Conduction and Radiation

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Lecture No. # 28
Solution to the RTE

So, in today's class, we look at solutions of the equation of Radiative Transfer. As you can very well imagine, the radiation, RTE is, that is, Radiative Transfer Equation is very formidable. It is very difficult to solve. Particularly, the left, the left, hand side has got a simple d I by ds, but do not underestimate that. Right side, there are lots of scattering terms, and if the scattering is not the same in all the directions, you have to use what is called anisotropic scattering, and the scattering, and the scattering, varies with the size of the particles.

We saw we the wavelength of the radiation which are considering. So, there is what is called the r by lambda or d by lambda parameter. Size ratio parameter is there. Depending on that, you can have a relay scattering. You can have Mie's scattering. You have to use geometric optics and so on. So, it is very highly, as I told in the earlier, one of the earlier classes, it is highly specialized field very pew, very few people work in that. Most of the engineering gas radiation is well understood, but astrophysics planetary, study of planetary atmospheres using satellites. The remotely measure rainfall and using satellites remotely capture the track of a cyclone.

Then, using satellites to infer the atmospheric profiles and then reconnaissance applications, military applications, spy satellites, so many other technologies. This will be (( )). What happens to this radiation is very very important. Even a simple night vision camera used by army is based on the infrared. Say similarly, the Swine Flu detection in the airport is based on the infrared which is emitted by. So, person having fever will have more surface temperature near the nose and there are the certain portions. Then they have a standard template. They will compare it to the reference benchmark.

If it, if it, is off beyond the certain limit, then he suspected, he suspected, to have Swine Flu, flu, and so on. Even you the CAT scan, the CT scan, and the CT scan is basically looking at x-rays and then looking at 3D. You look at, we take the pictures in various slices at various sections and then we reconstructed 3D. We reconstruct 3D image based on that. So, tomography, that computerized, computerized, axial tomography, that is, cat. That become CT scan and all that. Very advanced numerical method, advanced radiative transfer and all that, and non-destructive testing, ultrasonics, lots of applications.

Now, we will, because this is only of first level goes will see. Look at a simplified treatment to the equation of transfer. We will look at a one-dimensional. We look at something where we do not worry about spectral effects. We do not consider scattering and all that. A very very simplified solution for this was what we considered in the last class, where I completely knocked off the emission term and I told what happens to radiations which is absorb by of layer of water or in ocean and all that. What happens in the sunlight, as it reaches the bottom and so on. Now, we will formally look at some solutions of the equation of transfer and it will lead to some complicated integrals which will see as we go long.

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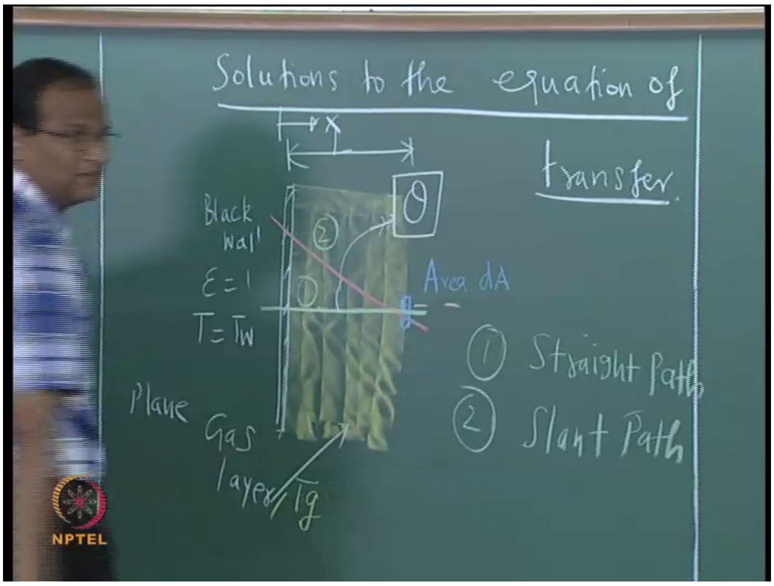

So, this is basically you are considering black wall. Basically, I am starting with black wall to otherwise that radiosity other things will come, and it will be epsilon sigma T to the power of 4; it will lead to other problem. So, reflection component will be there and so on.

Now, the temperature is T W. There is gas layer a plane glass layer, plane gas layer and not plane glass layer. It could be glass, but this is gas. Thickness is l. It is infinity in the, infinity in the direction. In this direction is infinitely deep in the direction perpendicular to the plane of the board. So, essentially we are looking at how I or the intensity varies in the direction x. x I have given some where now. So, this is x positive direction of x. That is the positive direction of x. The gas is at a temperature. T g the wall at a temperature T W, and the gas is going to absorb and emit radiation. So, it is gas radiation. Now, you want to solve the equation of transfer. By solving the equation of transfer, we want to find out how I or the intensity propagates with x correct. That is the goal. So, we have to write down the equation is already known to us.

