Conduction and Radiation

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Module No. # 01

Lecture No. # 27

Radiation in Participating Media

So, today we will start a new topic namely: radiation participating media, and the first 15 minutes, I will use power point because it is descriptive. I will make this available on module. You do not have to copy it down. After that, we will switch over to regular chalk and talk. We will have to derive lot of equations and solve problems and so on.

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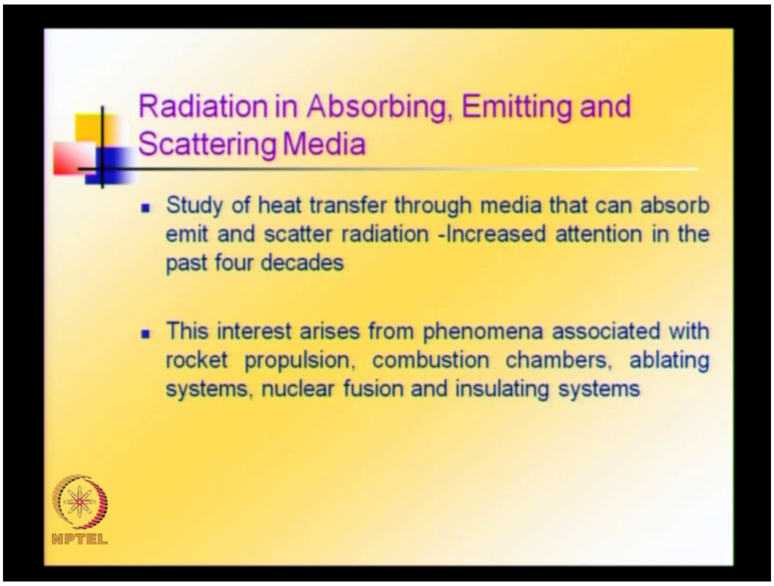

So, a participating media is a, is an observing emitting and scattering medium. You know that any particle which has a temperature more than 0 kelvin will emit that comes from provost law. So, it will emit. Absorption is something different for us like gas volume absorbs radiation, that is, if 100 watts per meter square come, what comes out may be less than hundred, will be less than 100 watts per meter square. It absorbs certain positional certain electromagnetic radiation. So, that is absorption.

So, what this, what it can do further is - it can receive the radiation and reflect it in several directions. That is called scattering. From a volume, we call it as scattering. This is called out scattering. Like that it can, there can be in scattering from other directions also. So, this out scattering need not be the same in all the direction. So, which means it is not isotropic; it is anisotropic. There can be functions, that is, this how much it scatters can be a function of theta. It could be f of theta and then you will have to integrate, what will be the total intensity of scattering in all that. So, it becomes very very advanced handling scattering media and all that.

So, in this course, we will avoid scattering. We will just have emission and absorption. So, this is alright for unless you have big suspended particles and all that. If you have very small particles or no particles, but you still have carbon di oxide and water vapor, you can consider that to be just an absorbing and emitting media. So, the study of heat transfer through media that can absorb, emit and scatter radiation has been receiving increase attention in the last 40, 50 years, and this subject was not there before 1940 or 1950. In fact, it gain momentum and process. Subrahmanyan chandrasekhar actually figured out the equation of radiative heat transfer - Nobel laureate Physicist from India. So, early work probably in the 30's and 40's; afterwards, engineers started looking at it because it had lot of engineering applications. I will let you know.

The interest arises from phenomena associated with the rocket propulsion, combustion chambers, ablating systems, nuclear fusion and insulating systems. In all these, in all these, cases, you have gases which are participating; gases which are not like air. In this room, radiation from the left wall will directly go to the right wall. Air will not participate. Air is at temperature, is at some temperature. Convection may take place all that is, but this will not create mischief, but if you, the moment you put carbon-dioxide and water vapor, it will do something. It will absorb and then something it will scatter and all this. That is why we have this global warming, greenhouse gases and all that, because they would not keep quiet and there absorption, absorption, and emission and all that, they spectrally dependant.

If it is spectrally dependent, that means it has to be dependent on the temperature, and if you, if you, look at the absorption of solar radiation by the earth, you can see that the radiation is incoming from a blackbody at 6,000 Kelvin, whereas the outgoing radiation is from a body at 300 Kelvin or 288 you worked out in the quiz; you worked out the earth's temperature.

So, basically the incoming is mostly in the visible; outgoing is mostly in the infrared, and if the gases which are in the atmosphere are such that, they permit the incoming and do not permit the outgoing. Then what happens is there is a constant build up of energy within the, at the earth. This is responsible this is very quick half a minute course on global warming. We can actually work out the spectral branch and all that and find out if the concentration of carbon dioxide in the atmosphere is increased from 380 ppm to 390 ppm, what happens and so on. We can do detail calculation, spectral calculation like what we did yesterday; it is spectral radiosity like that you can find out for the atmosphere and so on. So, there is lots of engineering applications for gas radiation.

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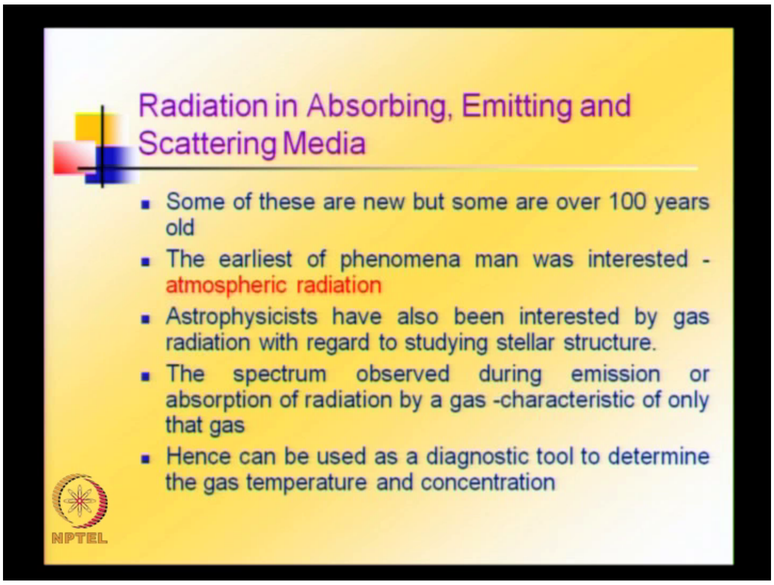

So, some of these are new but some are over 100 years old. The earliest of phenomena man was interested was in atmospheric radiation. Astrophysicists have also been interested by gas radiation with regard to studying this stellar structure. If you want to study the structure of stars, you have to find out the radiation which is coming out from the stars. You will have instruments and you will have bubble telescope, hubble telescope and all that, and you can then you get some radiation, then you use some equations, and then figure out what is going around. This is, this kind of activities basically or fascination to astrophysicists. That is what, that is a kind of work they do. So, they analyze of a spectrum coming from various stars other planets and all that, all that.

