Conduction and Radiation
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Module No. # 01
Lecture No. # 23
Enclosure Analysis

So, now we are at a very important position, in our study of radiation heat transfer, where we will actually do the enclosure analysis. So, so far, last two months, we have developed the background required to do the radiation analysis. Now, now we are ready to do the radiation analysis, that is if there is a n surface of enclosure, how do we find the net radiation heat transfer between the various surfaces.

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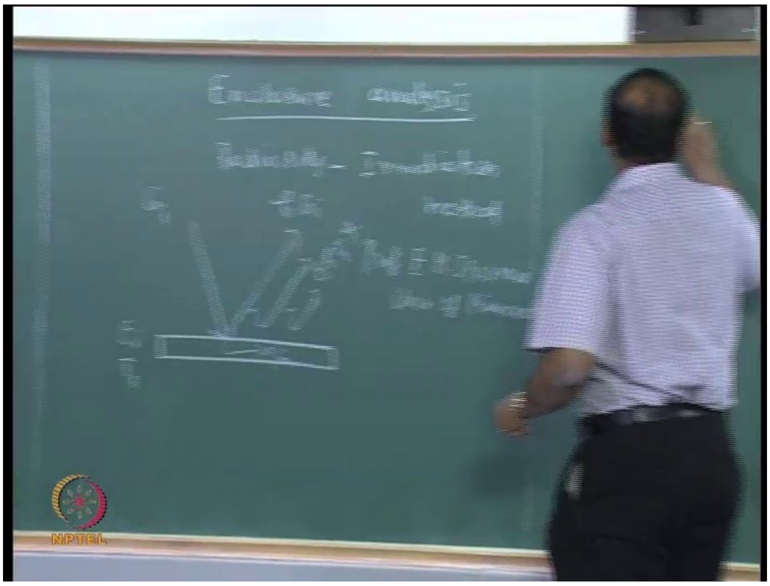

There are several ways of doing it, undergraduate texts. In undergraduate texts and undergraduate heat transfer courses, people must have taught you what is as the known as a network analysis, they will draw radiation resistance, and will put a series and parallel combination, and they will have an analogy to electrical resistance and solve the problem, but the problem is this network business is has the number of surfaces increases it becomes increasingly messy to handle all these resistances.

So, in this course I will not teach you the network method at all, I will teach you what is called the Radiosity Irradiation Method, which is valid from one surface to any number of surfaces. So, what we are going to do essentially is the Radiosity, please remember that the credit goes to professor F. M. Sparrow, university of Minnesota in Minneapolis, which used to be the mach of heat transfer, once upon a time it used to be the mach of heat transfer, all legendry names like Eckert, Sparrow, Goldstein, Edible, T. W. Simon all those people were who were there in this lab, even today Sparrow is there, of course professor Eckert died, Sparrow was also the editor of the ASME general of heat transfer for a longtime.

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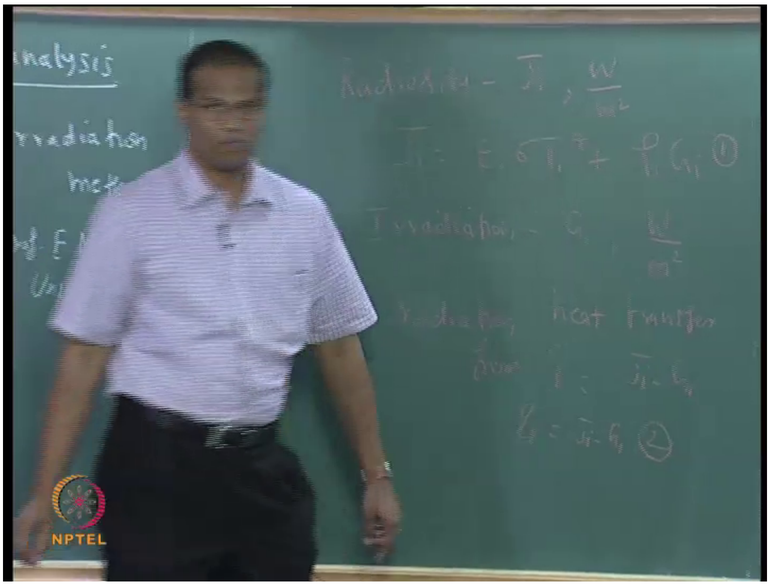

So, this was developed in the sixties, we have not got a method which is more powerful than this, and it is eminently programmable on the computer. Therefore, it blends itself easily with c f d calculation. So, combined heat transfer problem, Radiosity Radiation Method is very good, before that will have to define certain terms, before getting into the

Now, So, there is a irradiation G i falling on a surface, this surface has got emissivity, this is what emissivity is this now, be more specific hemispherical total, it is hemispherical total emissivity, this is the temperature of the plate. Now, out of this G i certain potion is reflected, that is rho i g i, because this fellow is at a temperature greater than zero Kelvin, he will also emit. Out of the incoming radiation G i certain portion is reflected, certain will be absorbed, alpha i, G i and epsilon i, sigma t i to the power of 4.

