Conduction and Radiation
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Module No. # 01
Lecture No. # 25
Enclosure Analysis Gray Surface

So in today's class we will look at, we will solve some more problems on enclosure analysis or enclosure theory, and one will be on cylinder within a cylinder, concentric cylinders, basically such problems are involved in insulation, insulation design is a current carrying wire and then you have insulation, and then you have vacuum in between, it is carrying certain amount of current.

So, what will be the maximum temperature or rather we have to work backwards, this is the maximum temperature which is possible, how much of current can it carry, like that you design for different thickness, if wires have different thickness their current carrying capacity is decided accordingly. In the second problem will be a three zone enclosure, where we look at a triangular furnace or triangular duct, where three surfaces are maintained at three different temperatures, two surfaces are maintained at two different temperatures, and we have one reradiating surface that will close our discussion on enclosure analysis for gray diffused surfaces. Tomorrow's class I will talk about enclosure analysis for diffused surfaces which are non-gray, suppose emissivity of this surfaces varies with lambda, then how do you go about solving this. There are surfaces where a properties spectrally dependent, that is they cannot be considered as gray.

So, we will give an approach, and I will just give one example. From Friday we will start gas radiation that will go on for four to five classes. So, before March 31 hopefully we will start conduction, and we will have about ten twelve classes max for conduction heat transfer.

Let us discuss those aspects of conduction which I am not being considered in other courses, all right. Now, we will work out this problem, problem number 34, a very long electrical conductor, 10 millimeters in diameter is concentric with a cooled cylindrical tube, 50 millimeters in diameter, whose surface is diffuse and gray with an emissivity of point 9 at a temperature of 300 Kelvin. The electrical conductor has a diffused gray surface then emissivity of point 6, and dissipate seven watt per meter length. Assuming that the space between the conductor and the tube is evacuated, determine the surface temperature of the conductor. Is the question clear?

We can say that it is an inverse problem in the sense that we know the heat flux, but we do not know the temperature, what is the story behind trying to find out the temperature then we will see whether this fellow can withstand the temperature and all that. So, inverse problems in fact, are more practical from engineers prospective, rather than the direct problem, many times we have to infer, we know some information about something, but we do not know what cause that behavior. So, that is why inverse problems are very important.

Now, you cannot straightaway solve this without first writing the radiosity relations and getting the formula. So, good practice would be, hang on do not start working and do not put epsilon sigma t to the power of 4 epsilon one. I would advice that you keep it as surface one and surface two, with areas A 1 and A 2, start with view factors, write its after all a two surface enclosure, write expressions for J 1 and J 2, get an expression for q 1. Now, in the very intelligent way, we can call these two areas as A 1 and A 2, so that, suppose in the exam, I ask a problem where there is a sphere within a sphere, this A 1 by A 2 can be put as 4 pi r 1 square by 4 pi r 2 square; now, this A is 2 pi r 1 divided by 2 pi r 1 into 1 or l divided by 2 pi r 2 into l 2. So, you keep it as A 1 and A 2. Now, you can see that, it will take 15 or 20 minutes for us to get through this, as algebra is very very painful, already we saw the parallel plate formula itself, I did so many things, epsilon 1 sigma t to the power of 4 minus epsilon divided by epsilon 1 epsilon 2 throughout and all that. Apart from all this epsilon 1 epsilon 2 confusion, we will have the A 1 by A 2 confusion here. So, there is ample scope for you to make mistakes, take a minus as plus and put A 2 by A 1 instead of A 1 by A 2 and so on. So, you’ll have to do it slowly and carefully, and we will get the expression for J 1 J 2 and q 1, and then substitute the values it will be straightforward.

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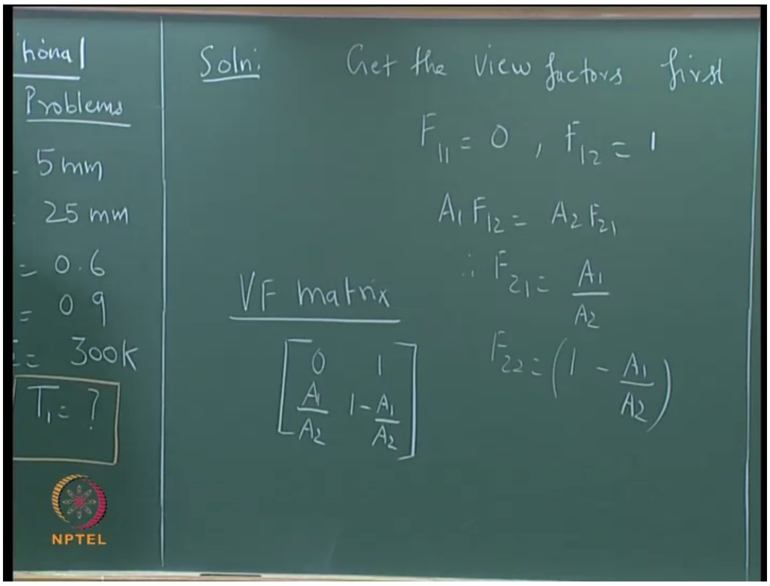

