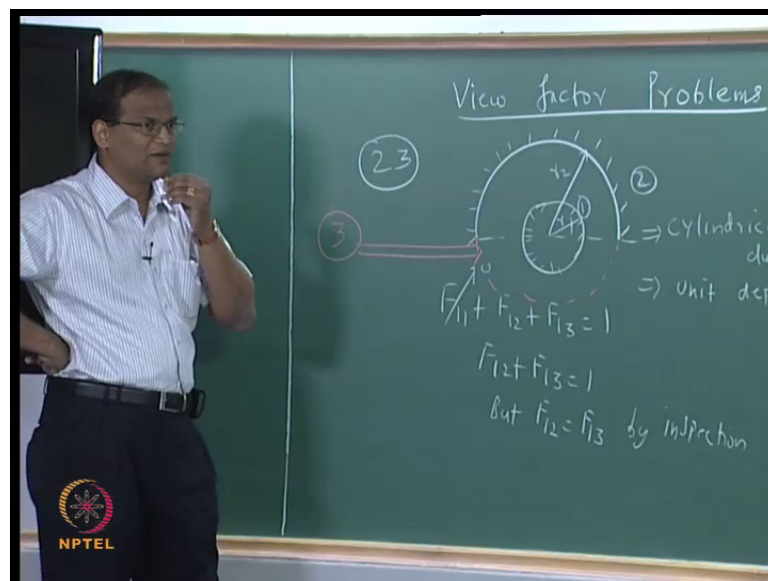


Conduction and Radiation
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Lecture No. # 21
View factor contd...

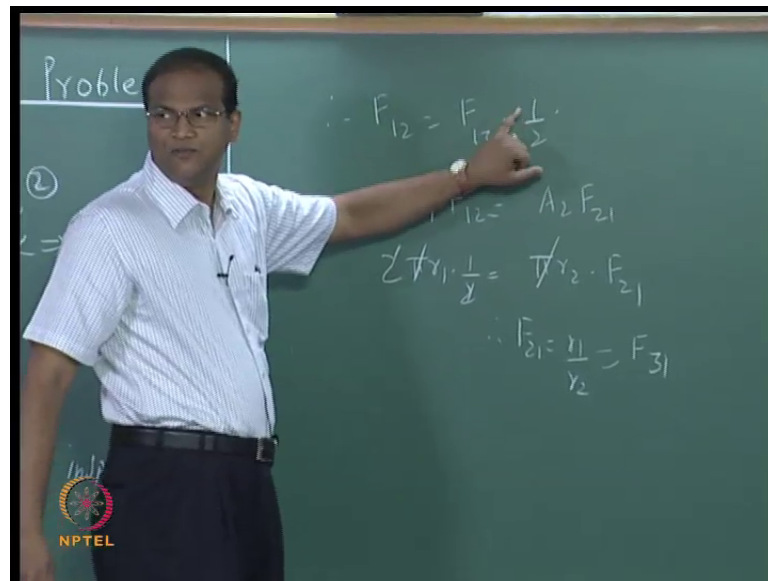
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So, in the last class, we were looking at the view factor problems. The last problem, which we took up in yesterday's class, looked like a relatively simple problem. There is a circular tube or a pipe, outside there is, outside there is a concentric pipe, but you have taken only half, so only the semicircle we have taken. So, the bottom is open, we wanted to get all the view factors. So, we put, we connected the bottom and called it 3. So, so now, it there are 3 surfaces, we started out with our view factor algebra. Now, as far as 1 is concerned, there is no self view factor F_{11} equal to 0. Now, 1 to 2, if you look at 1 to 2 or 1 to 3, 1 to 2 and 1 to 3, there is no preference, it is a same thing. Therefore, F_{12} must be equal to F_{13} .

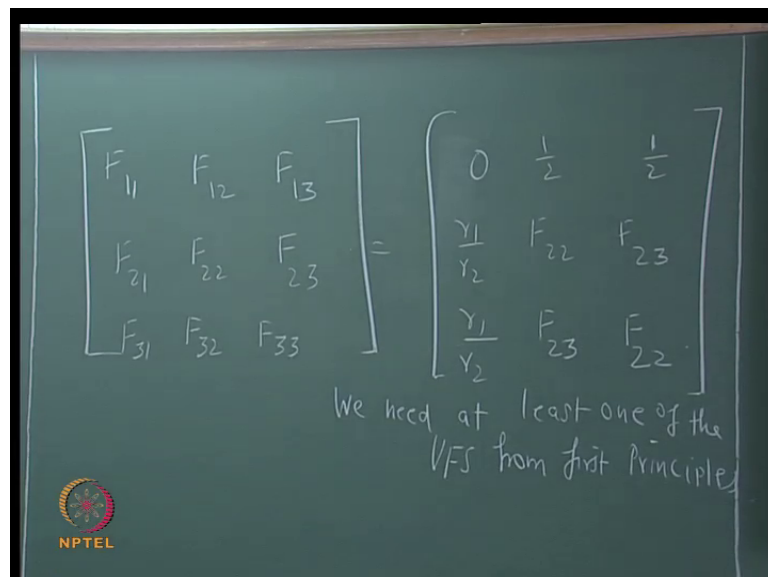
So, I have put sum rule, if you apply the sum rule for surface 1, you get this, F_{11} equal to 0, F_{12} equal to F_{13} by symmetry, therefore F_{12} equal to F_{13} by symmetry.