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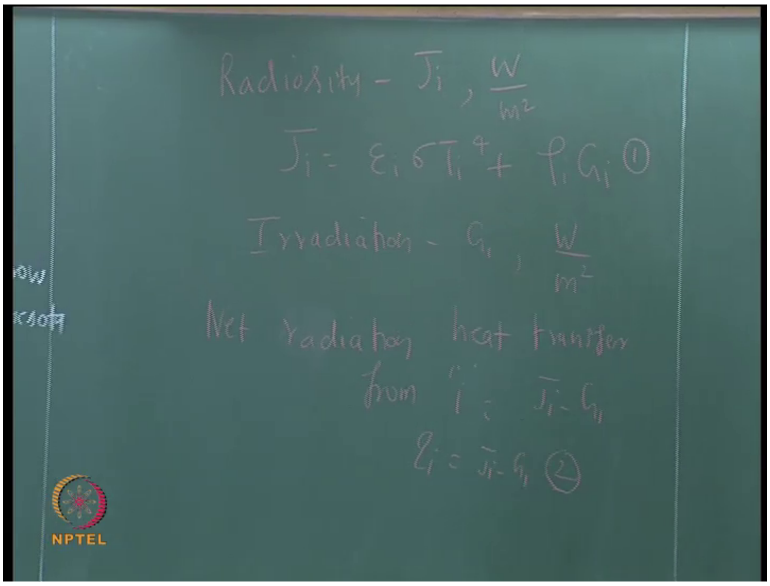

Now, Radiosity is the leaving flux or the leaving radiation this is equal to (()) The incoming radiation is called G, i am putting G i, because i can be any surface. For a single surface the subscript is meaningless, but if there are three four surfaces, G i refers to i can be 1, 2, 3, 4, it can be G 1 G 2 G 3 G 4. So, G is also in meters per watts per meter square. Now, net radiation, we call it as q i for an opaque surface, is this color not very good, so not good.

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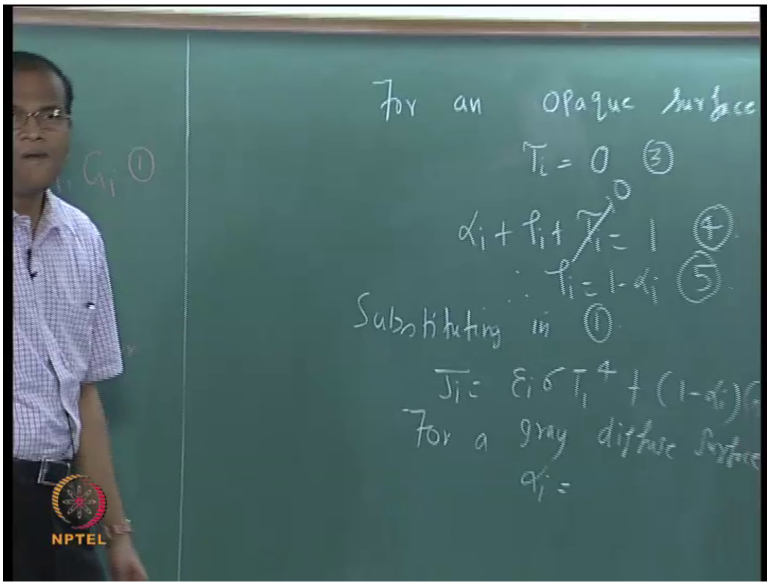

Let us see, for an opaque surface T i is equal to 0, the transmissivity is zero, alpha plus, rho plus. This is the fate of the incident radiation, the incident radiation could be absorbed, it could be reflected or it could be transmitted. For an opaque surface there is no transmission, this only absorption or reflection. Therefore, the substituting in one, that is the Radiosity relation, or the i th surface of the enclosure, would only one surface keep it as J or j 1 whatever. Now, for gray diffuse surface, alpha i equal to Epsilon i.

Therefore, if you want to calculate the radiosity from a particular surface, you need to know the emissivity, you need to know the temperature, and you need to know the irradiation falling on that surface. If the irradiation is because of radiation from several surfaces, then we have to worry about the Radiosity of these several surfaces, that is what the enclosures analysis is all about, but we are now in the preliminary stage, we are trying to first get the expression for net radiation heat transfer. So, that we are able to use when we eventually use the enclosure analysis.

Now, q i. So, if you are able to find just the irradiation and all the surfaces you have solved the problem, but it is not so easy to calculate the irradiation on any surface, because irradiation of any surface is because of the radiosity of other surfaces. So, therefore we may simultaneously solve either for the radiosities or for the radiation, we are now trying to develop relationships or expressions for this net radioactive heat transfer. Now, can we have some other expression, so G i is, i got an expression for G i, I can substitute for G i in 10.

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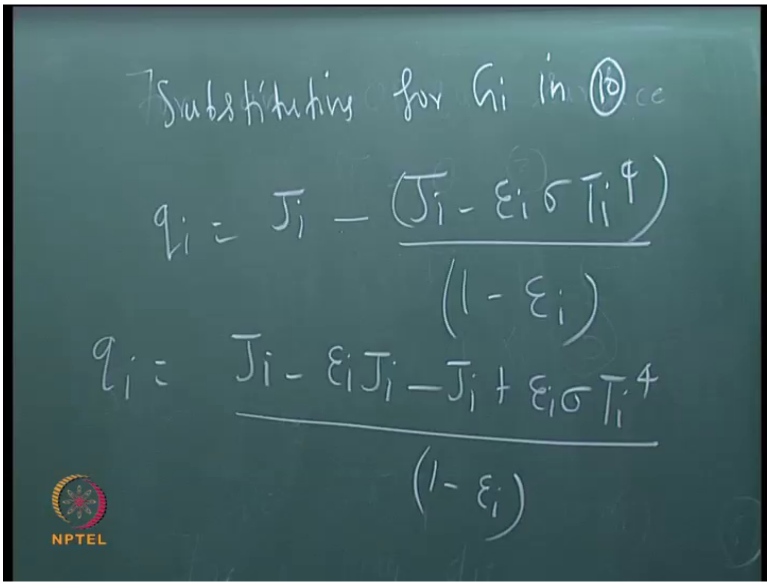

