Conduction and Radiation
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Module No. # 01
Lecture No. # 26
Enclosure analysis –Non gray surfaces

So, today, we will look at what is known as a spectral radiosity formulation. It is a slightly advanced topic, but since it is a graduate level course, I thought we will do that. So, it will take one hour for us to discuss briefly this concept and then work out a problem. So, so far our analysis of enclosures, where we try to do enclosure analysis with radiosity, irradiation method was restricted to enclosures which are having, which were having gray diffuse surfaces.

But if you, if you, remember before we started with view factors, we spends so much of time in trying to define spectral, directional, emissivity, hemispherical, total hemispherical, hemispherical, spectral emissivity, directional total emissivity, hemispherical total emissivity and so on. We also told, we also discussed that there are several surfaces which can exhibit non-gray behavior, that is, epsilon lambda need not be, epsilon lambda will in general be a function of lambda. We cannot, we do not have to automatically cut out by saying that epsilon lambda is not a function of lambda. So, everything is gray and diffuse, but please remember that this gray diffuse is a good approximation, but there are applications where this diffuse, this gray approximation does not work; the diffuse approximation also does not work, but we are not going to worry about the angular dependence. That is more involved.

So, if the gray assumption does not work, how do we go about handling the radiosity? Please remember, for the radiosity, we simply use radiosities emission plus reflection, and for the emission, we straightaway used epsilon sigma t to the power of 4. For the reflection, we put rho of that surface multiplied by whatever irradiation is coming on the surface. The irradiation coming on the surface is a consequence of the radiosity from the other surfaces multiplied by the corresponding view factors and then we worked out all the, all the, algebra and we worked out different expressions for getting the Q in terms of g, in terms of J, in terms of J minus g and so on.

Now, for a moment if you think, if you, if you pause and think, if you cannot use that epsilon sigma t to the power of 4 because epsilon lambda is a function of lambda, how do you think the formulation will get change?

Student: (( ))

Epsilon lambda is a function of lambda. So, I just cannot use epsilon sigma t to the power of 4 and have one radiosity. So, we have to use. So, Deepak is coming out with some answers. He is saying that we have to use the f function chart. So, if you have to use the f function chart, that means you have to use the f function charts for different ranges of the interval. Are you getting the point? If lambda is equal to 0 to infinity, if it, if there is a value of epsilon 1 for 0 to 3 micrometer, then you will use some f function chart and find out what is the total amount of radiation in terms of sigma t to the power of 4. How, how much of sigma t to the power of 4, then 4 micrometer to 7 micrometer 7 micrometer to 9 micrometer whatever up to infinity.

Therefore, necessarily you will have to introduce what is called as spectral radiosity, whose units will be watts per meter square per micrometer. Therefore, in a wavelength interval d lambda, you have to multiply this J lambda multiplied by d lambda, and then are you getting the point? So that you are able to find out so many watts per meter square in that portion of the interval and so on, and then you will have to do a sum for all these intervals to get your final answer. We will go through the rigorous mathematical formulation, but the whole point is its going to be computationally very expensive if the epsilon lambda versus lambda is very very painful. It is going to be computationally very very expensive, that is, if your, if d lambda has to be necessarily very small, then you will have so many spectral radiosities for each surface.

If you have n surface enclosure, you will have, previously you had n radiosities. For each surface, you will have m radiosities corresponding to how many intervals you are dividing epsilon lambda. Are you getting the point? Therefore, it will be computationally very very challenging. There are some approximation; there are some approximate ways of doing this. So, we will do this in today’s class.

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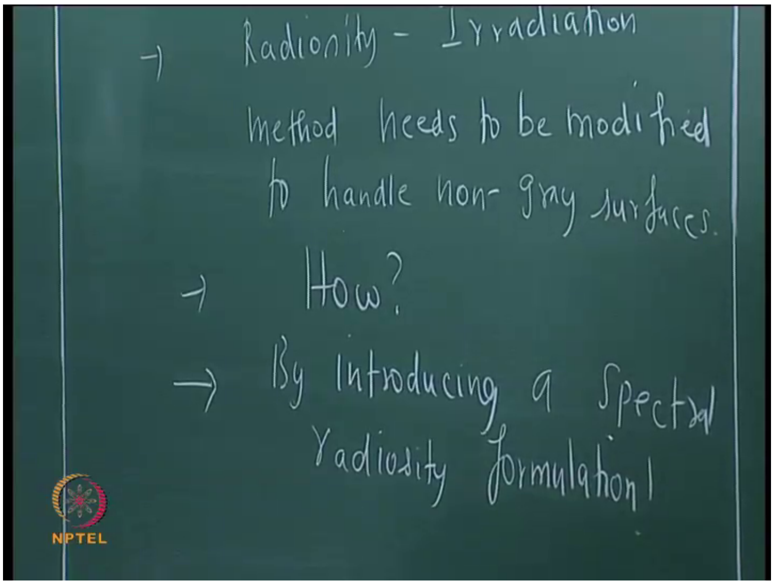

So, we now look at what is called the radiosity. The radiosity irradiation method needs to be modified. How? Correct? So, the need is establish. How? By using a formulation called the spectral radiosity formulation.

