Conduction and Radiation

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Module No. # 01

Lecture No. # 29

Concept of Mean Beam Length

So, we continue the discussion on gas radiation in the last few classes we have been deriving the equation of transfer or also called the R T radiative heat transfer equation then we started looking at analytical solutions the equation of transfer for some simple cases, that is for a gray gas and which is optically thin and we started looking at the analytical solution in yesterday is class and then, we came across a very interesting concept called mean beam length the mean beam length is a concept wherein we are we are looking at the average distance travel by the rays when passing through this participating gas before arriving at a particular area d A.

So, in the first part of today is lecture we will elaborate on this concept of mean length mean beam length we will try to figure out what this exactly means and then for a general case how do you get the mean beam length because a mean beam length as you will see shortly in today is class will be the starting point will be the starting point for your analysis of radiative heat transferring enclosures which contain absorbing and emitting gas which is in the previous weeks we looked at radiation heat transfers from an evacuated enclosure or an enclosure containing air which is radiatively transparent or radiatively nonparticipating.

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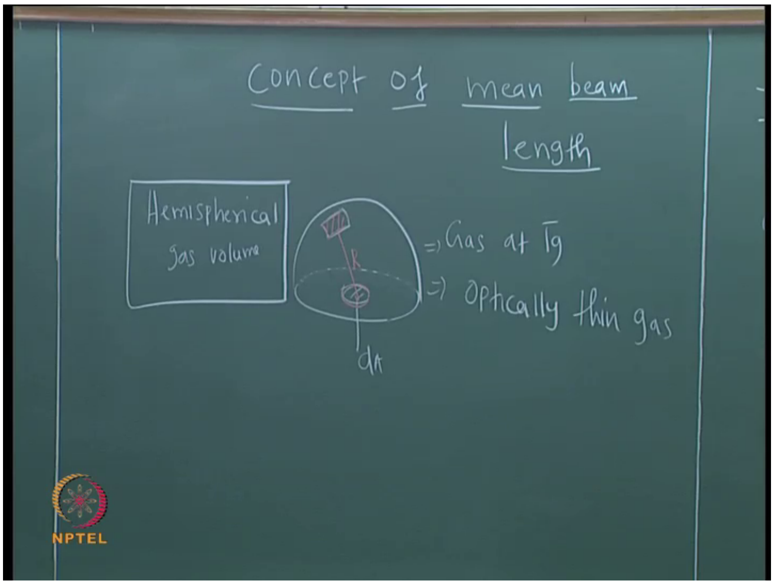

Now let us consider hemispherical gas. Hemispherical gas volume, the radius of the hemisphere is given by R there is a elemental area d A, at the center of the bottom surface of the hemisphere we are looking at this gas volume which is absorbing an emitting. I am looking at a small area on the top of this hemisphere and then, we are connecting this area to this area that is we are trying to find out what is the radiation from here which is arriving here that is radiation originating from this and then traveling through the gas volume.

Now, the gas is at a temperature T g it is an optically thin gas these are the two assumptions and this is a hemispherical gas volume the idea behind this exercise is to understand more about the concept of mean bean length. Now in yesterday is class we derived that for the optically thin gas for radiation from somewhere from a surface arriving at a length L the expression for the intensity at I plus of L where I plus is the positive direction of L is given by this. Now, what I am saying is instead of I plus from I plus L we can calculate the q also instead of I plus I am taking q minus because q minus is arriving at this I am considering R to be positive in this direction it is minus. So, I am saying q minus R will be pi times this intensity.

So, I have just use this intensity. Intensity expression I multiplied by pi. So, this is isotropic I also done one more thing instead of L I have used R there is no problem with this.

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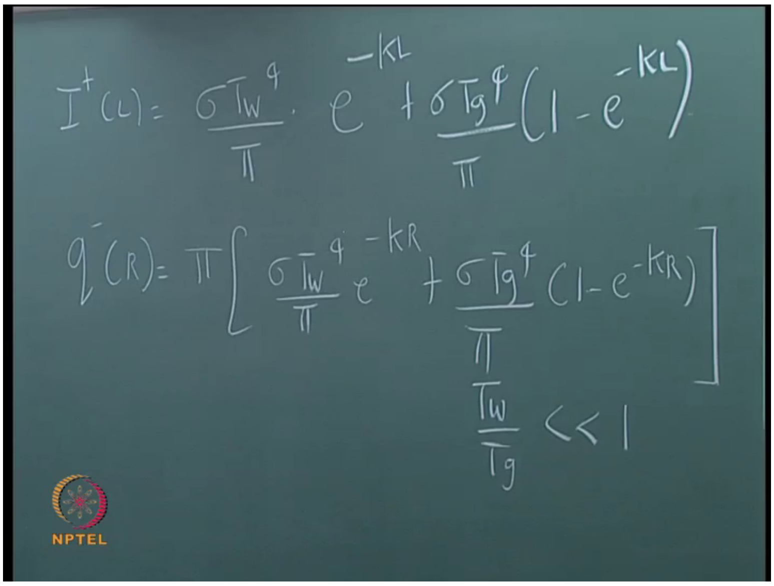

Now, it is possible for me to cancel the pi, one more thing what I am going to do is T w by T g T w by T g is much less than 1, it is a gas it is a gas which is contributing more to the radiation T w may be at 300 Kelvin the gas may be at 1500 Kelvin it may be hot combustion gases the walls are not very hot then what can happen to the first term knock it.

