

Conduction and Radiation
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Lecture No. # 22
View Factor Contd...

So, we will work out some more problems on view factors today and hopefully, today or tomorrow, we will start the radiosity irradiation method. So, we will specifically solve 2 problems, which involve the use of the charts. So, essentially, you have to use charts for three-dimensional geometries. If we recall, we started off with simple two-dimensional geometries, wherein we used the algebra or simple rules, like summation rule, reciprocal rule and then self view factor equal to 0 and all that, and then we algebraically manipulated and got the view factors.

Then, there are problems in which you have to use 1st principle in which integration, that is, you have to integrate from 1st principles and get the view factors. So, we chose a simple problem, where it was from an infinitesimally small disk, infinitely small area to cylindrical disk, it was easy. But if both were finite areas, then we have to make it $d \times 1 \times d \times 2$, you have to use double integral strip and so on.

And some people have developed techniques between 2 parallel rectangles, between 2 perpendicular rectangles, between 2 coaxial circles and so on, concentric, 2 coaxial disks. So, people have developed this integration and they have presented the results in the form of charts, as well as tables. I did not give you the tables, I have given you the charts, they are, accuracy is somewhat lower for the chart, but it is for this course. If you want to actually solve problems involving radiation and all that, you can, may be you can use the tables or you can write the program yourself and subdivide it, subdivide each surface into elemental surfaces and calculate the view factors.

Now, I have given you charts in which there are basically 3 charts, one is perpendicular rectangle, parallel rectangle and then, the parallel disk. So, we will now solve 2 problems, one involving perpendicular rectangle.

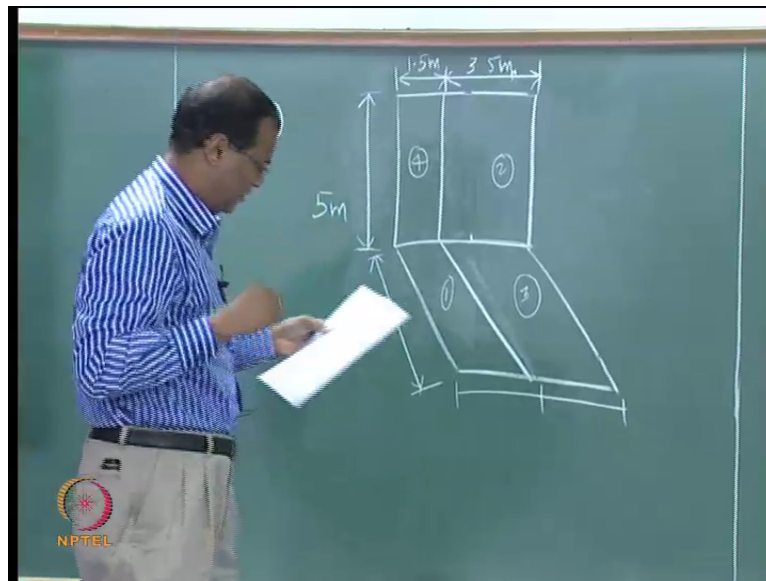
Student: Parallel rectangle.

Parallel rectangle. No, equal area, no, no, that base must be with a common base, that base must be common, but the area of the, one need not to be equal to area of 2

Student: 3 x by 1 and y by 1 (())

Yeah, yeah, so it has a common edge; it has a common edge.

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I, I, I got what you are saying, that chart cannot be used for this. Of course, we can do something, we can again subdivide and this thing and do all, all sorts of permutations and combinations. Basically, they, they will have a common edge, but A 1 need not be equal to A 2, there is no requirement. So, from the charts you will get only limited number of view, view factors, we will have to use repeated algebraic manipulations, we will have to do repeated algebraic manipulations, and in the quizzes and exams you have to be really fast, but somewhere some trick will be involved. So, I will ensure that some trick is involved; otherwise, we will go round, round. So, that smartness also is required, so in using this. Of course, as the computer gets more and more powerful, all these things will be relegated to the background. Whatever we are doing today, I may not teach it after 10 or 15 years, just like I am not teaching Bessel function and all that nowadays. Bessel function, all this not required, you can write code, script in Mat lab and solve some

equations and get the fin profile and all that, otherwise variable area, fin, we use to have Bessel function and solve it and, fine.

Now, what is a problem number?

Student: 30

28 and 29?

We have finished 28, 29.

We did not do.

Student: 29?

You cleared the thing, but left it out. What was the 29? F 1 4. F 1 4, you take it as homework exercise, we will go to 30. F 1, F 1 4 was what? No, I will not do that, I will go to some other problem, then I have to teach the whole 1 full semester on view factors.