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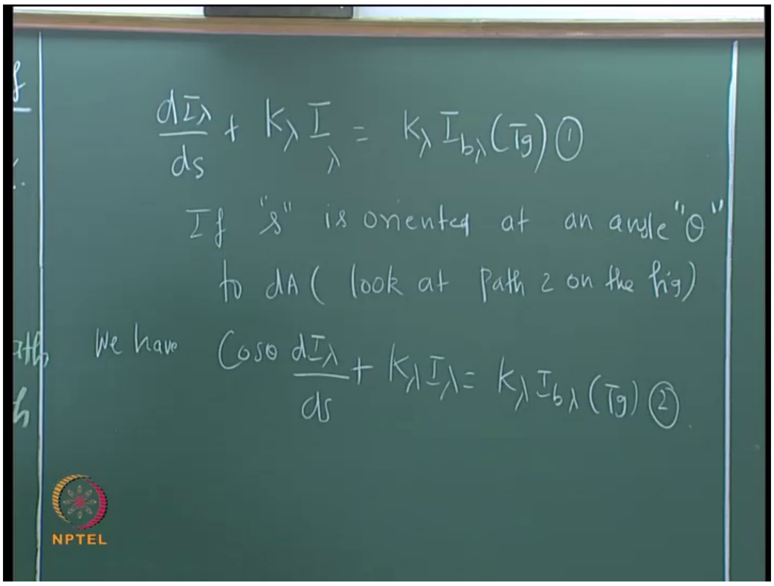

So, d I lambda by ds plus kappa lambda I lambda equal to kappa lambda of T g. Did you write like this? Correct. So, this is one. So, this is for the normal direction. If s is oriented at an angle theta to dA, look at path 2 on the figure. So, the equation gets modified as cos theta we have. Needless to say when theta equal to 0 equation 2 reduces to equation 1.

Now, we have to find out the story. At the radiation arriving at an element dA which is located at distance l, the gas layer thickness is l. Therefore, the radiation arriving at, arriving at a consists of two components - one is a radiation from the wall which is sigma

T W to the power 4 by pi, that is a I; q will be sigma T W to the power of 4. I will be sigma T W to the power 4 by pi which is going to going through the gas gets attenuated; that means it strength is reduce because the gas is absorbing or participating in the radiation and it is arriving here. Then there is also separately there is a component because of the emission from the gas which is also arriving here. There is transmission by the gas and there is emission from the gas.

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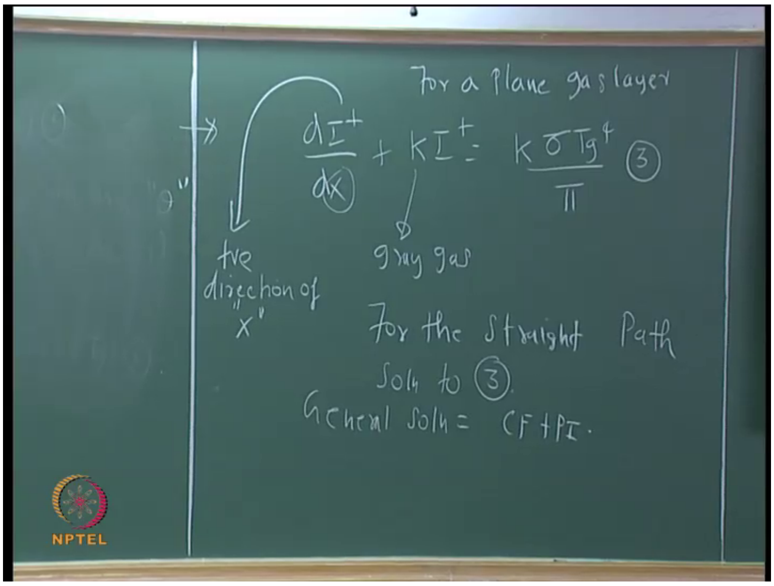

So, let us look at. Consider an area element. Now, let us simplify this. Let us say that d I for a, for a, plane gas layer dI plus dX. I have done several things here. Can you tell me 1 2 3? I have done several things here.

Student: (( )) gas is gray.

I am assuming that the grass, gas is gray first point, yes, because I knocked off kappa lambda and put it as kappa. So, kappa is equal to kappa lambda. Second, I am using something called I plus. What does it mean? I am looking at positive direction of x. So, what else? I am replacing the I b lambda by I b by sigma t g to the power of 4 by pi, nothing spectacular. There is nothing which was applied, which we did not know.

Now, we can write the, we solve the equation of transfer for the straight path and then will infer what the solution will be for the slant path. So, let us try and solve equation 3 for the straight path. The solution to three consists of two parts namely: the complimentary function plus the particular integral. The complementary function is obtained by setting the right hand side equal to 0 equation 3. Let me do that general solution to get the complimentary function.

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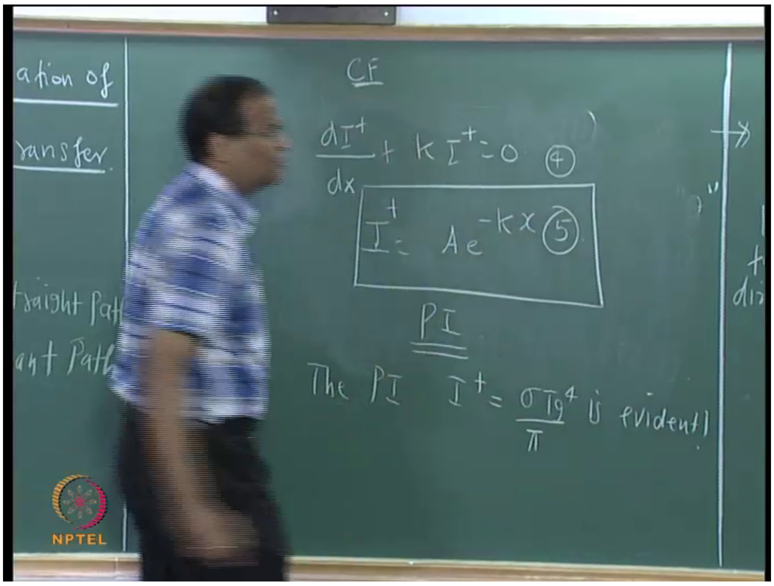