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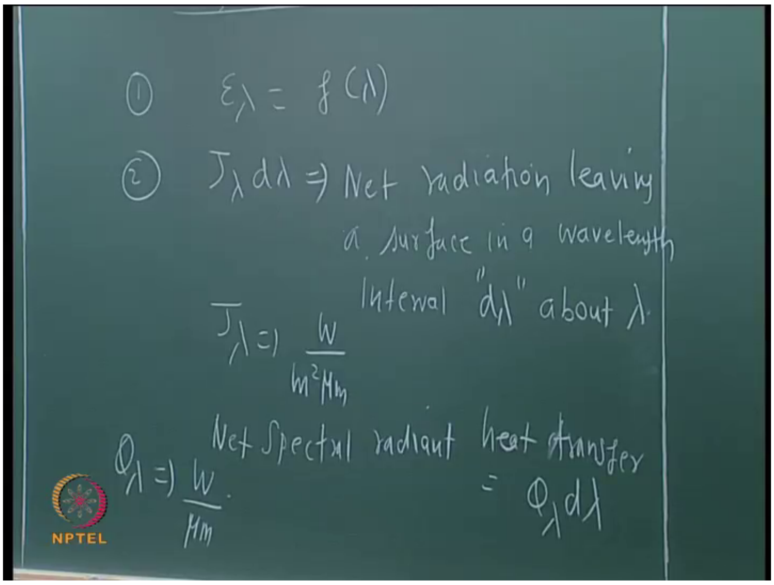

What are the key features? What are the key features of this spectral radiosity formulation? First, epsilon lambda is a function of lambda, good. First, epsilon lambda and needless to say this f is known to us. Otherwise, you cannot proceed. So, then it is an inverse problem. So, epsilon lambda is a function of lambda. I know what this f is. Now, instead of J, watch, watch here, instead of J, it is J lambda d lambda which is the net radiation leaving a surface in a wave length interval d lambda about lambda. What should be the units of J lambda?

Watts per meter square per

Student: Micrometer

Micrometer J lambda will have the units. So, so that when watts per meter square per micrometer is multiplied by micrometer. You will get watts per meter square which is the original unit for radiosity. Now, net spectral, so, net spectral radiant heat transfer is Q lambda. What should be the units of Q lambda?

Student: (( )) watt per (( ))

Watt per micrometer, very good. So, if I put small Q, it is different story; it is watts per meter square per micrometer. Since the whole wavelength interval lambda is equal to 0 to lambda is equal to infinity is not used. I come again. Since the whole wavelength interval lambda is equal to 0 to lambda is equal to infinity is not used. We are considering d lambda about lambda. We cannot use the Stefan-Boltzmanns law, but does it mean that we cannot proceed with calculations? Instead of the Stefan-Boltzmann's law, what will you use?

Student: (( ))

What is a original law which (( ))

Student: (( ))

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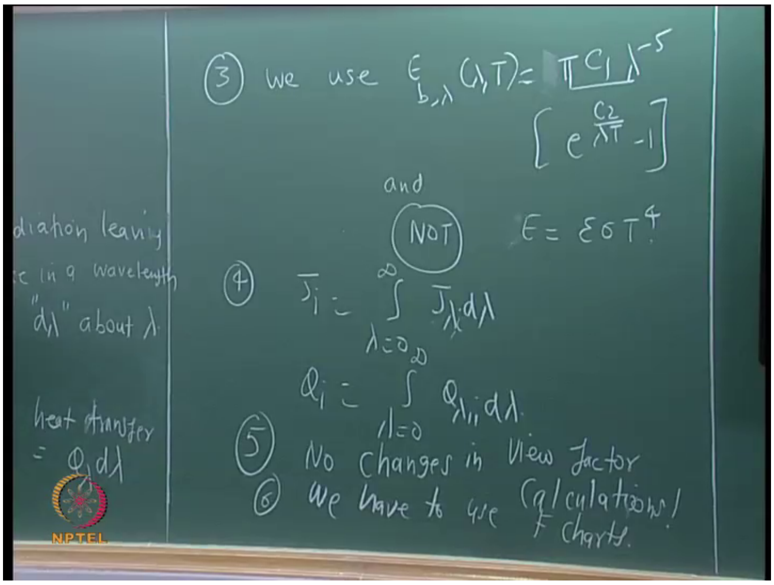

We will use the Plancks law. We use Planck, we use, did it put? Did we use 2 pi C 1 or C 1 straightaway for e? We multiplied that by 2 pi.

Student: C 1

Directly C 1

Student: (( ))

Yeah please check. No, no, I am not talking about I b lambda.

Student: (( ))

I hope you have not forgotten the difference. Quiz is coming up between the E b lambda to I b lambda. Lambda 2 of course, correct. Do not ask me sir why are you using E b lambda, it is not a blackbody. Anyway, epsilon lambda will come. Do not worry about that. Vikram, any problem?

Student: (( ))

There is a big story know.

Student: (( )) pi (( ))

Pi? Was it pi or 2 pi?

Student: Pi (( ))

No, no, it depends upon how I have defined in the earlier classes. If you all say it is pi…

Student: Pi I b lambda (( ))

Pii b (( ))

Then it is pi C 1. It depends on how we have done it, fine. And now, J i, yeah, please watch this. So, the J i will be lambda equal to 0 to infinity J lambda d lambda.