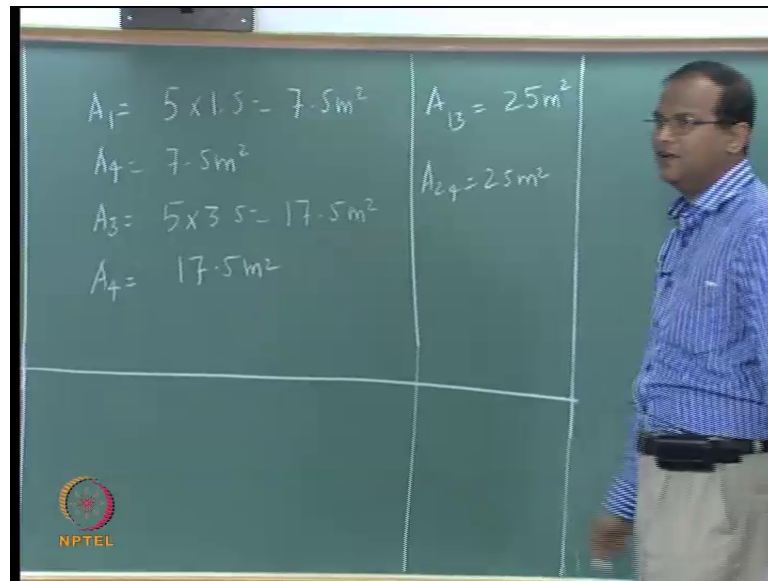
No I am having trouble. So, we will do the same F 1 2 3 4, but now 1 is here, 2 is here, 3 is here and 4 is here. This is the common edge, the perpendicular. I will give you the dimensions, I have not drawn it properly, I can still... This is 1.5 meter. So, the question is, get F 1 2.

The problem can read like this, consider the configuration shown in the figure, consideration, consider the configuration shown in the figure, 2 parallel rectangles with a common edge. Each of these rectangles is further subdivided into 2 rectangles, so that we have 4 areas A 1, A 2, A 3, A 4. Get the view factor between 1 and 2; get the view factor between 1 and 2.

Please start slowly, a good strategy would be to first write A 1, A 2, A 3 A 4 and then A 2 4 and A 1 3. Write the, write the 6 areas first, then calculate all the view factors, which can, find out all the view factors, which can be determined from the chart and then, start your algebraic manipulation. The other risky route would be, disregard completely F 1 2 3 4 is equal to F 1 2 3, F 1 you keep on doing till you get an expression for F 1 2 in terms of all the other things and then substitute A 1, A 2, A 3, A 4. This is high risk from a student perspective. It is better to just list out first the areas and list out all the view factors at, at least the teacher knows, that you know how to read the charts, how to get

the basic view factors. Then, you can start off with your algebraic manipulation. We can also post the question to get a general expression for F_{12} . I need not tell you the areas, get a general expression for F_{12} in terms of A_1, A_2, A_3, A_4 . So, I will start in parallel, please do not look at the board, you keep doing and then we will see, whether we get convergence.

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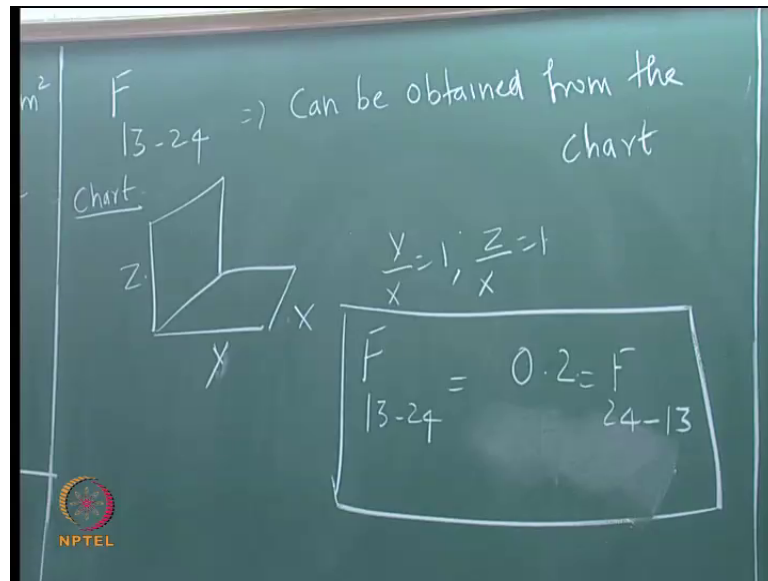


So, A_1 is, A_1 is 7.5 meter square, A_4 is also equal to 7.5 meter square, A_3 equal to... A_4 is also equal to 17.5. Latecomers, we just started on a view factor problem, it is self explanatory. A_{13} equals...

Student: 25

25 meter square, very good; A_{24} ... fine. So, we listed the 6 areas.