I also made one more change. I am using x instead of s. The original equation had s. I am using x. That is ok. This is right. I plus is a. I hope all of you remember this one way of solving ODE's. You can set the right side equal to 0, and first, get the CF and then you get the particular integral, and then combined the complimentary function of particular integral. Get the general solution; apply the boundary condition and get the constant of the integration. As far as particular integral is concerned, will you agree with me? The particular integral I plus equal sigma T g to the power of 4 T g to the power of 4 by pi is evident. How do you check it? Now, put this particular integral back into equation three. What will be the first term? dI by dX plus 0. Second term is K sigma T g to the power of 4 by 5. Therefore, this is a particular integral to this problem. Therefore, the general solution consist of the sum of both the CF and the PI. Seven, how do you get a? What is I at x equal to 0?

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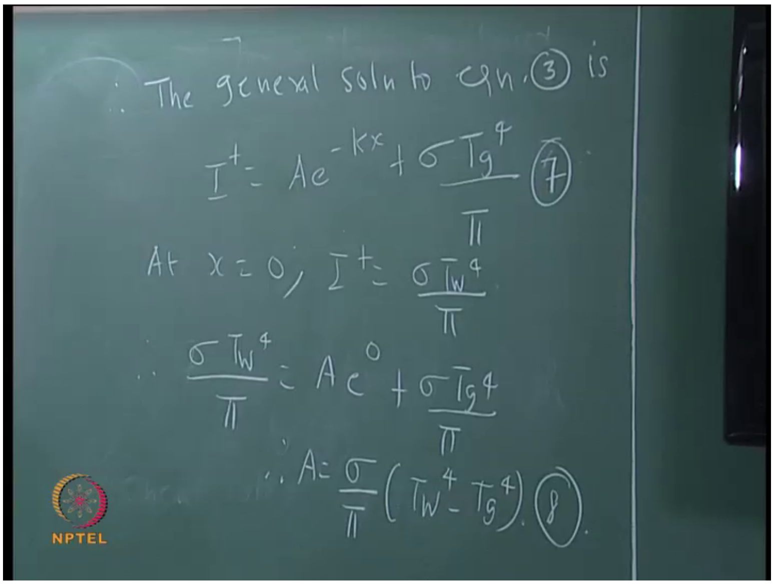

Student: (( )).

It is not sigma T W to the power 4. Be more accurate.

Student: (( )) T W minus (( ))

I, I am not talking about e.

Student: By pi

By pi; at x is equal to 0, I is basically that is originating from the wall. We are looking at the story. What is happing to the sigma T W to the power of 4 by pi as it goes through the gas layer? So, at x equal to 0.

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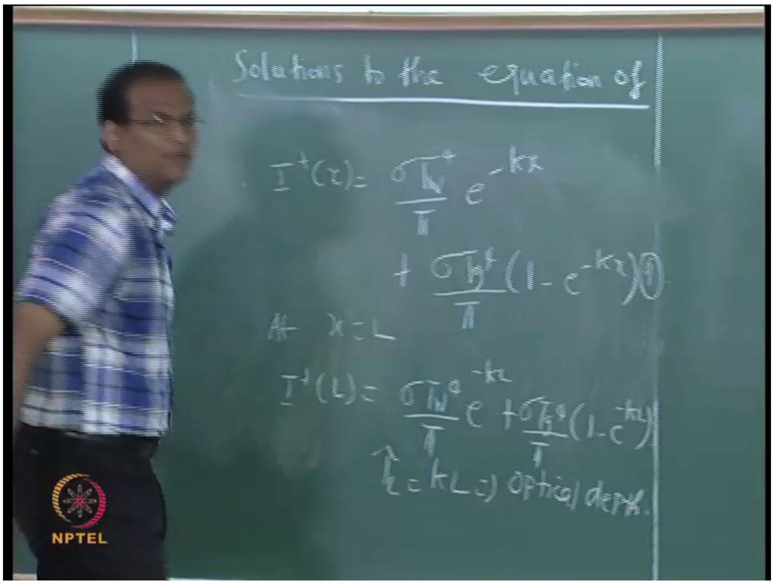

Now, therefore, I plus of x equal to is it ok? Is it fine? Jaydev, I just picked the a into a collected terms with T g common. Where did we start the exercise? We want to find out I plus at x equal to l. So, we will have to obtain the expression for I plus at x equal to l. Any problem? What are the units of kappa?

Student: (( ))

Meter inverse L meter. So, the product of kappa into L, kappa into x frequently appears in radiative heat transfer. So, it is called the optical depth k into L. So, if for two media, the optical depth of one medium is higher compared to the other, what does it mean? Its capacity to absorb gas, absorb radiation is much more.

An optically thin gas, an optically thin gas is one in which tau is very small; that means it absorbs minimum. An optically thick gas is something which is got a terrific tau. Little will emerge out of a optically thick gas, but beware optically thin or thick. The tau is dimensionless. I you are also tempted, you are almost tempted to believe that tau should have units of meter when I say optically thin and optically thick. Kappa is units of meter minus tau is dimensionless.