The spectrum observed during emission or absorption of radiation by a gas is characteristic only of that gas. From the spectrum, you can get what is a signature. That is a signature; we all put signature; that means you are saying that if you say Balaji, if we sign that means, I have seen it; I have read. It is me. Like that if you say Vikramhm like that this. Suppose you get emission spectrum from a gas, from the gas which is surrounding the, which is at the surface of a planet or a star, then it is, it is, indicative; it is diagnostic of what is, what is, a gas which is present on that surface. So, from that, we can figure out many things.

Then if you study over various periods of time, you can find out the gas concentration which is changing and then you can put some equations; you can figure out whether the, that star is moving away from us, whether it is moving closer. They will do all sorts of, they will apply some mathematical models; they will combine their experimental data with mathematical models they will; do such type of work

Hence, this spectrum can be used as a diagnostic tool to determine the gas temperature and concentration. Like that we have all these techniques. We go to sophisticated analytical instrumentations and facility safe which we have here such as scanning electron microscope, x ray diffraction. Based on some physical principle, it will give you some output. From that output is called the signature of that specimen or somebody is using ultrasonic and then you can get the structural health of something by remotely finding out since, since, some ultra ultrasonic waves.

Get the echo pattern. You send the echo, echo, into the abdomen and get the, receive the, send the signal; receive the echo and find out. It can even tell us whether liver cancer is there; pancreatic cancer is there and all, that is, so, that gives the signature of what is going on in the, what is a texture; what is a echogenicity; what is a texture of this thing, tissues in the liver whether it is normal, whether it is suggestive of carcinoma or something like that. Like that this spectrum can also be diagnostic of the gas temperature and then concentration.

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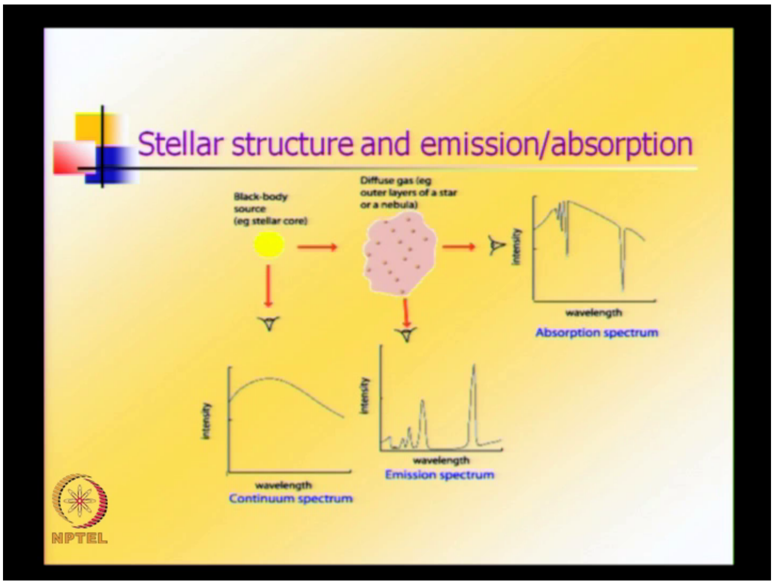

So, if you consider, so, if you consider a blackbody source, stellar means star, star core or a diffuse gas example outer layers of a star structure. You can look at two spectra. You can look at the absorb emission spectra or the absorption spectra and give a signature like this, intensity versus wavelength, intensity versus wavelength. From this, from this, you can have; you can figure out what will be the gas concentration, gas temperature. So, this is basically I what it is called as an inverse problem. From the output, you will have to figure out the input. So, there can be various causes for this spectrum. So, the goal of a successful inverse methodology is to correctly identify the cost which created, which led to that behavior. So, these are discrete spectra. These two, this is a continuous spectrum. This is just a depiction.

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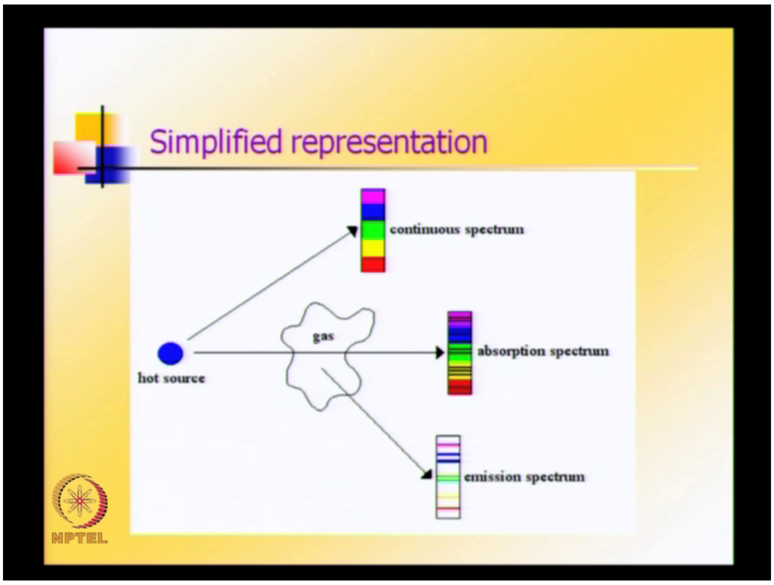

Simplified representation, you have a hot source. There is a gas and then you can either get a continuous spectrum or you can get absorption spectrum like this, emission spectrum. These are called absorption bands; these are called emission bands.