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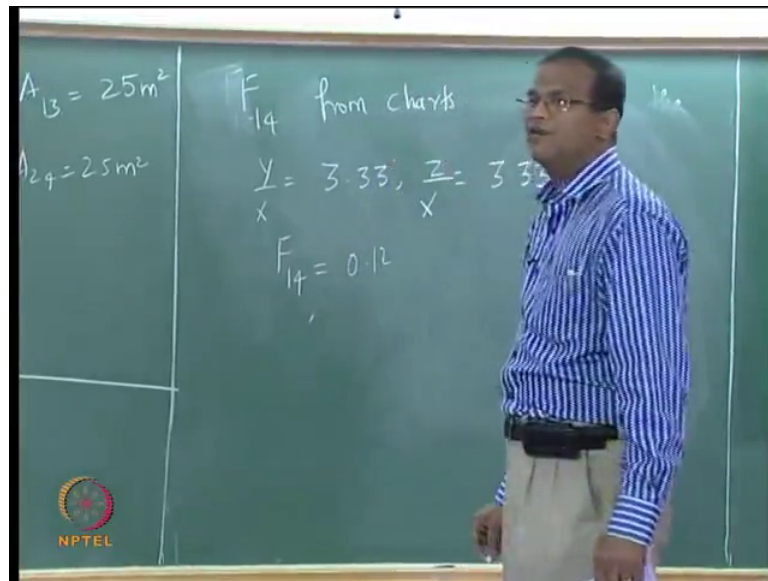


So, the easiest view factor to get is F_{13-24} , we can get it from the chart. So, you have to, so on this chart, so please take Y by X and Z by X ; please take Y by X and Z by X . Now, Y by X is 1, Z by X is also 1. F , so from the chart, you please read, read off from the chart, what did I get? I got around 0.2. So, is it ok? Is it the same as F_{24-13} ?

Student: Yes sir.

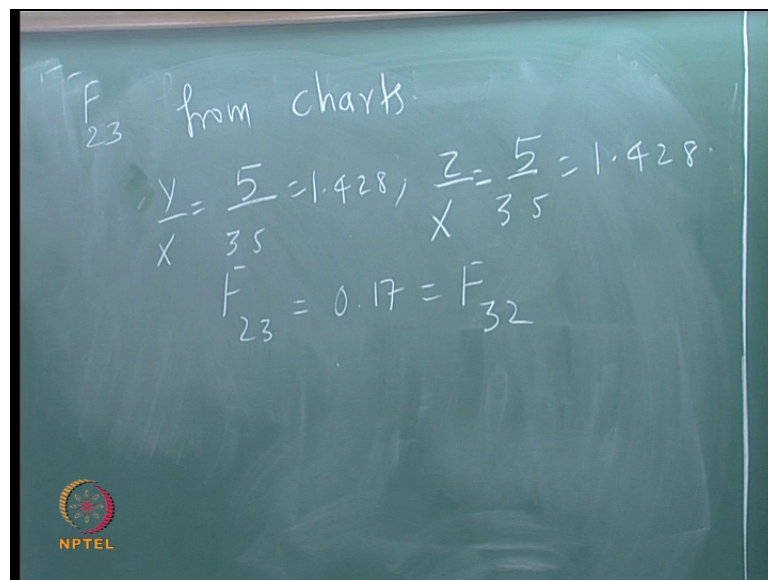
Yeah, so, we have just started, we are far off from the final answer, we are just warming up, lot of algebra is involved; F_{13-24} is fine, so far so good. Let us keep this fellow here itself, whatever we obtain we will write here. F , we will keep the figure intact, we will also keep these areas intact, we will, we will keep working on this side of the board. A view factor problem like this would be 20 minutes, 15 minutes if you are very fast, 20 minutes is a reasonable time; you after finishing, you can check it out. Now, what is a next thing?

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Now, before doing the algebra, we can get F_{14} from charts, correct; F_{14} can be obtained, yeah Y by X equal to 3 point...; Z by X equal to point, 0.12. Is F_{14} the same as f_{41} ? Yes, go back here and write.

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Let us also calculate F_{34} , yes, F_{32} . Yeah, yeah, F_{23} you want to write, I, we will write F_{23} , no problem. Y by X , 1 point...

Student: 1 point 2 8

Same

Student: Same

Point

Student: Point 0.17

17, yeah, yeah. We will, we will do it slowly, 30 to 40 percent is over, now. So, regardless of the difficulty level of the problem, all of you can do up to this stage. No great thing involved. So, if you do it systematically and at least, I, we will know, that you are able to follow. So, F_{2-3} equal to, now the manipulation will start, you have to apply the reciprocal rule and the decomposition rule. So, it is just a manipulation of reciprocal rule and decomposition rule, nothing else, some rule will not work because it is not an enclosure.

So, this reciprocal and decomposition rule, repeatedly you have to apply, use your common sense and then, we will have to crack the problem now, fine. We have got these basic view factors, not even us even 1 more view factor you cannot get from the chart, that is, it over, the use of the chart is over. Now, you, the chart can be relegated to the background now, you have to start working on that, fine. Now, we do not know where to start. So, I will start with F...