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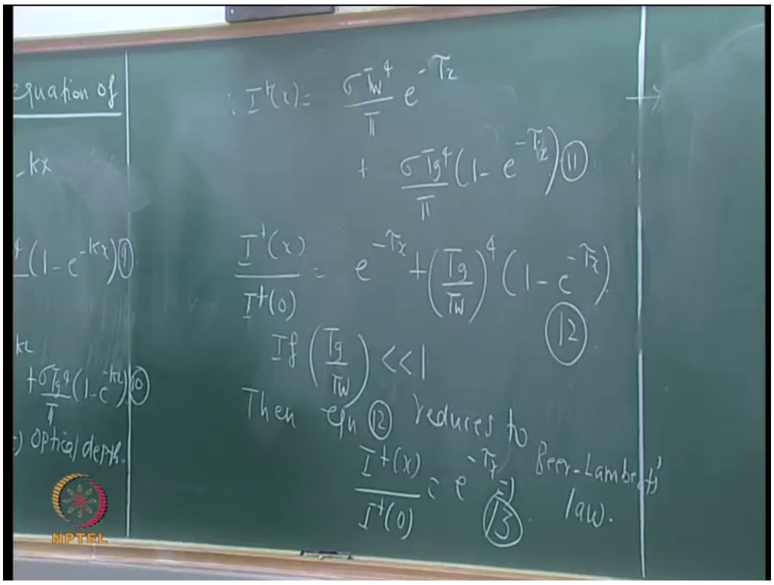

Now, so, therefore, in general, it is tau subscript l. So, this is tau x. So, what is this ratio? I want to find out the intensity at x divided by the intensity at 0. What is the intensity at 0? Sigma T T W to the power 4 by

Student: Pi

Pi. If you do that, what we get is, This is a performance metric for the gas layer of thickness l. If T g by T W is much less than 1, that is the gas temperature is, if the gas temperature is 300 kelvin and the wall temperature is 1500 Kelvin. Therefore, T g to the power of 4 by T W to the power 4 is very small. What happens in the second term and then this reduces to what?

Student: (( ))

What does it reduces to?

Student: Beer-Lambert's law

It reduces to the Beer-Lambert's law. Then, all right, fine. So, why should we always worry about the straight path? Radiation can arrive at that elemental area df from the other paths also, is not it? So, but we did not want, we did not want to work with the general path and get into the mess. Now, we worked out for a simple path one, and then we can by induction or inference or whatever, you can, you, can guess what this I plus will be for a slant path, for a slant path, no. mu is equal to 1 represent the straight path. Mu is very easy to handle. Normally in radiative heat transfer, we use the mu, and also when you are looking at, did you study fin heat transfer? If you have a variable fin area, you will get the cos theta frequently. Cos theta comes frequently in several areas of radiation heat transfer. I am not sure about stress analysis and all that. Cos theta is an important fellow in many works of life. Therefore, your treat cos theta as mu, and therefore, sin theta can be treated as...

Student: (( ))

Minus d mu? Sin theta d theta can be treated. Why am I worrying about sin theta and d theta, because I am looking at I plus arriving from all angles, and then for a hemispherical average quantity, I have to find out integral, integral, over the solid angle. So, I will get sin theta cos theta d theta d 5. Then I can convert it to d mu. Once you have d mu, already you can expect that. You have got e to the power of minus e to the power of something by mu d mu all that, and then beyond a certain point, it becomes un-integrable or non-integrable is not integrable. Then we have to resort to some table. These are called the exponential integral tables which are introduce shortly. So, this basically the general, this basically the solution of the equation of transfer. You can see that even for a simple, simple, case of there is only one wall which is black. There is gas layer which is gray, and we looking at one positive direction of x. It is quite formidable, but of course, we can do it with a chalk and talk. I am, we are able to do it with pencil and paper. For the more difficult things, we will have to program. Little bit of complications we can handle. Now, let us try to find out what will be the heat flux arriving at the area, elementally area dA. It is clear up to this stage.

Student: There should be (( ))

Which one?

Student: (( )).

I plus

Student: (( ))

I do not understand.

Student: (( ))

Yeah, yeah, but kappa is always, always there. Since kappa is a constant, I can call it is a function of tau x. What is the problem? I can change my variable, is not it? If x is the variable, I multiply x by K and call it a new variable x 1, I mean it is perfectly legal, because K is anyway is a constant. We have to worry about the heat fluxes.

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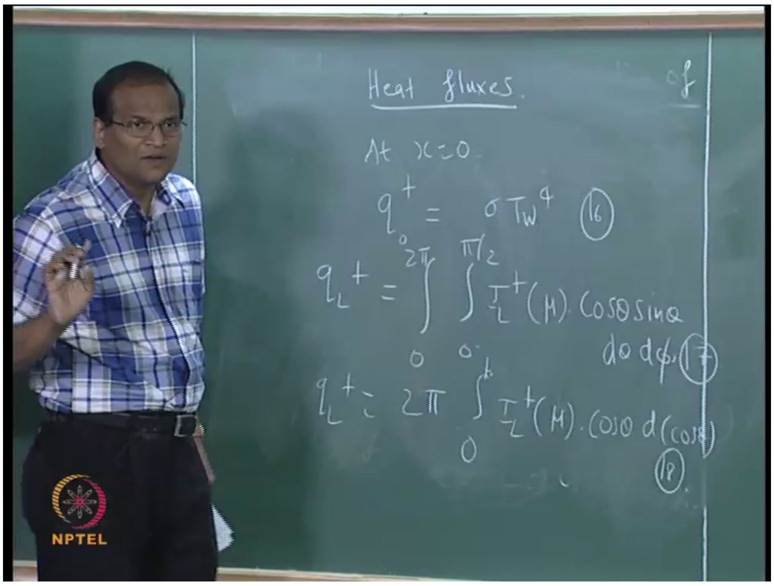

So, the heat fluxes are little more involved. Now, if you have to write the heat flux, heat flux at x equal to 0, which is going in a positive direction of x, is given by sigma T W to the power of 4. No problem, 16 q L plus will be...