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Therefore, sum is 1, therefore F_{12} is equal to F_{13} equal to half, there is no problem over this. Now, $A_1 F_{12}$ equal to 2, reciprocal rule. From the reciprocal rule, you get, that F_{21} is r_1 by r_2 , correct. This F_{21} must also be equal to F_{31} , correct. So, so far so good; there is no problem.

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Now, let us write the view factor matrix. The view factor matrix, there are 3 surfaces, 3 square, 9 view factors are there. It is like this, so 1st row we are able to fill up, 0, half, half. 2nd row, we have got r_1 by r_2 , F_{22} I do not know, F_{23} I do not know, F_{31} is

also the same as F_{21} by r_2 . Now, I can say F_{23} by symmetry F_{22} is the same as F_{32} , correct. So, instead of F_{32} , I wrote F_{23} and then, F_{22} is the same as F_{32} to itself. So, there are 2 unknowns, F_{22} and F_{23} ; that is it, we are struck here, we cannot proceed any further with view factor algebra, this is the maximum we can achieve.

So, I must tell you, that I am equally surprised, that such a simple problem, it is not possible for us to because it, I have not taken from somewhere. Yesterday, on the spot I, I thought we will work out a simple case and, and I am completely surprised, that we are not able to solve it using view factor, so let it be there. So, we cannot let it be like that. So, we cannot underestimate just because there are only 2 surfaces.

Now, there should be some way, you have to take $\cos \theta_i \cos \theta_j d_i d_j$ and you have to integrate by contour integration, you can get it. Of course, there are some asymptotic limits, that is, when this r_1 is very small compared to r_2 , then we can make some limiting assumption. Then, F_{23} to itself will be equal to 1 and so on, you can proceed like that, but right now, no further progress is possible, fine. So, let us leave it at this. That is all, we cannot get all the view factors; we can get some view factor. Let us solve another problem, there is a surface like this, is enclosed by a hemispherical, not hemispherical, semicircular disk, it is 1, now get all VFs.

Student: (())

No, no, if I get 1, I can use that sum rule and break it down.

Student: (())

Symmetry, you are not able to see the other one, (()) I wrote F_{32} as F_{23} , are you able to see that?

Student: (())

No, no, even if areas are the, areas are different, then more unknowns, unknowns will be there.

Student: (())

You are not understanding me, if you get one of the view factors, it is done.

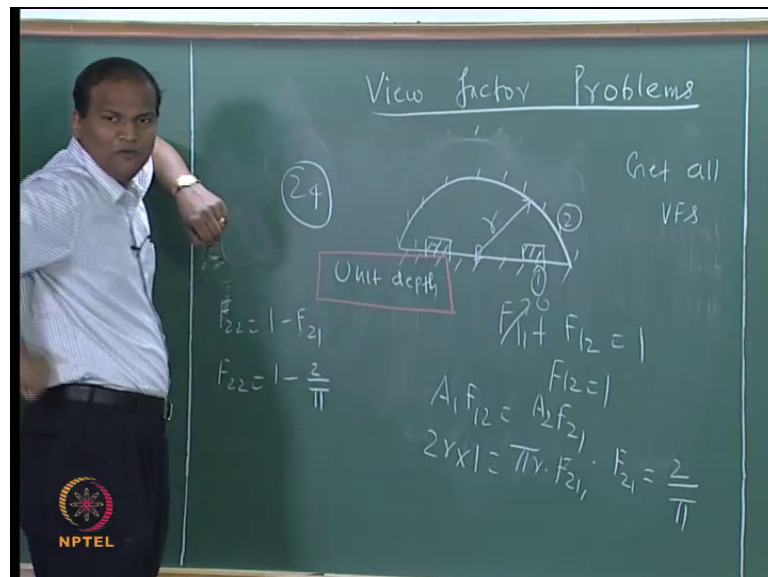
Student: (())

Even otherwise it is not, there is one, which is there, will be one, which will trouble if you try.

Student: (())

So, if you have (()) you can solve. Let us try something like that in next problem we will do, but right now, it is over, its gone, this problem is gone, it cannot be solved. Let us do this now, this one is pretty straightforward, is not it?

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F 1 1, what they are trying to say is, if A 2 is not equal to A 3, then you can solve it or that is, not. He is saying that more equations are required; I do not think this... Like, saying A 1 equal to A 2 and hence, they, if you see, if you apply... No, no, why are you complicating it further, is this correct or not? If I get one of view factor, is the problem over? That is what I am saying, what, what else is there in this problem?

Student: No, whatever if you take, that you will get some, I will get 2, sir.