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The image shows a chalkboard with four equations written in white chalk, numbered 1 through 4. Equation 1 is $F_{2-13} = F_{2-1} + F_{2-3}$. Equation 2 is $F_{13-24} = F_{13-2} + F_{13-4}$. Equation 3 is $0.2 = F_{13-2} + F_{13-4}$. Equation 4 is $F_{2-13} = F_{2-1} + 0.17$. An arrow points from the F_{13-2} term in equation 3 to the F_{2-13} term in equation 4. The NPTEL logo is visible in the bottom left corner of the chalkboard image.

$$F_{2-13} = F_{2-1} + F_{2-3} \quad (1)$$
$$F_{13-24} = F_{13-2} + F_{13-4} \quad (2)$$
$$0.2 = F_{13-2} + F_{13-4} \quad (3)$$
$$\text{From } (1) \quad F_{2-13} = F_{2-1} + 0.17 \quad (4)$$

No I will start with F_{2-13} is fine, F_{13} ... What did we get? F_{2-13} was, we cannot get. No, no, no, no, no, no, no, no, no problem. F_{13} F_{13} , no, no F_{13} to 2 also we cannot get, no problem. What did I do? F_{13} to F_{13} to 4. No, no, let it be like that, we cannot do any, we cannot do anything with that F_{13} to 2 4. No, F_{13} to 2 4 is equal to F_{13} to 2 plus F_{13} to 4. Now, we can do something with this. Left side is 0.2 you know; F_{13} to 2 4 is 0.2 we already got; therefore, on the first one F_{2-3} is known.

Can you write it like that? Can you write? F_{2-12} plus is not common, no.

Student: Yes sir now we have to

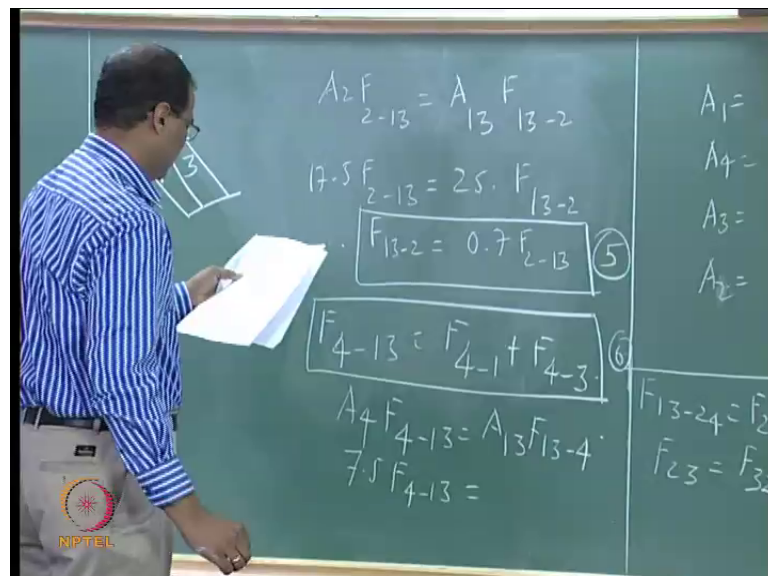
What, what, what is your problem?

Student: Sir 2 to 1, that decomposition.

No, no this is correct. From common area you have to give. I already proved in the last class, if you take F_{13} to 2, you cannot write it as F_{12} plus F_{32} if it is going from 2; it is for us to write F_{2-1} F_{2-3} , we already done it in the last class.

But the converse is not true you cannot split F_{13} to 2 like this, then it will be A_{13} F_{13} let 2 is equal to A_1 . And from 1... We have F_{2-3} , is not it, 0.17, correct. Now, I will try to link these 2, it is possible because A_{2-1} F_{2-13} is equal to A_3 , sorry, A_{2-1} F_{13} is equal to A_{13} F_{13} to 2. So, we will get rid of the figure now.

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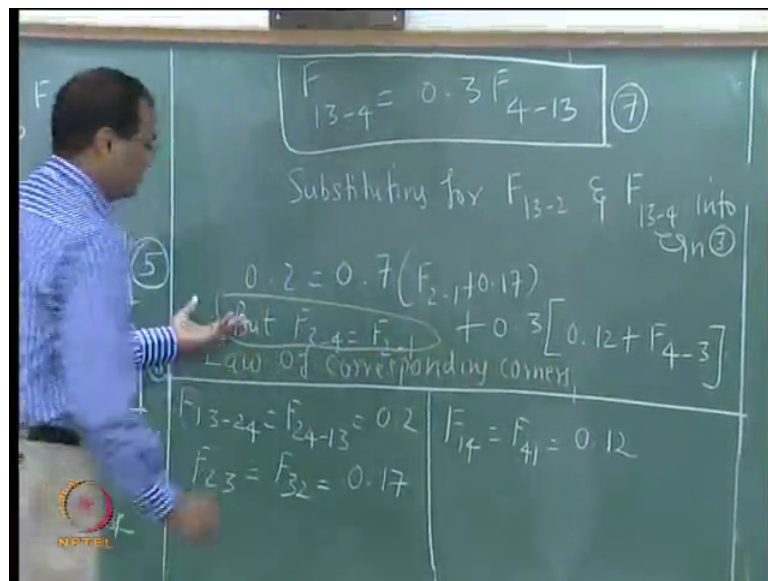
Just to, just to avoid being misled, I will just have it. Which is 1, 2, 3? This is 1, 2. Now, A 2, now what is A 2? What happened, we did not write A 2?