Simplify cross multiply and simplify further, is clear I am not doing any great stuff, i am just doing some algebraic manipulations that is all. Now, I can cancel J I, let us called this equation 12, what is the difference between 9 and 12, both are expressions for the heat flux, but nine is in terms of irradiation, 12 is in terms of radiocities, but our original expression is in terms of both radiosity and irradiation. If, we are evaluating the radiosity and irradiation for thousands of elements, if it calculation involves hundreds of thousands of millions of nodes, there is no point in simultaneously storing radiosity and irradiation, because one information in radiosity can be obtained from irradiation or vice versa. Therefore, it makes sense to store only one of the two quantities. So, some people just use the irradiation method where they will just solve for G. Some people use the radiosity method where they will just solve for J. Radiosity irradiation is general method, but if you have if you have too many nodes, too many calculations to be done, there is no point in storing both J and G, one word of caution can equation 12 be applied for all cases.

It cannot be applied for a blackbody, because there is a singularity, because the denominator becomes zero, not applicable so, we have to say that 12 is not applicable for a blackbody, but in a program if you want to, I have used it when I did my PHd and I wrote programs for all these radiosity irradiation, I used to, I still use this, but I will make blackbody point 9 9 5 or 9 9 9 something. It will still work, if you make 1 point 0 0 only this fellow will go mad. So, not applicable for. So, now, before starting the radiosity irradiation method, now I gave you expressions for the heat flux, now what remains is basically to determine the radiosities in n surface enclosure, but if before doing that we will do one last thing, is this clear up to this stage.

We have to look at what is called reradiating surface, from the name what do you thing a reradiating surface means. A reradiating surface is the radiation equivalent of an insulator, in an insulator the q equal to zero, that q could be q conduction or q convection, if q radiation equal to zero in a radiation problem then it is called a reradiating surface, we do not say adiabatic surface.

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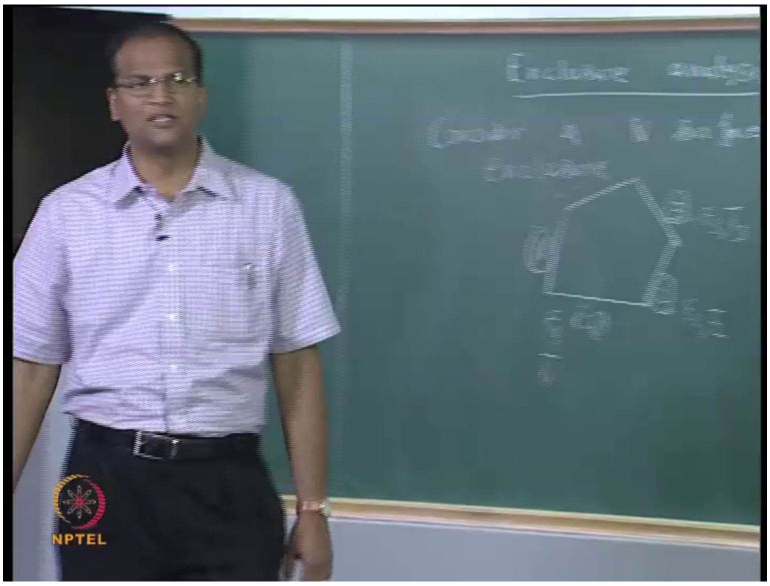

A reradiating surface is one, is equivalent of the insulated surface in conduction convection. How do we get that reradiating surface, you can have a surface and backside you make heavy insulation, so that the no radiation passes through, I mean there is no conduction or conduction heat transfer, and you prevent all possible opportunities for heat transfer to take place, then when you do that whatever is impinging on it must go out. So, J i minus G i equal to zero. Therefore, for a reradiating surface J i equal to sigma T to the power of 4. It is a remarkable result, because the radiosity of the reradiating surface is independent of emissivity, very very significant result. The radiosity of a reradiating surface is independent of the emissivity and it will come, and it will come to an equilibrium temperature of T i depending upon whatever irradiation it receives from the neighboring fellows, so it will come to some equilibrium temperature. This, sigmaT i to the power of 4 is also equal to G i. So, his temperature, whose temperature, the reradiating surface temperature is decided by his neighbors or by his friends, the other surfaces the enclosures will decide his temperature. So, if there is a three surface enclosure, one is a hot surface, for example heat treatment furnace. In surface one you are supplying the heat. In surface three you are putting some metal objects and heat treating them, and this is the intermediate surface, he is the mediator, he takes the heat from the hot surface and passes it on to the other surfaces. He will come to a temperature which is in between these two. So, such reradiating surfaces are frequently used in furnaces and enclosures, in combustion chambers and so on.