Yeah, that is what, out of that 2, 1 if you get, you can break down, you, you can break down because there is an additional rule. His question is, what are the total number of view factors, 9; how many sum rules, 3; how many reciprocal rules, 3. Then, self-view factor, 1 self view factor, only 1; F 1 1 is 0. So, 9 minus 7, 2; he says, he is arguing, that,

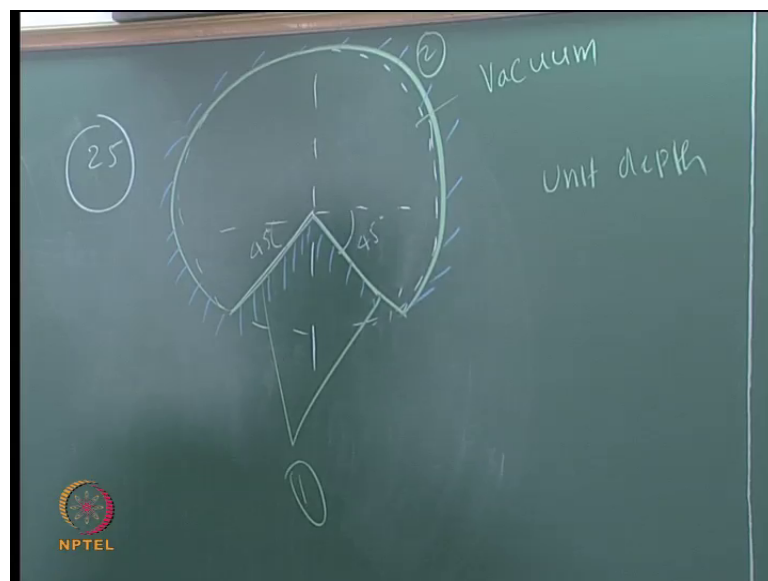
so 2 independent view factors have to be, have to be obtained why are he saying 1? But I have an additional thing, A_2 equal to A_3 and, and the geometric orientation is such that the view factor between 2, 1, 1 is same as 3 and 1, that is additional information, which I cannot simply get by n minus n minus n into n minus 1 by 2. I can use my domain or knowledge of the geometry to reduce a number of view factor, is that point clear, that was valid doubt, everybody. So, everybody through with this?

Now, this problem is, do not worry, that every problem will give rise to trouble and all that will, these problems are solvable, $F_{11} + F_{12}$ equal to 1. What is this fellow F_{11} ? Yeah, 0, therefore, F_{12} is A_1 . What is A_1 unit depth? Yeah, please note, that we are solving only two-dimension problems, you can, unit depth. So, $2r$, what is this?

Student: πr .

πr , correct, 2 by π . Such problems occur, cooling of equipment or you are putting a shroud, you are putting a shroud, you have some electronic equipment, your configuration may be like this, you may have some electronic chips and then, this may call 1, 2, 3, 4, then you have to get view factors and all that. In ISRO and all that, they frequently enclose this electronic equipment with shrouds and all, but this may not be of this shape, but, so to protect it from wind, dust and...

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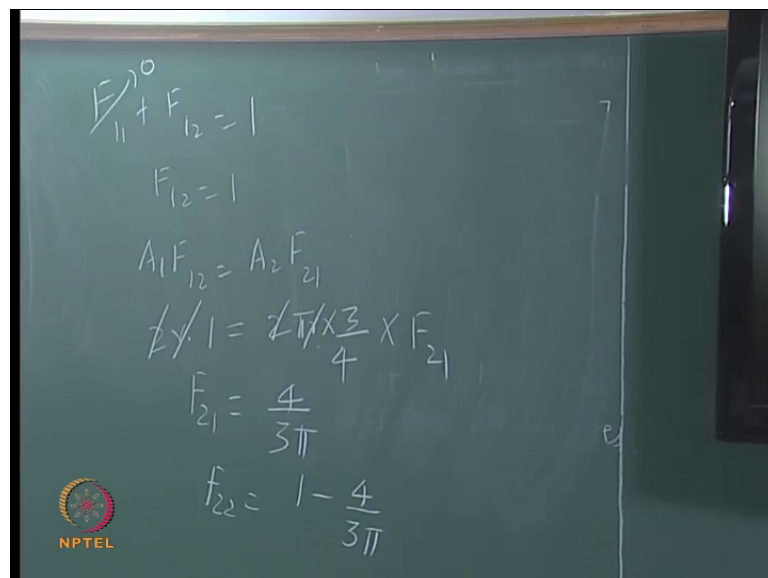


Now, this basically, all these are warming up exercise. Eventually, our goal is to do the enclosure analysis. So, I have configuration like this, vacuum, this plus this is 1, this is 2, get all view factors. 45 degrees, no, no, no, no, no, no, correct, yes, what is included angle?

Student: 90

90, very good. So, so, radius, of course, needless to say radius is r, you convert the problems appropriately for the given configuration with unit depth, determine all view factors, get all VFs.

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Student: F 1 1,

F 1 1 very good,

Student: A 1.

Very good, 2 r; A 2?

Student: 2 pi r.