Student: (( ))

Sure, I want to put it J lambda I. If you do not like J lambda I, you can use J i lambda. It does not matter. Lambda and I must come. Lambda is a spectral and I is a i-th surface. We just want to leave it as J lambda because we are talking about radiation heat transfer in a n surface enclosure. So, this, so, this makes it the spectral radiosity formulation, correct? It is a spectral radiosity. If you want, you get the total radiosity which we have defined earlier it has to be done. Similarly, Q i lambda equal to 0 to infinity Q lambda i d lambda, fine. So, this is for cutting the radiosities. What about view factors will they change? No. So, let, let us write that. No changes in view factor calculations. So, no changes in view factor calculations. So, how do you get the, how do you get the reflectivity?

Student: (( ))

It is spectrally dependent. Therefore, you have to invoke Kirchhoff's law to get the reflectivity. You have to inverse, you have to invoke Kirchhoff's law and say that alpha lambda is equal to epsilon lambda. So, this completes the formulation. This gives you the broad framework after going through these important rules. This is like (( )) prerequisites. Once we have all these, then we write the formulation. I will write the original radiosity equation which we, which we, have been using in the last few classes on the spectral radiosity formulation equations, and then see this integral lambda equal to 0 to infinity as engineers sometimes integral is replace by summation.

If it has to be replace by summation, so I will have an approximate epsilon lambda versus lambda instead of you having stupid curve. That is an approximation; an approximation is introduce there, that we will see. It is called the band approximation. We will introduce that band approximation and see how to solve this problem. So, the problem, suppose if I have, if I have, a three surface enclosure and each surface, I will give you epsilon 1 epsilon 2 epsilon 3 for three different reasons of lambda. You are sunk. So, you will calculate all this for the first 0 to lambda 1; then you will calculate for lambda 1 to lambda 2; then lambda 3 to infinity and so on.

Then finally, you can add up all the radiosity; add up all the Q. That is a linear addition will work. Now, I think quite a few of you must have got an idea of how the eventually equations will look like, but we will write out the formulation; we will solve a problem, then everything will become clear. We will just take the simple parallel plate formula, but now, epsilon 1 and epsilon 2 are functions of lambda. I will just give a simple step function. Now, shall we write the, and then most importantly, since I am using this, since I am using this and then I am telling you that epsilon lambda is a function of lambda. In various wavelength intervals, I have to use what? I have to use f charts. So, this is the background required for a spectral radiosity formulation. So, for hemispherical total quantities, what was the original radiosity? We will first write the whatever we are familiar with and then we will write the J i equal to epsilon i sigma 1 (( )) yeah, yeah, yeah. They look similar or J equal to 1. Now, the spectral radiosity formulation, watch this very carefully I will write it with orange chalk Please see whether it is all right. If you do not like this notation, yeah, no, no, it is fine; it is fine, no; notation is good. So, so, the J lambda i is epsilon lambda E b lambda i. I cannot put sigma t to the power of 4 because this is J lambda. It is not J from 0 to infinity. So, I have to put epsilon lambda this plus 1 minus epsilon. View factor is the same and J lambda of J. For an N surface enclosure, we will have N such equations for…

Student: (( ))

Every wavelength, every wavelength lambda, and for every wavelength lambda, the N equations can be simultaneously solved to get J lambda. Once you get that J lambda, you can integrate it to get J i. Then you can similarly get J a for all the surfaces and J j i minus g i is the q i. So, what is additional labor involved? You have to do all that. You have to first get the view factors and all that, write out the radiosity equations, but radiosity equations have be have to be written for each and every wavelength, but the wavelengths and the surfaces are not coupled; that means for every wavelength, you can independently solve. Are you getting the point?

For every wavelength range, you can take the N surfaces enclosure and solve the spectral radiosity equation independently. You do not have to all, you do not to solve, you do not have to solve N equations for m number of wavelength intervals simultaneously. You can solve from 0 to 3 micrometer. Get all the values of J lambdas 3 to 5 micrometer, 5 to 8 micrometer, 8 micrometer to infinity. So, J 1 itself if one is, the first surface is one. Let us say we can use superscript four wavelength intervals – 1, 2, 3, 4. Then we will have J 1 1 plus J 1 2 plus J 1 3. Are you getting the point? All these refer to the wavelength bands.

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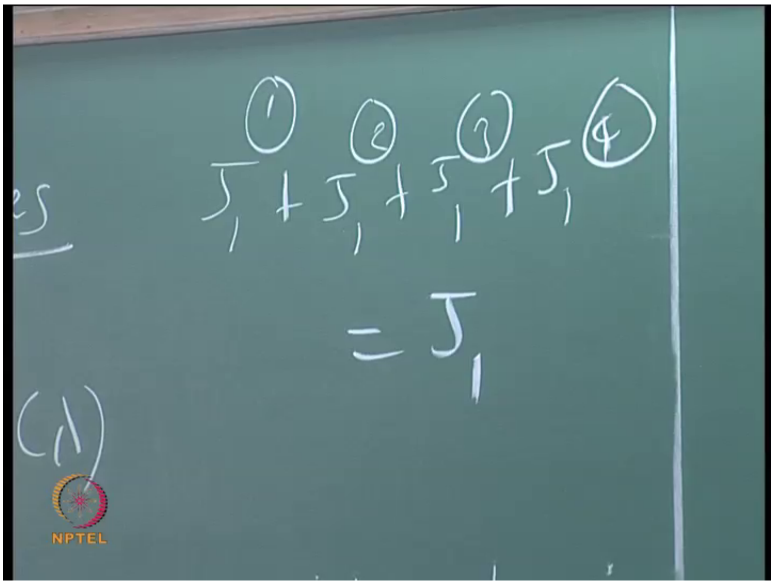

