Conduction and Radiation
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Lecture No. # 32
Conduction - Introduction

So, we will complete the gas radiation problem involving a spherical cavity, where we had a mixture of carbon dioxide, water vapour and nitrogen, it was a two atmosphere.

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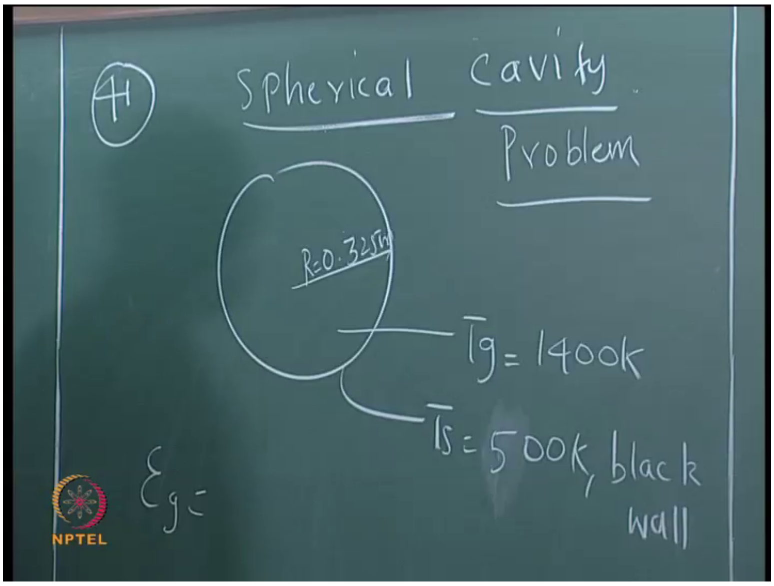

41 problem number

Student: 41

41400 Kelvin right

Student: (( ))

Is a black wall right, I gave it to be a black wall right

So, yesterday we looked at the charts repeatedly and arrived at the epsilon g, that is the emissivity of the gas, so epsilon g was 0.36, now we will have to, so we have to get alpha g.

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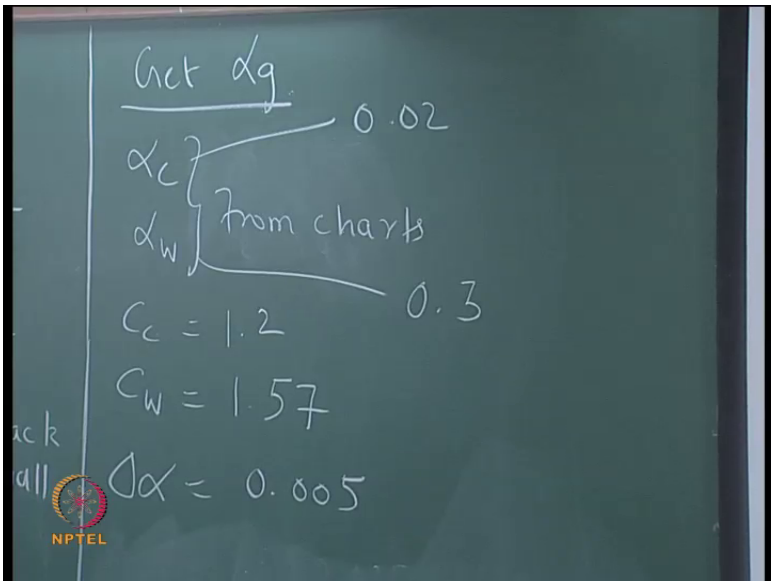

So, first step is to get can you tell me all these values, so you have to basically, employ the same procedure, except that there will be normalizing factor called T s by T g, takes care of the ratio of the two temperatures, and you have to apply the use the same charts. Shall we go one by one, what was the absorptive of the carbon dioxide; did we work it out yesterday?

Student: (( ))

0.025, the correction factors are the same right.

Student: (( )) same correction factors

Same correction factors should be used no, what are what are the correction factor 1.1 and 1.5 (( ))

Student: (( ))

So, we got this delta alpha we worked out yesterday right that was easy

Student: (( ))

Just get me the alpha of the water vapour and we are done, so delta alpha is almost 0 right.

Student: Yes sir

Very small we can use the, so we use the same charts, gas (( )) for CO_2 the same charts we use for getting alpha of CO_2 page 2, page 3 we get alpha of water vapour. So, you have to find out the p c into p w into l m, and then get the gas temperature and then get the alpha w.

How much was it alpha w

Student: 0.15

0.15, is it correct?

Others please check this

Student: (( ))

No, no no no I did not want you to tell me that, this alpha c from the chart, I want this from charts, do not go to the from charts what are the values?

Student: Sir r equals to

0 2 alpha whatever we should be high, I expected to

Student: (( ))

0.3, I expected to be high 0.3, Vikram I am not I do not want the post process values I do not want the post process values, we are (( )) step by step. So, show me what formula did you write yesterday?

Student: (( ))

No, that C c was not there, but we are already calculating no based on that, no no there is a notational problem here, I gave you alpha c equal to epsilon CO_2 plus into T g by T s to the power of. So, shall we put this as star, this no problem, this actually epsilon of CO_2 and this thing got the corrected temperature that is all.