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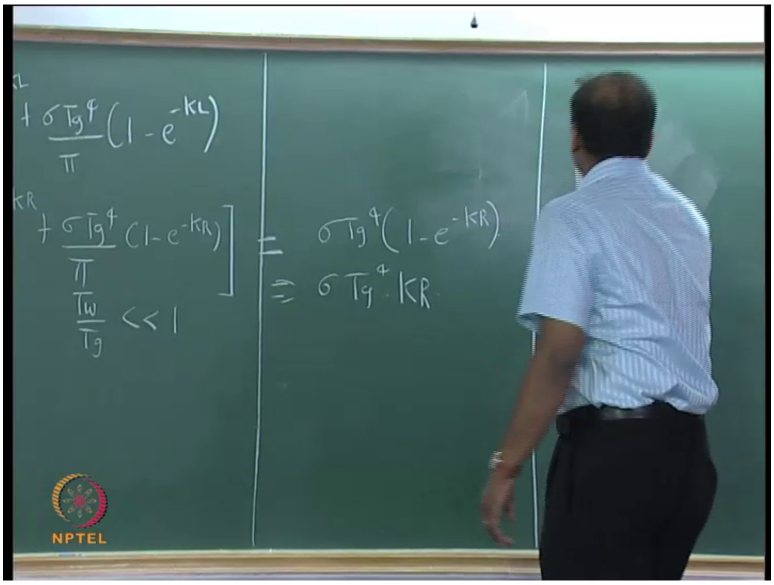

So, this will now become now for an optically thin gas kappa R is much much small compared to quite small compared to one therefore, 1 minus e to the power of minus kappa R can be expanded as K R.

K R simply, K R 1 minus e to the power of minus point naught one is just point naught one correct. So, this can be written as sigma T g to the power of 4 into K R.

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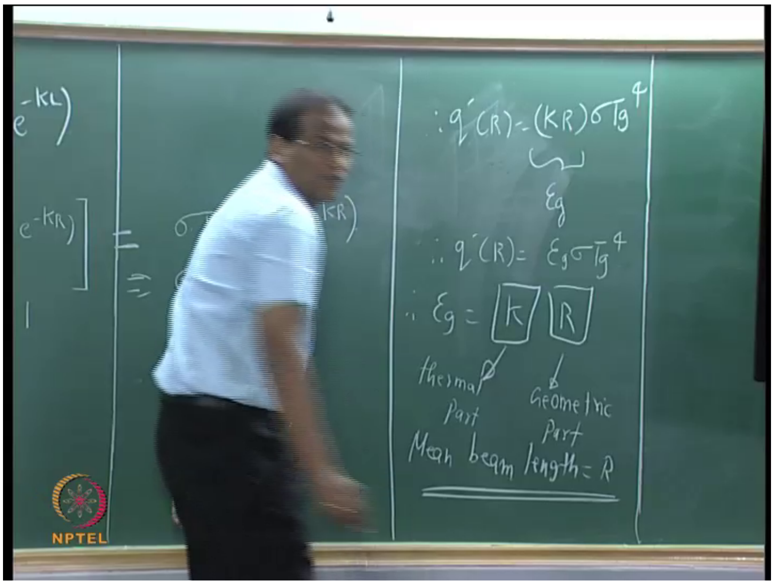

Therefore, if I am able to write a flux as sigma T g to the power of 4 multiplied by some K into R whatever is within the brackets can be called as an equivalent emissivity correct is it therefore, please note that please look at the assumptions under which this analysis is valid. Now this is just to therefore, epsilon g. So, this is a thermal part. So, when we considered a plain wall which was black which was at a temperature T w surrounded by an isothermal gas at T g we figured out that the mean beam length was equal to

2 L, here the mean beam length equal to Very good. So, mean beam length is a equal to R. So, for a plain gas layer of thickness L the mean beam length is 2 L for a hemispherical gas volume the mean beam length is R, however, these types of analytical ways or analytical ways of deriving this mean beam length cannot be done for geometry after geometry certain mean bean mean beam length have to be calculated or evaluated.

Therefore, what is the next logical step I will give you a table which can be used, which you can use in the exams or while solving problems where mean beam length for various things is given in that the hemispherical gas volume and the parallel plate will also be there, but instead of 2 L it may be 1.8 L that is a small difference instead of 2 L it is 1.8 L because of the expansion of E 3 of T in the vicinity of T equal to 0 there may be a small because it is not exactly equal to half.