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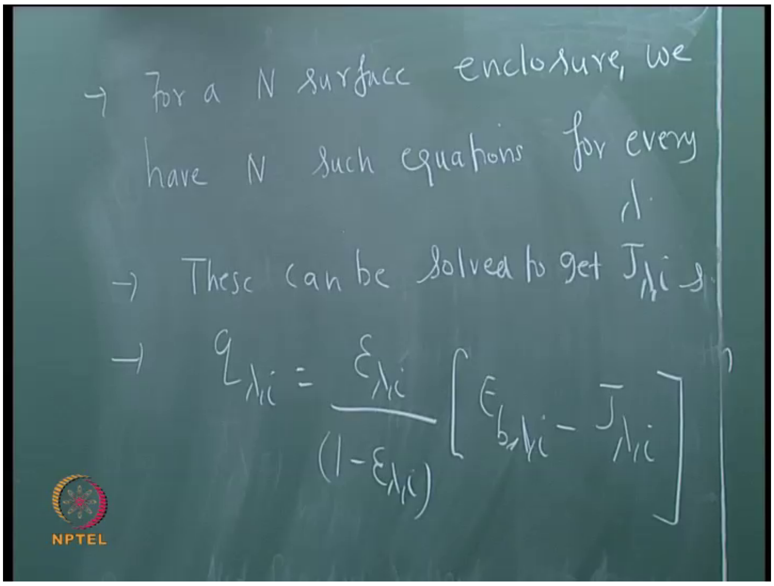

So, now, we will write the formulation completely. This can be solved to get J lambda i. Now, So, the q will be, q lambda i will be epsilon lambda i by 1 minus epsilon lambda i into E b lambda of i minus J lambda i. What is the E b lambda i? Will be the f function in that wavelength interval multiplied by sigma t to the power of 4, only that much. That E b lambda I will become sigma t to the power of 4 if the J lambda is replaced by 1 general J which we use in the previous classes.

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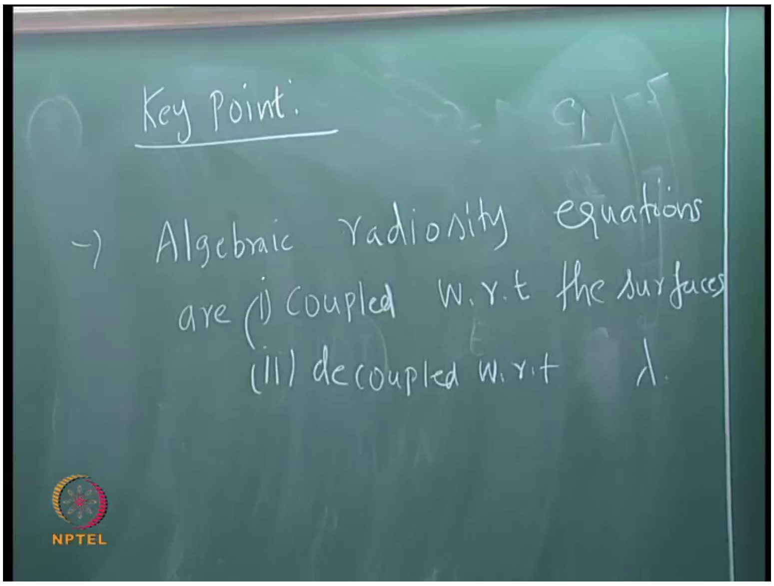

Yeah, what is the key point here? What is the key point here? Algebraic radiosity equations, algebraic radiosity equations are coupled with respect to? Come on, algebraic radiosity equations are coupled with each other with respect to what is that?

Student: (( ))

Algebraic radiosity equations are coupled with respect to the surfaces, with respect to the surfaces; d coupled with respect to…

Student: (( ))

That means if an n surface equation, you will do the same procedure as what you did before, but you have the luxury of doing it wavelength interval after wavelength interval. So, we can carry out. So, you can carry out all these calculations sequentially over various lambdas. Therefore, if the epsilon lambda versus lambda is such that no approximation is possible that you will have to necessarily divided into 100 wavelength intervals 200, 300 wavelength intervals, it is computationally very very costly; spectral calculations are very very costly. Do not think that only CFD, CFD is time consuming. You come to radiation. We will show you on (( )) c x 1 we have. I can put some radiation solver, solver and I can make the whole c x 1 struggle the supercomputer.

So, the challenge is not only, not only, in dou u dou x. It is there u dou u dou by dou x plus v dou. Once we enter the realm of spectral calculations, you are sunk. We do what is called line by line calculation for inside 3 d and all that. Line by line calculations, infrared portion of the spectrum. You have to define, you want 0.01 centimeter inverse. That is a wave number; that is a finest with which you want to do calculation.