Student: (( ))

Yeah yeah and then, but then we again multiplying with T g by T s

Student: (( ))

Yeah yeah you guys are, no no scaled up, I want only the directly from the chart, first directly read from the chart with T s by T g and tell me, then we will go step by step.

Student: (( ))

Directly from the chart, what do you get?

Student: That is 0.04

Which one?

Student: After that, we multiply the T g by T s.

No no leave it

And then correction factor

No no I do not before multiplying the correction factor, before everything 0 4, this one

Student: (( ))

Do not multiply anything, because somebody else is seeing the program, so first read it from the chart, what is the value, we will multiply step by step yeah

Student: (( ))

That is (( )) whatever I have given, that has to be multiplied

Student: (( ))

Correct, corresponding to the alpha (( ))

Student: (( ))

Point naught

Student: (( ))

Point naught, which one

Student: (( ))

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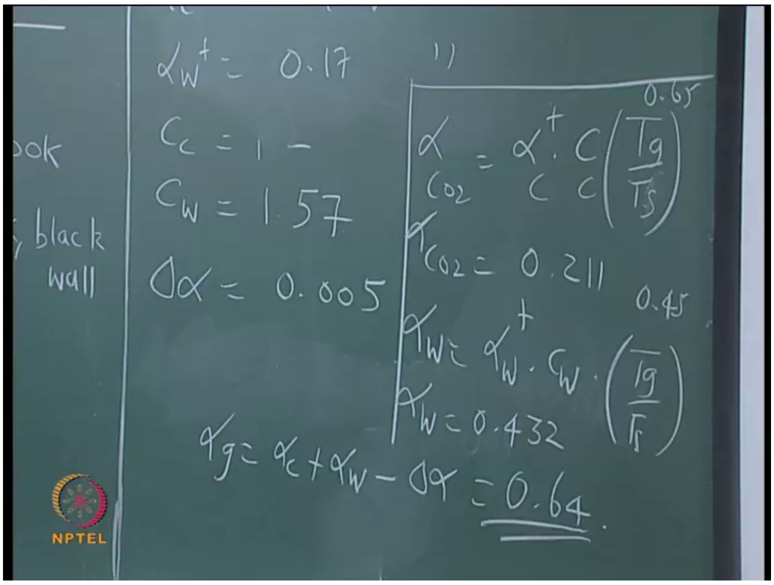

Alpha w developed these charts. Now, we can do solve the equation and transfer numerically and get this, but this was done 1945, lot of things developed during world war two, including war right. So, now we got the, what is the cooling rate required to maintain this (( )) Kelvin or 500 Kelvin.

Student: **(( ))**

500 Kelvin that means, we want 4 by R squared is that correct here, this is the cooling rate required to maintain it at 500 Kelvin.

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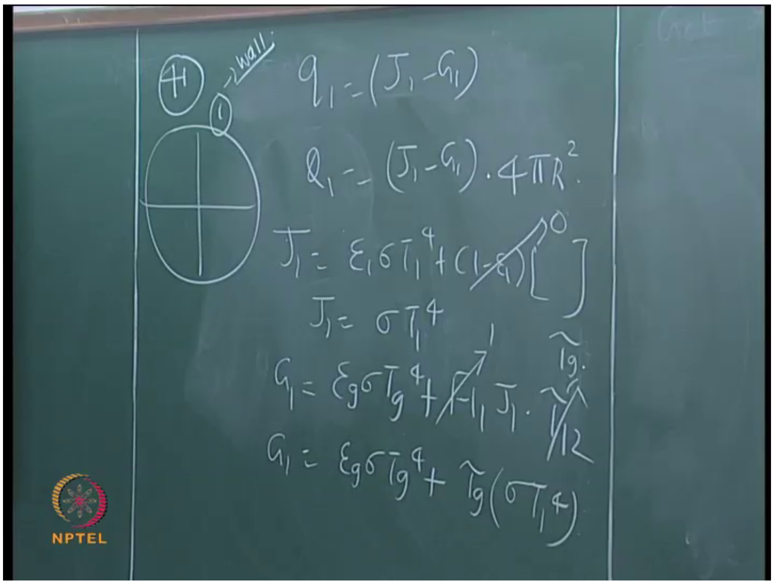

Now, J 1 is easy 1 is the wall, there is no two surfaces is enclosure, is only single surfaces is enclosure, I gave black wall is it not, it fine; how much is this, leave it we will shall we derive the generic generic formula first we will derive the, now what is G 1 it is gas radiation.