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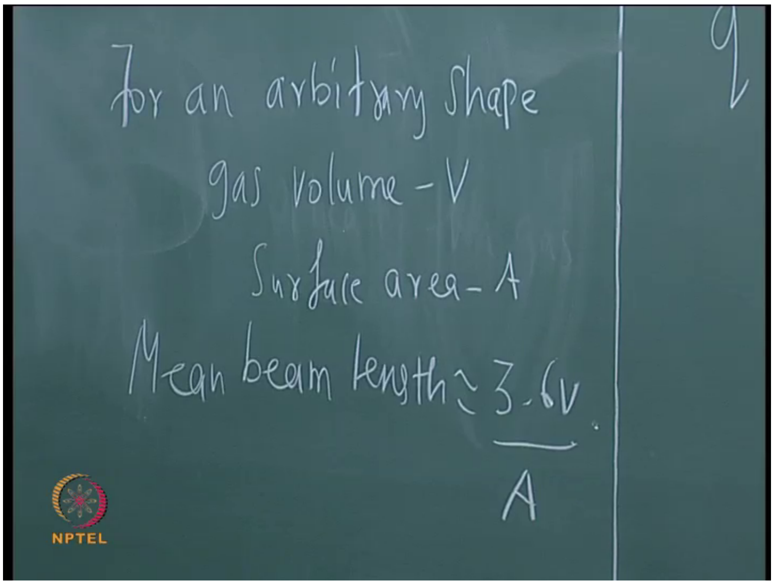

So, now, for an arbitrary shape if it is not a volume for an arbitrary shape what is a mean beam length. (( )) mean beam length is 3.6 times v by A if you apply the 3.6 v by A for the plain gas layer of thickness L what will you get 1.8 L that is why I said it would not be 2 L it will be 1.8 L this comes from calculations and I mean evaluate the exponential integral and get it.

Now, I will give you this contains additional information also, which will be used for solving problems the lectures I am going to deliver next week. I asked somebody to take 2 3 copies they have put that and with that they have taken the Xerox sympathy you take it and yeah now we are looking only at the first sheet your problem, there are 4 sides to this please look at this mean beam length for radiation from the entire gas volume in common geometries Mudhith you got it? you have problem what is happening yeah you can keep that there should be 4 sides Alok Srikanth you got it? Rohit you have any spare copy you want is it?

Student: (( ))

There are 4 sides do not bother about sides 2 3 4 there emissivity of carbon dioxide and water vapor, which we will use in the next weeks classes. So, the mean beam length for various geometry serial number geometry. So, sphere of diameter d radiating to inside surface 0.65 d hemisphere 0.5 d correct we worked with hemisphere today is class its R its 0.5 d then between 2 parallel plates look at point number 7 serial number 7 it is 1.8 l. So, arbitrary shape 3.6 v by A. So, with mean beam length you can proceed one. Now, we will solve some problems problem number 30.

Student: (( ))

36

Student: (( ))

37. problem number 37 consider a gray gas with an absorption coefficient of consider a gray gas with an absorption coefficient of kappa equal to 0.1 5 meter inverse, that is 1 by meter. It is maintained at a temperature of 400 Kelvin consider a gray gas kappa equal to 0.15 T w equal to 400 Kelvin and is it is maintained at a temperature of 400 Kelvin and is 0.4 meters thick 40 centimeters thick.

Student: (( )).

(( )) T g

Student: T g 4 (( )) T g (( )).

It is maintained at a temperature of 400 Kelvin and is 0.4 meter thick the gas layer is 0.4 meter thick no problem, and is 0.4 meters thick it is not the wall a black wall at 500 Kelvin is at x equal to 0. Sorry if I confused you T w is 500 Kelvin T g equal to 400 Kelvin thickness L equal to 40 centimeter kappa equal to 0.5 1 5 meter inverse everything is fine, data is fine determine the intensity at x equal to 0.4 meter, for determine the intensity at x equal to 0.4 meter for a determine the intensity at x equal to 0 for a a straight path determine the intensity for a or that is 1 first part of the question determine the intensity for a straight path b slant path at slant path at 60 degrees determine the intensity at x equal to 0.4 meter for a straight path or if you feel it this confusing I can reword it as determine the intensity at x equal to 0.4 meter for a straight path as well as for a slant path at 60 degrees. Use that small T approximation as well as the exponential integral for evaluating the heat flux at x equal to 0.4 meter and comment on the results. So, you do not require the mean beam length for this right you can do because analytical formula we have Pranai anybody else who came late who is without the you have what is that.

Student: (( ))

Fear I did not get fear now

(( )) here.

Ah if this substituted 3.6 v by a it will come 0.6 0.60.

No that small differences will be there, but, this 3.6 v by A do not try to check whether that formula is correct this is a general formula which is approximate see, mean beam length is approximately if you do not know anything use this for sphere and other things we can exactly calculate that 0.6 5 is accurate. Yeah it is just a test of how you understood the exact solutions exact solution, when that is all simple and straightforward I am asking you to solve in two ways use this small T approximation where E 3 of T equal to half minus T that is first part the second part you do not do use this directly use E 3 from the tables should take less than ten minutes. It was 38 problem number 38 or 37 (( )). Correct.