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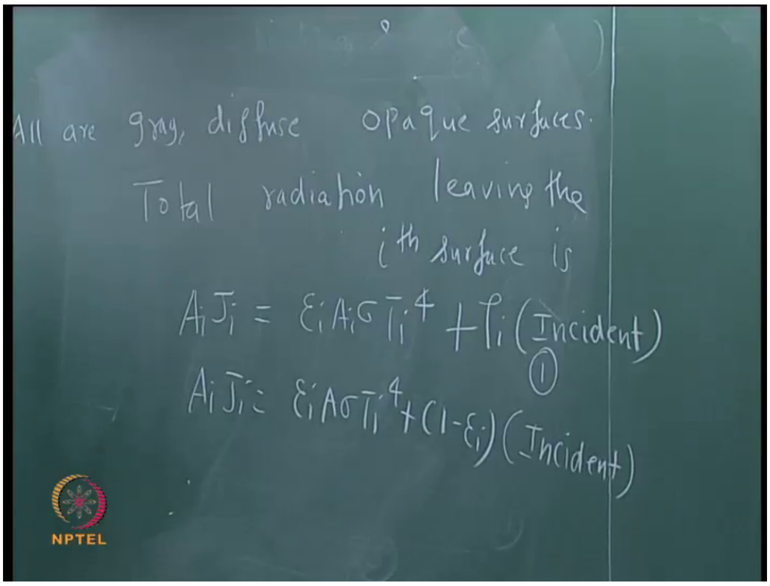

Now, let us go to the enclosure analysis. So, consider an n surface enclosure, each of these surfaces is characterized by hemispherical spectral emissivity and temperature, all of these are gray diffuse opaque surfaces. So, what is a total radiation which is leaving the surface i, the i th surface of an enclosure, what is the total radiation which is leaving, total radiation leaving the i th surface of an enclosure. Total J i is still the flux, J i into area.

J i into A i, total radiation leaving an i th surface, this must be equal to first part, emitted plus reflected, what is the first part, epsilon i A i sigma T i to the power of 4, what is that question mark. Not an incident. Is a reflected, Is a reflected part, the reflected part will be rho times of whatever is incident. So, we can say that this is correct; we can work on this further.

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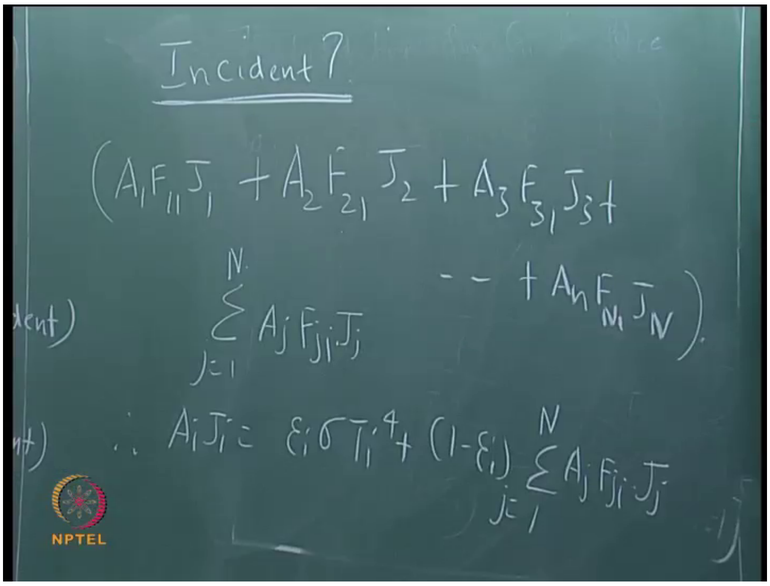

What is this incident coming from other surfaces; this is the question we have to answer. Sum of radiosities into view factors. Sum of radiosities into view factors, good enough, fair enough multiplied by the respective area, because I have put watts per meter square into. So, I will say incident could be, let us start with A 2. If, there is a self view factor it could be A 1 F 11 J 1 plus A 2 F 21 J 2 plus A 3 F 31 J 3 plus A n F n 1 J n. So, can we write it in a very compact form sigma.A j

Student: A j

F j 1 For the i,th surface,

Student: J i

J j summation.

Student: J equal to 1 to n

J equal to 1 to n. Is that clear to everybody. But what is A j f J i.

A i F i j, so one two.

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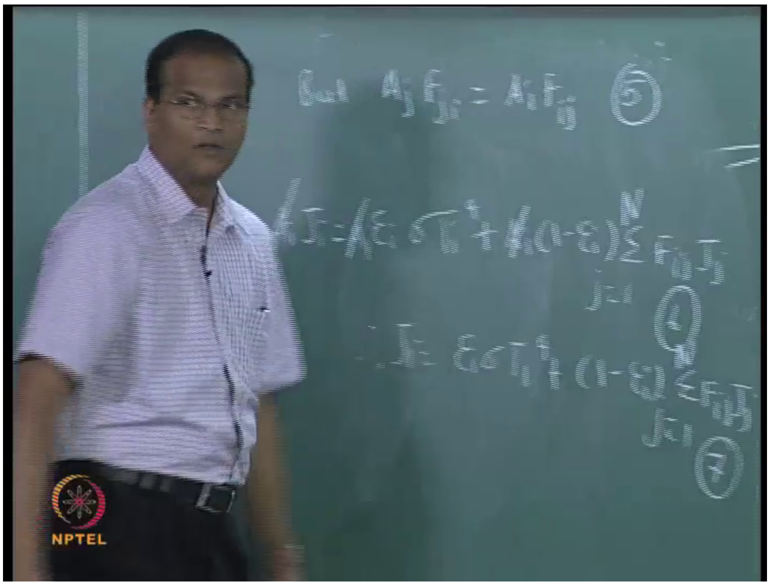