Therefore, you will have to do millions of calculations, millions of spectral calculations, that is, I want j lambda at unlet's say 1.00004 micrometer and 1.00041 micrometer, 1.00042 micrometer, 000043, then 00001011. If you want that, then a supercomputer will struggle, why, because we are having an instrument which will capture these radiances. Then that instrument will have; the instrument will behave. It will have a response which varies with wavelength. I have to integrate this using the Planck function the response of the instrument to this. So that I will know between these two wavelength intervals, what is the total energy which is captured or received by the satellite.

So, if I have 1000 of channels on my satellite which is called a hyper spectral instrument, I will have to do this line by line calculations million times and then convolve it or integrate it and put it into the various 2000 or 3000 channels and then I will have to do inverse problem. If this is the reading, what will be the, what, what, should be the condition of the atmosphere which gave this. So, computational radiation is very very expensive. Computational radiation is a craft which is practiced by very few people. So, there is lots of challenges in computational radiation.

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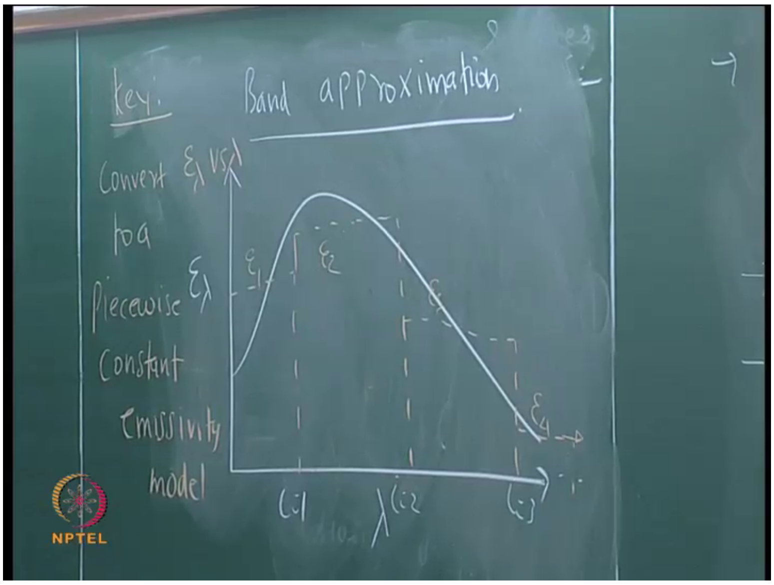

Band approximation I have epsilon lambda by epsilon lambda versus lambda varies like this. Let this spectral, what is that? Let the spectral hemispherical emissivity of a surface be like this. I want to use a spectral radiosity formulation. I cannot use a gray body. It is far from being a gray body. It is having multicolor; it is not a gray body. Now, in the light of what I told you previously, I do not want to do a million calculations; I do not want to do j lambda write thousand of j, j lambdas and all that. I want to make the problem tractable. Can you help me out? Is this clear? I want to, I want to, take one step forward which will simplify this. Come out with something, you have learnt so much. So, you have learnt so much of mathematics and…

Student: (( ))

Make steps, very good. So, how do you make the steps?

Student: (( ))

No, no, I will, I do not know. I may do. I do not, I do not know whether it will work, but it is I will do something like this. So, I then I will say this is epsilon 1 epsilon 2 epsilon 3 epsilon 4 and let us say it goes up to infinity. So, I can call this as I is equal to 1, j lambda I, I equal to 2, I equal to 3 and so on. What is a major achievement? (( )) instead of integration, we can do some summation (( )). Instead of integration, we can do some summation all that. Hit the bull's-eye.

Student: (( ))

What is it? Mathematically what have we done to a continuous function?

Student: (( ))

It is a piecewise constant emissivity model. So, a continuously varying function has been converted to a piecewise constant emissivity which can be easily handled and I know I can use the f function chart. Now, this sort of epsilon lambda versus lambda. So, many problems you have solved is it not correct? So, the key here, to a piecewise constant emissivity model, so, piecewise, first piece, second piece. So, it is a piece wise constant emissivity model. Of course, you can have piecewise linearly varying emissivity model. That will be little more complicated.

So, when do you think the complexity will increase? You can, as I keep talking and note down the point also. The complexity will increase if you employ more bands. Accuracy will also increase as we employ more bands, but it is a trade of between your computational cost and accuracy. So, accuracy will increase with number of bands; complexity will also increase with number of bands. Accuracy is also critically dependent on how good this piecewise model approximates the non-gray body behavior. That is the final point which I have said is very very important. How good this piecewise model? Though, finally the accuracy of the calculation depends on how good this piecewise constant emissivity model mimics the actual non-gray behavior of the surfaces under question. Problem number, problem number 30, 35

Student: Sir, is, is this graph is same for all the surfaces (( )) enclosure?

No, no, it is it need not be. If it is same, very good for us. Then, can we just calculate the total hemispherical (( )) and do it with the previous method. I do not, I think we have to check this. It would not come. What he is saying is every surface has like this. Can we just calculate the hemispherical emissivity once you know the temperature? It will give some answer, but I do not know whether it is 10 percent, 20 percent how far, how far, it is off.

Student: (( ))

I do not, we, we will have to do an exercise, then find out.