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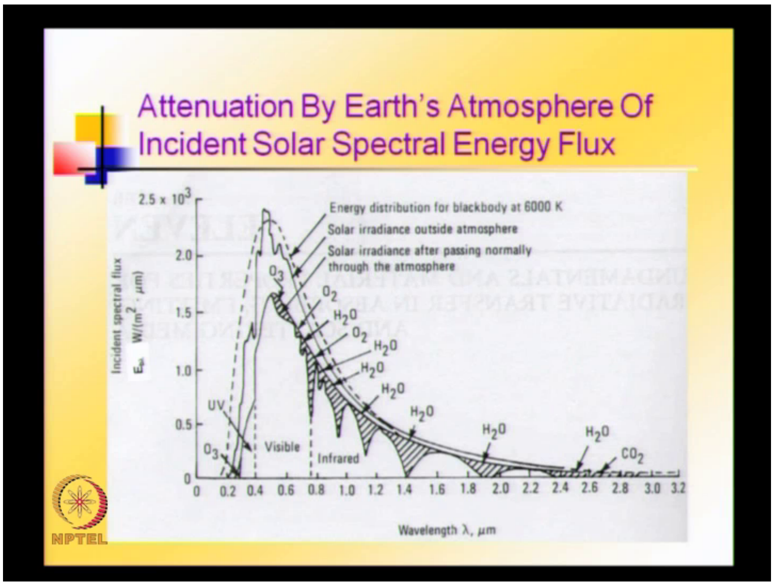

If you look at the incident solar energy flux which is coming on to the earth, then you can see. So, this is basically e b lambda watts per meter square per micro meter versus lambda. Now, this first line, dash line gives the energy distribution for a blackbody at 6000 Kelvin. Now, this is the actual solar radiance, solar radiance outside the atmosphere. So, then solar radiance irradiance after passing normally through the atmosphere, now you can see that it has got zigzag, zigzag variation, because mostly there are absorption bands available, oxygen molecule diatomic, diatomic, triatomic, oxygen, water, water vapor and all this, carbon dioxide bands, because of this, there is a signature. So, you can actually figure out the concentrations by looking at, by having an instrument which will exactly which will have a sensor which will capture between two particular wavelengths and so on. So, you can see that ozone is very much; this ozone absorption is there very much in which part of the spectrum?

Student: (( ))

This carbon dioxide?

Infrared

Infrared, this water is everywhere, water is everywhere. That is why this water vapor you can design an instrument to measure the water vapor by having a multiple channels. In a, in a geostationary orbit, you can put a satellite. So, it is called multi spectral, is called a multi spectral instrument, is called multi spectral sounder. So, the INSAT series of this thing or which we are also working next one is INSAT 3 d series, which will have 18 channel. We basically want to look at, you basically want to get the water vapor and, water vapor and, temperature profile, water vapor and humidity. So, we are actually coming out. So, launch will be any time this year.

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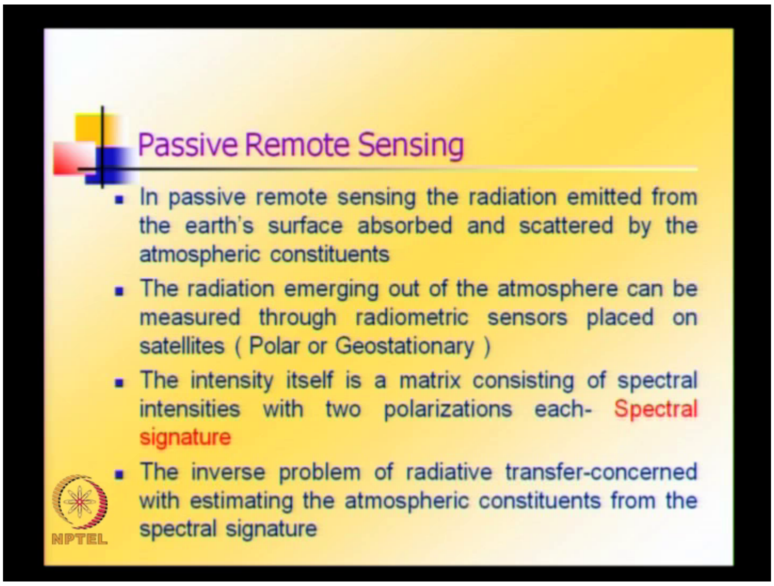

In passive remote sensing, the radiation emitted from the earth's surface is absorbed and scattered by the atmospheric constituents is measured at the top of the satellite. It is called the top, top, of the atmospheric radiance. The radiation emerging out of the atmosphere can be measured through radiometric sensors, sensors placed on satellites; it could either be polar or geostationary.

The intensity itself is a matrix consists, consisting of spectral intensities, that is, you can measure it at just like yesterday we saw with various j lambdas. You can get the if j lambda or the q lambda in, as a function of lambda. It would not be as a function of lambda; it will be a discrete values of lambda. The inverse problem of radiative heat transfer is concerned with what are the atmospheric constituents which gave rise to this q lambda at various lambda. So, what will be the logic? You will assume some atmospheric concentration. Solve the equation of transfer and find out q lambda at various spectral in, at various lambdas. You will match your prediction with what is absorbed by satellite. Generally, it will be not be agree. Then you will iterate. You will give a new guess. You will keep on guessing till they match closely.

So, this guessing is a big; this guessing is easiest than done. What I told you in one word is guessing is the field of inverse problems. How do you scientifically guess? That is a inverse problem. So, many things based in markov chain in montecarlo, hybrid montecarlo, so many regularization, tikhonov regularization.

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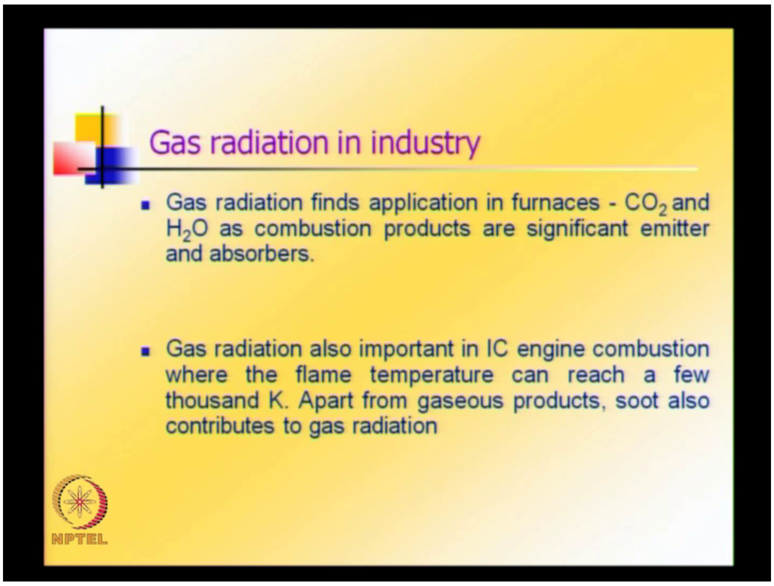

So, whole big field of inverse heat transfer is there. There are journals specifically devoted to inverse, inverse problems in science and engineering. There is one journal. What about gas radiation in industry? Gas radiation finds application in furnaces, carbon dioxide and water vapor, combustion products. They are significant emitters and absorbers. These are, these gases are found in furnaces combustion chambers IC engine. They are also there in IC engine.