So, let us start, get the view factors first F 1 1 equals to 0, F 1 2 is equal to 1. So, first write out the view factor matrix 0 1. So, part A is over, first part is over. Now, we will write the radiosity relations.

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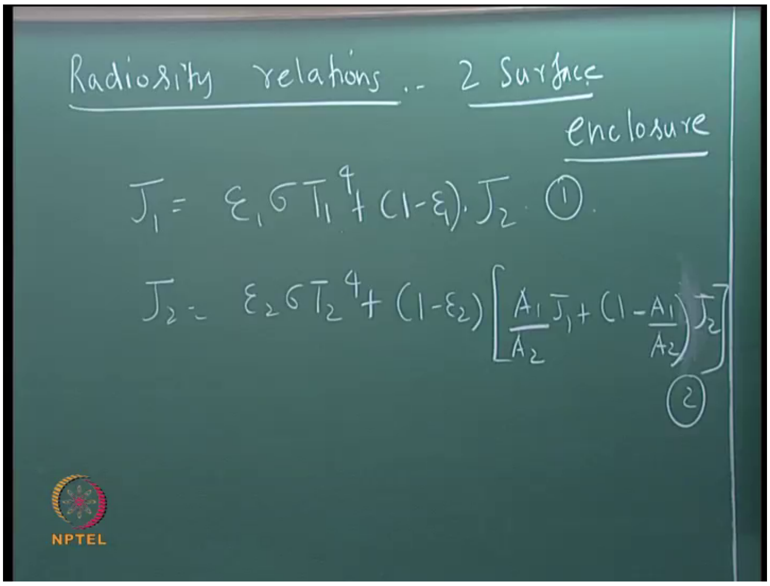

Now it is a two surface enclosure, the conductor is surface one and the tube is surface fluid is evacuated in between. So, you have to write two radiosity equations, we can write down it is a 2 surface enclosure. So, we will get the J 1 and J 2. I am already incorporating the view factors. It is just a question of solving two equations in two unknowns, straightforward, but the algebra is somewhat messy. So, you have to do it carefully without making mistakes, conceptually it is not great, two equations and two unknowns everything is known, any mistake which one is not known.

Student: T 1

Pawan raised a very important point T 1 is not known, I am just getting the general expression. Finally, I get, I will write an expression q 1 2 is equal to sigma T 1 to the power of 4 all that, I will get an expression, in that expression everything will be known except T 1. Hopefully I will reach that level stage, in another ten minutes. Now, what do we do now, the easiest thing is to substitute for J 2 in equation one, and slowly collect the terms involving J 1, put them to the left hand side solve for J 1 and then proceed further to get J 2. So, I will do this, now I want to have the full board, let me use the full board.

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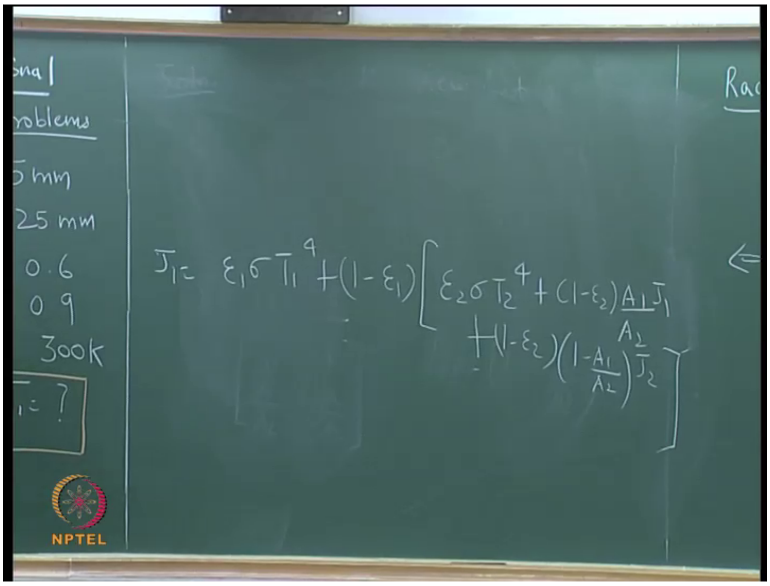

So, J 1, what is that 1 minus, 1 minus epsilon 1 is already there, epsilon 2; is this correct. Three terms from J 2, so what is your problem, it is fine; if there is a problem you want to tell me, because the error will propagate, is it fine. Now, I will have to do some simplification. So, J 1 is it taking me anywhere?