Now, problem 35 or 36? 36. Consider two infinite vertical parallel plates. Consider two infinite, consider two infinite, vertical parallel plates a temperatures t 1 equal to 1000 and t 2 is equal to 500 Kelvin. Consider two infinite vertical parallel plates at temperatures t 1 is equal to 1000 and t 2 is equal to 500 Kelvin respectively. The space between the two surfaces is evacuated; the space between the two surfaces is evacuated. The hemispherical spectral emissivity distribution for both the surfaces, the hemispherical spectral emissivity distribution for both the surfaces is given in the figure. The hemispherical spectral distribution of emissivity for both the surfaces or the two surfaces are given in the figure. Compute the net radiation heat transfer from one; compute the net radiation heat transfer from surface one. So, I will be decent; I will give a simple so that you do not have.

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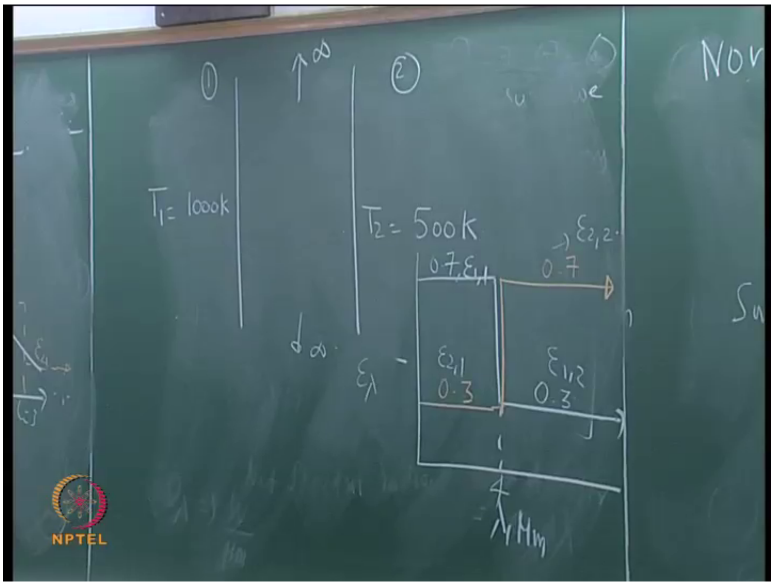

Please take out your f function charts. If you do not have, then share it with your 0.7, 0.7, 0.3, 0.3. So, this is epsilon 1 1; this is epsilon 2 2; this epsilon 2 1; this epsilon 1 2. and you can add the J's. You can add the q's and get the total q whatever you want. Now, now, better to use the F function charts. How many times we will use this?

Student: 4

4 times, very good. We will use 4 times. Why four times? Yeah, yeah, correct 0 to lambda, 0 to lambda, and lambda to infinity, 0 to lambda and lambda to infinity for the surface 1 and also for the surface 2.

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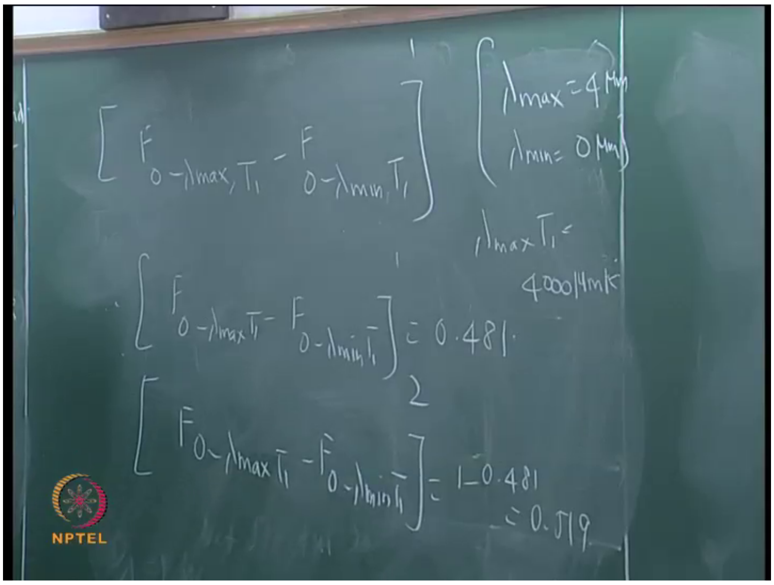

We will first calculate this fine? First surface - band 1. You got the F, correct, 0.48. Then there is no need to calculate. This is automatically 1 minus, correct, 1 minus 0.481. I hope everybody is following this. Now, I have to do that for the T 2 first band.

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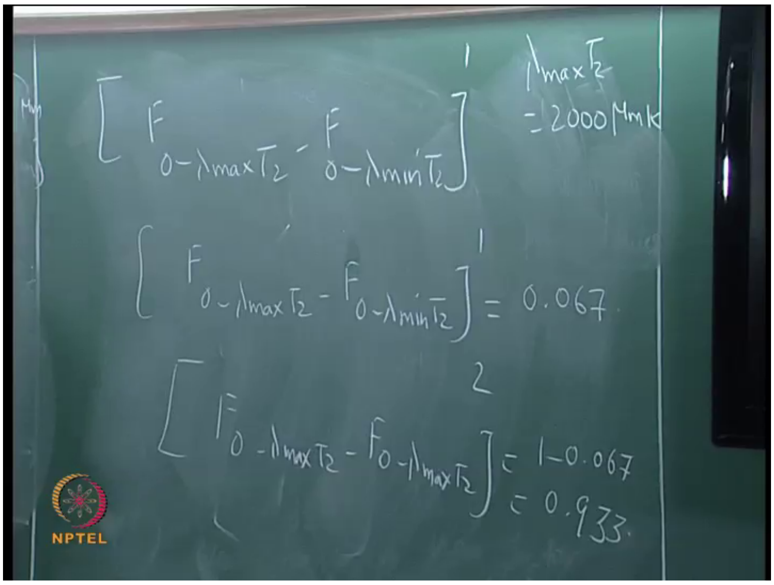

So, what is the story there? Lambda max T 2 equal to 2000 micrometer Kelvin. Therefore, there is F of 0 to 2000. How much is it?

