

**Conduction and Radiation**  
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**Module No. # 01**

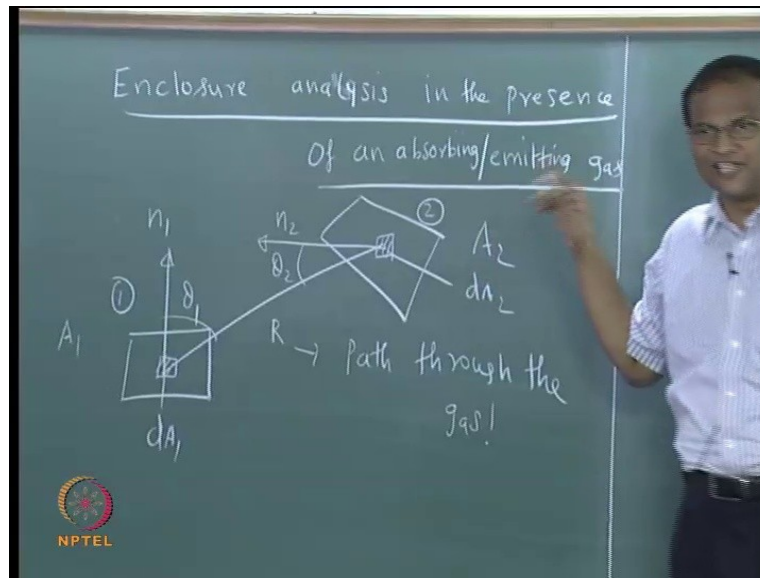
**Lecture No. # 30**

**Enclosure analysis in the presence of absorbing/emitting gas**

Okay, so, today we will look at enclosure analysis in the presence of an absorbing and emitting gas. We already looked at the enclosure theory for evacuated enclosures, which is also applicable for enclosures with air or some transparent medium, okay, just like enclosure analysis what you have studied can be applied to find out the radiative heat transfer between the various walls of this room, okay. However, if you have absorbing emitting gas, then we will have to modify. Some concepts of what happens to radiation in the presence of an absorbing and emitting gas we have already seen. We derived the equation of transfer to introduce the concept of emissivity, transmissivity, mean beam length, then we looked at some tables for calculating the mean beam length, for the general case it was  $3.6 \sqrt{V/A}$ , then we solved problem of radiation between 2 parallel plates kept at different temperatures, but intentionally I kept the emissivity equal to 1. So that you did not get the reflected term and all that. But it is very hard, almost impossible to get surface with emissivity equal to 1. The general norm is emissivity will not be equal to 1. So, in those cases how do you do the, how do you modify the enclosure analysis- that is what we are going to see.

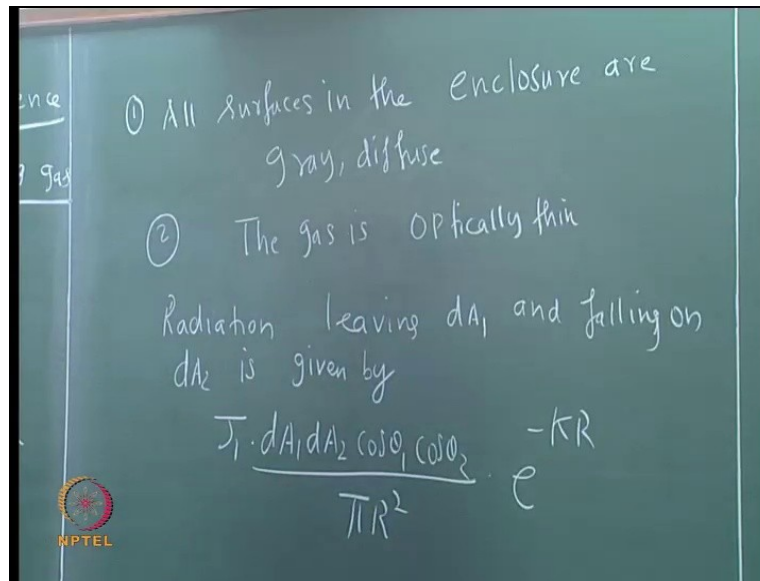
Now, as usual we will take two surfaces  $A_1$  and  $A_2$ . In the surfaces  $A_1$  and  $A_2$  we will take an elemental area  $dA_1$  to  $dA_2$ . We will find out what is the radiation going from  $dA_1$  which is intercepted by  $dA_2$ . We will integrate it over the whole area. But now, this will get attenuated because of, because of absorption by the gas. Then we include also the emission of the gas, if these two components are added in the other parts of the radiosity radiation method including the view factors will be the same. We formally discuss the procedure in today's class, and apply it to the problem which we have solved, we will extend the problem for the case of nonblack walls.

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Let us consider 2 areas-  $A_1$  and  $A_2$ , and 2 elemental areas-  $dA_1$ ,  $dA_2$  and  $R$  is the distance between the 2 areas and of course, their unit vectors-  $n_1$ ,  $n_2$ ; the angle subtended by this  $R$  with respect to the unit vectors of  $\theta_1$  and  $\theta_2$  respectively, and this is surface one and this is surface two. But but what is the difference between this  $R$  and the  $R$  we considered in the whole of February and March? This  $R$  is the path is the path in the through the gas. This  $R$  is path through the gas which is not going to keep quiet. Previously that  $R$  was through the vacuum or some transparent medium. So,  $R$  is basically path in the gas. Please keep it in mind.