Student: A 1 by (( ))

Student: Which one,

Student: First equation inside the bracket, square bracket.

No, even before this, tell me what is a problem.

Student: A 1 by A 2 into J 1, it has been multiplied by whole thing.

Which one

Student: (( )) inside that bracket, common to A 1 A 2

Where is this, no this epsilon 2 sigma is fine, no this is fine, but I have to do work on this further. Substitute for J 1 in equation 2 (( )). That is better, so substitute for J 1 in equation 2.

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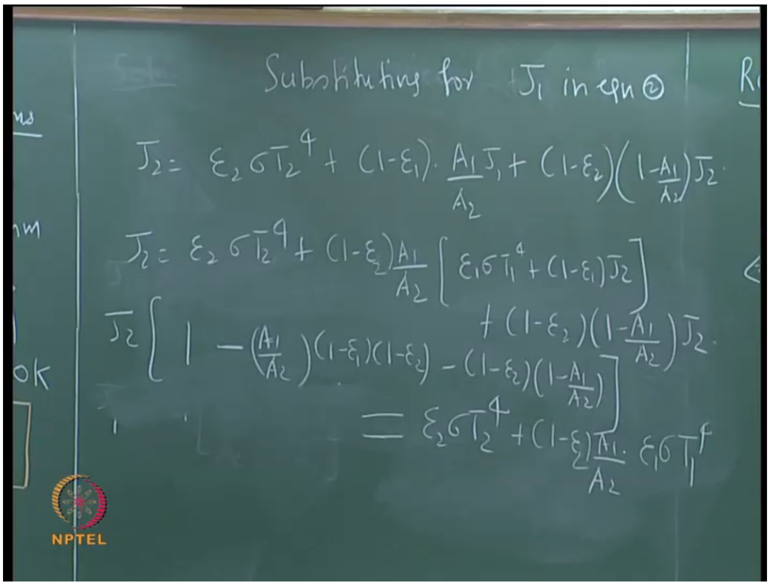

Now, this is equation 2, I am not substitute so far, is it fine. Now, J 2 into 1 minus, is this correct first, please tell me, Vikram is it okay; otherwise you will not get the solution. So, right side, right side is epsilon 2, T to the power of 4 plus, into epsilon one. This should be 1 minus epsilon 2

Here, I have written correct. What about this, 1 minus epsilon 2 square, no this is different, epsilon 2 sigma T 2 to the power of 4 plus 1 minus epsilon 2, epsilon 2 sigma, that is it, anything else, there are only two terms. How, many terms are there totally; one two three four. So, you have taken two terms on the left side and two terms on the right side should be fine. Let us see, but once you have this in the exam you can use it straightaway, you do not have to derive it, because it is all open notes, but we better derive it now, any hope of simplifying possible. Hang on I will just finish.

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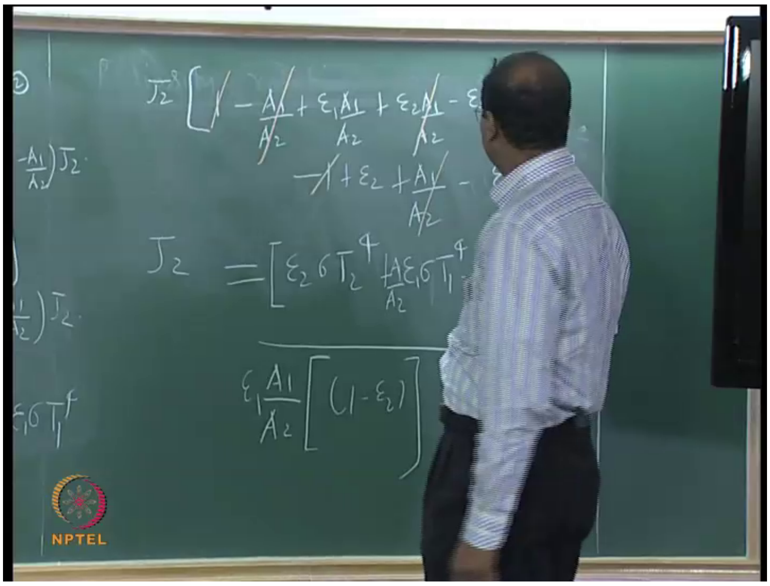

So, left side 1 minus, what can we do now A 1 by A 2, is it fine, no I have not completed, I have just expanded this, you have to just help me in identifying that minus and plus, minus 1 plus epsilon 2, Plus A 1 by A 2 minus, minus epsilon 2, A 1 by A 2.