Student: (( ))

I L plus, correct? I am saying I L is function only of mu. I am not having the function with the respect to... I am not saying it is a function of x because I am specifically evaluating the integral at x equal to L. Once I have L, I can make a general x. Let us not get confused. We should not, we do not want to handle so many variables at a time. Why, why cannot is I l be taken out? I l is a function of mu guarantee because we saw that tau x by mu and all that. So, it cannot be pulled out of the integral, but if it azimuthal, if you have azimuthal symmetry, then integration with respect to d 5, the azimuthal angle can be done. So, the first simplification is I will pull the d 5 out and put the 2 pi. Can I do that? Let us do that. So, q L plus, Is it allowed? Is it allowed, and cos theta can, no, no, this is what?

Student: (( )) it is correct sir (( )) will come minus (( ))

(( )) d f cos theta means

Student: Cos theta minus sin theta (( )) minus sin theta, no, (( )) is 0 to 1, no.

It is not 0 to pi by 2, cos theta ranging from. So, there was a minus, minus, was taken care 0 to 1 everything is done now. Several things have got into this, correct? The sin theta, only two things, correct. The sin theta was taken as minus of d of cos theta. Therefore, the limits where changed for instead of we swapped from 0 to pi by 2 to pi by 2 to 0, and because it was instead of theta, it was cos theta. We change it 0 to 1 and we pi by t to 0 again we swapped and we took care of the minus.

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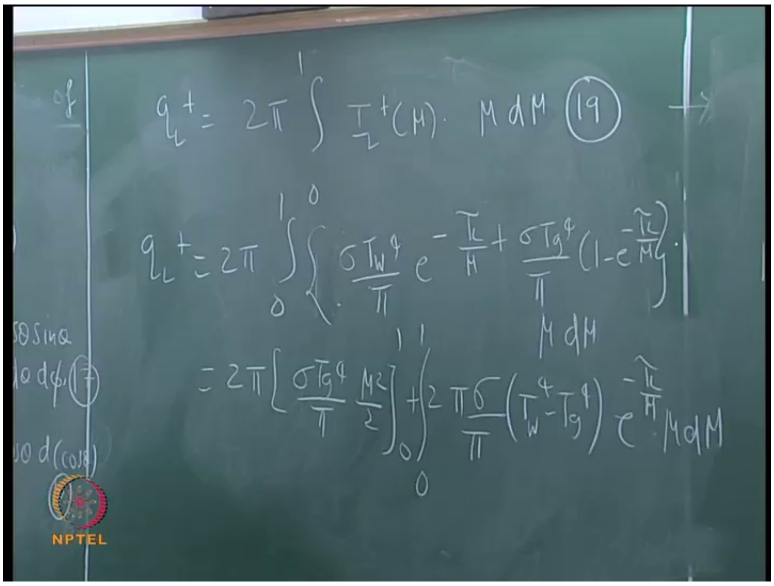

Now, instead of d of cos theta, I can put it as d of mu. Therefore, q L plus equal to... Now, we have to substitute for I L of mu. We have to substitute for I L of mu and try to accomplish the integration. As engineers ultimately were interested in the flux so many watts per meter square, why are you worried about watts per meter square per micrometer per steradian all that. So, we have, we have, to get this q L plus equal to 2 pi plus correct? Fine? Is it correct? Am I doing all right? I took the second term. Yeah, is it ok? Have you written it out properly? Now, so, first term therefore...

Student: (( ))

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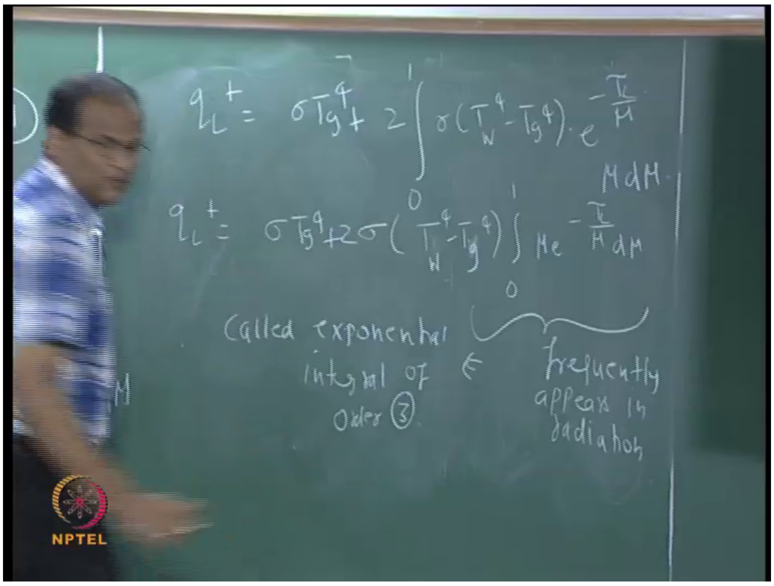

That is all plus 2 sigma as 2. I can take the T L to the power of 4; T g can be taken out. So, you get mu e to the power of minus tau L by mu d mu. It cannot be integrated. Do whatever you want, do whatever you want integral u dv u v minus integral v d u. Then it will e to power e to power e to the power. It will give period whatever all g e stuff will not work, none of it at all, nothing of it all. So, the integral mu into e to the power minus is an integral which frequently appears in radiative heat transfer is called an exponential integral. So, what happen to that? I left a pi.