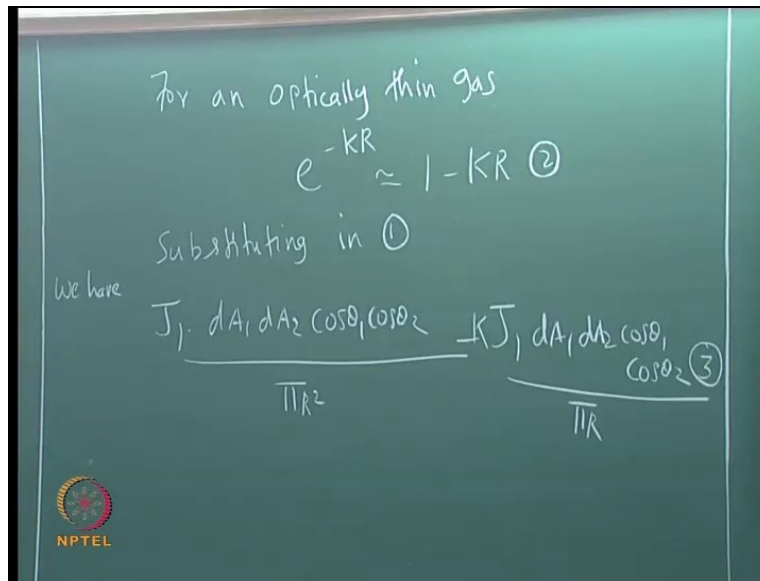
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Now how do we start? Let all surfaces be, okay? So, you have to start, consider two surfaces A 1 and A 2, which are part of a general enclosure. In A 1 and A 2 we take two elemental areas  $dA_1$   $dA_2$ ,  $dA_1$   $dA_2$  and the configuration is shown. Now all surfaces in the enclosure are gray diffused. The gas is optically thin. The gas is optically thin. So, the radiation which is leaving  $dA_1$ , which is falling on  $dA_2$  is given by.

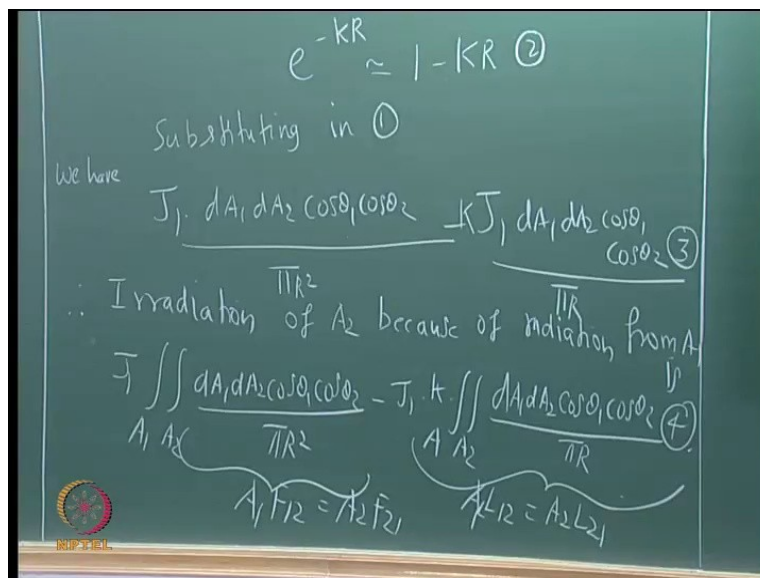
Yeah, can you tell me?  $J_1$  multiplied by?  $J_1$ ?  $dA_1$ , yeah. Okay? What is that extra funda I have put? Attenuation. Exponentially it is going  $e$  to the power of minus kappa. Do not say to me ask me sir what is happening to emission? I will come to it. We will first handle the absorption.

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So, for an optically thin gas,  $e$  to the power of minus kappa R can be written as? 1 minus kappa R.  $e$  to the power of minus point naught two is point naught two, correct? No, no 0.98, sorry.  $e$  to the power of minus point naught two is point 98. Okay. Now, substituting in one, we have J 1 correct?

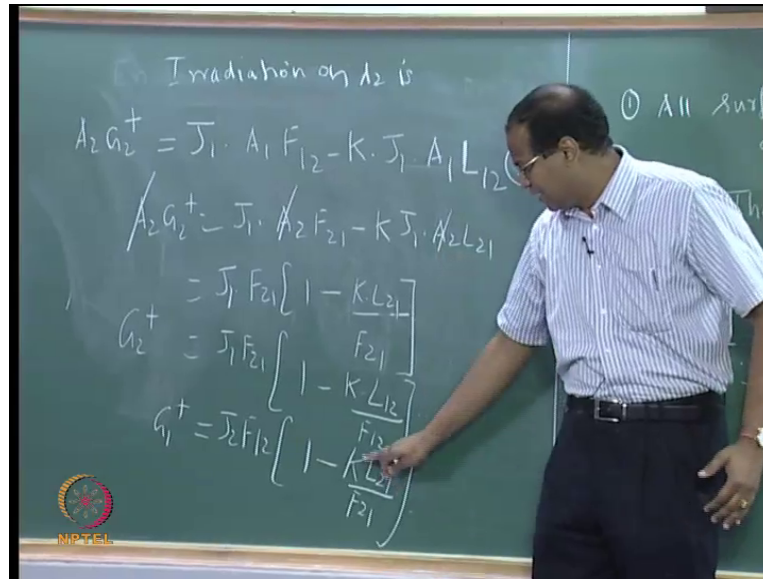
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What is the first term, J 1 into? What is the integral? It is not view factor, be precise.  $A_1 F_{12}$   
 2. View factor is 1 by  $A_1$ , okay? So this is, okay. That is known to us. We did problems; we