Gas radiation is also important in IC engine where the flame temperature can reach a few 1000 Kelvin. Also the one important thing, there is also soot; there is also a soot in the IC engine products of combustion; soot also is luminous. So, soot also contributes to gas radiation. How do you handle soot? So, some group of scientists are work soot modeling. That is another that is another, separate field. There are some big stalwarts in Chalmers University, Sweden and all that, where whole life time, they have done soot model, whole life time they and they wrote lot of things. All your commercial course, all the course will use this. The moment they are found to be inaccurate, somebody code is found to be more accurate. That will be incorporated. That is how science and technology progress.

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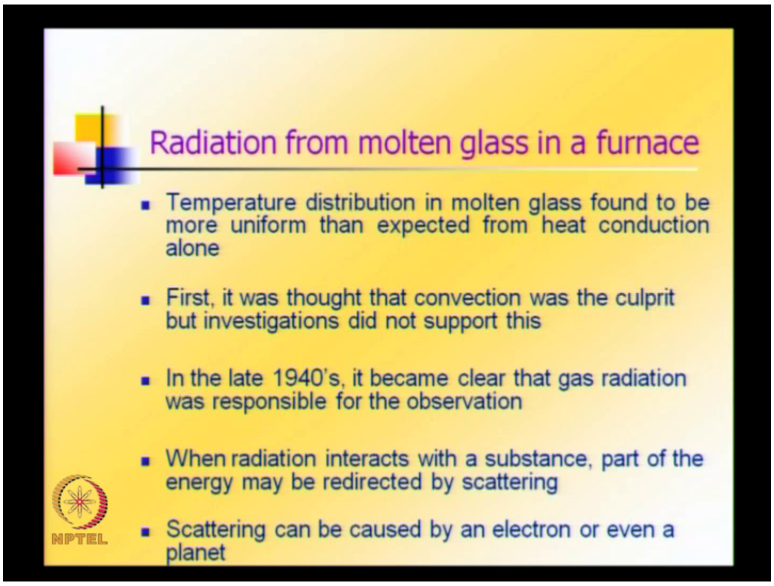

So, if the origin of all this is basically the classical problem of radiation from molten glass in a furnace. So, glass making is a, glass blowing is basically a very traditional and very old technique. So, temperature distribution molten glass when they measured it, they found it to be more uniform than expect, than expected from heat condition alone. When they did the measurements, when they solve the heat condition equation if they, if there is a theoretically expected temperature distribution, the actual temperature distribution was much more uniform, then they started investigating why is it like this; why is the conduction equation not able to satisfy this accurately.

Then, they thought that the convection is the culprit. There may be some convection at the boundary. Then you put q is equal to h a delta t minus k d t by d x is h a delta t at the boundary and all that. They could not get agreement. Finally, they found out within the glass itself, glass itself has got an emission and absorption characteristics, and therefore, you have to integrate this conduction with the equation of transfer.

So, in the late 40's, it became clear that gas radiation was responsible for the observation. So, when radiation interacts with substance, part of the energy may be redirected by scattering. Scattering can be caused even by an electron or it can be caused by a planet also. So, the, these are the length scales which are involved in scattering. Depending upon the length scale, I will use different theories of scattering. If it is very small, I will use only Rayleigh scattering. Rayleigh scattering studied by Raman and that is the, that is why the sky is blue now and all that.

Next, if I have suspended, if I have ice, water, rain in the atmosphere, I cannot use Rayleigh scattering. I will use what is called the Lorenz-Mie. I will use, I will use, the Lorenz-Mie scattering theory. That is what we do in our work little more advanced after some level if the particles are big enough and they are non spherical big big ice particles, then all these theories will fail, because Lorenz-Mie means scattering basically assumes the particles to be spherical. Then all these will cup; then you have to use geometric optics. Finally, it will become optics, electromagnetic theory optics. That is the escalating difficulty level associated with studying gas radiation.

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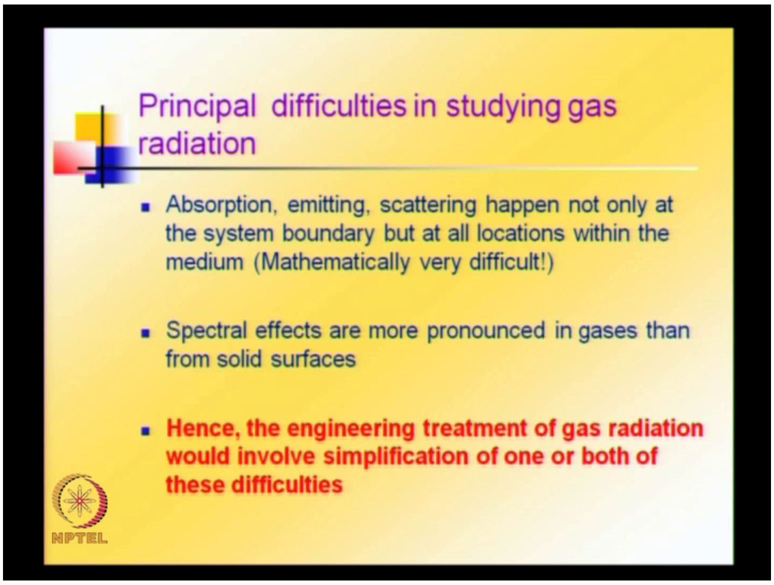

What are the principal difficulties in studying gas radiation? Everything is happening from a volume; it is not happening from a surface like before. Everything is happening from a volume, and as usual the problem is there, this absorption emission and scattering there function of lambda that is also there. So, absorption emitting and scattering happen not only at the system boundary, but at locations within the medium which makes it mathematically very very difficult; Azimuthal, Zenith, all angles you have to be considered, and this fellow may behave differently; it may behave crazily in different directions though. These have to be accounted.