I am also doing it live along with you, because my solution I started J 1, Ketan told me sir you'll start with J 2. So, it is possible that I end it, but let us hope that it works. So, I want to pray to god that some terms must cancel, are they cancelling out? I do not like the 1 first. 1 will go 1 goes, we will use the red, 1 should go. A 1 by A 2m. This is also not bad at all, is it correct? We do not know, so it is too simplified, why 1 term is only epsilon 1, other term is epsilon 2. I do not know whether we are falling into (( )) A No it comes out. It comes out Finally, it is simple I know that, right side, epsilon plus minus epsilon 1 epsilon 2, A 1 by A 2. T 1 to the power 4. It is very similar to what we got previously,

Student: (( ))

Tell me, tell me the denominator,

Student: A 1 by A 2 minus,

A 1 by A 2 can be taken outside,

Student: Epsilon 2

Why we can take out, epsilon 1, Epsilon 1 is not taken, epsilon 2 term. Epsilon 1 of 1 minus. No is it ok. No, there is something there I cannot do this, let me keep it like this, so epsilon 2 minus. This is more like it, hope now; this equation 3 after all this is just 3, can we write J 1 by inspection, should be possible, why? Because 1 1 A 2 is concave surface A 1 is a convex surface. Then where is the hope now, Substituting in the first equation.

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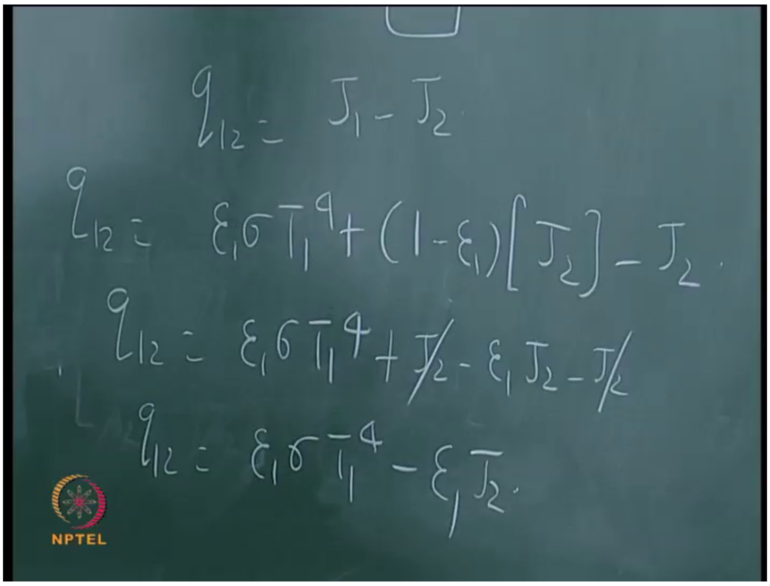

Substituting in equation 1. One would be J 1 minus J 2, so in equation 1 if you just subtract it on both sides. We can get the q straightaway, but that is what, we will write this, we can get or. Therefore, is that correct, that was good Vikram, otherwise you can find J 1 also, I cannot believe this, is this correct. I am now flying an autopilot I do not have this in my notes, let us see you should be able to see the runway.

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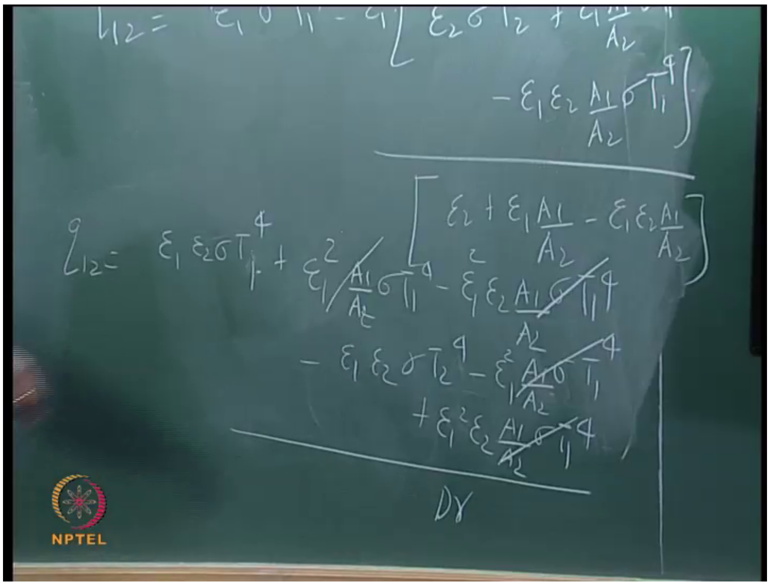