solved so many problems with this; we solved so many problems without this. So, that is  $A_1 F_{12}$ . So, the second term should also be something, okay. So, the second term... second term. I call it as  $A_1 L_{12}$  or  $A_2 L_{21}$ , okay? Correct?  $A_1 L_{12}$  is equal to, all right? Therefore, irradiation... this is irradiation  $A_2$  just only from the radiation coming from  $A_1$ , there could be other surfaces in the enclosure. Okay.

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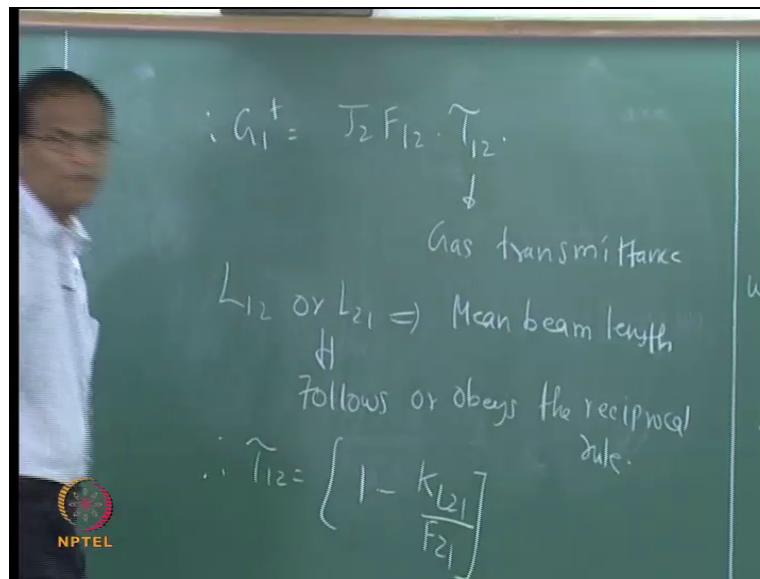


So, this is nothing, but kappa times. Okay. Now, this is equal to irradiation falling on, irradiation falling on the area two, basically from the wall element, we have not considered the irradiation coming because of the gas radiation. So, this will be equal to  $A_2 G_2$  star, correct? Because everything is in watts, right side is also in watts.  $J_1$  is watts per meter square,  $F$  is dimensionless,  $A$  is meter square. But now I do not like left side I have  $A_2$  right side I have  $A_1$ , I do not like this. So, what can I do? Can I do something? Yeah, I can use the reciprocal rule. So, ( $J_1$ . Okay? Huh? Ah  $F_{21}$ ).

What about  $L_{21}$ ? Kappa, huh. Kappa? This is also equal to  $L_{12}$  by  $F_{12}$ ? Okay?  $G_2$  star. By the same token I can write  $G_1$  star, right? Therefore,  $G_1$  star will be?  $G_1$  star will be?  $J_2$ ... ah, any of the two things. Correct? Any of the two things. It will be the same, right? For two surface, watch carefully, for two surface enclosure problem, please remember this parallel plate formula: the  $G_1$ , the  $G_1$  will be  $J_2$  multiplied by the view factor  $F_{12}$ . So, now we are getting next question where it is  $F_{12}$  multiplied by some factor. So, that factor can be deemed to be the transmittance of the gas. If the transmittance of the gas is one,

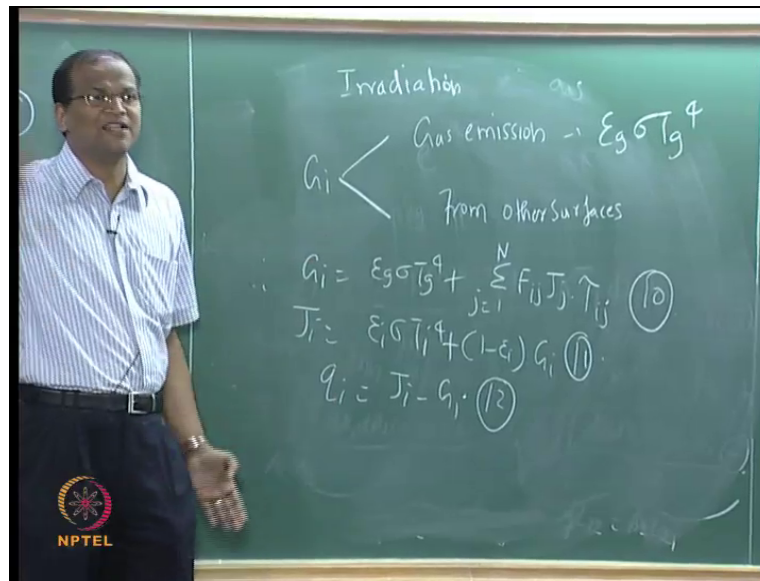
then we have what is called the evacuated enclosure or transparent medium. So, whatever is within the bracket will have a value less than 1 consequent upon the fact that kappa is not equal to 0. Kappa is the, kappa represents the amount of mischief caused by the gas. Even it is, it is an optically thin gas only that is why we are able to write e to the power of minus kappa, even then it will come. If kappa is equal to 0 then there is no need to consider this, consider gas radiation at all. Therefore, this  $1 - \kappa L_{21}$  by  $F_{21}$  can be considered as the tau of G, that can be considered as the tau of i j - transmissivity of the gas.