Student: 0.067

0.067, very good. Now, F of 0 to lambda max T 2 minus of 2 equal to 0.933. Now, we got the important things here. The other things are straight forward. Now, we will write the band wise equations.

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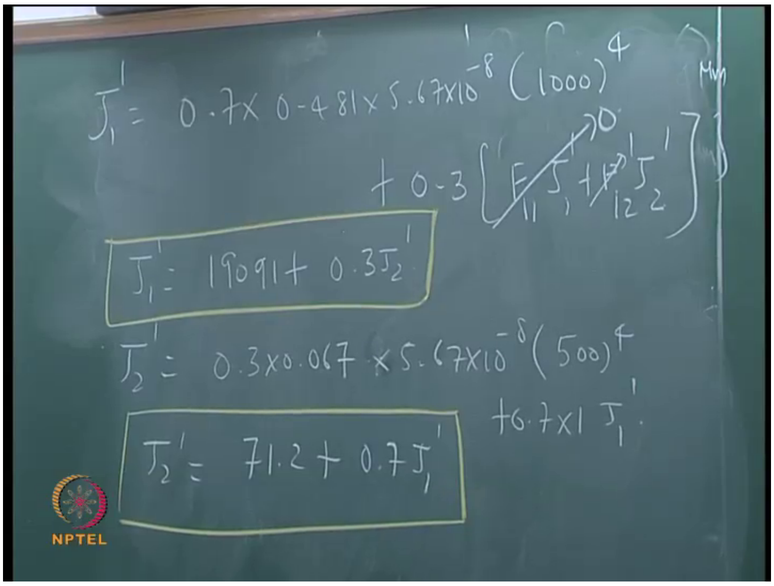

So, J 1 1 0.7 to 0.481 to 5.67 10 to the minus 8 to 1000 power of 0.3 into view factor. Is that correct? F 1 1 J 1 plus F 1 2 J 2 Are you getting this? Correct? (( )) did you get it? So, J 2 dash please check if I have made any algebraic errors. That 0.3 into 0.067 everything is fine? I did it last night.

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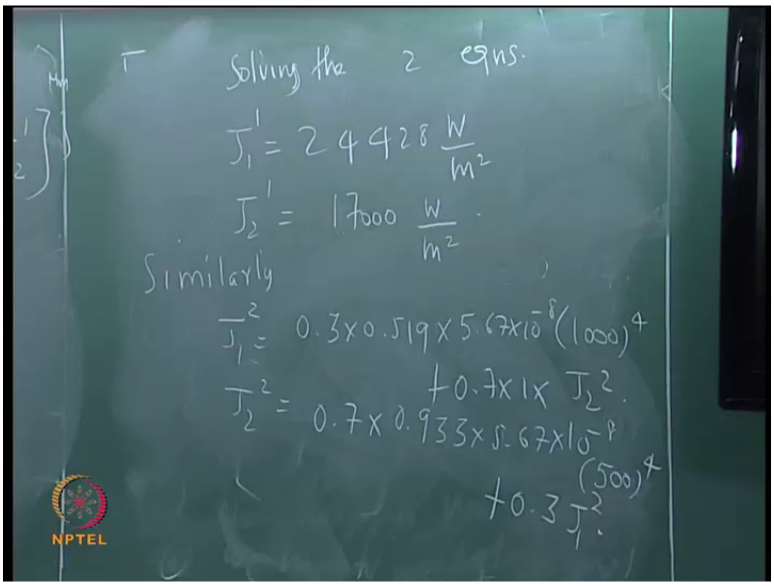

So, I have not checked it again. It could be wrong. Is it fine? Both are fine? Solve for J 1 J 2; solve for J 1 J 2 of 1 that is band 1. Solving the two equations, any problem with that 24,000? Abhishek, did you get it? Ramanujam, you have got it?

Student: (( ))

24

Student: (( ))

24,000

Student: (( ))

428

Student: (( ))

24428 watts per meter square J 2 dash and J 2 1

One point (( ))

17,000, yes, exactly similar procedure. You will get J 1 and J 2 of band 2. You have got J 1 and J 2 band 1. So, similarly, see, you have to simple system suppose here three surfaces and you had self view factors, it will be very messy. Then you have to do the same thing again and again for how many ever bands you have plus correct, yeah. What we get on solving this J 1 2?