So, q 12, I erased the J 2. So, epsilon 1 epsilon 2. Sigma T 2 the power of 4. Plus epsilon 1 A 1 by A 2, sigma T 1 to the power 4 minus epsilon 1 epsilon 2 A 1 by A 2 sigma T 1 to the power 4, It is correct. Divided by Epsilon 2 plus epsilon 1 A 1 by A 2 minus epsilon 1 epsilon 2 A 1 by A 2 q 1 2 epsilon 1 epsilon 2 sigma T 1 to the power of 4, plus epsilon 1 epsilon 2 A 1 by A 2 sigma T 1 to the power of 4 minus epsilon 1 epsilon 2 A 1 by A 2, you are getting epsilon 1 square.

This epsilon 1 square is it not? What am I doing epsilon to epsilon then epsilon 1 square A 1 by A 2 sigma, minus epsilon 1 square epsilon 2, is it correct? No, if you cross multiply, let us see, epsilon 1 square plus by we will just say D r. Now, tell me what are the things which get cancel. The first and fourth Epsilon 1 square get cancelled, then Second and fifth What remains, it is nice, now I know I am getting the answer.

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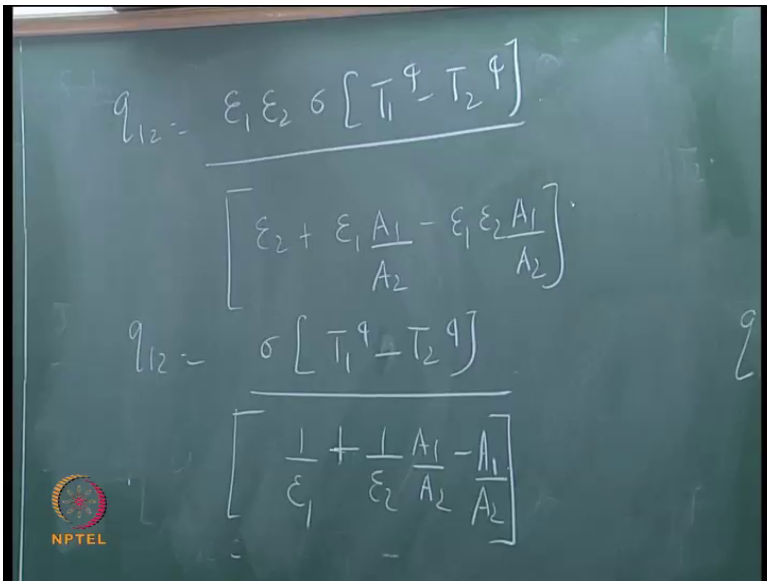

So q 1 2, 1 by, now what you want to do 1 by. A good check of this formula is if A 1 by A 2 equal to 1, it will reduce with the parallel plate formula, the numerator will be the same, the denominator will be 1 by epsilon 1 plus 1 by epsilon 2 minus 1. So, this formula displays asymptotic correctness. Therefore, it has to be corrected, it can be correct, at least it is not wrong, it is necessary, but not sufficient, but when we get the whole point is like this, when we have done all our search method algorithm, and finally we are trying to minimize we will not end up with the maximum, I use to say that in the optimization course, like that when it is such a nice answer it cannot be wrong, it could be possibly incorrect.

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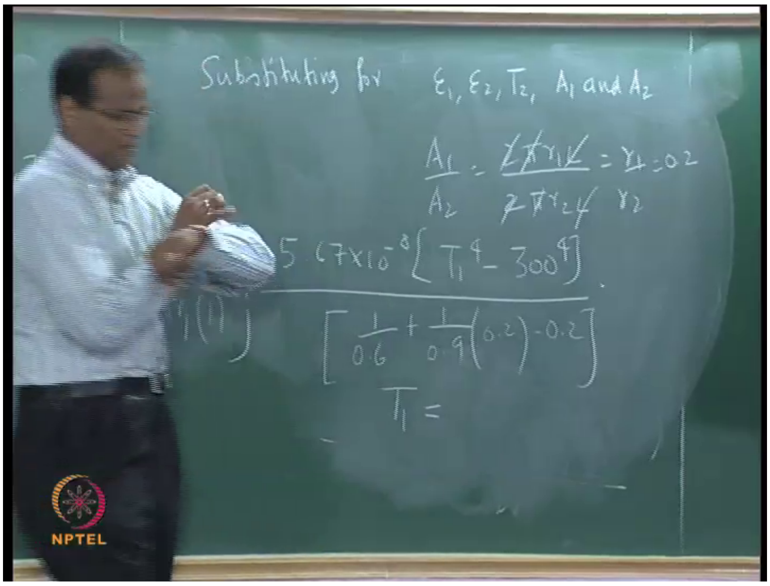