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So,  $G_1$  star. Okay? What is this  $L_{12}$ ? This  $L_{12}$  or  $L_{21}$  is the mean beam length, okay? Because we are able to write this as a view factor and from by inspection we are able to figure out that  $A_1 F_{12}$  is equal to  $A_2 F_{21}$ , since this expression is also analogous except for the fact that instead of  $\pi R^2$  I have  $\pi R$ , it follows that  $L_{12}$  or  $L_{ij}$  also follows the reciprocal rule. Correct? So, okay. So  $\tau_{12}$ . So, when you have to work, already you are able you you can conjure up visions of how to solve this gas radiation problem. You'll write the area first, you will calculate the view factors, after calculating the view factors you'll put in the mean beam length and calculate the tau i j's. Then the tau i j will multiply the  $F_{ij}$ , it is titrated or attenuated or reduced because of the influence of the gas, okay.

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Now, we have defined this tau of the gas. So, this is basically, I can call it as gas transmittance. Now, having defined this I have to modify the irradiation term. So irradiation  $G$  of  $i$  consist of two components. This irradiation from... from other surfaces we have a way of handling this, okay. You can do it for every tau  $i j$ , depending on how many surfaces are there in the enclosure. What about gas emission? Yeah, what is a gas emission?  $\epsilon_g \sigma T_g^4$  to the power of 4 into epsilon  $g$ , right? How do you calculate epsilon  $g$ ? From the mean beam length. Correct? So, the starting point is a mean beam length. In fact, the tau will be 1 minus epsilon and tau will be related in such a way that epsilon plus tau is 1, something you will get, right? Fine?

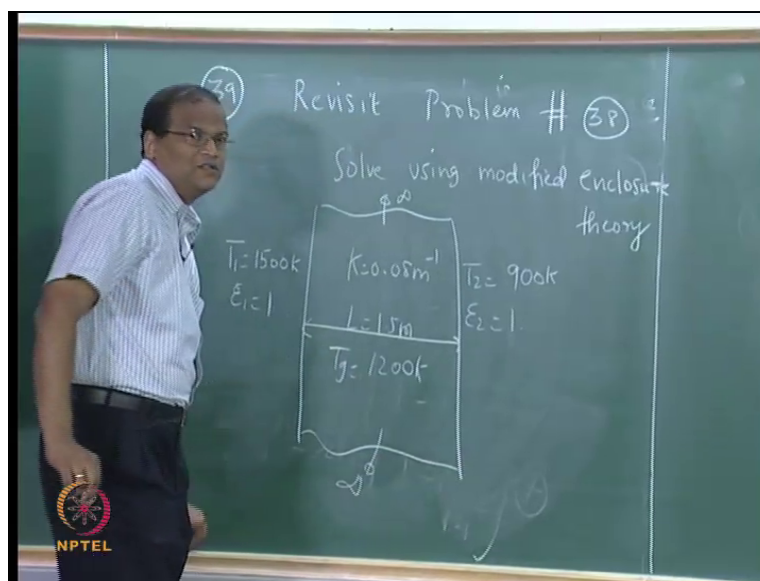
So, now, you are able to calculate the gas emission, you are also able to calculate the irradiation of other sources therefore (( )) that is it. There are two critical changes, which we have done to the irradiation, what are those? There are two important changes. The first change is we have an emission term in the irradiation also. That is we have to account, we have to account for the gas emission. So, there is an emission term in the, I think we will have to start numbering so. There are two changes with respect to the theory of evacuated enclosures- the irradiation term consist of an extra emission term and then within the summation I have the gas transmittance included by tau  $i j$ . The gas transmittance consists of the  $L_{12}$ . The  $L_{12}$  is basically simplified representation of a solution to the equation of transfer. All this [FL] we are able to do because we are doing some optically thin and we are

able to get mean beam length and all that. But if it is not an optically thin gas all these things have to be buried; you have to solve the equation of transfer and then it will be full blown radiative heat transfer calculation.

So, this is to aid, it is to help, it is the simplified way, just like you have lumped capacitance system- you say the whole body is at one temperature. It is also valid under some limiting conditions, Biot number-  $h l$  by  $k$  is less than 0.1, like that optically thin gas. This is valid. Optically thin gas only when gas, it is not for a gas mixture. Now,  $J_{1-2} = \epsilon_1 I_{1-2}$  correct? So, the formulation is exactly the same as before, except that the irradiation is, the irradiation has some extra terms- one extra term and one modified term. Extra term in the emission, the modified term is the  $F_{ij}$  is that okay? Fine. So, and what is the mean beam length for an arbitrary ray?  $3.5 \epsilon$  by  $a$ , all right?

So, this is basically the theory of evacuated enclosure, applied to applied to a problem for an absorbing and emitting gas. Now it can be pretty straightforward. For example, if the  $\kappa$  equal to 0, that is the gas is not at all participating, if  $\kappa$  equal to 0  $\tau_{1-2}$  will be equal to 1. So, this will become? No this will become 0, okay this. This you do not worry. So, this you do not worry about.  $\epsilon_g$  will become? 0, correct. This is 0, this will become 1. So,  $g_i$  will be just  $F_{ij}$ , this will be the same and you will get the same answer as before. Therefore, there is an asymptotic correctness associated with this. Right.