Student: (( ))

Sigma so, frequently appears in radiation. It is known as the exponential integral of order 3.

Student: (( ))

There that, correct, what is minus?

Student: (( )) T W (( ))

T W minus T g (( )), T W minus T g, is it?

Student: Yes (( ))

No

Student: (( )) T W minus T g

Yeah, yeah, I do not want to write what I do not believe in. Let us see. So, what happen?

Student: (( )).

Morning when I worked out at 5 o'clock, it was T g minus T W. I think I made a mistake. Now, this mu e to the power minus tau L by mu d mu frequently appears. It is called exponential integral of order 3. So, what does it mean? There are exponential integrals of other order also, other orders also.

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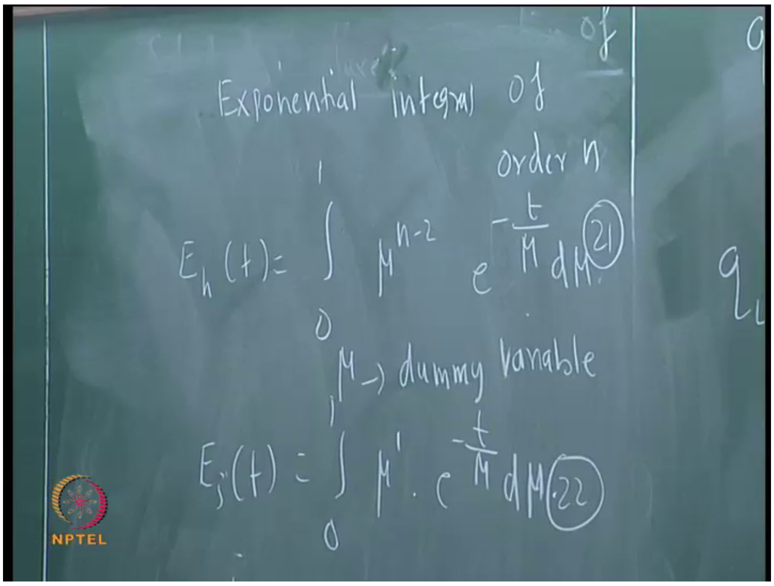

So, so the general exponential integral is given like this mu to the power of n minus 2 into e to the power of minus t by mu. That is the general exponential integral of order n. Mu is basically a dummy variable, mu is basically a dummy variable. So, this is what we got. No, I think we are, it should be 20. The exponential integrals have been worked out and they are tabulated and I am going to give table now. Yeah, please, the left most now number has not come out well. It got cut in the photo copying but it is.

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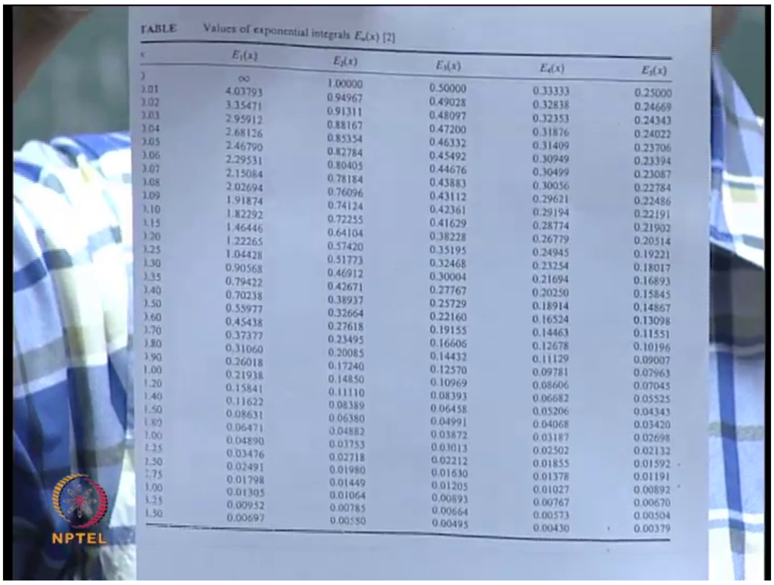

TABLE Values of exponential integrals $E_n(x)$ [2]