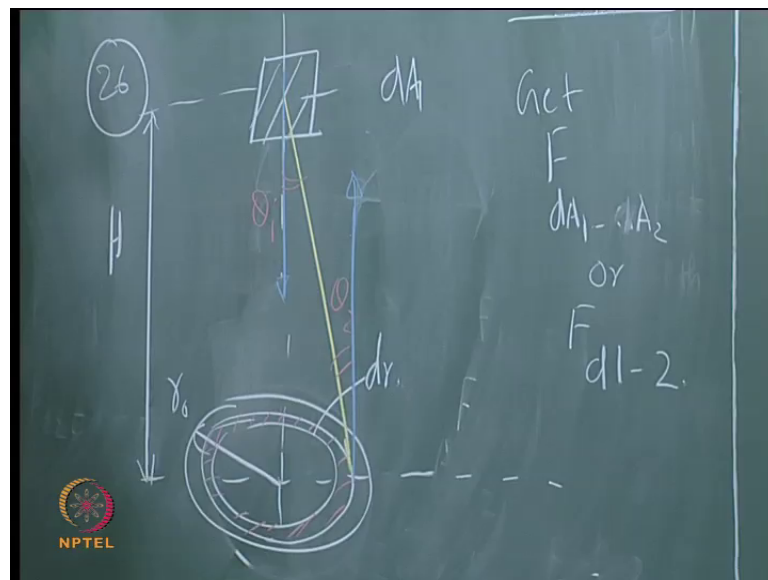
2 pi r. Three-fourths of pi d, correct, into F 2 1; 4 by 3 pi r, you can substitute for pi, 3.14 and simplify all. So, like that, you, exploiting algebra, we can keep on solving problems,

but we have to take a call and we have to close the discussion on this. I am just seeing whether I am missing something.

Now, we will take up a problem where we will have to necessarily get the view factor by direct integration, but since direct integration of 2 finite areas will be very painful, for demonstration purpose we will take a simpler problem. But even a simple problem, some fundas are required, you cannot just blindly use the formula. Let us look at problem where you have to get view factors with 1st principle integration.

I hope because of, because we have worked at so many problems in the last few classes, as far as view factor algebra is concerned, I hope all of you should be comfortable by now. If you recall the problem, we, where we got struck up, you must always suspect a problem in which concave and convex surfaces exist and both concave and convex surfaces exist together. You have to see, whether something is missing or that, that is, those kinds of problems will lead to, are potential trouble maker. In any case, if you have a concave surface itself, it has to be handled with caution.

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Now, problem number 26. I have a, I have differential area dA_1 or simply $d1$, I have a finite area here, does not look alright, I have finite area here, a circle. This fellow, the height is h , this fellow's outer radius is r_0 , it is a circle, it will look like this, it is not an ellipse. Now, so the problem can be posed as, determine the view factor $dF_{1 \rightarrow 2}$, but in a differential area above a disk and a finite disk of radius r_0 , if you want I

will... Determine the view factor $F_{dA_1 \rightarrow A_2}$; determine the view factor $F_{A_1 \rightarrow dA_2}$. Why, why did I say $dA_1 \rightarrow A_2$, that one is a differential area? So, I qualified as $dA_1 \rightarrow A_2$ or else, you can take it as $dA_1 \rightarrow A_2$ into A_2 , whichever way. So, determine the view factor $F_{dA_1 \rightarrow A_2}$ between a differential area, between a differential area above a disk and a finite disk at a height of h , and a finite, and a finite disk and a finite disk at a height of h , from dA_1 having a radius small r naught, having a radius small r naught. You do not have to worry about dA_1 . What is area and all because it is, it is a differential area, A_2 I have given.

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The image shows a chalkboard with the following handwritten equations:

$$F_{dA_1 \rightarrow A_2} = \int_{A_2} \frac{\cos \theta_i \cos \theta_j}{\pi R^2} dA_2 \quad (1)$$

$$\theta_i = \theta_j = \theta; \quad R = \sqrt{r^2 + H^2} \quad (3)$$

$$\cos \theta = \frac{H}{R} \quad (4)$$

$$dA_2 = 2\pi r dr \quad (5)$$

An NPTEL logo is visible in the bottom left corner of the chalkboard image.

Now, what is a formula for view factor from a finite area to a, from infinitesimally small area to a finite area $F_{dA_1 \rightarrow A_2}$ or integral over A_2 , is it correct. Yeah, this is what we derive. Now, you have to convert the, watch very carefully, you have to convert the problem into one in which the variable is small r , that small r can take a value 0 to r naught. Therefore, first you have to take an infinitesimally small area on that disk, take an element, take a thickness of dr , so that, that area will be $2\pi r dr$. Wherever dA_2 is there, you can put $2\pi r dr$, then if it is at a radius r , from fundamental trigonometric principles, you can find out, what is that capital r square, that would not be equal to h ? From that disk to this, it will be root of H square plus r square and the limits of the integrations small r can go from 0 to r naught, it is, the problem is not over $\cos \theta_i \cos \theta_j$, you have to represent in terms of H r r naught, whatever. If you do all this, you can accomplish the integration. I can, you can do it in 10 minutes, can we start? Ketan finished? Is the problem clear? So, what do you do first?