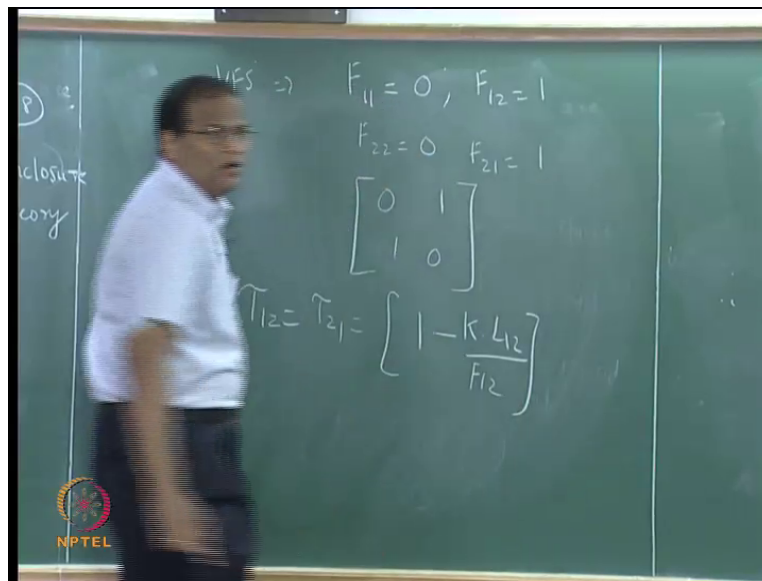
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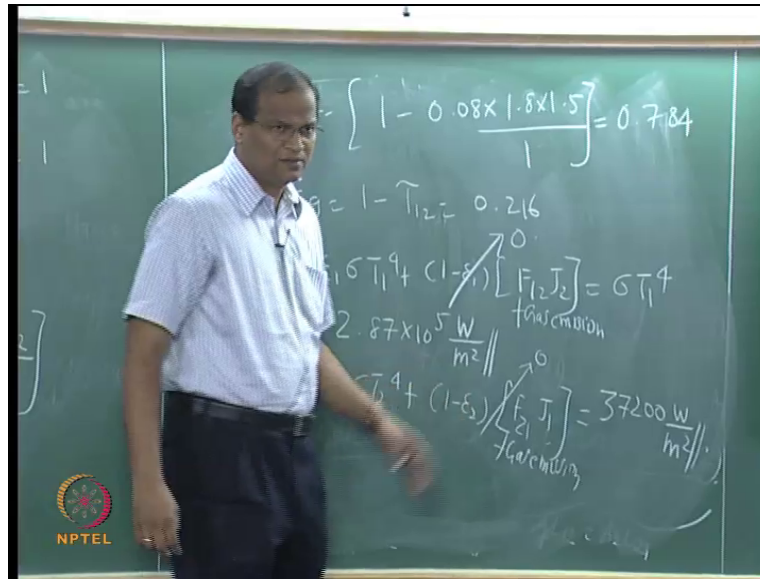
Now, revisit the problem which we did yesterday, and solve it using the theory of evacuated enclosure, right? That is two parallel plates- problem number 39. By modified enclosure theory, I mean enclosure theory modified for an absorbing and emitting gas. Okay? We shall get the same answer as before. So, let us take this. What is  $T_g$ ? 1200 K, correct. 900 K? Same as before? Now, instead of solving that, instead of using the solution to the equation of transfer and exponential integral, we want to see whether we can use this theory of evacuated enclosure.

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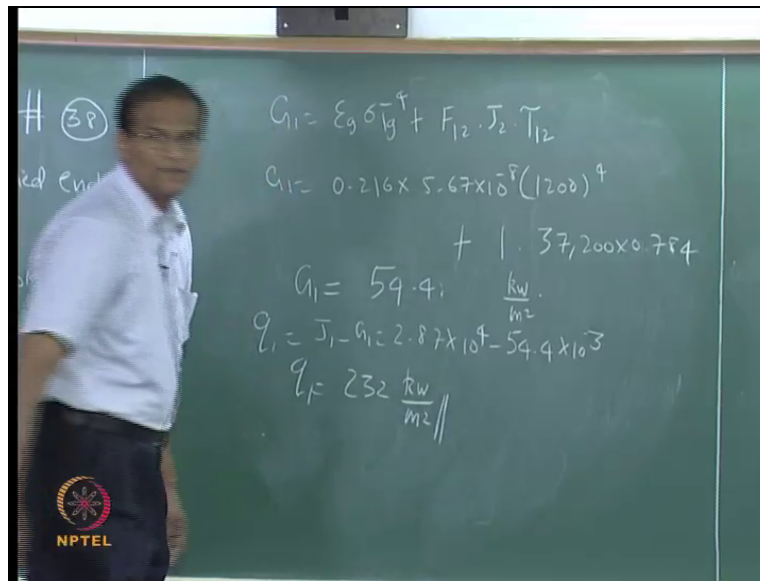
Now, let that formulation stay. Now the first step is view factors-  $F_{11}, F_{22}$ . No problem? Next step.  $\tau_g$ , okay  $\tau_{12}$ .  $\tau_{12}$  equal to  $\tau_{21}$ ? Yes. Is everybody ok with this? Correct? What is  $L_{12}$  for parallel plates? We'll take the 1.8? Or you want to take two? 1.8 is given in the tables? We will take the 1.8, okay? What is  $L_{12}$  now? 1.8 into 0.784.