Can I cut the A i throughout. Therefore, the radiosity of the i th surface J i equal to epsilon sigma t to the power of 4 plus one minus epsilon sigma F i j J j, sigma F i j J j should not get confused, do not put F J i there, J i equal to epsilon sigma t to the power of 4 plus one minus epsilon sigma F i j J j, that F i j can include F i i, if you will be in trouble, if have a concave surface.

Otherwise the first term F 1 1 for the first surface, the second term F 2 2 for the second surface, the third term F 3 3 for the third surface will all be knocked out, if you have plane or convex surface. Suppose, I give you sphere within sphere cylinder within a cylinder, then some F 2 2 may be nonzero then you have to consider. Once you have this, but what was our nomenclature for this incident radiation, what did we call this incident radiation as G i.

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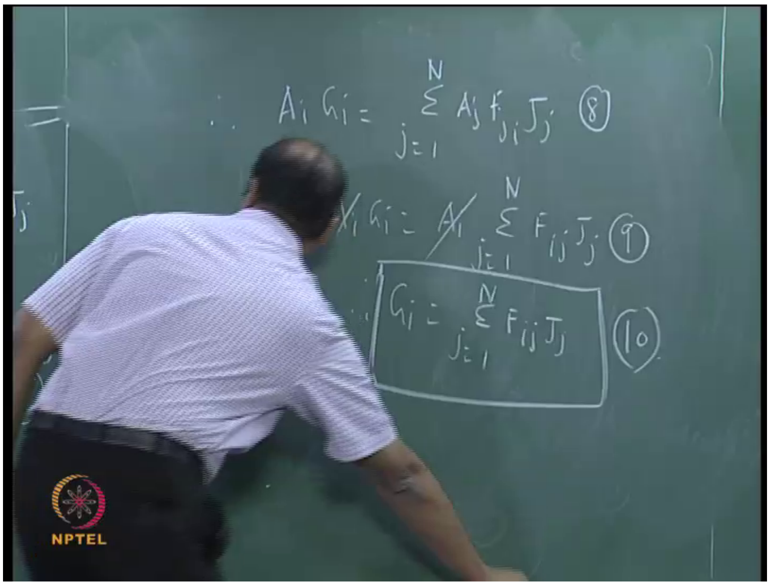

So, this term is nothing, but therefore, if you want to solve for the radiosity in n surface enclosure, the first part is the emission part, therefore you must know that the stefan-boltzmann law is at work.

So, it is just not sigma t to the power of 4, it is epsilon sigma t to the power of 4. So, you need to know the hemispherical spectral emissivity. Now, you need to know exploit the Kirchhoff's law, and first figure out that there is an opaque surface then for a gray diffuse surface, rho can be written as one minus alpha and then one minus alpha can be written as one minus epsilon. So, your knowledge of radioactive properties comes into effect there, and then when you write sigma F i j J j, your knowledge of view factors comes into picture. So, you all that you have studied is incorporated in one single equation J equal to epsilon sigma t to the power of 4 plus one minus epsilon f sigma F i j J j.

In an n surface enclosure there are n such radiosity relations, if you have n equations in n unknowns they can be easily solved, you can get all the radiosities. Once you get all the radiosities, you can straightaway use the formula for the net radiation heat transfer from any surface in terms of radiosity, otherwise if it is only three surfaces, if you have the time and the patience, you can individually evaluate the G 1 G 2 G 3 for all the surfaces from if the J which you have obtained by simultaneously solving, then get J minus g for 1 2 3 surfaces, then essentially you have solve the problem of radiation heat transfer in an enclosure where there is no conduction and convection, if conduction and convection are there you will write the additional energy equation and solve, but this will be the radiation portion of the solver;

However, here the enclosure is evacuated or is filled with a medium which is not participating in the radiation. This is the enclosure theory developed by professor sparrow and his colleagues at the University of Minnesota, Twin cities in Minneapolis. Now, up to three surfaces or max four surfaces, you can solve these using hand calculations, if a number of surfaces in enclosures exceeds four, you have to use the computer and since it is system of simultaneous equation, it is eminently solvable with iterative gauss-sidle method, you do not have to invert. So, you can use the gauss-sidle method. So, you start with some assume J 1 equal to J 3 equal to j 4 equal to 1000 watts per meter square start with that and iterate, then once it converges, then you get all the J's, you get all the G's and you the get the net radiation heat transfer, this is how the enclosure analysis is used.

Now, let us let us have up warming up exercise by considering just the net radiation heat transfer from simple one plate, there is only one surface, there is no enclosure, but there is no problem, we know that even this even though the second surface is not there, we can always close it by hemispherical basket, shall we do that and see whether the enclosure theory works, and will get our familiar relations q equal to epsilon sigma T to the power of 4 minus T infinity to the power of 4, let us consider one surface enclosure. So, this is basically the enclosure theory.