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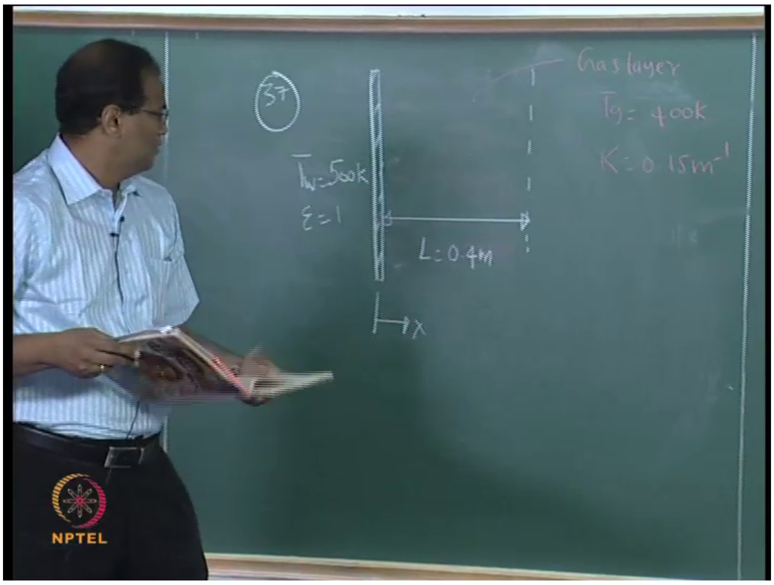

So, there is a schematic of the geometry under consideration, now we can straightaway start getting the solution for theta equal to 0.

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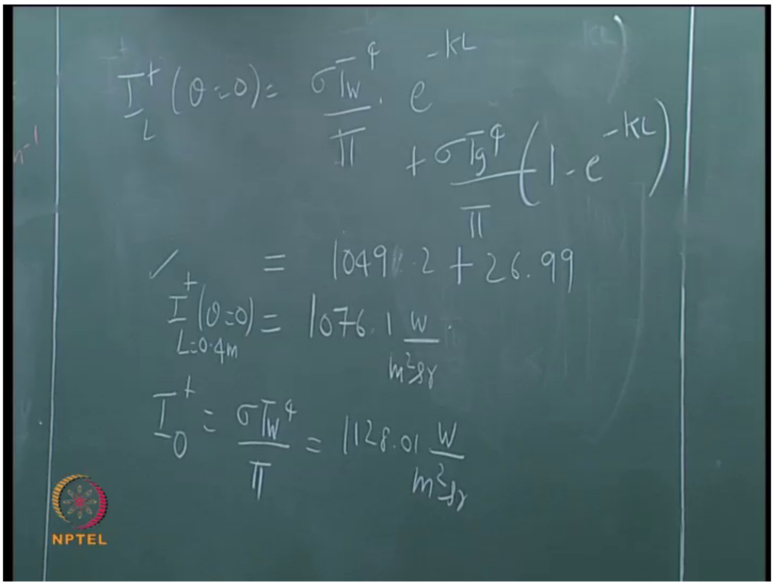

I plus L sigma e to the power of minus K L 1 0 7 6.1 (( )) fine now what is vinay is it what is I naught at what is I naught plus I naught plus how much is it? what is the intensity coming out of the black wall 1 1 point this is not asked in the question, but I am trying to use this to get some more inside into the problem right this is not asked you got this first part. Now I am asking you to look at the ratio of please look at the board I at L divided by I at 0 the I L at divided by I at 0 is almost equal to point 98 or something else 98 or 99 percent therefore, the gas is not absorbing much therefore, it is an optically thin gas, this is the sort of inside you have to get just blindly you can substitute the formula and get the result, but now if you do this now if I naught at if I at L divided by I naught is 0.5 or 0.6 then it is a very questionable assumption our, but, we have not made we have not made any assumption. So, far the optically thin gas assumption will come, when you are using that in the q correct this is right.

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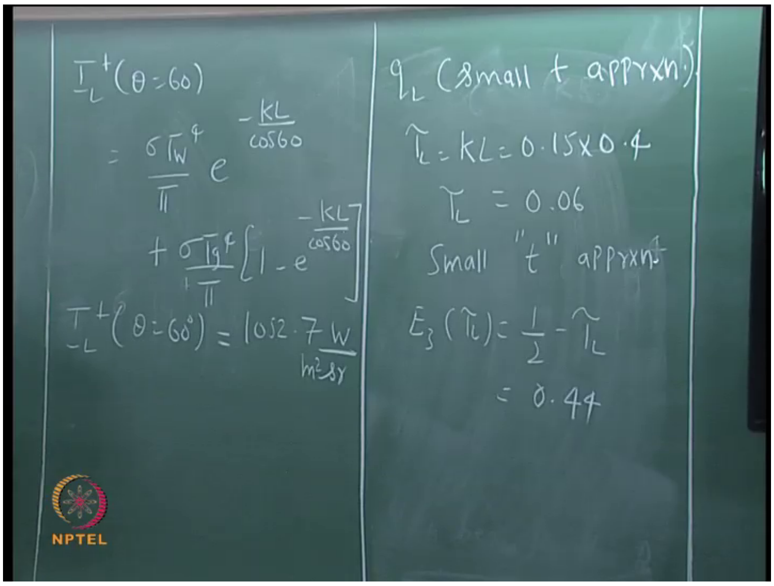