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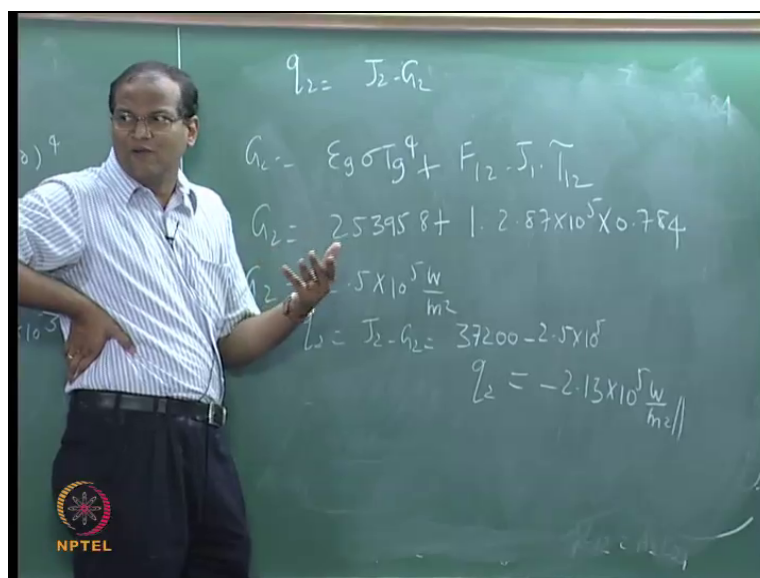
What is it physically represent- 0.784 means? The gases allowing eighty percent of the radiation to go through, only 20 percent it is absorbing. Correct? What is epsilon g? (( )) Correct? Okay, now, you have to use the radiosity radiation formulation. J 1, because it is a black wall, right? Correct? What is J 1?  $2.87 \times 10^5$  to the power of 5, correct? We worked out in yesterday's class. J 2? 37200, no? No, no one point? 37 37200. I am not calculating that 1 minus epsilon. No, no, what you are saying is plus. That does not really matter, this fellow is already... correct, correct, correct. In order to be clinically correct I will say plus gas emission and then make it 0 correct. Fine?

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What is G 1? Plus? Is this clear? If it is not clear please ask me. These are fairly advanced. You would not see them in many books. You would not see this material in many books only a few books. 54? Yeah 54.4. (( )) Is it correct? How much is this? You should get the same answer as yesterday what you got yesterday. 200 and... 232 kilowatt per meter square. Did you get the same thing yesterday? Correct? Please revisit 38, problem 38. Umesh, what did you get yesterday? Okay, two two kilowatt is okay. Transmission and distribution losses. That is the technical term for pill phrase.

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Now, we will also do the  $q_2$ . What is the first term? 25395.8, what is it? No, how is it more? (( )) 2.250 2.7 kilowatt (( )) He is already going to the next point. 25395.8 something, plus 1 into  $G_2$  is equal to two point (( )) minus? Two point (( ))

Must be the same as what we have got yesterday. So, if you are given a, if you are given a black enclosure or a problem involving two black surfaces, parallel plate formula. You can either use the analytical method, I mean get the solution use the solution get the exponential integral, or you can use the theory of evacuated enclosures to solve the problem. Most important point is, as common sense will tell you, radiation is leaving the left wall and is arriving at the right wall. Therefore, the, therefore the net heat flux on the left wall will have to be positive because that is the wall at the highest temperature that wall is the giver. The gas is also at higher temperature, the wall is also at higher temperature compared to the right side wall. Right side wall is at the temperature of 900 kelvin therefore, it must receive radiation both from the wall as well as from the intervening gas, both of which are at higher temperature compared to this. Therefore, intuitively, one would expect that  $q_2$  is negative.

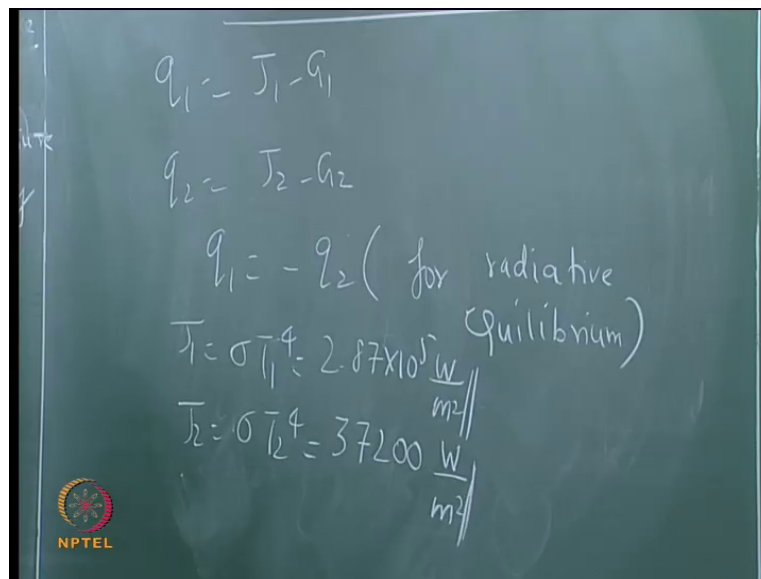
But the beauty is  $q_1$  is not equal to  $q_2$ .  $q_1$  is not equal to  $q_2$ , what does it mean?(( )) The situation is unbalanced. You do not have an equilibrium here. Therefore, the radiative transfer equation is not an expression of the law of conservation of energy. At that instant at that instant of time what is the net balance of what is the net heat flux which is going? Therefore, we have to combine it with some energy equation if you want to, if you want to solve for the temperature distribution? Are you getting the point this is not the energy equation. Alternatively I can pose the problem like this - for equilibrium to prevail, for equilibrium to prevail between these two parallel plates, what should be the gas temperature? That means, I will put  $q_1$  is equal to minus  $q_2$ , but then I will keep  $T_g$  as unknown, and I will find out the resultant temperature. You are going to do that and we will close the, we will close the class with that.