Spectral effects are more pronounce in gases than from solid surfaces. This gray, this gray, gas assumption is more a myth than a reality, but many surfaces we put gray diffusions proceeded, is not it in the last one month? Hence, the engineering treatment of gas radiation; however, as engineers, we want to solve the problem. We do not want to keep simply say the problem is very difficult, very difficult, and keep quite. We want to make some products, and so, we want some approximate solutions.

Therefore, the engineering treatment of gas radiation would involve simplification of one or both of these difficulties. What will I say? One, one, I can say that scattering is negligible; that means, I am handling point one or I will say that in the region of, in the spectral region of interest, it is gray. So, I will do one of the two one of the one or both of the simplifications in order to make the problem tractable.

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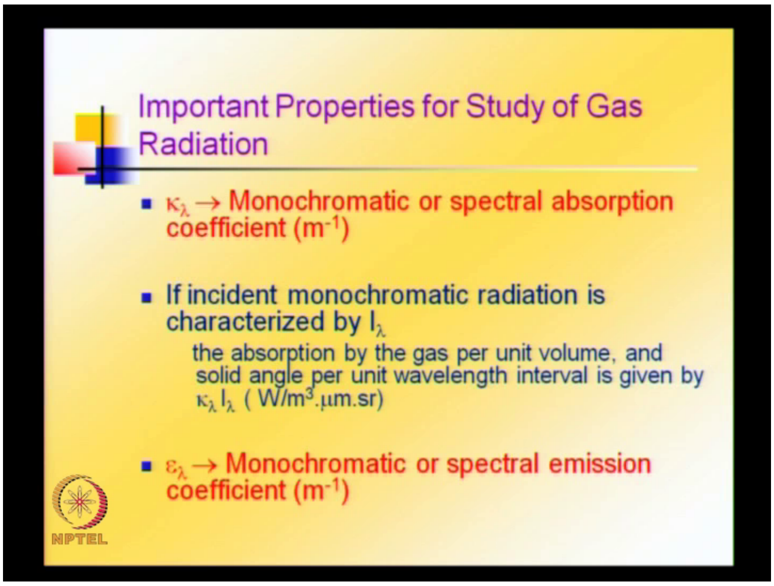

Now, what are the important properties for study of gas radiation? Kappa lambda, kappa lambda, is the monochromatic or spectral absorption coefficient. Kappa is the absorption coefficient. Therefore, if incident monochromatic radiation is given by I lambda, the absorption by the gas per unit volume per unit solid angle per unit wavelength interval is given by I lambda kappa lambda. There is a short; there is a short form also known as.

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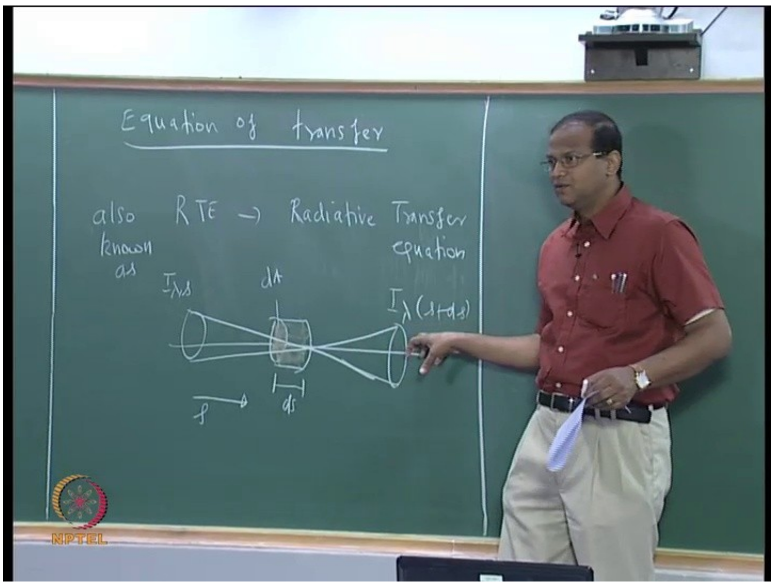

Student: (( ))

Radiative (( )) transfer

No, no, that is ok. There is short form for also known as

Student: (( )).

(( )) So, we are not worried about the short form of a k a. The long form of RTE, the long form of RTE is radiative transfer equation. So, Professor S. Chandrasekhar is credited with this mathematical development associated with the equation of transfer and brilliant book. It is very difficult to follow up beyond 10 for first 10 pages. It is there in the library principles of radiation or radiation heat transfer is, mathematical level is so high. So, S. Chandrasekhar our Indian born, Indian Scientist who studied in the Presidency College, Madras near the beach. He did his B sc physics there.

So, his book is there, his, all his, so, every ten years, he used to change fields - radiative transfer, stellar structures ten years, hydro dynamics ten years. Every ten years, he will conquer a subject, write a book and move on, and he says, he said that the greatest invention of mankind is god. So, you can, so, you can find out his leanings. That is what he said. Now, let us, these are gas volume cross sectional areas dA. This is ds. Radiation is I lambda at s, I lambda at s plus d s. Radiation is traveling in this direction. Generally, direction is s. Cartesian coordinate system we can just later on replace the s by x and so on. In order to make it less restrictive, we can use a notation s.

So, some I lambda is coming and it is going out. So, it is basically going through a distance ds. The area dA is normal to the direction s. The radiations, the radiations is traveling in the direction normal to the area. Now, we are trying to find out what is a rate of change of this intensity. What is the rate of change of the intensity of the radiation when it is passing through this gas valve? That we can write is, write it using calculus dA by d lambda whatever.

Then, we have to find out this rate of change of intensity not being equal to 0 is because of what all factors. We are neglecting scattering. There could be only two things - there could be absorption within the gas volume and there could be remission from the gas volume. So, the balance between emission and absorption will lead to this d I lambda. If you write this equation, write this three steps, that is the equation of transfer and it looks like a it is innocuous; it looks like a very simple very unassuming equation, but that it is a, we cannot say it is innocuous; it is deceptively innocuous, because when you have to solve it for three dimensional cases it becomes extremely formidable and difficult to solve.

