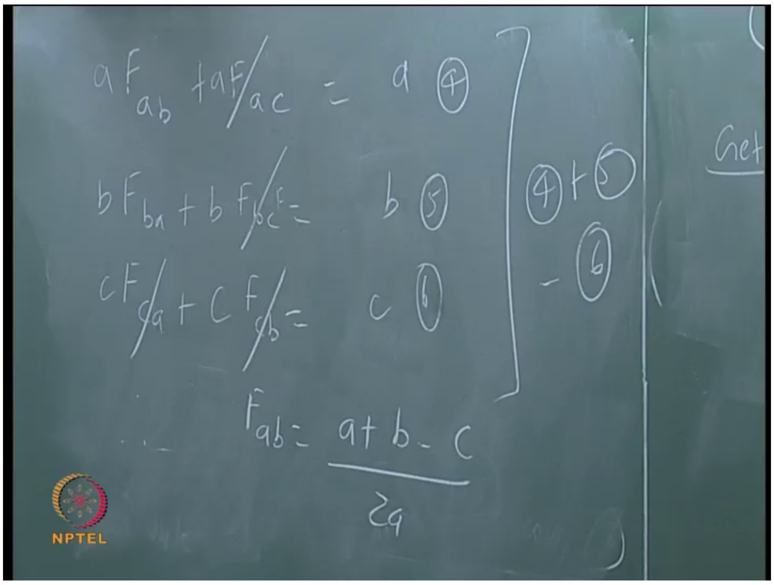

Now, please look at the board, please look at the board. I multiplied the 1st equation by a; I multiplied the 2nd equation by b; I multiplied the 3rd equation by c. I call this equation 4, 5, 6. I did not do anything great now. Now, I will do four plus five minus six, what is the beauty? aF ac equal to cF ca, I am adding four and five and subtracting from six, therefore these two will go, bF ba equal to, cF b F bc equal to cF bc, this will go, aF ab equal to bF ba, that is equal to two times…., correct, that is it.

People who do not believe what I am saying, consider an equilateral triangle. What do you expect, all the view factors to be, equilateral triangle? Equal, equal means what?

Student: 0.5

0.5, are you getting? Let us, let us take 1 meter, 1 meter, 1 meter; 1 meter plus 1 meter minus 1 meter by 2 meters, 0.5. There are some complicated ways of doing, but do not get (( )). I am teaching for many years, so it is taking many years for me to come out with such things. So, it can be easily done. Even if you do not, do this trick, cheeky algebra, by, by elaborate manipulation you can get the same result.

So, tomorrow we will solve several puzzles. We will look at cylinder within a cylinder, sphere with a sphere, cylinder in, cylinder within half circle and all that, get view factors and we will have to do one or two problems where the integral is also involved, so that you know how to handle that.