Problem number 40. Is it okay? That is called, what I am talking to you about is, radiative equilibrium, okay? Problem number 40. Revisit problem 39, revisit problem 39 for the case of radiative equilibrium, for the case of radiative equilibrium. Determine the gas temperature  $T_g$ . Determine the gas temperature  $T_g$  for this case. Determine the gas temperature. Is the question clear? Needless to say you can write all other parameters really fixed as the previous problem. If you want you can, all other parameters are same as before. All other parameters are same as before. So, a solution to radiative transfer equation does not guarantee

equilibrium. So, the gas is getting heated is it? Is that correct, what we got? Okay, because 235 is coming out from the left wall. 232 is coming out from left wall, but this fellow is able to absorb only 213 kilowatt. So, 19 kilowatts every second, 19 kilojoules per second of energy is being absorbed by the gas. The gas has to get heated up. So, my equilibrium temperature should be above or below 1200? (( )) Ahn, it should be above 1200. Just see whether you are getting it. I hope so, okay? It is interesting to note that I have not solved this. So, we solve it together.

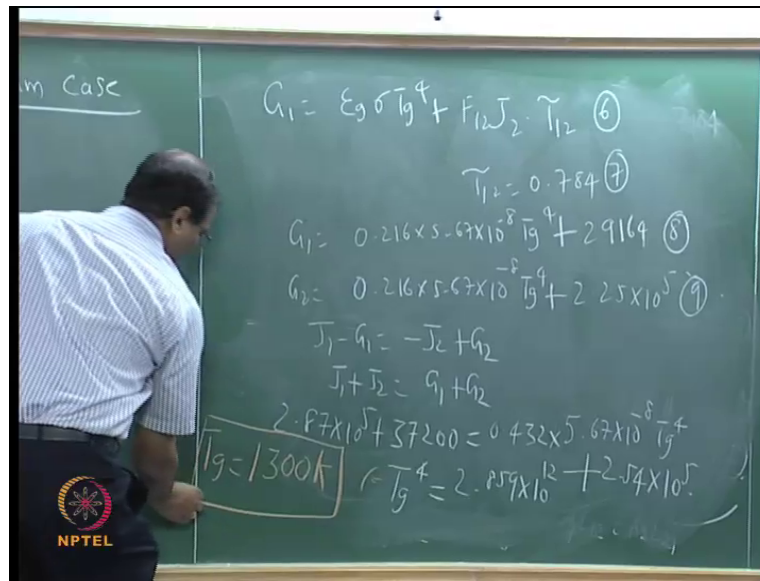
Then you are saying the gas could be at any temperature. The next noble in physics. Yeah, see from common sense point of view you know that you are wrong isn't it Deepak. So, mathematics will be wrong, the physics cannot be. Let us work it out, okay. I think most of the other things you can keep, right? For tomorrow's class please bring the charts - emissivity and emissivity emissivity chart or water vapor and carbon dioxide. I will quickly introduce the theory; we will solve one or two problems involving mixtures of carbon dioxide, water vapor. Radiative equilibrium case.

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Arjun what happened? Ah. It will be like (( )). Finally, what it is? 1300 something. 1300 we will do that. So, what is J 1? Same as before? Same as before? Okay. J 2.

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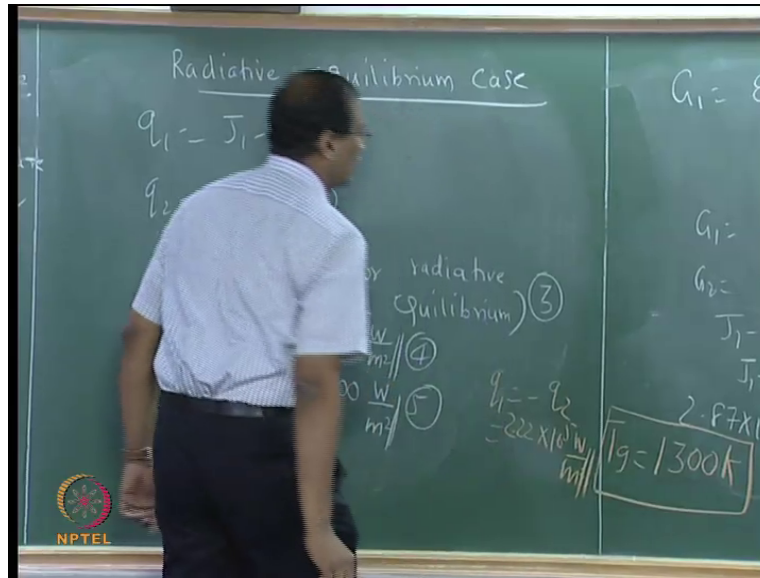
So,  $J_1$   $J_2$  is known. What is  $G_1$ ? Plus  $F_{12}$ ? Fortunately the mean beam length is not dependent on temperature. If the mean beam length is dependent on temperature you get into an infinite, you'll get into a spin, you get into a loop. So, I made a statement on one of the earlier classes, we are able to separate the geometric and thermal parts. You did not understand the significance of that statement. If the thermal and geometric parts are not separated, then it leads to tedious iterations. Fortunately view factors are also not dependent on temperature. View factors mean, beam length, tau are not dependent on temperature, which is good news for us.