Now, point 2 isn't it, by r 2 1 is point 2, therefore 7 Why? 7 by 2 pi r 1 into l, because that is 7 watts per, no I get 7 watts per meter length, its correct, no I gave 7 watts per meter, that was meter length. Therefore, I will still have 2 pi r 1, now this is what is 1 by epsilon 1. Per meter length. No, but this q is in watts per meter square, therefore 7 into 1 by 2 pi r l.

This is correct, you go; do not worry about that formula. So, what I gave you 7 watts, per meter length means, I am saying l is 1, I am not saying r 1 is 1, r 1 is there. Now, the 2 pi r 1 is correct. Now, 1 by epsilon 1 is point 6, epsilon 2 is point 9, T 1 equal to 3000

Student: 348

348 point 4 1, can a wire withstand the temperature, it can withstand, suppose there we get 1343 or something, then we have to check our calculation, what is this 348 is how much 75 degrees, 75 degrees wire can withstand. So, there is no problem.

So, if I give you concentric spheres A 1 by A 2 will be 4 pi r 1 square divided by 4 pi r 2 square, you can apply this formula for sphere within a sphere also. So, you can see that whenever the view factors are not straightforward, it involves the r 1 r 2 fundamental dimensions, then the analysis becomes little complicated, but towards the end you will get a very simplified answer.

So, we will close our discussion with one more problem, where the view factors are pretty straightforward, is this clear to everybody. So, we get about 348 kelvin. There should be another method of doing (( )) You use the electrical analogy, use resistance network, but the resistance network will be very painful if you have more number of surfaces. So, the advantage I told you is, it can be scaled up point one, and point two it is eminently programmable, we can easily write radiosity equation in the computer, and very powerful techniques like gauss-sidle technique are available where simultaneous equations can be solved. So, in mathematicians have developed, and engineers have developed good techniques for solving system of linear equations, we exploit that.

Now, in thirty fifth problem, consider a very deep duct; that means, it is very deep in the direction perpendicular to the plane of the board; that means, it is two-dimensional, consider a very deep duct made of diffused gray walls, each of which has a width of 1 point 5 meters, the temperatures of surfaces one and two are 1200 and 800 Kelvin respectively, the corresponding emissivities are point 4 and point 6 respectively, surface three is completely insulated, and has an emissivity of point 5. For this two-dimensional enclosure, that is for this 2 d enclosure; A, determine the net radiation heat transfer from surface one, B; determine the temperature of the insulated surface three and part C; if epsilon 3 is change, will your results change, what is the answer to part c, why, because surface three is a reradiating surface, completely insulated means reradiate, first draw the figure, all the view factors are very simple man, it is all point 5, it is an equilateral triangle.

So, view factors are straightaway obtained, and J 3 is also very easily obtained. So, though you have a system of three equations you have to solve only for two, because J 3 can be written in terms of J 1 and J 2. So, this will take about 10 minutes, we will solve it.

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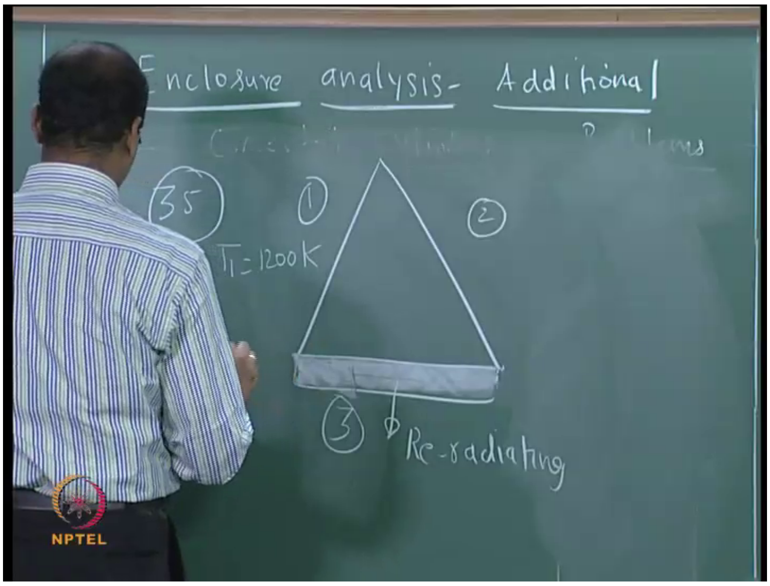

Surface one, surface two, this fellow is reradiating 1200, first see, where all the conditions written on the board correspond to what has been given in the problem. Consider a very deep triangular duct, consider a very deep triangular duct. So, the word triangular was missing.