Epsilon g

Student: (( ))

f y j j j tau 1 tau y j there is only one enclosure, there is only one surface, the other one is gas. Therefore, and F 1 2 is not there, F 1 1equal to F 1 1equal to 1, this sees itself fully and tau one two can be taken to be tau of the gas correct, you can call it as tau g therefore, plus 43, so now what is this tau g, we cannot put 1 minus (( )) l 1 2 by because, that was for gray gas and all that. 1 minus alpha g

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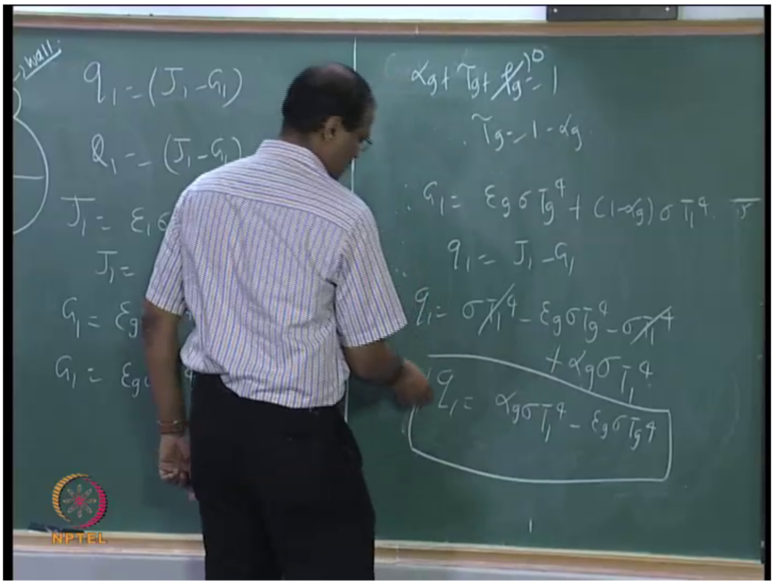

One minus alpha g we did so much so much struggle, we undertook to calculate alpha g right, so we can say that, because alpha g plus tau g plus rho g equal to 1 for the gas also we do not worry about this fellow now, therefore, tau g equal to 1 minus alpha g.

Therefore, so if it is single surface enclosure involving gas radiation, which is the mixture of these gases right, in the exam you can straight away use this final formula, you do not have to derive. Alpha g sigma T to the power of 4 when is epsilon g sigma T g to the power of 4, this is the standard formula it will be given in the several text books, but we derived it using enclosure theory in today's class correct.

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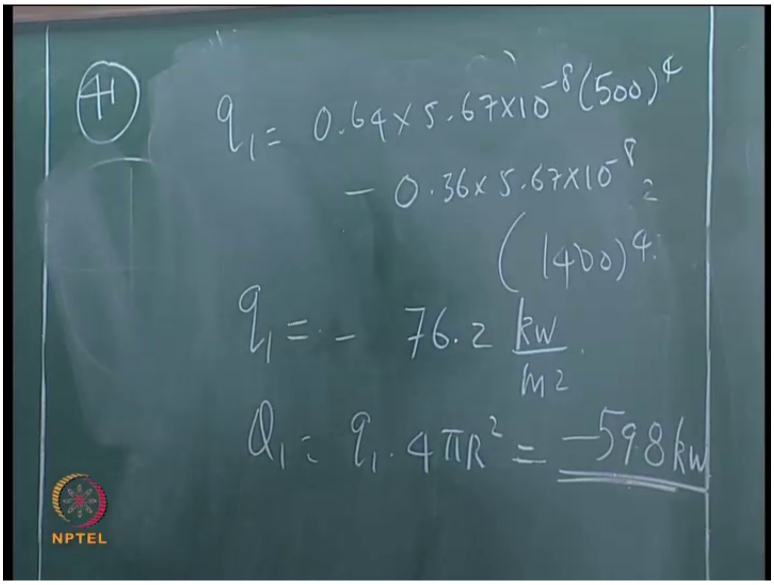

If you see (( )) you will work out radiation, gas radiation problem straight away by taking this formula what is alpha g 0.64, you got alpha g plus epsilon is equal to 1, (( )) that is strange, need not happen always yeah, kilowatts minus of course, it is minus is it not, it minus sure, kilowatt per meter square is it correct, (( )) q 1, you are not getting that (( )).

Student: (( ))

1400 minus 76 point, so q equal to boundary condition or a the the boundary condition will come because of m V squared by 2 m V squared by 2 is stopped and is converted to kinetic converted enthalpy on the surface.

So, these applications the body is moving, can you give me some applications where the source is moving, if you are welding and moving the welding rod, that is where the heat sources is there, so it is a moving a heat source problem, it is a moving heat source problem. So, there are problems in which the heat source or the body moves, but these are exceptions in conduction heat transfer.

The general rule is the body is not moving the particular laws are three laws, the Fourier's law of heat conduction Fourier's law of heat conduction, basically for conduction heat transfer, the Newton law of cooling for convective heat transfer. The Stefan Boltzmann law of radiation for the irradiative heat transfer, all these are particular laws which are not universally applicable right, they are they are applicable only in specific situation.

So, if you if you apply the Fourier's law of heat conduction, and the Newton second law on to your first and second law of thermodynamics, it is possible for you to derive the general heat conduction equation. Then finally, you can say that the body is not moving that will give rise to your Poisson equation, Laplace equation, then one-dimensional heat equation and so on.

It is possible to apply Fourier's law of heat conduction and Newton's law of cooling to the law of conservation of mass first law, second law and Newton's second law of motion, what we get? You get the Navier-Stokes equation you get the Navier-Stokes equation. And the equation of energy radiation we do not radiation is separate please remember, that the equation of transfer is not an energy equation, even the problem which we solved in the last class, there is no equilibrium there that means, the (( )) equation is not a standalone equation; it gives rise to a source or a sync.