So,  $G_1$ ... epsilon  $G$  is there, right- 0.216. What is this? Please tell me that value- 37200 into 0.784. 29164. So, okay? So,  $G_2$  plus... this will be too much, how much is it? Ah. I will erase this, you have to calculate this -  $F_{12}$  is one,  $J_1$  is  $2.87 \times 10^5$ , this is 0.784. This 2.25. Now since  $q_1$  is equal to minus  $q_2$ ,  $J_1$  minus Deepak they are not getting, cancel. Will get added, okay. So, plus 2.54 into 10 to the power of 5. (( )) Yeah, yeah, that is okay. I know how to do that. You think  $J_1$  plus  $J_2$  left side into factor of, I have already I am already committed to this, So. Shrikant what did you get? No calsi? 2.54. Two point 2.5 (( )) Hm. So,  $T_g$  to the power of 4 is? Yeah? Ah? Point 85... point 859. 10 to 12. 1300 and 1300 (()).

So, if you want to be under, if you want the gas to be under radiative equilibrium it should be at a temperature of 3 1300. Or left to itself, if sufficiently longtime has elapsed and you are

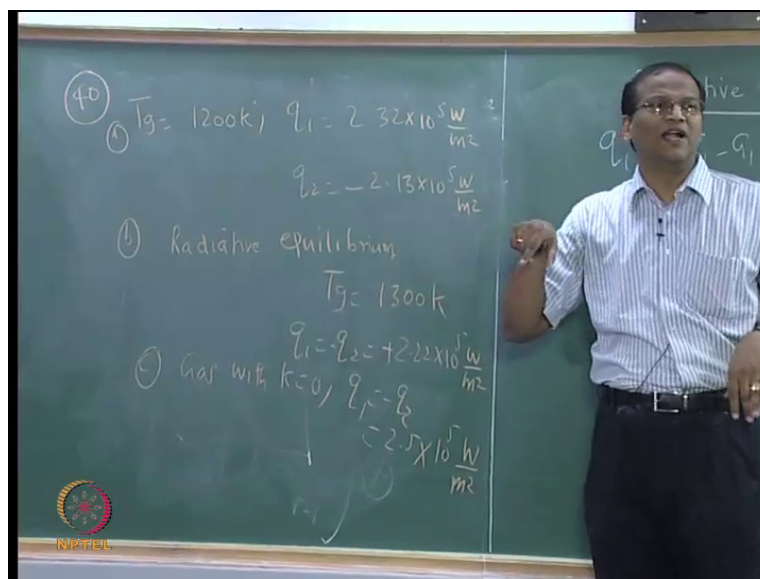
not controlling the gas temperature, the gas will come to this temperature. Then, whatever is coming from the left side will go to the right side. In the absence of the gas what will be  $q$ ?

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Now what is  $q$ ? What is  $q$  for this now? What is  $q$  1? 232? No, no, just check, tell me. What is  $q$  1 for this? 240? No, no, no, with the gas. I am now, can you calculate the  $q$ ? Please calculate the  $q$  and tell me. Ketan what? What happened? 222. Okay?

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Now, I want you to appreciate the three cases where all the temperatures are specified then you have no control over the energy balance. Second you talk about energy balance in equilibrium case, where  $q_1$  will be equal to minus  $q_2$ , and the third case will be the  $kappa$  equal to 0 and Arjun its correct 2.5. Now, for the even for the radiative equilibrium case because the gas has got a  $kappa$  which is not equal to 0, you are getting a flux which is lower than what would you have got if you had you had transparent gas or had you vacuum in between those, in between these two place. So, from our working of these problems yesterday all all the many of the concepts which we studied in gas radiation become more clear, right? So, the because of the absorption of the gas, though it is emitting the gas is at a temperature in between the other two walls, the net effect is it it it retards the flow of heat from one wall to the other wall.

There could be the other case where the gases are very hot and you want the heat transfer to the walls. Where do such situations occur? In the fire tube boiler and all that where, from the hot gases, on the walls you have tubes where water is sent it becomes steam. In this case the walls are hot and the gas is getting heated up because of the presence of the wall. The wall definitely reduces, sorry, the gas definitely reduces the radiative heat transfer between the two walls. Is this clear? We can keep on escalating this problem, another two hours I can solve. Now I will say instead of one and one take epsilon is 0.9 and 0.6, and for this case find out the heat transfer. For epsilon is 1 equal to point and epsilon 2 is equal to 0.6, what will be the equilibrium temperature, we can keep on doing, but will close the discussion here, as far this thing is concerned. Tomorrow's class we will just take up the mixture of gases and solve one one or two problems depending on the time and on Friday we will start conduction heat transfer. Okay?