This, got d r. Now, what is that and what is that? cos theta i or theta i 1, theta 1, theta 2, what is this? 1st theta 1 equal to theta 2 and we can just call it as theta, theta 1; what is cos theta? r by (()) is r H square root.

Student: r equal to, cos theta equal to...

H by capital R, very good; d A 2 equal to, d A 2 equal to...

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$$F_{d1-2} = \int_0^{y_0} \frac{H^2}{R^2} \cdot \frac{2\pi r dr}{\pi R^2} \quad (6)$$

$$F_{d1-2} = 2 H^2 \int_0^{y_0} \frac{r dr}{R^4} = 2 H^2 \int_0^{y_0} \frac{r dr}{(y^2 + H^2)^2} \quad (7)$$

2 pi r d r, very good. 1, 2, 3, 4, 5, substituting, substituting for cos theta r and d A 2 in equation 1, we can get... What is the limit of integration for r? 0 to

Student: (())

R naught, very good; 0 to r naught H square by R square pi r square, is it correct? 2 H square. What is r itself? Root of H square plus r square (()) is it ok. Let us assign some numbers.

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$$F_{d1-2} = \int_0^{y_0} \frac{H^2}{R^2} \frac{2\pi r dy}{\pi R^2} \quad (6)$$
$$F_{d1-2} = 2H^2 \int_0^{y_0} \frac{y dy}{R^4 (y^2 + H^2)^2} \quad (7)$$

Yeah, you can directly integrate or you can still use 1st principles integration, you can do substitute in terms of r sine theta. What I will do basically is, let r square plus H square be y . Therefore, so I will have a half, is it ok. Yeah, yeah; no, no; H square. H square; yeah, yeah, by 2. By 2, by 2, by 2, y square. This is correct. 1 minus... something is wrong.

Yeah, yeah, it is fine. So, this minus, minus will get cancel. Yeah, you get an elegant result, but please remember 1 is infinitesimal, both are finite areas, it will become difficult and now, if it is not regulated disk and all that, it is going to be very tedious. So, lot of people in the 60s, 70s and 80s, different groups in the world worked on developing view factor relations solving these integrals. It was considered a very important activity because the view factors are repeatedly used in the (()) calculations also in multimode heat transfer problem. So, view factors were also one of the reasons why double precision was invented and all that you are, you need view factors up to 7th decimal or 8th decimal accuracy because if you have 100, 1000 view factors, we just cannot leave with 2nd decimal or 3rd decimal. When you are doing a convective solver, if it is a convection and radiation together, then it will lead to lots of errors. So, lots of people will, you must, you can see, papers were by then SPV's written papers, on accurate determination of view factors where they look at 4th decimal, 5th decimal, 6th decimal and all.

So, view, so lot of groups were preoccupied with the determination of view factors, but now with the computational resources becoming more powerful and more programs being available and software, like fluent directly using this radiation, this activity of research is, is not that prominent nowadays because these are, these are considered as well-settled problems, I mean, we know how to get view factors. There are of course, still few situations, for example you want to do a static, this thing, for example, G GSLV or PSLV, you put it horizontally, you hold it somewhere and you do firing in Sriharikota. Then, when it, when it is generating hot gases, what, what, what is a radiation load in the nearby structures, on the electrical cables, wiring, nearby men, material and all that, for that you have will have to do all these view factor analysis, radiosity and all that and find out. So, this, regularly the testing is, testing takes place, before launch we will have to find out. So, they will test all these motors in Sriharikota because the, basically, that is the place where, what is it called, Satish Dhawan, Satish Dhawan launch centre or research centre, whatever.

Satish Dhawan space centre, that is, where they do all these launching business. So, for all these space related things, this radiation becomes very, very difficult. For example, now, we are looking at lunar model and all that; we want to put a man on moon and all that. For the lunar model also, you will have to keep the pressure at 1 bar, you have to keep the temperature at 24, 18 to 24 degree centigrade. Then, there will be nearby electronics and all that, then there will be solar radiation, not to this spacecraft, and electronics will be there. Then, you have to do cooling, you have to do heat load calculation, there it is again radioactive heat transfer, which will be very, very important. So, in all this exhorting and fancy applications, it is eventually the radiation alone that matters; convection is not there.