So, it has to be solved in conjection with a conjection with the equation where some properties conserved, are you able to (( )). So, in a combined conduction radiation problem, you have to solve the conduction equation there will be a source term for radiation, if you want to calculate that source term, you have to stop the conduction calculations and do the radioactive calculations and go back.

So, if you combine this particular laws with general laws, you get new subjects, if you combine this laws with this you get conduction heat transfer, if you combine this with this you will get conductive heat transfer, if you combine this with this and you will say that d a by you, have the equation of transfer you have radioactive heat transfer (Refer Slide Time: 29:40). So, basically so these are what these are what are called the constitutive equation, constitutive relations in, so far as the subject of heat transfer is concerned these are general laws.

So, every every subject will have a field of will have some experts, who take, who draw upon the basic laws of nature, apply equations applicable to their media and then come out with a science, but still it is all Newtonian in physics. So, we can still say, that heat transfer is still mechanics mechanics and thermodynamics we want right we are not still using we are not still trying to solve Schrodinger equation or quantum mechanics of course, in radiation we you do that. Most convective and convective heat transfer still Newtonian physics.

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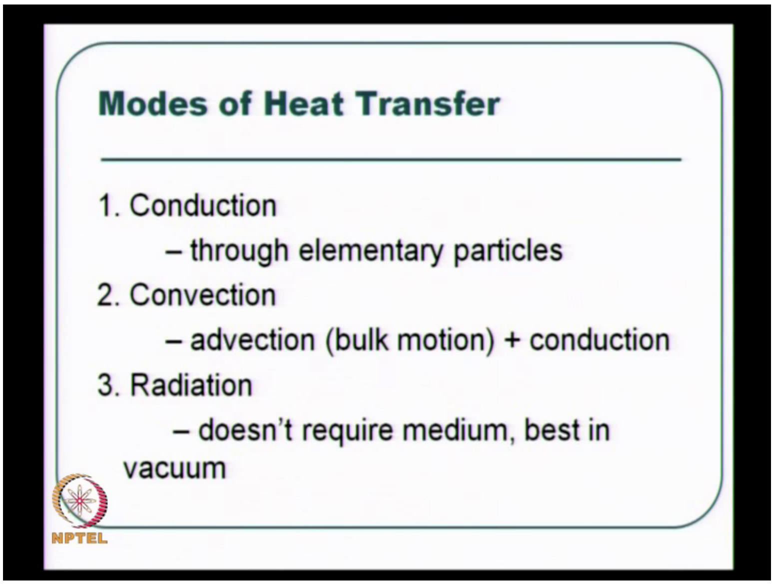

So, you we should not say this fag end of the course everybody knows that, there are three modes of heat transfer conduction through elementary particles, convection is advection plus bulk motion, but advection plus conduction, radiation does not require a medium it is best in vacuum. So, some people skip number 2 and say that that only 2 modes of heat transfer conduction and radiation, the elementary particles at high temperature regions at higher energy level oscillate.

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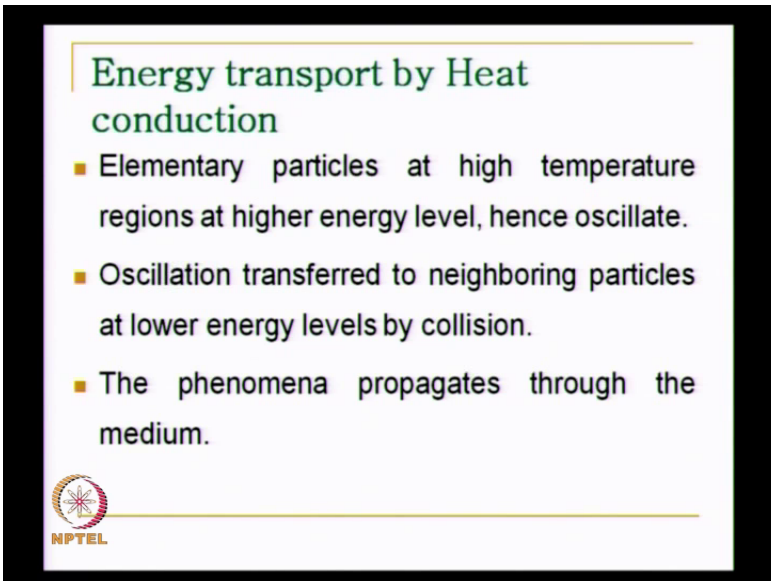

So, then there will be a microscopic movement of molecules, so at the particular layer some molecules from top can come to the bottom, from bottom they can come to the top, but there is a temperature difference, so though the number of molecules coming from the top to the bottom or the bottom to the top microscopically, even though their microscopic movement is the same.

The energy carried by the particles to the higher temperature is more compared to the energy carried by the particles with a lower temperature. Therefore, there is a net positive transfer of energy in the direction of decreasing temperature, that is a mechanism of conduction which is an which can be easily explained in the case of gases.