Now, what is the second part you have to do it for (( )) please look at the units watts per square meter per steradian fine. So, this is the answer of the first part of the question. What is the intensity at x equal to 0.4 meter for the two paths straight path and the slant path now coming to the second part of the coming to the third part of the question what is a q L small T approximation what is this small t approximation? E 3 of x equal to half minus T, where now how much is it 0.0 6 correct that is my small T.

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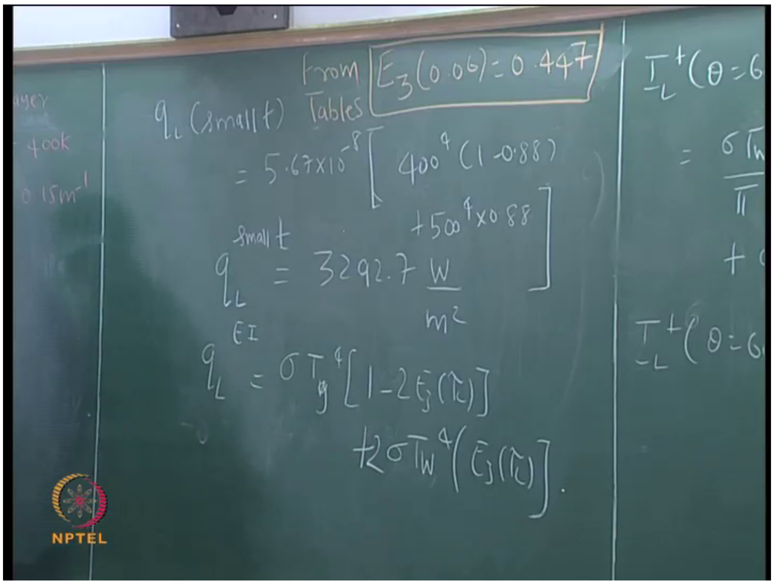

Now if you do the now I will do it here. (( )) Is it correct plus 400 is this correct. So, what do you get now 2 6 2 3 did I make any mistake, where is the factor of 2 let us see.

Student: (( )) 2 (( ))

Let us see what is the final solution

Student: Sigma.

Sigma (( )).

Student: 2 sigma (( )).

Sigma p g to the power of 4 0..8 8.

Student: (( ))

This one also ah.

Student: (( ))

Did I do that? No

No T g is what is it correct now, what is answer?

Student: (( )).

Watts per very good. So, so this is q L small T now q L E I this is plus into is correct.

Student: (( ))

2 bothers us throughout 2 sigma T w E 3 now what is E 3 of 0.5 E 3 of 0.0 6

Student: (( ))

0.4 4 7correct.

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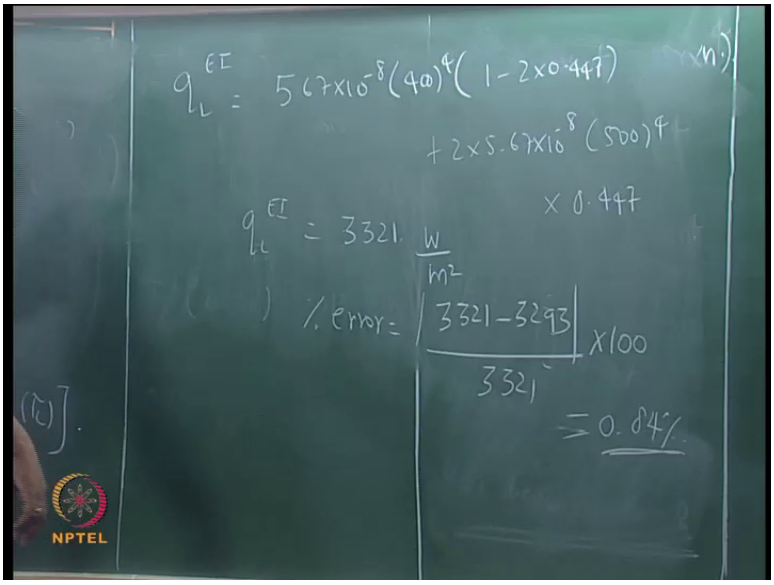

Now you can use this q L what was T g 400 what is this

Student: (( ))

Point

Student: (( ))

Less than 1 percent yeah. So, good man point how much error

Student: (( ))