Student: Sir Last 1 is A 1 A 4

No, no, what is A 2?

17.5 F 2 to 1 3 equal to 25 F 1 3 2. Therefore, F 1 3 to 2 is equal to 0.7 F 2 to 1 3, this is equation number 5. We may finally get only F 2 1, if I get F 2 1 I can go home because A 1 F 1 2 is equal to A 2 F 2 1. Now, F 4 1 to 3 is equal to F 4 to 1 plus F 4 to 3, correct. And this is 6 as far as F 4 1 F 4 to 1 3 is concerned. I am having F 1 3 to 4 here and the relationship between F 1 3 to 4 and F 4 to 1 3, so straightforward, reciprocal rule. So, A 4... So, 7.5 is equal to 25. Now, I can remove this.

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So, F... So this I will call it as 7. So, I substitute for F 1 3 to 2 and F 1 3 to 4 into equation 3. So, 0.2 equal to... is it ok? 0.7... plus, what did I do?

Student: F 4 1

Plus 0.3

Student: F 4

into

Student: 0.1 2

0.1 2 plus

Student: F 4 3

Correct.

Student: (())

No, no, no, no, no, we have to be careful.

Student: (())

F 3 4; F 3 4 equal to F 1 2, that is the killer step.

Student: (())

F 3 4 equal to F 2 1.

Student: (())

F 3 4 equal to F 2 1, it is the same 90 degree. This fellow is 3.5 into 5, this fellow is 3.5 into 5, this fellow is...

Student: (())

1.5 into 5, this fellow is 3.5 into 5, this fellow is... Unless you get, unless you make use this common sense, this thing, this problem cannot be solved. A 1 F 2, you will keep on doing that is where you reduce a number of view factors by 1. Now, but the critical step now is, but F 3 4 equal to F 2 1. So, shall we give a name to this?

Law of corresponding corners; this is called a law of corresponding corners. It is important to give a name so that you do not forget. Now, we can simplify. So, I told you it will take 20 minutes.

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on simplifying

$$0.0415 = 0.7 F_{2-1} + 0.3 F_{1-2}$$
$$A_1 F_{1-2} = A_2 F_{2-1}$$
$$17.5 F_{2-1} = 7.5 F_{2-1}$$
$$F_{2-1} = 0.428 F_{1-2}$$
$$0.0415 = 0.7 \times 0.428 F_{1-2} + 0.3 F_{1-2}$$
$$0.0415 = 0.5996 F_{1-2}$$
$$F_{1-2} = 0.07$$

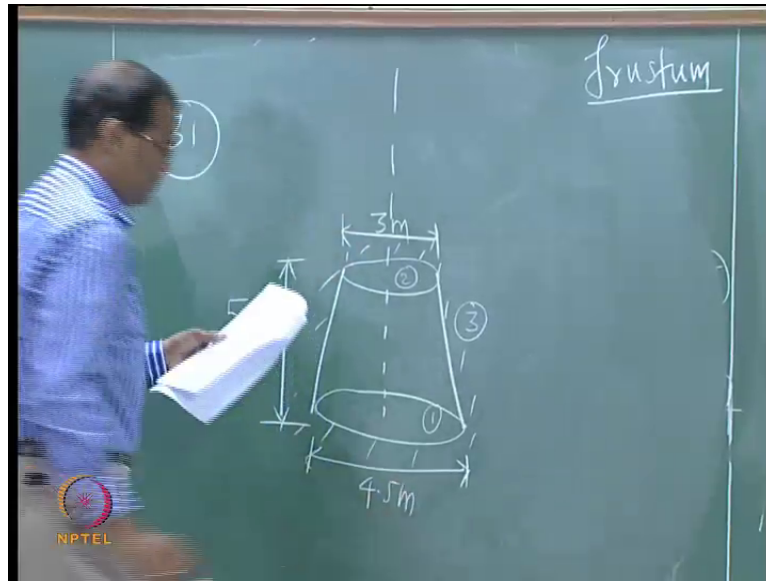
On simplifying what we get Vikram, F_{1-2} or F_{2-1} ?