You can also extend this to liquid for solids its more difficult valence electrons, and all those things will and will come into picture. So, if you look at energy transfer by heat conduction that is the thing oscillation transferred to neighboring particles, phenomena propagates through the medium.

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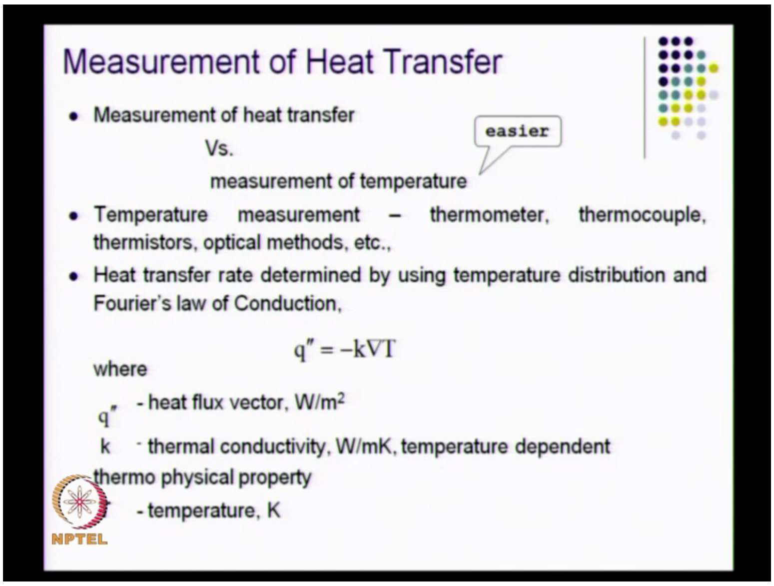

What about the measurement of heat transfer, why are we always talking about the measurement of temperature and not about measurement of heat flux, it is possible to construct heat flux gages, come up with the heat flux gages and so on. But, it is lot easier to it is a lot easier to measure temperature, because the various ways of measuring temperature you can use thermometer, thermocouple, thermostats optical methods, one of my student is working on liquid crystal thermograph.

In liquid crystal thermograph what you do is, you have a sheet you have a sheet and place it on the surface whose temperature is changing, where there is the temperature distribution, the the colour of the sheet will change depending on the temperature. And you take a picture in a high speed camera or CCD camera, then convert the colours to temperatures and then you get a temperature field, from that you solve additional equation you solve inverse problems and so on, that is liquid crystal thermograph right.

So, there are various temperature is lot easier lot easier to measure, so then what we do is, sometime we want to know the temperature, we want to know whether the temperature, the maximum temperature in the laptop, respects the respects the standards set by the manufacturer, what I mean by respect the standards is, if it if it should not exceed 80 degree centigrade, it should not it should not exceed 80 degree centigrade. But, often times you are interested in the flux, if you are interested in the flux what you have to do is, you have to first measure the temperature and and link this temperature gradient to the heat transfer rate.

So, this to the Fourier's law of heat conduction, when q q double prime equals to minus k delta t it is basically, a rate law applicable to conduction, it is not such a great law that it cannot be, in the (( )) it cannot be proved from first principle. Whenever conduction heat transfer takes place, the heat transfer rate is always proportional to the temperature difference is inversely proportional to the area, and if you replace a proportionality constant by equality you get what is called the thermal conductivity. Therefore, the thermal conductivity of a medium is nothing but, the heat flux and the gradient of the temperature is unity that is all.

So, the Fourier's law of heat conduction gives you an operational definition for k, it it gives you a way to measure k. So, when you are actually trying to measure thermal conductivity in a lab, did you do this thermal conductivity measurement no, when you are actually trying to measure thermal conductivity in lab, you use what is called guarded hot plate apparatus.

There are two plates, they maintain different temperatures, and then you send a certain amount of heat flux and find out the two temperatures or for the two temperatures what is heat flux, which is occurring. Then you take the delta d can be put as dT by dx delta d by delta x, you estimate the k. So, this simple this simple measurement of k, what you are actually doing is you are solving an inverse problem because, if you know the thermal conductivity you can take the conduction equation and predict the temperature distribution or the temperatures at the two boundaries. Then what do you doing is the reverse, I am giving you the two temperatures turn around and tell me the thermal conductivity. So, this is very simple case where in a straight forward way you can get the k, but if the k itself is k itself varies k x is different from k y is different from k z, you require lot more work.

So, this comes under the subject area of inverse heat transfer, so this is the heat flux vector, this is thermal conductivity this is thermo physical property, and t is the temperature.