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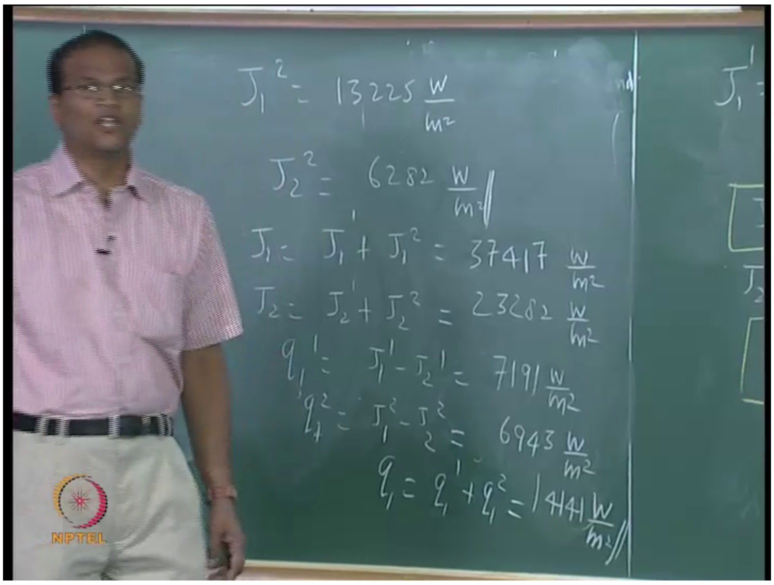

Student: (( )) two zero(( ))

Is it?

Student: 13,000 (( ))

13,225 that is what I get, check. So, J 2 2

Student: (( )) 6000 thousand (( ))

Is it ok? What happened? Equations are right? Now, yeah, get the expressions for J 1 and J 2.

Student: (( ))

No micrometer. (( )) already been (( )), already been integrated. When you use the F function chart, it is already done. How much is it?

Student: (( ))

Yeah, you can get the radiosities and then q 1 1. I am able to write q is equal to J 1 minus J 2 because q equal to J 1 minus g 1; g 1 for this problem is J 2. We have done so many times. When you study for quiz, you should not get confuse how did I write q equal to J 1 minus J 2. It is obvious. J 1 minus g 1 g 1 for this problem is F 1 1 J 1 plus F 1 2 J 2; it is j 2. Therefore, I can write. Now, please fill this J 1 is…

Student: (( ))

Directly you can write, but for the sake of completeness J J 1 J 1 is

Student: (( ))

3765

Student: (( ))

37650, yeah, j 2

Student: (( ))

23,000

Student: Two (( ))

282, yeah, q 1

Student: 7,428

7,428. q 2? q 1 2?

Student: (( ))

6

Student: 694

694. So, the sum is…

Student: ( ))

Yeah, 24,428 is correct or…

Student: (( ))

24,192, yeah. So, what all will change? This is?

Student: (( ))

Yeah, then q 1

Student: (( )) 7 (( ))

7191. So, total is now 15,000. How much is 14,000?

Student: 14,000

14,000. You can check that q 1 must be equal to minus q 2 because that is the energy balance. Whatever is coming from one must go to two and vice versa, because there is no other surface which absorbs emits or participates in the radiation. View factor is exactly one between the two surfaces. Now, you got an idea; you can complicate by I can make epsilon lambda versus lambda such that I can give two three places where it is changing and I can give epsilon for the surface to I can give some other distribution. So, it will be an extremely painful procedure. This is simple.

Student: Sir, if they are not changing at the same lambdas?

Then we can still do.

Student: (( ))

We can now…

Student: (( ))

You have to split, and if you are not splitting at the same place or if I have some, if you want to have fun, then we can do like this; then it involved.

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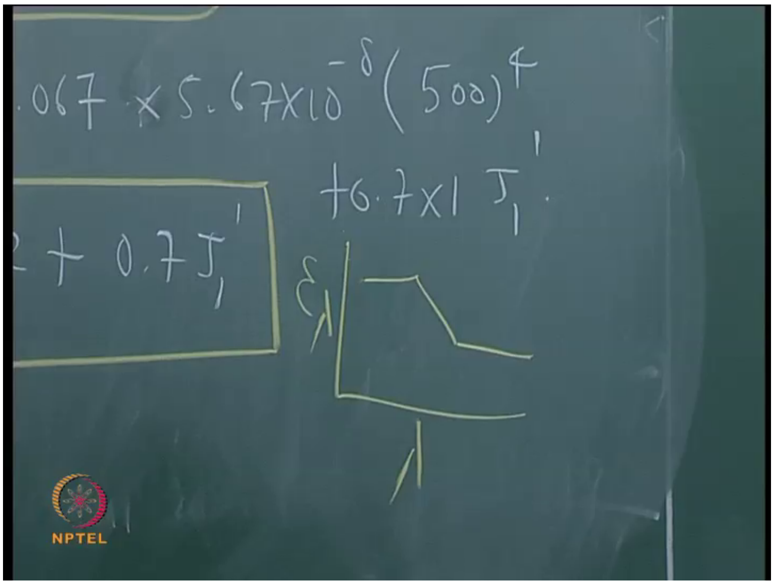

So, you, you, may get stump for a few minutes, but you can, you can, recover and you can proceed. This gives an idea of… So, I told you that these calculations are important. Do not think it is a just a academic interest. When you are working with atmospheric radiation satellite technology and all that, all the spectral formulations there. Apart from this spectral formulation, you will have spectral formulation and participating atmosphere that makes it very complicated.