0.8 4 percent very good simple, what can I infer from this, if tau is equal to 0.05 0.06, if it is less than around point 1 then this optically thin gas is a good assumption. So, whether it is optically thin or thick not only depends on the absorptive it also depends on the length. So, it is a product which matters it can be a very heavily absorbing gas, but, its only 1 millimeter or 2 millimeter it does not matter, it can be a poorly absorbing gas, but, if you are taking a large thickness L K into L will be the trouble will create the trouble; however, if you are treating it as an insulation or something or you want this radiation to be attenuated it is good for us correct, but if you want if you if you do not if you do not want the gas to create lot of mischief then you do not want it to be optically thick gas correct, but, these are all we have made. So, many assumptions it is a gray gas it is only one gas, it is a gray gas it is only one gas it is a isothermal many of this assumptions can be can be questioned most importantly this analysis cannot be used in I c engine or furnace simply because there is no one gas.

So, the next level of complexity will be how to handle how to get the epsilon g for a mixture of gases what will happen if the walls are not black. So, we will do we will go to up to a certain level and stop beyond that it is research. So, we will now look at a modified enclosures theory when the walls or not black, but, still it is optically thin it is still the single gas one gas it is gray then are there some recipes available if there is a mixture of gases yes carbon dioxide and water vapor because, they are they are used they keep coming in power plants that is the in power plants I c engines and all that carbon dioxide and water people have done experiments and at 1 atmospheric pressure and they have given the charts which incidentally are given in pages 2 3 4 of the handout.

So, there is a recipe available I will we will look at the recipe in the next class and then using that you can calculate, but t beyond that, if it is other pressures or other things you will have to solve this spectral radiative heat transfer equation and get all, but this is not the end of the story it is still an absorbing and emitting gas only, if it is a mixture of gases it is absorbing emitting and scattering an isotropically, it is really difficult are able to appreciate the complexities involve.

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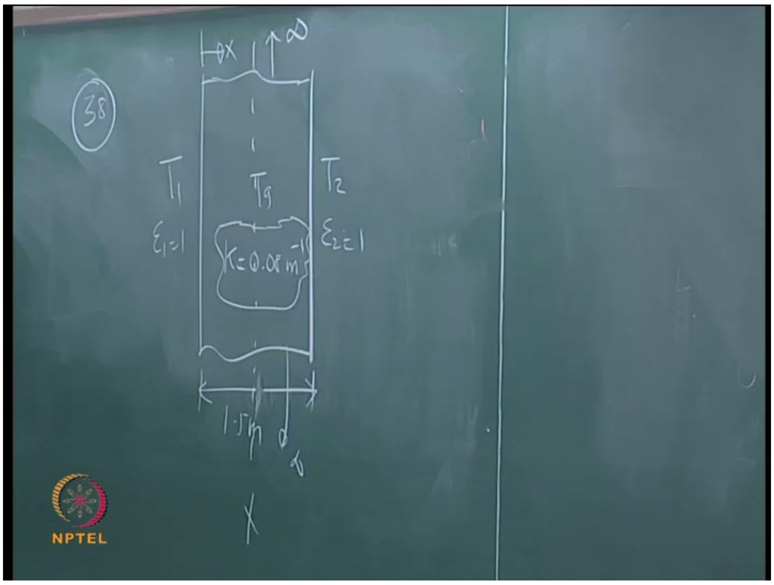

We will do now next we will do one more problem whether a 2 parallel plates (( )) suppose there is a black wall there is a black wall there is a gas layer a gas layer is having a temperature which is different from the black wall we figured out how to solve the problem, right side I put 1 more wall which is parallel to this that is a parallel plate formula, but, between the two parallel plates I am putting a gas which creates mischief it is not vacuum, but, I am saying it is optically thin how will you figure out the heat flux at the two ends and heat flux at the center of the center of this arrangement that is what we are going to look at right.

Problem number 38, two infinitely long vertical plates are parallel to each other two infinitely long vertical parallel plates no two infinitely long vertical plates are parallel to each other. both the plates are black and are at temperatures, both the plates are black and are at temperatures 1500 and 900 Kelvin respectively. And are at temperatures to be more clear T 1 equal to 1500 Kelvin and T 2 equal to 900 Kelvin respectively, the spacing between the two plates is 1.5 meter. The spacing between the plates is 1.5 meter and is filled with the gray gas at 1200 Kelvin with an absorption coefficient of 0.0 8 meter inverse. Determine the heat transfer rate at each of the two boundaries. (( ))

Student: (( ))

Determine the radiative heat transfer rate, what else could be there conduction is not there convection I did not tell you there is a (( )) fluid natural convection is there or I did not say air is something is blow blown (( )) there is a fan or blow air, now it is pretty interesting because, from left side also me activity is there, from right side also me activity is there superposition can be used we have to just combine the 2 solution, but for left to right axis in the positive direction for right to left it is a negative direction.