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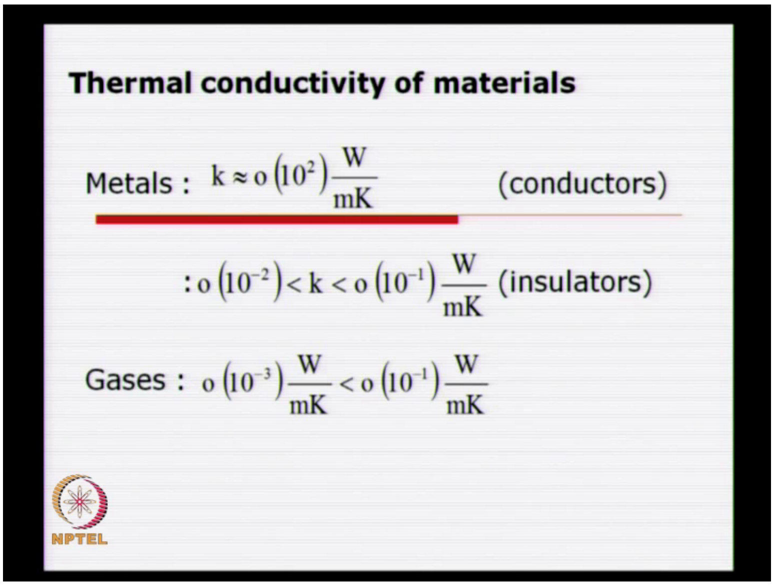

So, for metals the thermal conductivity is order 10 to the power of 2, for insulators its varies from 10 to the 10 to the minus 2 to 10 to the minus 1, gases it varies from 10 to the minus 3 to 10 to the minus 1, I will give you a table.

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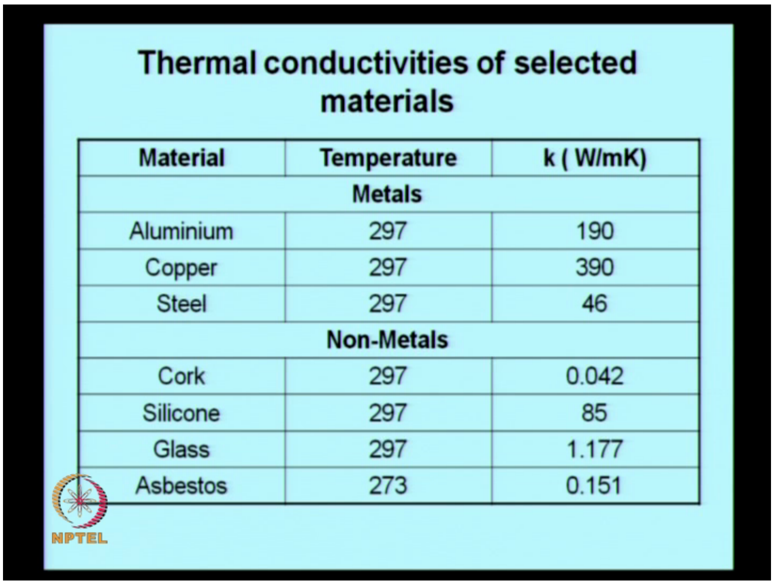

Thermal conductivities of selected materials

Material	Temperature	k (W/mK)
Metals		
Aluminium	297	190
Copper	297	390
Steel	297	46
Non-Metals		
Cork	297	0.042
Silicone	297	85
Glass	297	1.177
Asbestos	273	0.151

So, thermal conductivity of selected materials, aluminum pure aluminum is to is about 190 watt per meter per Kelvin, normally we use some aluminum alloy, so safely you can assume usually around 200 aluminum is about 200 watts per meter per Kelvin, copper has a terrific thermal conductivity 390 watts per meter per Kelvin. As far as heat transfer is concerned it is (( )) for us, copper copper is a benchmark in several cases right, copper cables, we use copper everywhere.

It also, that is why it also gets stolen, that brass also Indian institute of technology, the Indian was stolen, because Indian can easily be resold in gate, stolen main gate, let us see when metal. So, there is a story about metals, steel the thermal conductivity the thermal conductivity of steel, steel I cannot put steel there are, so many types you must have studied in metallurgy courses, you got mild steel, stainless steel, and high carbon steel and so many raw carbon steel, there are many varieties of steel, it is a ball park an approximate figure of 46 watts meter Kelvin, 46 watts per meter per Kelvin, nonmetals cork is terrific cork is almost close to that of air.

So, it is an excellent insulator cork, 0.042 thermal where is 0.03, so it is almost close to air, but you can I have put silicon basically, because you should not be under the impression that any nonmetal will have a very poor thermal conductivity. Silicon has a good thermal conductivity, glass has around 2 1.2 1.2 asbestos is 0.151, asbestos is no bang because it is carcinogenic (( )) you cannot use.

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Thermal conductivities of selected materials (contd.,)

Gas	Temperature	k (W/mK)
Air	297	0.026
CO_2	297	0.016
O_2	297	0.027
H_2	297	0.182

Air air from kinetic theory of gases you know, that the thermal conductivity of air will keep changing with temperatures, so this is for some particular temperature 297, 0.026. Carbon dioxide is 0.0167, O2 0.016, oxygen 0.02, hydrogen is 0.182, so if we see air CO2, O2, they are all 0.0 something they are very poor conductors. So, the expression for heat flux is very important.