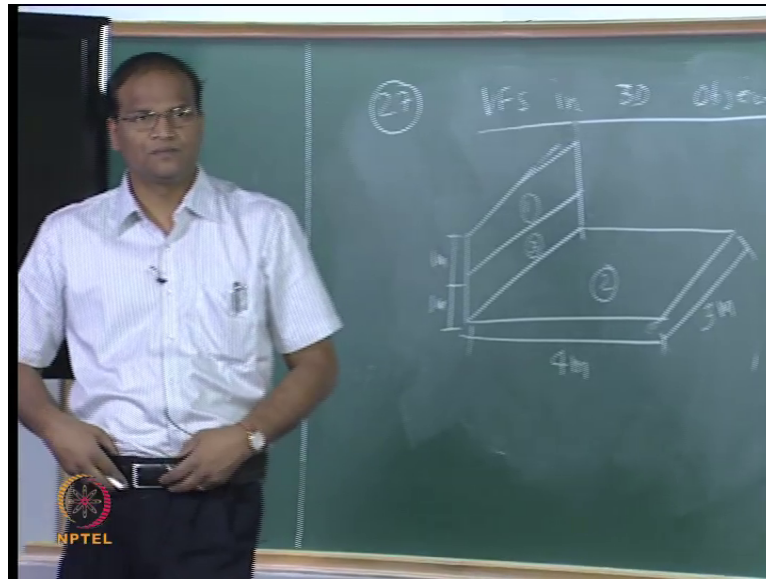
Now, what is a physical interpretation of the, when r naught is much small or when H is much, much larger compared to r naught. Of course, when H is infinite, F will be 0, infinitesimal disk, when some other disk, let it be at few thousand Kelvin. If it is infinitely far off, then it has no influence, but if r naught by H is much, much less than 1, we can reduce it to r naught square by H square and it confirms the basic thing, like the view factors. It, it goes as inverse square law, it goes inversely as the square of the distance between the two, 1 by H square, H is the distance between the two, so that is one indirect validation for this.

Now, so far we restricted our attention to 2 dimension surfaces, it is nice to work with two-dimension surfaces because we can show it in the, we can demonstrate it in the classroom, you can solve simple problems, it will boost your confidence and you are very happy, that you are able to solve all these problems and all that. But unfortunately, most of the real life surfaces, 3-D surfaces, once you have 3-D surfaces, just by algebra you cannot get because of the sheer finiteness.

Therefore, people have done, developed elaborate techniques. One of the most important techniques is called the contour integration, dA_1 is $dx_1 dy_1$ dA_2 is $dx_2 dy_2$, therefore 4 integrations are involved. So, they, they considered elemental strips on a, one elemental strip on a 2 and then, they will find out $\cos \theta_i$, $\cos \theta_j$, integrate it and do it, it is not a great thing, but basically, the question of labour and the computational cost. So, these are fortunately available in the form of charts, I will distribute the charts. Now, we will solve 1 or 2 problems with the charts, so please look at these charts. So, view, 1 view factor is, view factor between 2 parallel rectangles, in the 2nd one is between 2 parallel disks and on the backside will have perpendicular rectangles. These are some 3 simple configurations; you can do other configurations also. But with this simple configuration, if you have a perpendicular rectangle and parallel rectangle, then you can solve a furnace problem. If it is cuboids or something, all the, all are either parallel or perpendicular, then you, other view factors you have to get by algebra.

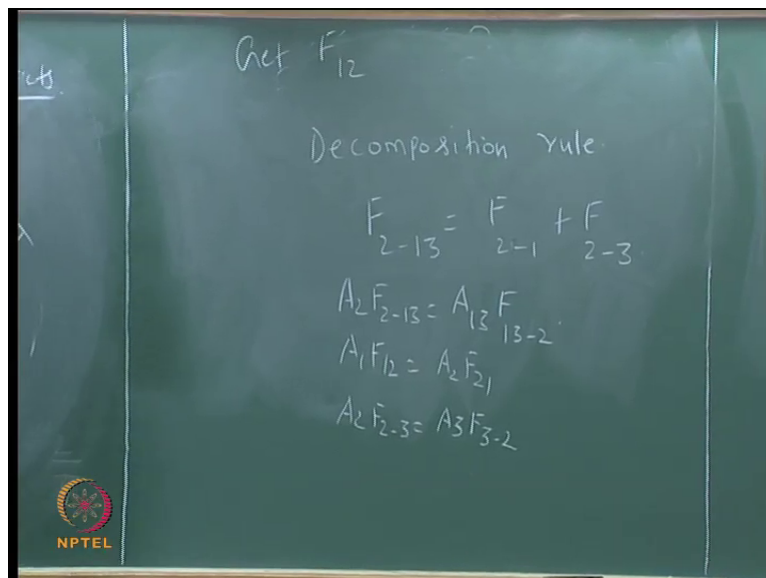
Now, as far as disk is concerned, if your furnace is in the form of frustum of cone and all that you need, from top to bottom, if it is unequal areas, general formula for unequal areas, 2 disks of unequal areas; equal area will be a special case. So, there are some, so r_i will be on the x-axis for the 2 coaxial parallel disk, then the other variable is basically r_j by l , please look at it carefully. If it is not, you can make correction now itself, you are going to use it in the exam quiz, second quiz as well as the exam, fine. Now, let us use this and solve some problems.

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Twenty, 27, let us consider 2 perpendicular rectangles, it is not really, I have not drawn it... This is 1, this is 3, this 2, 1 meter, 1 meter, for all the dimensions given, get F 1 2.