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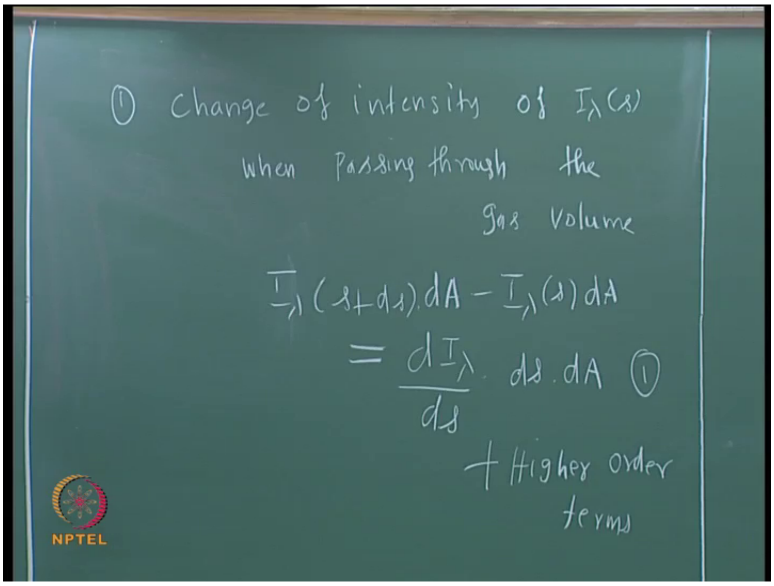

Now, let us say. So, let us consider change of, change of, intensity when passing through the gas volume is I lambda s plus d s into…

Student: (( )).

I lambda has got the units of?

Student: (( ))

So, we have to multiply by dA. Do not multiply by dA ds, dA minus, correct? What is going out minus what is coming in? Using a Taylor series expansion, how will you write this?

d I lambda (( ))

d I lambda by ds.

Into d I (( ))

plus plus

d square i.

plus higher order term.

So, we set the higher order terms 0. That is error associated with this approximation. If the ds is very small, this approximation is alright.

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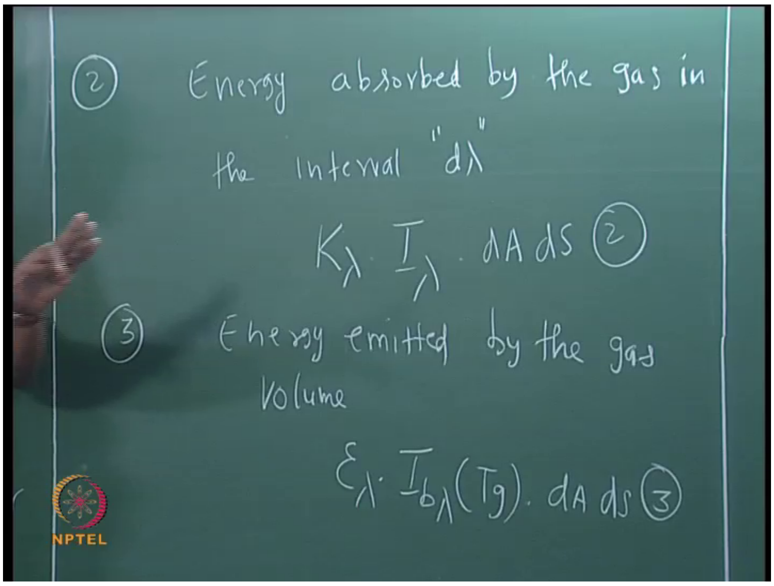

Now, energy absorbed by the gas. Please be reminded that it is the spectral formulation. I am retaining by watts. I am retaining by watts per micrometer per steradian. It is a spectral formula. So, what will be, what will this be? I introduced two important properties - kappa lambda and epsilon lambda. So, please tell me kappa lambda I lambda multiplied by…

Lambda dA, dA

dA, very good; d lambda is any were there know. It is there in I lambda itself two.

Student: (( ))

We looked at energy absorbed; we also look at energy emitted by the gas volume.

We cancel dA ds throughout, correct? When you cancel dA, d A, when you cancel dA ds throughout, it is implicit that dA into ds cannot be 0. Therefore, these equations are not valid at mathematical points. That is a control volume.

A mathematical point has no area, no volume, nothing. So, these are valid only a around, around, a small area or volume. That is, that is, very important for us. Otherwise, you are, you are not authorized to do that. Then you can take that 4. How do you, how do you, prove? 4 equal to 6 4 into 0 equal to 6, equal to 6, into 0. Therefore, 6 l h is equal to r h.

Student: (( ))

What is that? Now, you tell me. When it is absorbing, I am saying that I lambda will decrease. I have a sheet of water; I am shining a radiation on this; obviously, as the radiation goes through, the bottom layers will get less radiation. In that case, what will happen to I lambda from the surface? It will exponentially decay, correct? So, it decreases the intensity which is coming out because some of it is observed. If 100 enter, 95 comes out; 5 is held back, but that 95 can be compensated by, it, it can become 97, 98 or even one not 5 if there is an emission from that gas. What is, that is a change, yes. Now, therefore, no, some people would not write it like this. Yeah, any problem? This is the RTR equation, RTE. This is the so called radiative transfer equation.

Student: (( )).

You have to have in scattering and out scattering. So, d I lambda will be equal to, d I lambda by ds will be equal to emission minus absorption plus out scattering minus, out scattering minus in scattering that you will you take care of the algebraic, but these out scattering in scattering also will involve some integral terms, because this scattering may have integration which is different in different angles, correct? If you include scattering, you may have to use an integral term because integral with respect to that solid angle because it can scatter differently in different direction. Then what will happen is the left hand side will have differential term and the right hand side will have integral term. Then it will become inter go differential equation.

The classic radiation equation is an integral differential equation. Already differential equation is so difficult to solve. Integral equation also, integral equations also, are very difficult to solve. So, the integro differential equation make the radiative heat transfer extremely difficult. Fortunately, we have numerical techniques and methods now available. Peoples struggled, you please look at the book methods of applied Maths by Hildaabraham. They variables have separable, kernel and all that. The kernel of integration and so, it was consider big craft solving an integral equation. I did not tell you the integral equation at all. I just quiet quietly moved on.