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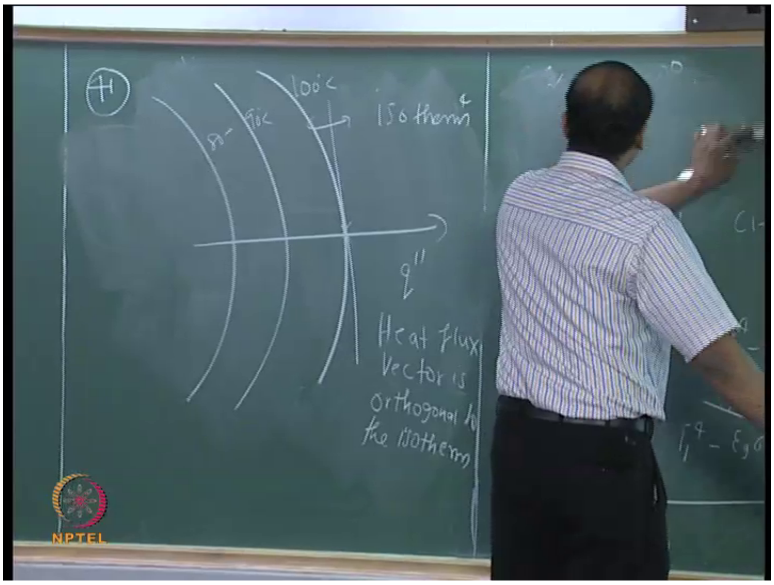

So, so if this is your isotherm, that is you have cross the temperatures, I mean to 100, let us say 100 degree centigrade, 90 degree centigrade, at this point you draw a tangent, so the heat flux vector will be like this. So, (( )) so the heat flux vector is normal to the, the heat flux vector is orthogonal to the isotherm lets say conduction in a plate, what should be the temperature here, the only problem no heat generation nothing constant properties, what you expect the temperature to be 75, so this will be T of x. So, isotherms are like this, this is not properly drawn everything should be vertical. So, this is the 100 degree isotherm, 100, 90, 80, 70, 50, 40, on the 100, 90, 80, 75, 65, 60, 65, so it should be like this. So, the heat flux vector is orthogonal to the isotherm.

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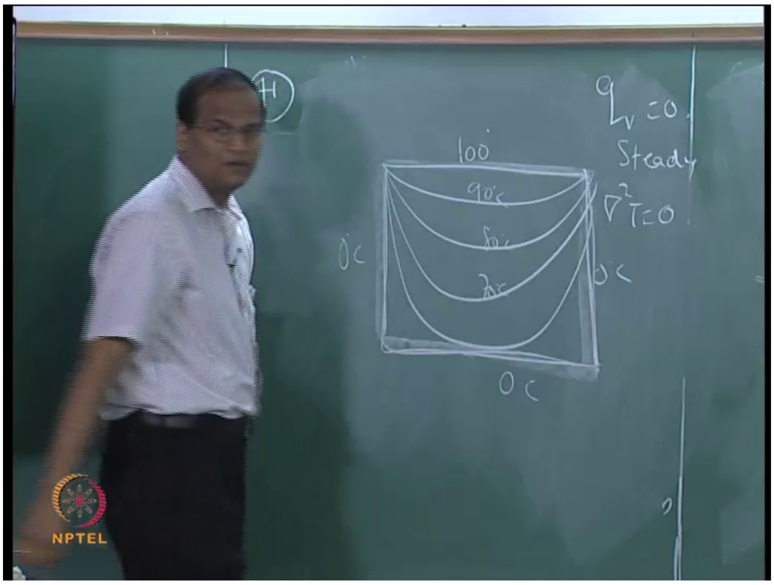

So, if you consider a problem all of you have studied basic heat transfer right, let us consider a square slab correct, the governing equation is del square T is equal to 0, what is this Laplace equation. Suppose, I say this wall is at 100, all the three walls at all the three walls at are 0 degree centigrade, how will the isotherms look like isotherms, not other way.

Student: (( ))

So, we will start like this please follow me, the 100 degree isotherm is this, the 0 degree isotherm is this, all temperatures are between 0 and 100 in this problem correct. So, the 90 degree isotherm will be like this, the 80 degree isotherm will be like this, 70 and finally, this fellow is 0 (Refer Slide Time: 42:27), so 90, 80. Now, I leave it as an exercise to you, please plot the is heat flux lines on this, because the heat flux lines have to be orthogonal to isotherm fine.

We can discuss it at the end of the class.

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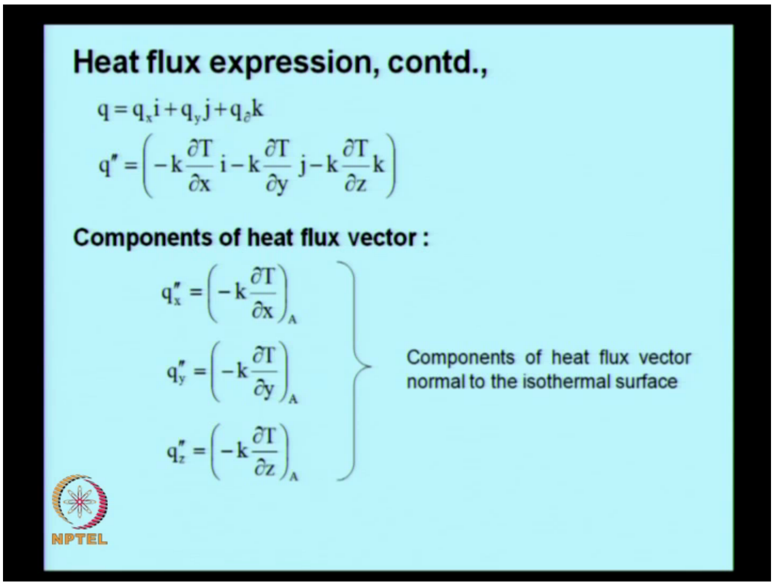

So, the heat flux is evaluated like this orthogonal, and the heat flux is a vector, temperature is a temperature is a scalar; heat flux is a vector, so it has got three components q x, q y and q z. So, the q x is given by minus k dT by dx minus k dT by d y minus k dT by d z.