x	$E_1(x)$	$E_2(x)$	$E_3(x)$	$E_4(x)$	$E_5(x)$
0	∞	1.00000	0.50000	0.33333	0.25000
0.01	4.03793	0.94967	0.49028	0.32838	0.24669
0.02	3.35471	0.91311	0.48097	0.32353	0.24343
0.03	2.95912	0.88167	0.47200	0.31876	0.24022
0.04	2.68126	0.85354	0.46332	0.31409	0.23706
0.05	2.46790	0.82784	0.45492	0.30949	0.23394
0.06	2.29531	0.80405	0.44676	0.30499	0.23087
0.07	2.15084	0.78184	0.43883	0.30056	0.22784
0.08	2.02694	0.76096	0.43112	0.29621	0.22486
0.09	1.91874	0.74124	0.42361	0.29194	0.22191
0.10	1.82292	0.72255	0.41629	0.28774	0.21902
0.15	1.46446	0.64104	0.38228	0.26779	0.20514
0.20	1.22265	0.57420	0.35195	0.24945	0.19221
0.25	1.04428	0.51773	0.32468	0.23254	0.18017
0.30	0.90568	0.46912	0.30004	0.21694	0.16893
0.35	0.79422	0.42671	0.27767	0.20250	0.15845
0.40	0.70238	0.38937	0.25729	0.18914	0.14867
0.50	0.55977	0.32664	0.22160	0.16524	0.13098
0.60	0.45438	0.27618	0.19155	0.14463	0.11551
0.70	0.37377	0.23495	0.16606	0.12678	0.10196
0.80	0.31060	0.20085	0.14432	0.11129	0.09007
0.90	0.26018	0.17240	0.12570	0.09781	0.07963
1.00	0.21938	0.14850	0.10969	0.08606	0.07045
1.20	0.15841	0.11110	0.08393	0.06682	0.05525
1.40	0.11622	0.08389	0.06458	0.05206	0.04343
1.60	0.08631	0.06380	0.04991	0.04068	0.03420
1.80	0.06471	0.04882	0.03872	0.03187	0.02698
2.00	0.04890	0.03753	0.03013	0.02502	0.02132
2.25	0.03476	0.02718	0.02212	0.01855	0.01592
2.50	0.02491	0.01980	0.01630	0.01378	0.01191
2.75	0.01798	0.01449	0.01205	0.01027	0.00892
3.00	0.01305	0.01064	0.00893	0.00767	0.00670
3.25	0.00952	0.00785	0.00664	0.00573	0.00504
3.50	0.00697	0.00580	0.00495	0.00430	0.00379

So, this is the, you will take it. This is the exponential integral you can see the first column. You see the value of x. The second column is E 1; third column is E 2, E 3, E 4. As far as this course is concerned, we are worried about the first column and the fourth column, fourth column.

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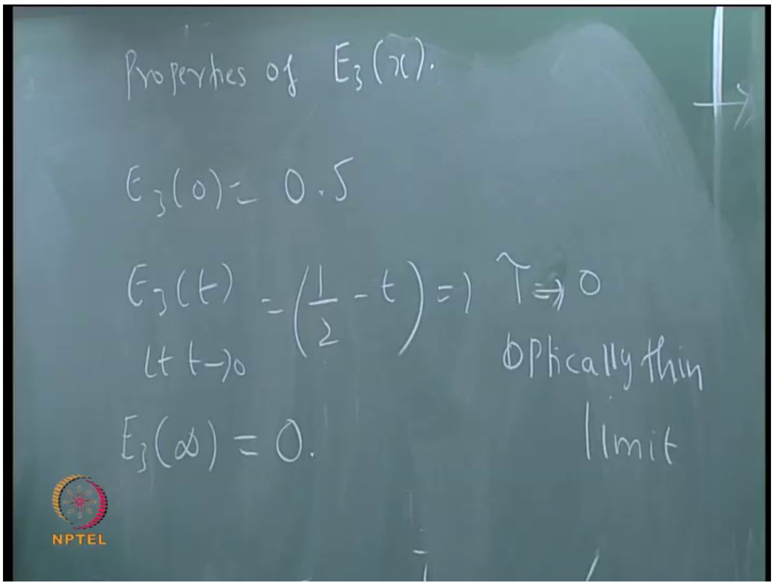

Fine, what are the salient properties of this E 3 as observed from the table? E 3 of 0 is 0.5. Please check. So that means that t is very small. What is our T? Our T is actually tau optical depth. So, the optical depth is very small. Then this exponential integral reduces to half minus t which is a great boon for us. So, the limit where the t approaches 0; that means the gas is not so heavily absorbing is called the optically thin limit or radiation. It is the optically thin limit; it is optically thin enough to allow that approximation, but it is not thin enough to neglect gas radiation 1 by 1 plus 10. The 10 is very, the 1 is very small compared to the 10. Therefore, 1 by 1 plus 10 equal to 0.1, but we cannot say 1 is very small compared to the 10. Therefore, whole thing is 0. There are two different stories. It is like this. It makes it handle able and e 3 of infinity at 3., what happen that 3.5 itself it become, it became what? 005. Therefore, E 3 of 10 20 30 will tend to, very good.

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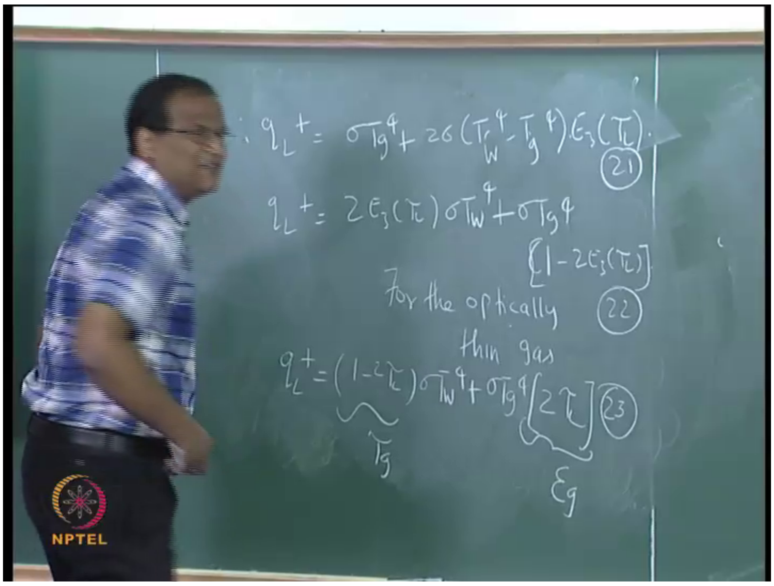

So, we have these properties. Now, you can write the general expression for q L plus, can you write? Therefore, q L plus the sigma E 3 of...