Suppose we consider a three surface enclosure, in which the temperature of each of the surfaces is varying with respective x. Then I cannot use the uniform radiosity assumption. If I cannot use the radiosity, uniform radiosity assumption, I have to divide each surface into sub parts and then I will have j x d x or j j 1 itself will have several things. Therefore, instead of summation, I have to use that integral.

Therefore, the radiosity equation which I told you where I put j equal to epsilon sigma t to the power of 4 plus 1 minus epsilon sigma f I j jj will be integral. So, the radiosity equations per say or by themselves or integral equation. I did not want you to get scared. So, when I learnt it as a student, professor SPV thought me all this integral equation. Exam questions were there that integral how to solve kernel. There are two types of integral equations - integral equation first kind second, second kind world tera integral, Friedman integral. That is the separate field. I do not teach my students all this now. We can straight away put sigma and use Gauss Seidel method and proceed. So, radiation is very deadly.

Now, are you happy with this equation? It is fine. It gives you the rate of change of this thing blah blah whatever, but I am having two properties. We try to do something where we relate the properties. Previously, we try to cut out, cut down the number of properties we want to independently evaluate. Can we do something like this?

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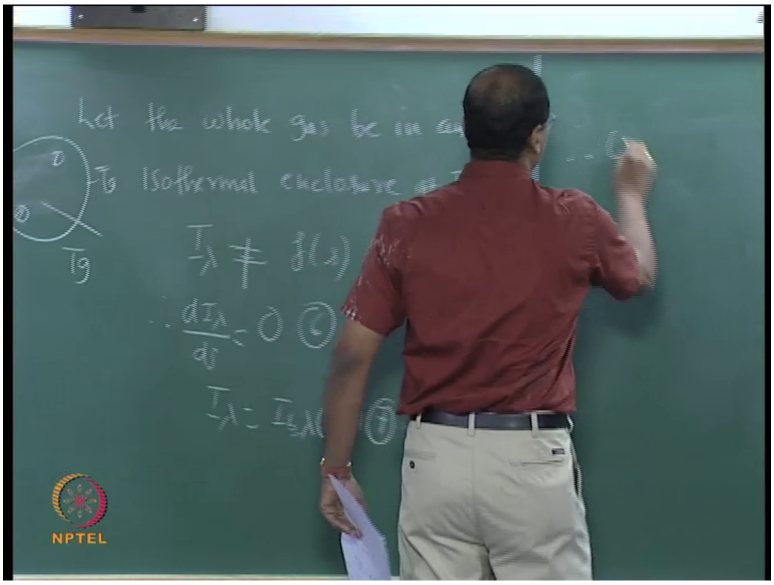

So, I will do a some small trick. Let the whole, let the gas be contained in isothermal enclosure. I will put forward an argument like this. Let the whole gas, let the whole gas, be in a isothermal enclosure. Then, what is the story now? Is it correct?

Student: (( ))

What is the funda involved in an isothermal enclosure? Gas, gas, and wall all are T g. I lambda anywhere at any point will the radiation be the same. Is it an isotropic radiation? Even if it is not black the surface, it is multiple internal reflection. So, the I lambda is not a function of s. If I lambda is not a function of s, what is d I lambda by ds?

Dhawal, you are not following. What I am saying is - if it is isothermal everywhere, then it is like a (( )) if you put a hole, radiation coming out will be analogous to radiation coming out from the blackbody. So, you got more or less a whole drum or a blackbody here. Therefore, if I take a sample, if I take a gas volume and find out its I lambda in a particular wavelength interval, if you take it here, will it be different. Therefore, dI lambda by ds equal to 0 anywhere within the, within the, isothermal cavity. What additional simplifications we can make and I lambda will be equal to…

Student: (( ))

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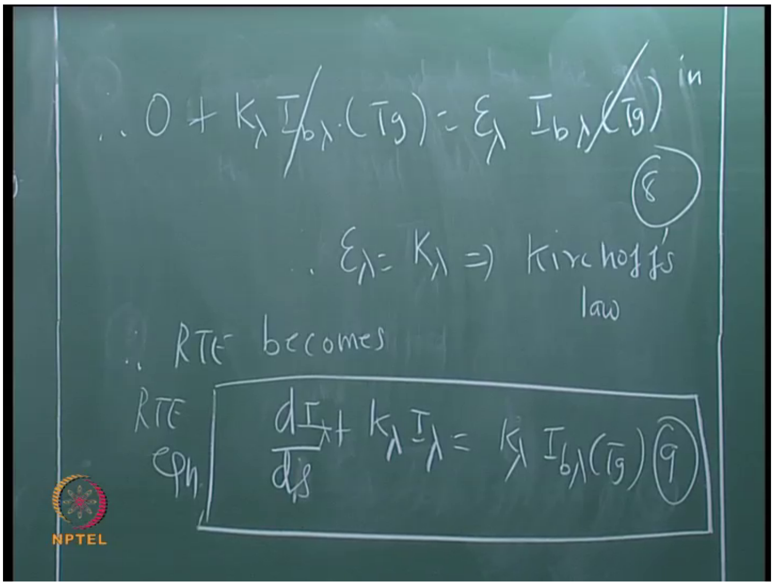

I lambda will itself be equal to I b lambda at T g, correct. Therefore, 0 equal to, no, in simple English, epsilon lambda equal to, what was this law? Kirchhoff law, correct. We already know this, but I try to derive it from first principles because you may object, sir, previously epsilon and alpha where dimensionless. Now, they are having meter minus 1, how do you prove. So, somebody will ask – sir, if it is not an isothermal enclosure, will it be valid, or I used the isothermal enclosure concept only to get the relationship. It is universally applicable. You can measurements and check it.

Now, therefore, no, This is RTE equation (( )) no way, dI lambda by ds, correct? Did you make more mistake?