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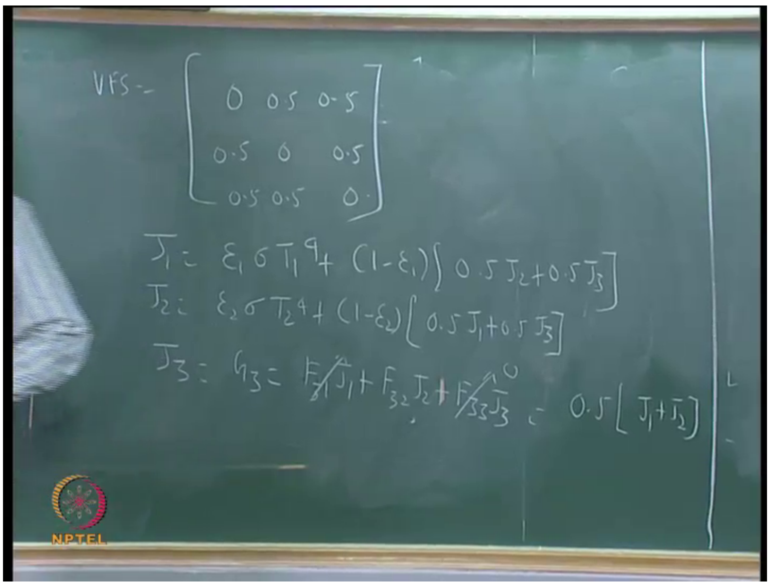

Now, because it is a equilateral triangle; F 1 1 plus F1 2 plus F 1 3 equal to 1, F 1 1 is 0, F 1 2 equal to F 13, therefore 2 F 1 2 equal to 1, F 1 2 is equal to half F 1 2 is equal to half, A 1 F 1 2 is equal to all, I do not have to do that if it is equilateral. Now, J 1; watch this please, look at the board look at the way I am writing J 3, I am saying J 3 equal to G 3, G 3 is the irradiation on the surface three, it is equal to F i j J j, i equal to 3 here. So, F 3 1 J 1 plus F 3 2 J 2 plus F 3 3 J 3, this is 0. So, this is just point 5 J 1 plus J 2. So, this is the crucial step in an enclosure involving reradiating surface even if you have n surfaces, you'll have to solve for only n minus equations, because for the reradiating surface, the radios ties can be directly expressed in terms of the radiosities of the other surfaces. So, instead of solving three simultaneous equations you have to solve for only two, it becomes straightforward know, you can substitute the values. So, wherever J 3 is there, you will substitute point 5 J 1 plus point 5 J 2, here you substitute.

So, these two equation just become equations in J 1 and J 2, J 3 can be straightaway eliminated from the two equations. Solve for J 1 and J 2 get q 1, because once you have J 1 and J 2 you can write J 3 get q 1 and that is it, and then J 3 divided by sigma whole to the power of point 25 will give you T 3, can we do that.

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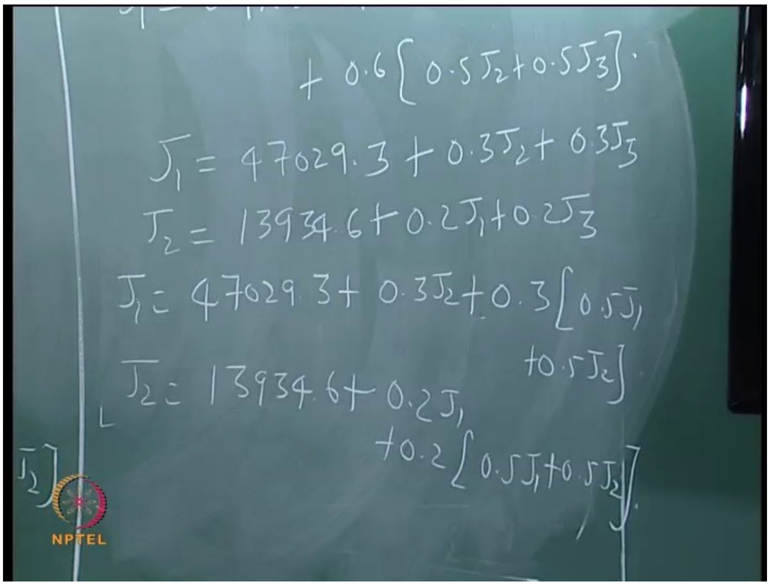