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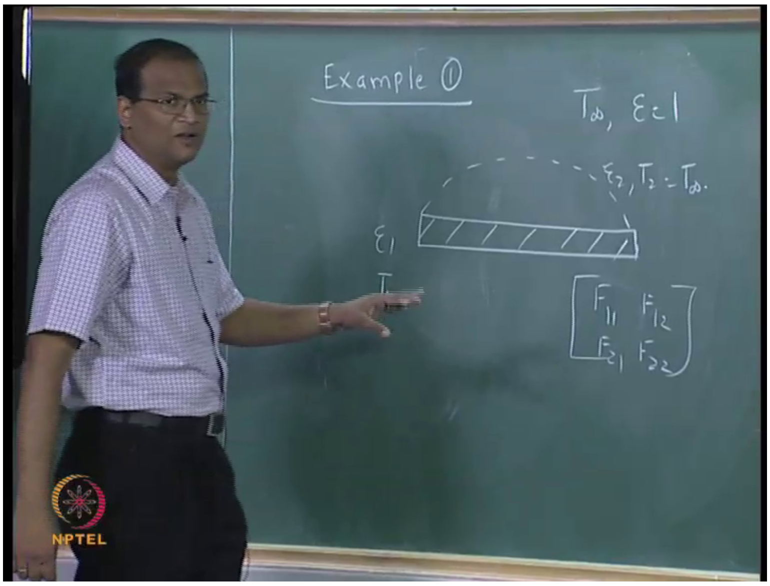

We want we can, we do not call it 31 no problem, we will call it as an example. Let, us say it is epsilon 1 and T 1 or say it is T infinity, the outside is considered to be perfectly or it is a perfect emitter, zero reflectivity surroundings.

So, how many surfaces are there, two surface enclosure, there is no need to determine all these view, what is F 1 1 is 0, F 1 2 is 1, we can stay like this, because I will tell you why, it is enough to stay like this.

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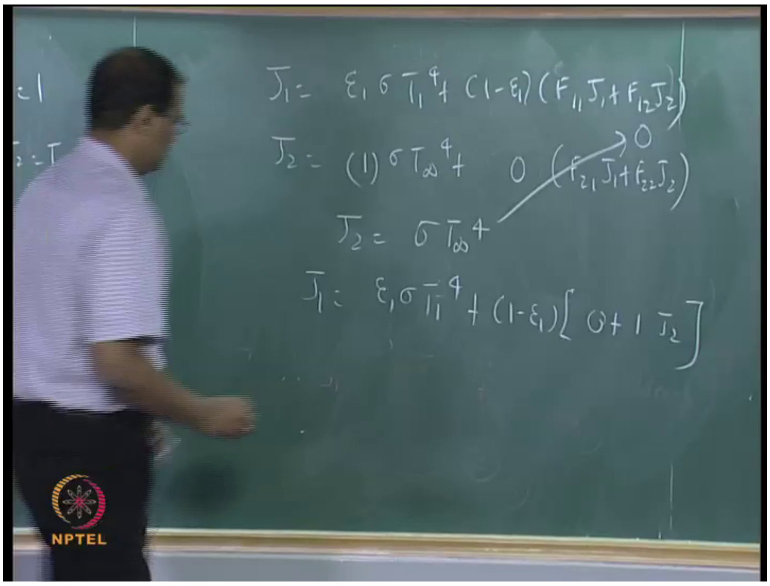

J 1, this is 1 that gets naught of, because the emissivity of the surroundings is considered to be 1. So, it is a waste of time to get F 2 1 and F 2 2 and all that, whenever you have surroundings like this do not waste time in calculating those view factors. Now, therefore, J 2 is straightaway J 1, F 1 1 is 0.

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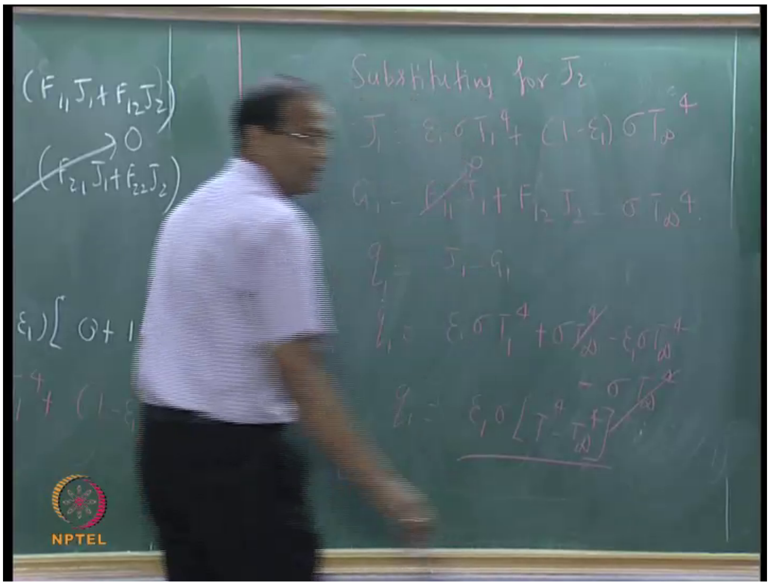

Now, substituting for J 2, what is G 1, what is a irradiation on 1, what is F 1 1 is 0 F 1 2 is 1, J 2 is sigma T infinity, what is q 1, J 1 minus G 1. So, we have used the radiosity irradiation method, it can be used even for the simplest of problem, this is a formula which we learnt probably in a high school or maybe you learnt it again in the third year heat transfer course, this is one of the most abused formulae in heat transfer. Now, having gone through this course, you must know what are the limiting conditions under which this is valid; single plate at a uniform temperature, the plate is having a uniform radiosity, it is characterized by one hemispherical spectral emissivity, the surroundings temperature is constant, the temperature of the surrounding does not change, there is no irradiation from any other object, there is no other object in the vicinity, under these conditions, it is a net radiation heat transfer from that body, simply we cannot use.