So, you have to take the algebraic some of the heat fluxes and the heat flux expression should be asymptotically correct if the kappa is equal to 0 what is the expression you have to get q 1 2 equal to sigma T 1 to the power of 4 minus T 2 to the power of 4 divided by 1 by epsilon 1 plus 1 by epsilon 2 minus 1 here epsilon 1 is equal to epsilon 2 is equal to 1. So, we will get a simple expression sigma of T 1 to the power of 4 minus T 2 to the power of 4. So, first its good practice to just draw 2 parallel plates T 1 T 2 put a gas layer at T g get the general expressions for heat flux at the x equal to 0 and x equal to L, then towards the end just substitute all these values and get them, (( )) you can take the general solution T w to the power of 4 1 minus E 3 of tau L all those expressions you can use you do not have to start from d A by d x equal to yes, what is this is x how much was this.

Student: (( ))

Kappa

Student: 1 0 (( )).

All this is infinitely deep this infinitely deep it is infinitely it is infinitely long and infinitely deep. So, the best thing would be to take a section x find out what is the heat flux arriving at station x from the left side from the right side algebraic sum is the net radiative heat flux at x, then put x equal to 0 and x equal to 1.5 meter your home is it instead of x I may use tau of x that may be better for me, because the E 3 is available for in terms of tau instead of x I can use tau right.

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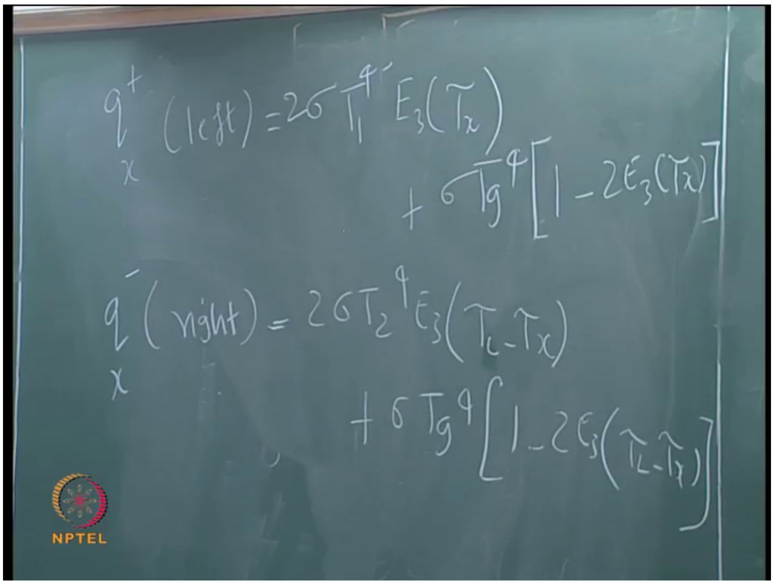

Now I will say q q plus at x coming from left side correct what is the expression for this sigma T 1 to the power of 4 into

Student: (( ))

2 E 3

Student: 2 E 3,

2

Student: (( )) sigma T 1 2 sigma T 1 0.4 (( )) 2 E 3 (( )) from the left

From the left I am taking from the left x is from one

Student: (( ))

2 sigma

Student: (( )).

E 3 of tau of x not it is not over then,

Student: (( ))

Plus one second no T 2 will not come now.

Student: (( ))

2 no 2

Student: (( ))

Sigma

Student: (( ))

1 minus.

Student: (( )).

Towards the end we will say, that it is a optically thin gas and then use the small T approximation now we will give the general solution, now what is the q x

Student: (( ))

Man point 1 only otherwise you stick with this. So, let us look at the 2 x minus from right side 2 x 2 sigma

Student: (( ))

Very good 2 sigma.

Student: (( ))

What is that?

Student: L minus (( )).

L minus is it?

Student: (( ))

That is a distance now that is kappa L minus x no problem is that correct plus, but, what is the direction in which this fellow heat flux is a vector correct what is the direction in which this fellow is coming he is coming from left to right or right to left.

Student: (( ))

Right to left therefore, if you want to have the net flux you have to take q 1 q x left minus q.

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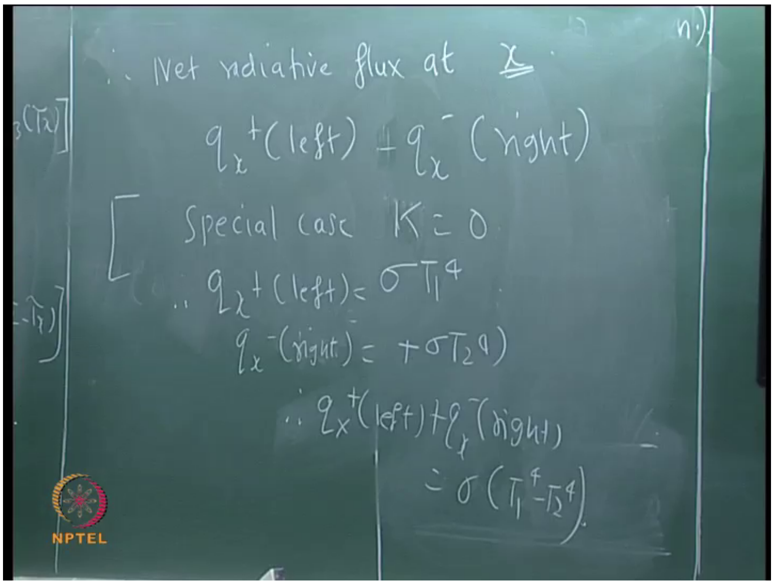