Student: (( ))

That is all. E 3 of, correct, no, I am making mistakes again and again. This is, yeah, plus 1 minus, all right. Now, what can you say from this 1 minus? Can you interpret something from this? Can you interpret something from this? Let me see. Now, we will apply for the optically, now, 20 for the optically thin gas. What is E 3?

Student: Half minus T

Half minus T. Therefore, this will be...

Student: (( ))

1 minus 2 T

Student: (( ))

1 minus 2.

Student: (( ))

Tau 1 by 2.

Student: (( ))

plus tau l, 2 tau l

2 tau l. The 2 is not leaving us know. What is the big deal man now? What is the big deal? The radiation arriving at, the radiation arriving at x equal to l consists of two part - the radiation which is coming from the wall, which is attenuated by the gas, and the radiation which is directly coming from the gas. Fortunately, for us, now we have an expression which gives two independent things. The first seems to be, the first seems to concern the radiation from the wall. The second seem to concern the radiation from the from the gas. Therefore, this tau L can be, if we assume if we say this tau L is an equivalent gas emissivity, then the second term is epsilon g sigma T g to the power of 4. What is that? What is that radiation which is emitted by the gas which is arriving at the elemental area at x equal to l. So, this can be consist, treated as a transitive of the gas.

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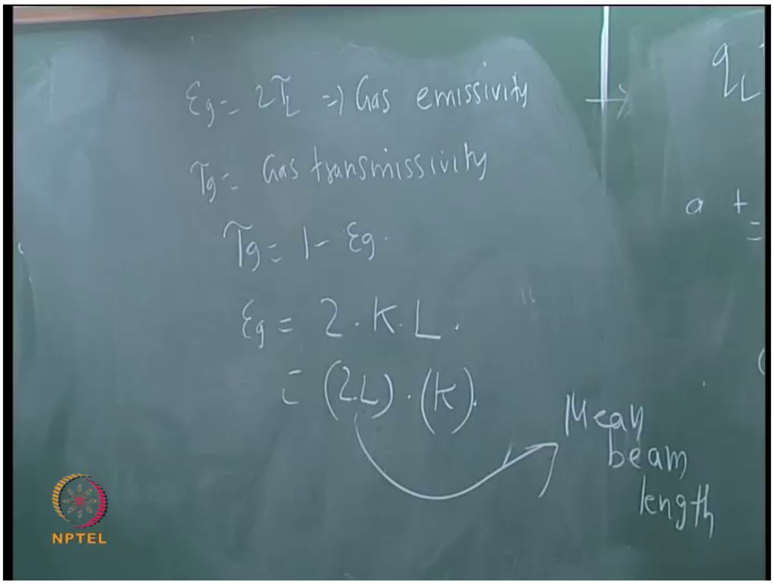

So, no, no, no, what is the relationship between this? Tau g is 1 minus, 1 minus, epsilon g. Epsilon g itself two times kappa into L is 2 time kappa, 2 times L into kappa. Now, comes the killer stroke. For all this, the goal of anything is to finally simplify. The goal of anything is finally to simplify. Now, at the end, we, we, are able to define equivalent gas emissivity which consisted of two parts 2 L into kappa. I put intentionally I put 2 L within brackets and kappa within bracket.

The 2 L is basically related to the geometry. The kappa, the kappa, is related to the capacity of the gas to absorb. Therefore, when I combined this geometric part and the thermal part, I am able to get then equivalent gas absorptivity with which I can use my radiosity formulation, which I, which I use for evacuated enclosure. I can modify it and use it for this. What is this 2 L? This 2 L represents the mean path travel by all rays in arriving at the area element a which was located exactly at a minimum distances of L from the wall, because for cos theta equal to 1, it will be just L. For all the others, it will be L by cos theta and L by cos theta keeps changing.

So, this 2 L is some sort of an average or a mean length which rate travels before hitting this area element dL area element dA. So, this is called the mean beam length. So, this is the terrific progress we have made in the last 50 minutes that we started out the equation to transfer and it all mathematics exponential integral, but I did so many things. We did optically thin gas and all that. We arrived at a.

Now, the formulation is reach the critical stage where the gas, the gas, emissivity can be composed into two distinct parts and the thermal part can be can be completely separated from the geometric part. So, this 2 L is the, 2 L is the, mean beam length for a plane gas layer. This mean beam length will change for a cylinder for sphere and so on. So, if you are able to calculate the mean beam length and if you also know the kappa, you have handle; you can calculate equivalent gas emissivity, and then from the gas emissivity, use this relation and gas and calculate the gas absorptivity and you can proceed further.

In tomorrow's class, we will see how we can use the theory of evacuated enclosure, modify this and then solve problems of enclosures with absorbing, absorbing emitting gases. Anyway, to reinforce our concepts and the solutions of RTE. Tomorrow we will solve two problems involving basic plane, plane layer, plane gas layer and then proceed to theory of evacuated enclosure.

Thank you.

I will just...