Student: Only F_{2-1} sir.

Yeah, I also got F_{2-1} I think. F_{1-2} , F_{1-2} equal to F_{4-3} ; F_{1-2} equal to F_{4-3} , correct. If you get point naught 6 or point naught 8, it is ok. If, but if you get 0.7 or 0.6, it is wrong. There may be some small differences because when you read the chart, there may be some small and the error will be more if the view factor is smaller.

So, we can see, with 4 surfaces it takes 25 minutes. Now, come with your, convince, he was not happy that F_{2-1} we put it, as if there was any mistake, there we would have got F is negative. It will, it would have been a fatal flaw. Now, of course, your fluent software can handle all these, you can divide it and it will, S to S it can get the view factors, it will the counter integration and get the view factors. But still, if you have n number of surfaces, wherever n is only 3, 4 or 5, we can still do these hand calculations. Is this clear?

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Yeah, let us go to the last problem on view factors. So, problem number 31; please take down this problem. An enclosure is in the shape, problem 31, an enclosure, an enclosure is in the shape of a frustum of a cone, an enclosure is in the shape, an enclosure is in the shape of a frustum of a cone, that is a slice of the cone not the full cone, lot of people think it is a frustrum, it is frustum, r is not there, you can check up with the dictionary, the frustum of a cone. The dimensions of the enclosure are given in the figure, the dimensions of the enclosure are given in the figure, figure we will follow, dimensions of the enclosure are given in the figure, determine all the view factors.

So, so this is the enclosure, this is surface 1, this surface 2, this is surface 3, outside there is no activity, the whole thing is hollow. So, radiation is taking place inside. Let us not worry about the emissivity temperature that will follow after we learn radiosity and irradiation method, enclosure analysis, we will solve for radiosity and get the heat flux. But now, we are interested in the view factors. Now, this height is now 5 meters and this is about 3 meters, this is about 4.5 meters. If an enclosure problem is given, it is a good idea to first state: how many view factors are to be determined, how many sum rules are there, how many reciprocal rules are there, how many self view factors are there. Then, find out how many view factors have to be independently determined, then you will get an idea whether you have to use the chart or by manipulation because sometimes, we, we are, sometimes we are confident or even overconfident, that by just algebraic manipulation we can get the view factor.

Now, how many surfaces are there?

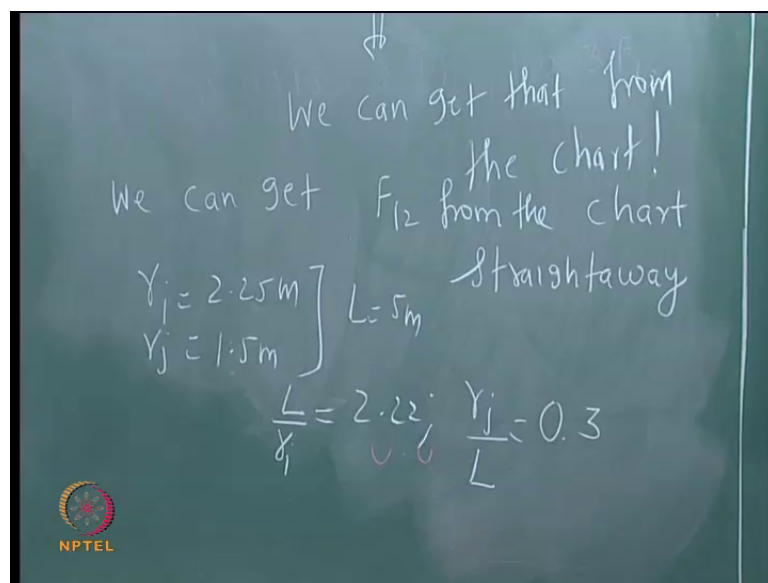
Student: (())

3; total number of view factors?

Student: (())

9

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And sum rules, reciprocal rules, self view factors, total? So, 1 view factor has to be independently determined. Fortunately, that 1 view factor is this fellow for which I gave you the chart, you determine that fellow and then, using your view factor algebra and other relations, get all the view factors now. Hence, for those rectangles and corresponding corners, you do not have to apply this, this algebra applies only for enclosure problem, we can get that from the chart. So, if you look at the chart, so the chart is given like this. So, view factor for coaxial parallel, coaxial parallel disk. So, r_i and r_j are the radii of the 2 respective disks and height is given as l , that is separation distance between them. Now, we have to use the charts and get this now.

So, we can get F_{12} from the chart straightaway now, correct. Now, r_i therefore, l by l equal to l by r_i , how much? 2 point, 2 point 2 r_j by l . Point 3. So, based on this, based

on 1 by r i, based on 1 by r i and r j by 1, based on 1 by r i and r j by 1, you can straightaway get the, how much did I get? I got 8.08.