So, net flux at now shall we use a small T approximation you want to see chart yeah then proceed. So, I will do something on the board let me take this special case tau of x kappa equal to 0, what is E 3 of tau of x 0.5 therefore, because the second term this fellow E 3 is half, half into half 1 this fellow gets knocked off. So, the contribution from the gas the gas volume term is 0 for both the left and the right. So, this is consistent with our understanding of the problem. So, therefore, whatever the formula which we derived must be correct you found out now, yeah now we can get the values of q x left and q x right please tell me.

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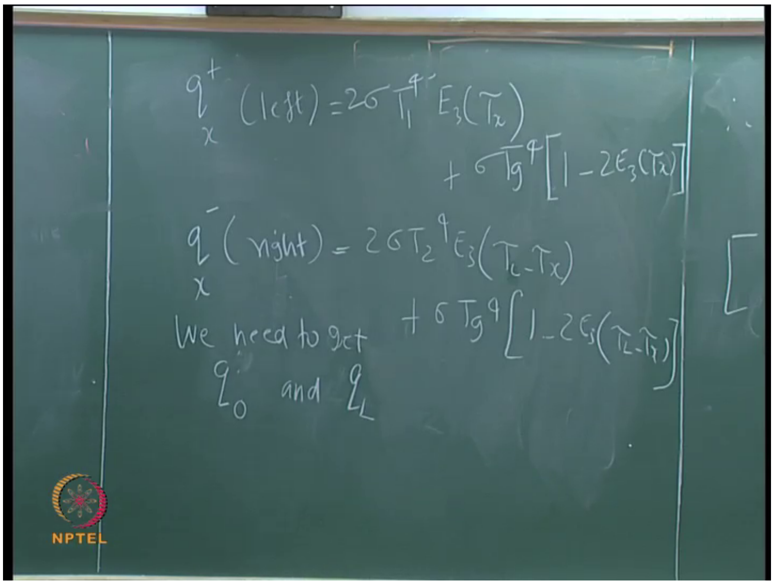

Now you have to substitute correct. So, we need to get q at 0 and q at L what are you doing now 0 or L or you doing at the center.

Student: (( ))

0 and L, because I left it open at x is everybody through with the procedure. So, it involves more calculations because there are 2 you have to look at the E 3 twice or may be 4 times because I want you to evaluate that x equal to 0 and x equal to L just give me one answer x equal to 0. How many of you are doing x equal to 0 yeah just we will just finish this and then I will distribute the assignments.

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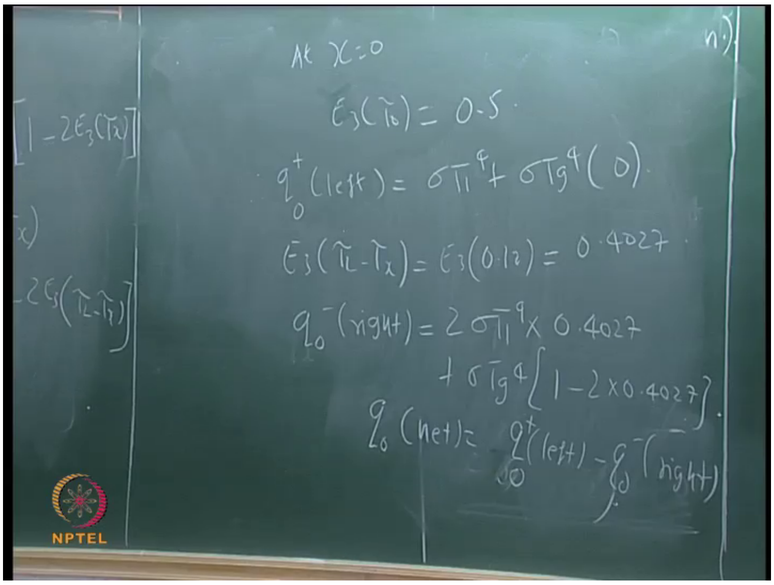

So, tau of x equal to (( )) why 0.1 2 0.0 8 into 1 point (( )) the E 3 of tau of x is 0.1 2 E 3 no (( )) at x equal to 0.

Student: (( )).

Equal to 0.5 therefore, into 0 all right, E 3 of tau L minus tau x is E 3 of tau L minus E 3 of tau x that is correct what is this 0.1 2.

Student: (( ))

(( )) please complete this, what is the answer

Student: (( ))

Kilowatts 10 to the power of 5.