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The beauty is, F 1 2 cannot be obtained directly from the charts, F 1 3 to 2 can be obtained or F 2 to 1 3 can be obtained. So, before starting off, you have to remember a very important thing, is called the decomposition rule. Yeah, please look at the board, this is the decomposition rule, F 2 to 1 3, 1 3 is a composite, which consists of 1 plus 3 together, F of 2 to 1 3 equal to F of 2 to 1 plus F of 2 to 3.

Now, let us work on this. Agree? Yes, is everybody through with this? The decomposition rule says, that the view factor from, say, one surface to composite surface is equal to $F_{2 \rightarrow 1} + F_{2 \rightarrow 3}$. I just wrote 3 reciprocal relations. So, I will substitute for $F_{2 \rightarrow 1}$, $F_{2 \rightarrow 3}$ from these equation and let us see what happens.

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$$\frac{A_{13} F_{13-2}}{A_1} = \frac{A_1 F_{12}}{A_2} + \frac{A_3 F_{32}}{A_2}$$

$$\cancel{F_{13-2}} = \cancel{F_{1-2}} + F_{3-2}$$

So, $F_{2 \rightarrow 13}$ is basically $A_1 F_{13 \rightarrow 2}$, correct, $F_{2 \rightarrow 3}$. How many of you did not follow what I wrote? I did not do any magic here, I assume, that this is correct first, then wherever $F_{2 \rightarrow 13}$ was there, I am writing $F_{2 \rightarrow 13}$ as $F_{13 \rightarrow 2}$, $F_{13 \rightarrow 2}$ divided by $A_2 F_{2 \rightarrow 1}$, I am writing it as $A_1 F_{12}$ divided by $A_2 F_{2 \rightarrow 3}$, I am writing it as $A_3 F_{32}$ by A_2 . Now, very profound result, please look at the board, this is where most students make mistake, $A_1 F_{13 \rightarrow 2}$ is $A_1 F_{12}$ plus $A_3 F_{32}$. $F_{13 \rightarrow 2}$, here it is valid because it is originating from one surface to the remaining 2 surfaces, but if it is from 1 to 3, you cannot break it because 13 has got one area, are you getting the point. So, $A_1 F_{13 \rightarrow 2}$ is equal to A_1 . If you still want to use the decomposition rule, it should be used like this because this comes from basic energy balance, thermodynamics cannot be violated when we are using decomposition rule; you have to be careful.

Now, solve the above problem, solve the problem with the decomposition rule. Now, let us take the chart, you have to look at the chart view factor for perpendicular rectangles. So, what is Z by X , what is Z by X ? 0.67, very good and then, what else is there?

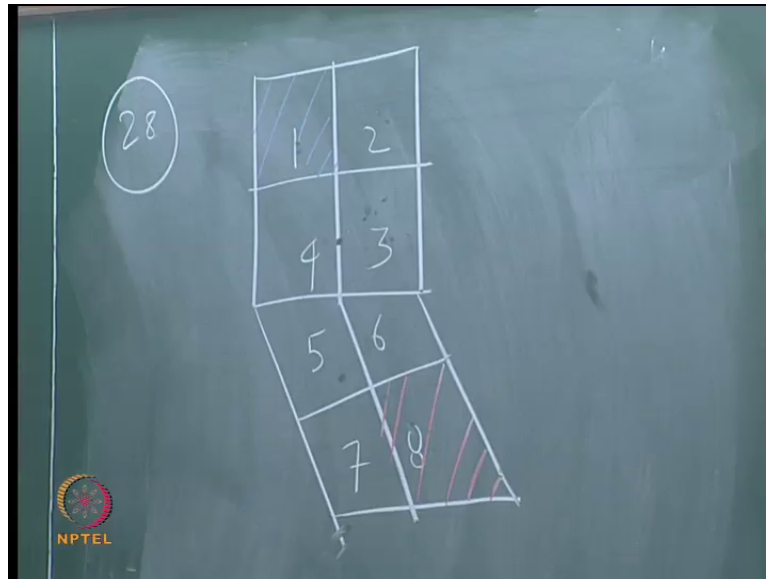
Y by X, how much was it? One point, what is Y? 4. Now, F_{ij} will be F_{21} , correct. F_{21} is F_{12} , what is the value? 0.1... Is it, did everybody get this? You are free to interpret the graph like this and use otherwise also, Z, Y or whatever; you can take the, whichever surfaces, alright. Now, (()) did you get, Tejas all of you have the charts? Mahindra did you get it? Nisha?