Student: (( ))

Ya, it is ok. Do not think, assume that this equation is for isothermal cavity. Isothermal cavity there is no RTE man. Isothermal cavity I lambda is constant everywhere. Why do you solve RTE? You solve an RTE you write an RTE and solve it because I lambda is changing. You want to find out what happens to I lambda when it goes through some gas volume. So, you can see it is formidable; it looks so simple. Sir, its only d I lambda; its after all ordinary differential equation, and the note is now when you actually, the next class you will see the mathematics which comes out of it. It will lead to something; it will lead to an integral which cannot be evaluate, and then people invented what is called the exponential integral function. This simple equation will lead to so many complication. That we will see in Wednesday's class, but before that, I want to give you sneak peek into what happens when the absorption is much more important than emission. So, let us consider a case. I will look at an asymptotic case still have 5 minutes.

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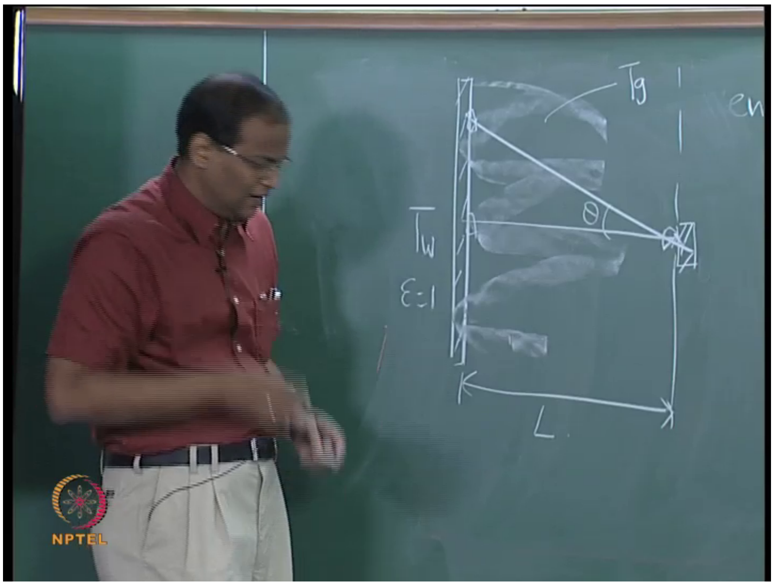

So, let us consider, let us consider, a wall, let us consider a wall which is black. It is at a temperature T W. Then it is surrounded by this. There is this gas at T g. This radiation is coming out of the wall. So, the radiation which is falling on, let say I have a receiver or a receptor. The radiation falling on this which is at the length L will depend on two things or it is made up of two contributions. What are these two contributions? The first contribution is the radiation from the wall which is given by sigma t t T W to the power of 4 by pi. That is I lambda; that is a i. So, I lambda that.

So, sigma t to the power 4 by pi which is originating from the wall which is modified or attenuated by the presence of this participating media and which eventually reaches this. That is not the only radiation which is arriving at this. There is also the radiation from the gas which is arriving at this, but what will happen is the radiation falling on this can also arise from this portion of this wall. So, it could be at an angle. So, we have to derive the formulation for general angle theta, and then when theta is equal to 0, it is for the normal angle.

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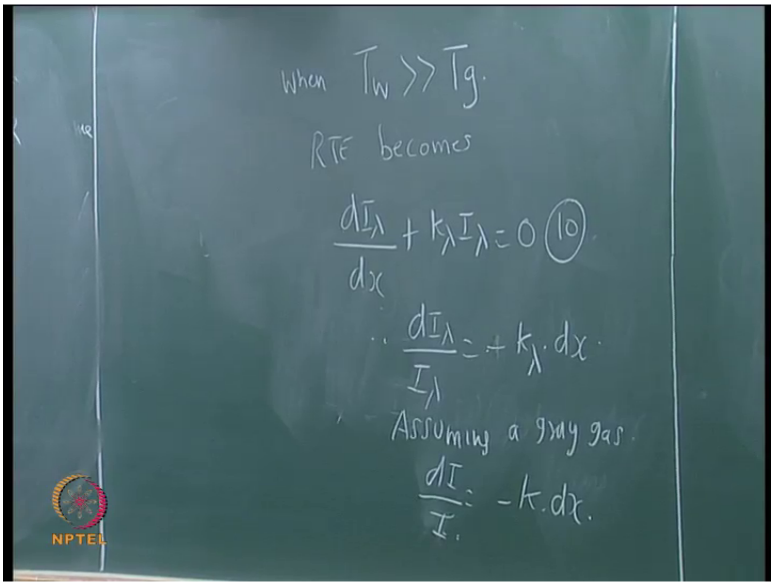

Now, so, now, slowly I am introducing the complication. We will work this out in the next class, but now, I want to look at this simplified case. When T W is much much greater than T g, what happens to the equation? Which term can be neglected? The epsilon lambda I b lambda T g is much much smaller compared to the other terms in the equation because emission component is very less. I have a 1,000 kelvin valve and the gases at only 300 kelvin. When RTE becomes, I will say that this is, I will say that this is x direction and I will say ten. Therefore, assuming a gray gas. correct? So, the radiation decays exponentially as it passes through the depth. So, this is called the, is called the Beer's law or Beer-Lambert's law or the Lambert's law. So, the solar radiation which is impinging on the ocean waters. So, let say it is

I not at the surface of the water. Then as it goes, it will be I not into e to the power of minus kappa x. So, as you go to the bottom layers of the ocean, the radiation which is, the radiation which is received at the bottom layers of the ocean is much less compared to whatever is received at the top, because the radiation received at the bottom is much less.

The bottom layers are at a temperature which is lower than the top layer. Therefore, the top layer will be at 30 degrees; the bottom layers, the temperature for example, at 1 kilometer, it may be something like, 1 kilometer depth, it may be something like 10 degree centigrade, and then this is a stable temperature gradient, because the warmer lighter water stays at the top. This is also responsible for maintenance of aquatic life and all that, and then if you exploit this temperature difference, you what is called the ocean thermal energy conversion called otec, and you can see that it is very difficult for organisms to survive deep below because no light is, light is, not available for photosynthesis.

So, some simple radiative transfer equation can be use to explain so many phenomena, but this is because the radiation is coming from blackbody at 5800. You do not worry about epsilon lambda, I b lambda of T g because that is only ocean temperature about 300 Kelvin or something, but the general case, when the gas emission cannot be neglected, then we are in trouble. Then the integration becomes very difficult if you evaluate the heat flux. We will, I will introduce the exponential integral function. I will give you tables and you have to use these tables to compute the heat fluxes. So…