So, J 1, can you tell me the first term epsilon 1 is point 4 plus. So, what is this, first term point 4 into 5 point 67 into 1200, how much is it, Vikram can you tell me the first one first term. 47000 29 point 2 5 . Point 7 plus. J 2 is equal to epsilon 2 is point 6, can you substitute, tell me that first term, 13000, 13000, 9 34 point 6 plus is it correct, 1 minus epsilon was point 4, then I multiplied by the view factors and absorbed them, is it ok, did you make a mistake, Shrikanth is it ok, point 21 point 2 are all right, priti what is your problem? Point 6. What ? Nothing.

I have multiplied F i j by 1 minus epsilon. Now, you can write the two equations in terms of J 1 and J 2 alone. Similarly J 2 point 3 J 1. So, 1, people who already have this, simultaneous equations solver can tell me J 1 equal to 77 point 9, J 2 point 5, J 3, because it is an equilateral triangle, point 5 J 1 plus 59166. Please see today we have solved two problems it takes again 50 minutes, in quiz there will be three problems for 50 minutes, but you do not have to derive 1 minus A 1 by A 2 is already there in the notebook. So, do not underestimate a three surface enclosure or two surface, even a two surface enclosure if it involves self view factors it will take twenty minutes to 30 minutes, if you do not have the formula, a three surface enclosure, because of the algebra it will take 20 to 30 minutes.

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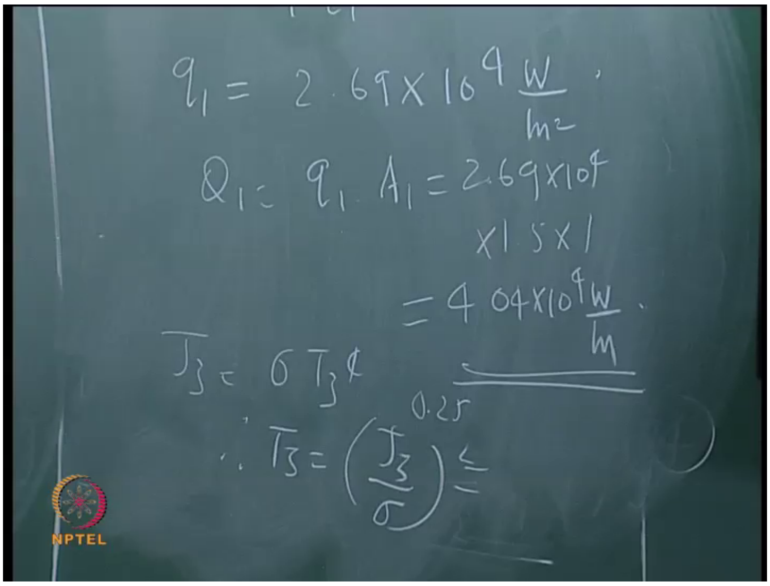

Now, q 1 is equal to, this is correct, we got the expressions for q in terms of G 1 as well as J 1, now you can write the expression for q 1. So, what do you get q 1, 2 point 69 into 10 to the power 4, Watts per meter square, this must be multiplied by 1 point 5 into 1, is 1 point 5 meters wide is it not? 4 point, 4 point 04.

Are other people getting the same radiosities, Shrikanth did you get the same radiosities? Ketan these radiosities are ok. Now, please get T 3, you already seen that J 3 is equal to

G 3, if J 3 is equal to G 3, I have worked out a formula specifically for the radiosity of a reradiating surface which is equivalent to that of a blackbody at that temperature. So, the radiosity of that surface is independent of its emissivity, you have already proved this mathematically. So, the answer must be between 800 and 1200, any other answer is incorrect. 1010 point 7

1010 point 7, so what is this surface doing; this surface is receiving heat from this fellow and is putting it back. So, he has no net radiation heat transfer, what is going out is equal to what is coming in, that is why it is a reradiating surface, and the answer to the part C is, if the emissivity is change it has no bearing. So, the emissivity equal to point 5 for the surface three is superfluous are redundant information, which is never used in the calculations, without that also we can do the calculation. So, the good thing is if you have a reradiating surface you do not need to worry of about its emissivity.

So, this T 3 is actually called the equilibrium temperature of the reradiating surface. So, we will stop, tomorrow we will look at the concept of spectral radiosity, what happens when epsilon is a function of lambda, it is a very difficult thing I will just solve it for the parallel plate formula and we will solve one problem, and Friday we will go to gas radiation.