Student: (())

0.07, ok, no problem.

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$$F_{11} + F_{12} + F_{13} = 1$$

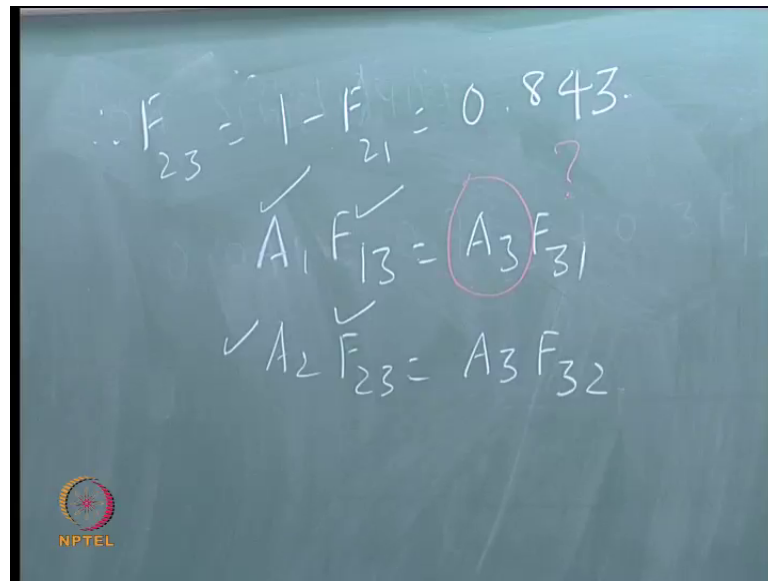
$$\therefore F_{13} = 1 - F_{12} = 0.93$$

$$A_1 F_{12} = A_2 F_{21}$$

$$F_{21} = \frac{A_1}{A_2} F_{12} = \left(\frac{r_1}{r_2}\right)^2 F_{12}$$

Now, F_{12} , so the 1st view factor we got. What is F_{11} ? 0, correct. So, we got 3 view factors now. We have solved for all the view factors for the 1st, like way 2nd surface also we can do, but when you come to 3rd surface, we are in trouble, it is only minor trouble, $A_1 F_{12}$. So, this F_{21} is even smaller. Is it, which one? r_1 is, r_1 is small, no problem. F_{21} is how much? Point, let me see, F_{21} I did not calculate at all.

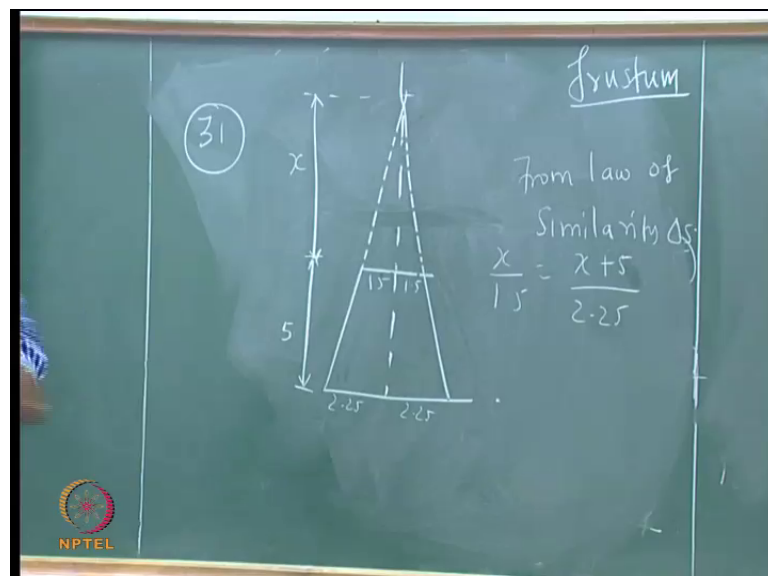
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F_{21} plus F_{22} , F_{22} equal to 0, therefore, F_{23} ... very good. 6 view factors are over, 6, 6 view factors are over, now we can also go a little further. Left side is completely known, A_3 is not known.

A_3 you have to, you have to find out from your fundas, from your knowledge of geometry, please do it. If, if you get A_3 , you can straightaway get F_{31} and F_{32} . Once you get F_{31} and F_{32} , F_{31} plus F_{32} plus F_{33} equal to 1, F_{33} is 1 minus F_{31} minus F_{32} , then it is over. In order to get this... You have to connect.

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So, this is, what, what is this? 1.5. Now, I call this as x. The law of similarity triangles, x divided by 1.5 is equal to x plus 5 divided by 2.25, is that correct? x divided by 1.5 is equal to x plus 5 divided by 2.25; Vikram is it ok? Now, 2.25, x, x equal to 10, huge, x equal to 10 meters. Now, the surface area of the cone is pi rL where L is the slant length. So, we have to find out the slant length now. So, L is a slant length. Now, there are some fundas involved, it is not the full cone, so pi rL of the big L minus pi rL of this will give the pi... So, you can take it as pi rL 2 minus L, no, no

Pi r 1. Pi, pi, pi r 1 L 1 minus, correct.