Surface is (( )) that will lead to some other, that story will be different. So, when you look at that formula, you must see, what are the limitations under which that formula can be used, what are the situations under which this formula cannot be used, this must become clear to you.

Emissivity 1 also (( )). Emissivity 1, it will work what is the problem. Emissivity 1 it will work, sigma T to the power 4 minus T infinity to the power 4 which emissivity is 1, surrounding, surrounding emissivity is 1 that is, because we are treating it as an enclosure. Now, let us go to equally, so is this fine.

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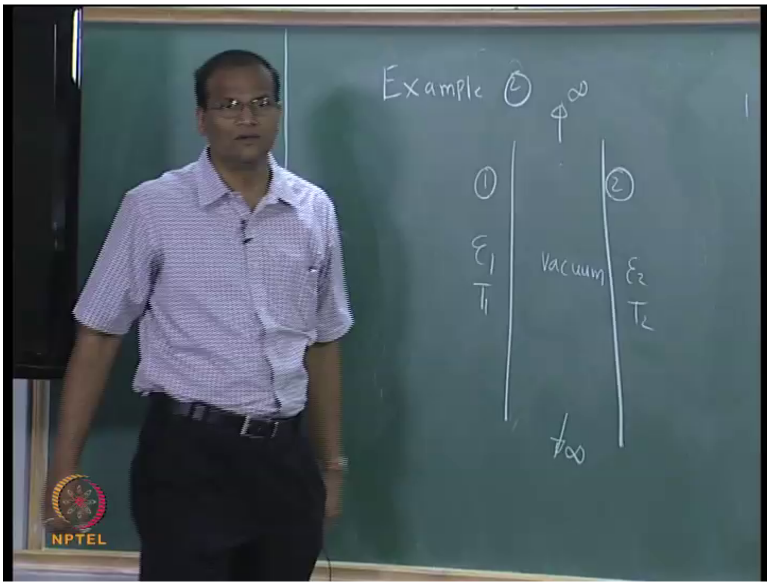

Consider two parallel plates, they are infinitely deep in the, they are also infinity in extent in the direction perpendicular to the plane of the board. Now, epsilon 1 T 1, there is a vacuum, this will happen in insulation. For example, a double plane glass which you see in your air-condition coach of Rajdhani, Shatabdi express and all that there are two glass sheets, probably you can model it using this or you want to model insulation, there are several layers of insulation, there, is an air gap in between and you feel that the thickness is so small that natural convection will not set in, then you can use this. So, it need not be completely evacuated, this is surface one, this infinity in extent is basically to help you figure out that F 1 2f is equal to 1,1 F 1 1 equal to 0.

Now, the story is very simple, find out the net radiation heat transfer between the two surfaces using enclosure theory. The question is, determine q 1 to 2 using enclosure theory. So, first write out the radiosity relations for these two. In the radiosity relations you will have view factors. The view factors are so obvious in this geometry, in other geometry first step is to get the view factors, second step is to write the radiosity relation, third is to manipulate the radiosity relation and get the radiosities and the fourth step would be to get q equal to J minus G.

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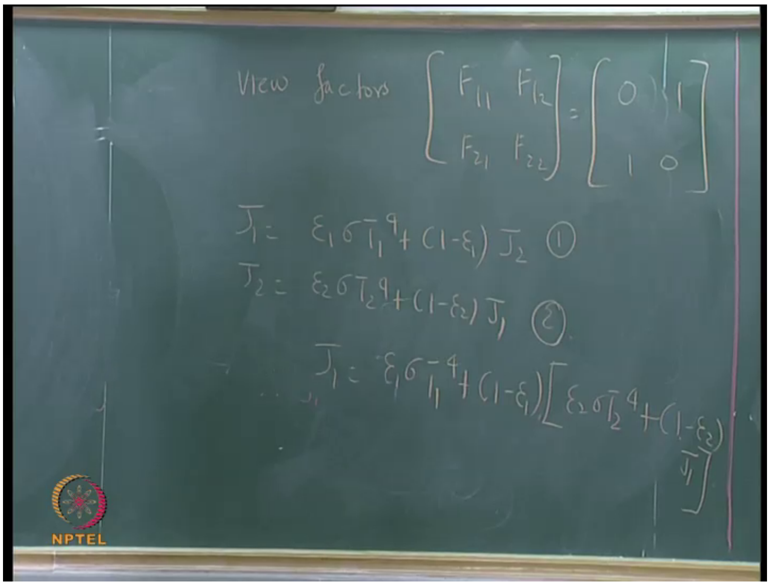

All of you look at the board, first step it is so simple everybody knows this.