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$$\begin{aligned} \therefore 0.14 &= F_{21} + 0.07 \\ F_{21} &= 0.07 \\ A_1 F_{12} &= A_2 F_{21} \\ A_1 &= 3\text{m}^2 \\ A_2 &= 12\text{m}^2 \\ \therefore 3 \times F_{12} &= 12 \times 0.07 \\ F_{12} &= 0.28 \end{aligned}$$

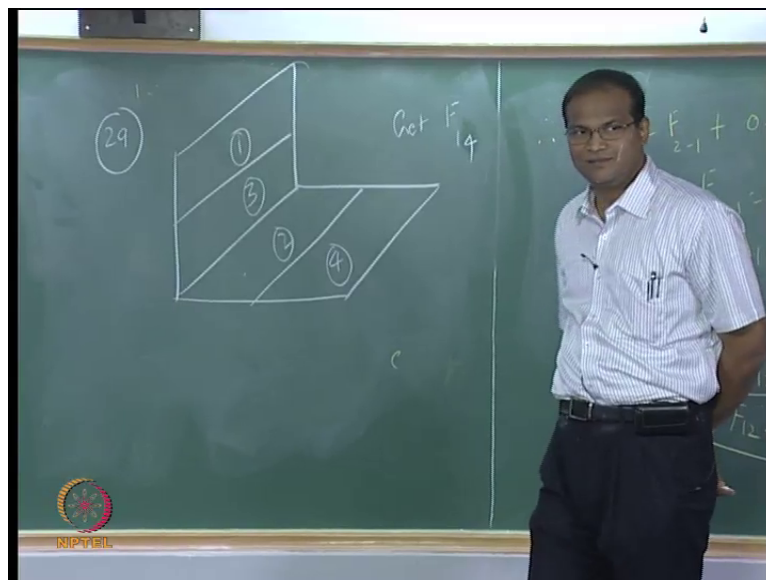
0.14 is alright. Now, we know decomposition rule, F_{21} , again from chart, correct. Sorry, F_{21} to 3, yeah, for F_{21} to 3, what is the story now? Z by X is point, 0.33. Y by X is the same, so F_{21} to 3 will be point, 0.7, means, I, we have to go home. 0.07, therefore what is A_2 , but they are same, know, yeah. I am sorry, so only 1 and 3 are same. What is A_1 ? 3 meter square, $A_2 = 12$, therefore this is correct, 0.28. Now, we can keep on escalating, you can keep on escalating the complexity level.

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So, we can take a problem; 1, 2, 3, 4, 5, 6, 7, 8, get F 1 to 8, over, quiz 2 over, but it is possible to get.

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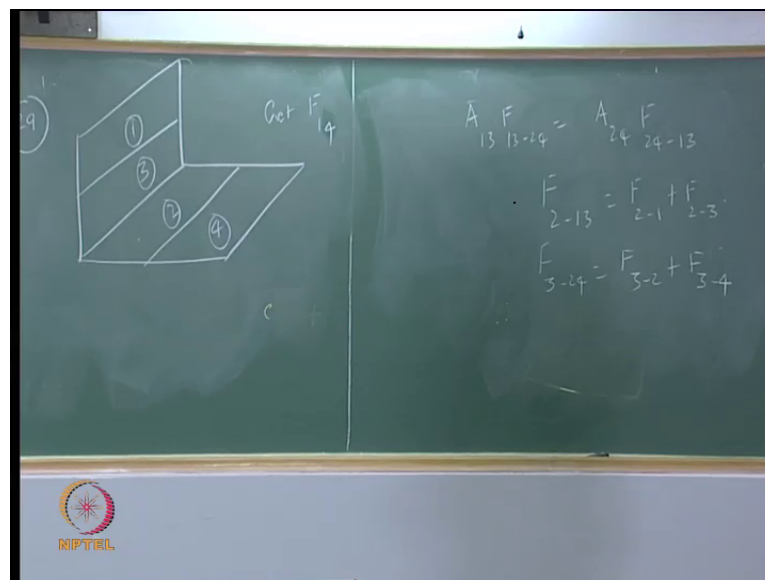


Yeah, yeah, you have, what we can do is F 1 2 3 4 to 5 6 7 8 you can get, then F 3 4 to 5 6 you can get, F 3 4 to 5 6 7 8 you can get. Then, what happens is, then some fundas you have to use. Vikram you, you have to watch this, F 1 to 8 is same as F 2 to 7, that is called the law of corresponding corners. F 1 to 8 is the same as F 2 to 7, if it is like this, if it is properly oriented and then, depending on the size, depending on the size and all

that, now you can use your common sense and reduce the, breakdown the complexity. And this, of course is, I think Professor SPV's book on heat transfer, I think he has derived. He can also derive it from 1st principle, but it will be a long story. We will solve one more problem on view factors, but that may be, I will reserve it for the next class. So, I will, I will give something more decent. So, why did I put 1 3 2 4? Get F 1 2, leave the dimensions. How will you do the geometry? Can you, can you work out the formula.

1 4, no, what I want, wait, I want 1 4, yes, 1 2 there is no fun, get F 1, get F 1 4, get F 1 4. So, maybe we will start out, we will solve this problem in the next class, but just 1 or 2 steps we can do.

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So, A... is it correct? This is a dangerous splitting; no, no, this is 1 and then, F 13 to 2, the exact you can slowly write and 15, 15 minutes I think you can solve. Can you solve it in 15 minutes problem like this in the quiz?

Yeah, not this, this corresponding corner will be difficult or I can give you a furnace, furnace I can divide it, I can ask you to find the view factor from this bottom to this top practical problem. One side is heated, other side you are keeping some material for heat treatment, what is the view factor.