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$$\begin{aligned} \therefore F_{23} &= 1 - F_{21} = 0.843 \\ A_1 F_{13} &= A_3 F_{31} \\ A_2 F_{23} &= A_3 F_{32} \\ 15.9 \times 0.93 &= 59.6 \times F_{31} \\ F_{31} &= 0.248 \\ 7.06 \times 0.843 &= 59.6 \times F_{32} \\ \therefore F_{32} &= 0.10 \end{aligned}$$

So, L equal to root of 225 plus 2.25 square; is it correct? 15 15 square, 15 square is 225, 225. How much is it? 15 minus... So, let us call this L 1 and this fellow is L 2. L 2 equal to 25 sorry, 100. No, no, 10.1, therefore...

Student: 59.6

Yeah, what? No 2 pi r, what 2 pi r? The circumference perimeter will be 2 pi r. It is pi r, pi r L is cone, you can derive it if you want, but that is not... Which one? Which, which comes out to the same, it has, that is the only correct. 2 pi r L.

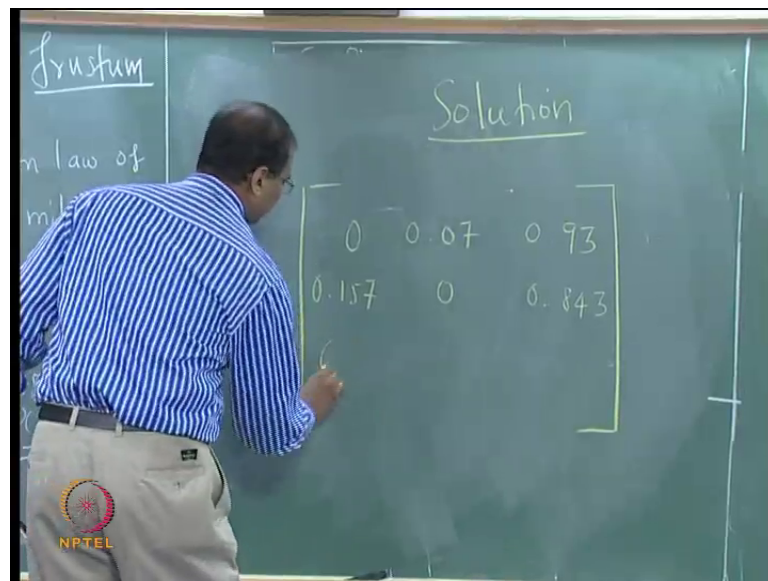
Student: Sir, you take the area of the triangle and do half base.

What is the $2\pi r$? What, what I have done is correct, there is no... Now, A 1 F 1 3, what is A 1? What is A 1? 15 15.9 into... F 1 3 was, what was F 1 3? Point, 94; F 3 1, point, very good. Similarly, A 2, 7 point, 7 point.

Student: F 2 3

F 3 2 equal to point 1...

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So, finally, you have to put the solution like this, which can be entered into your computer program. If you are doing the enclosure analysis because it is 3 by 3, you are able to do it with hand, if it is 10 by 10, 20 by 20, you will write a program to determine the view factors and store it once and for all. Then, when the temperatures are dynamically updated, this fellow will not change because this fellow is a function only of the geometry.

However, if you have a problem in which the surface is also changing, the dimensions are changing, which can happen if you have an ablating surface, a reentry vehicle enters the atmosphere and because of ablation, few millimeters of the surface is getting evaporated, sublimated away, then the geometry will change. So, geometry linked view factor calculations will be there, that, that is the state of the art; that is the state of the art if you want to do radiation calculation at a level, which you will make it general worthy.

Now, as far as our problem is concerned, so point naught 8, no, no, 0... point... Yes, this is a decent enough view factor problem.

So, we just solved 2 problems today that is quiz; quiz I will ask 3 questions. So, this is the difficulty level, this is the difficulty level with the classroom environment. When you actually solve your problem for your project or research, it will be much more complicated. People have thousands of elements and in fact, in convections, always you have millions of elements and the radiation is, we do not, people normally do not update radiation in the every iteration because convection is, radiation is well-behaved compared to convection. So, once in 10 or 20 iterations of convection we will stop and then do the radiation calculations and then proceed. Even though it is 3 to the power of 4, you think it is non-linear. The convection is $u \frac{d}{dt}$ by $\frac{d}{dx}$ or $u \frac{d}{dy}$ by $\frac{d}{dx}$ is terribly non-linear compared to this. The convection, the advection because direction dependent if, if, if it is a recalculating flow. If the direction changes, then the u , v , all that will struggle to convert. So, we will stop.