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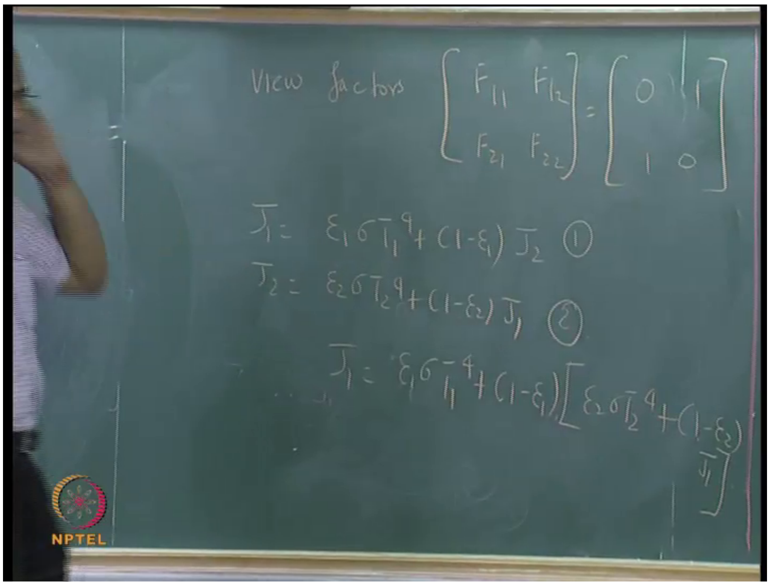

Now, write the radiosity relation, can I write like this, I have already incorporated the information I have on the view factor, we just mathematical manipulations that is all, is it correct, yes. So, what is the beauty of that expression 4. The beauty of the expression 4 is, by inspection I can write an expression for J 2, J 2 will also be epsilon 1 sigma T 1 to the power of 4, because epsilon 2 sigma T to the power of 4 minus epsilon 1, epsilon to sigma T 1 to the power of 4, denominator will be the same, by inspection, if you people do not believe me please go through the algebra and get the same result.

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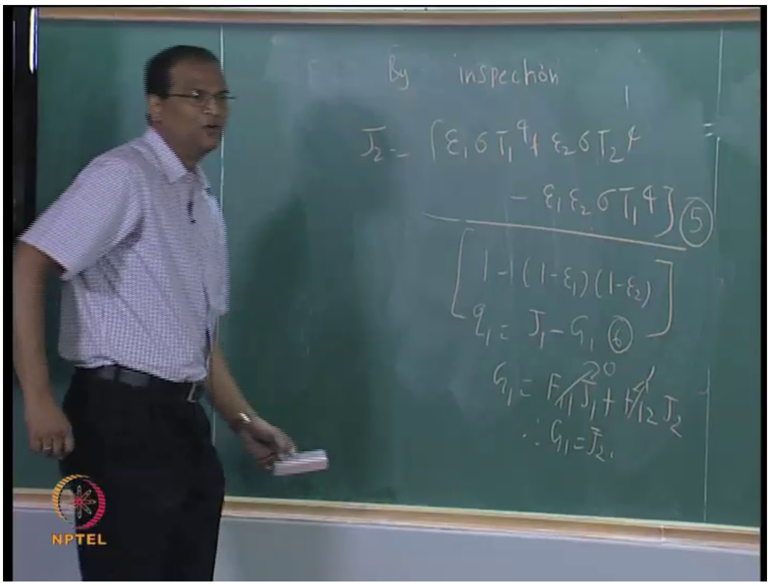

What is q 1 after all, J 1 minus G 1, but what is g one itself, J 2. F 1 1 is 0 F 1 1 is 1. Therefore, G 1 equal to J 2, therefore if you just get J 1 minus J 2 you got q, this is called the parallel plate formula, a very powerful result in radiation, so sigma T 1 to the power of 4 minus T 2 to the power of 4 divided by 1 by epsilon 1 plus 1 by epsilon 2 minus 1, if epsilon 1 is equal to epsilon 2 the denominator will be 2 by epsilon minus 1. The most important thing, if eventually you want to become researches, is to always look at what is called as asymptotic correctness of the result, what I mean by the asymptotic correctness of the result is, when I apply to an extreme case does it work, I will show you in a minute how to check for the asymptotic correctness of this expression. If the surface two is at surroundings at T infinity, what is epsilon 2. 1 by epsilon 2 is 1, that 1 by epsilon 2 minus 1 will get cancel; T 2 will be T infinity.

Therefore, q will be epsilon sigma T to the power of 4 minus T infinity to the power of 4, therefore in the asymptotic limit of the second surface being the ambient at T infinity this formula works. So, each and every time whenever you approach to any subject should be like that, when you deriving some results time and again you have to see whether the asymptotic correctness, whether it make sense, this can be worked out with paper and pencil, before going to the computer you must check all this.

So, in tomorrow’s class we will take up some interesting examples, suppose I put a shield in between these two in order to reduce the heat transfer rate, whether the radiation heat transfer will reduce, what will be the equilibrium temperature of that shield, I do not settled down with one shield, I put two shields, three shields, four shields; what will be the heat transfer rate, how do we get the equilibrium temperature under those conditions, we will solve some problems and then go to regular enclosure problems, where I give you numerical values and will work out.