Why is the minus coming, to take away the fact that heat is always flowing from a

Student: (( ))

High temperature to a low temperature

So, this q can be broken down as q x I, q y j, q k, q z of k and where q x, q y, q z of the heat flux components in the three directions, and it is given like this, so it is the components of the heat flux heat flux vector normal to the isothermal surface.

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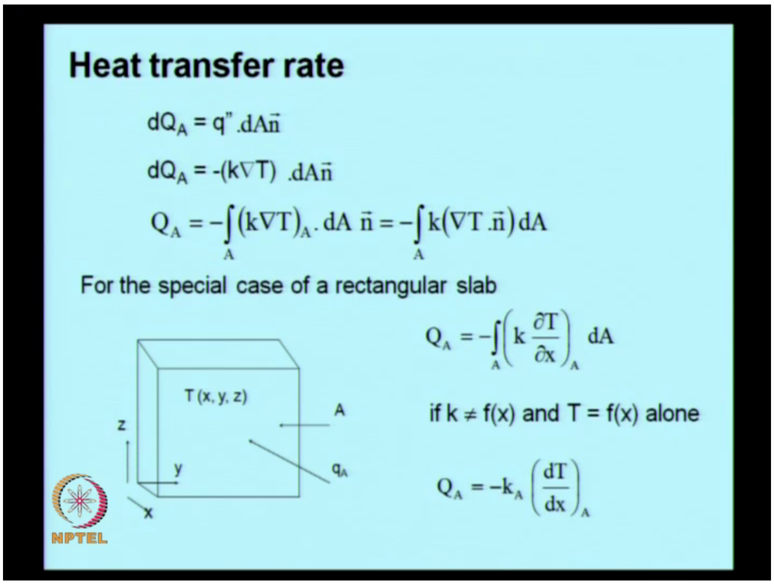

So, you can so you can say that the d Q A is basically this is watts watts per meter square, so the elemental heat transfer rate is equal to the flux into the elemental area. So, q can be replaced by k del T using the Fourier's law and then therefore, q must be integral d q over A, then you can do this k into del t into n, where n is unit vector.

Then for the special case of rectangular slab is a temperature distribution is one-dimensional that is T is the function only of x this will become k dT by dx and this k is not a function of x k can be taken outside. So, the q becomes minus k a dT by dx under the special case of temperature distribution is linear, this can be written as k a delta T by delta x. So, this is a simple expression which you have used, I think from high school onwards you learn this.

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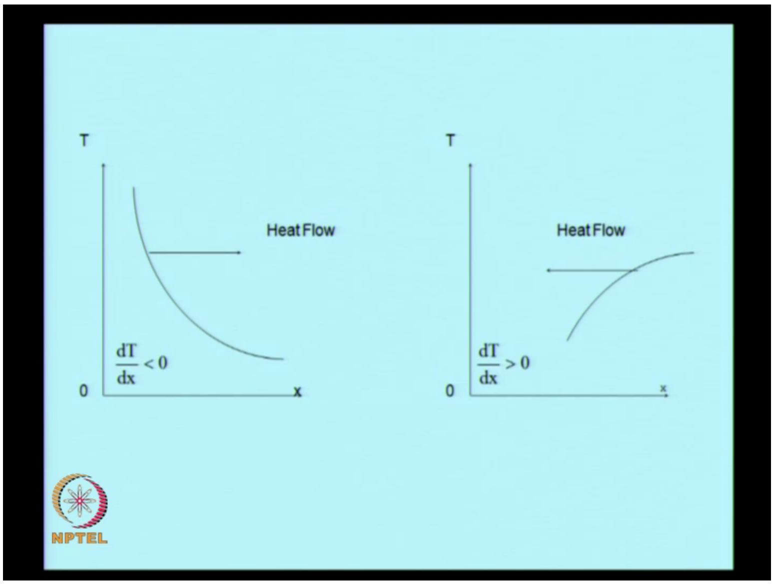

So, please note this very very important, if the temperature is decreasing along x and the heat is flowing from left to right, if the temperature profile is like this, heat is flowing from right to left, so you must get an idea of the direction. So, the direction is given by the second law of thermodynamics, which forbids flow of heat from a temperature, low temperature to high temperature without the application, without the aid of frictional work.

So, please note this dT by dx is less than 0, heat flow will be please remember this problem, temperature is decreasing along x therefore, heat must be flowing from left to right and this is the exactly the opposite for the other cases.

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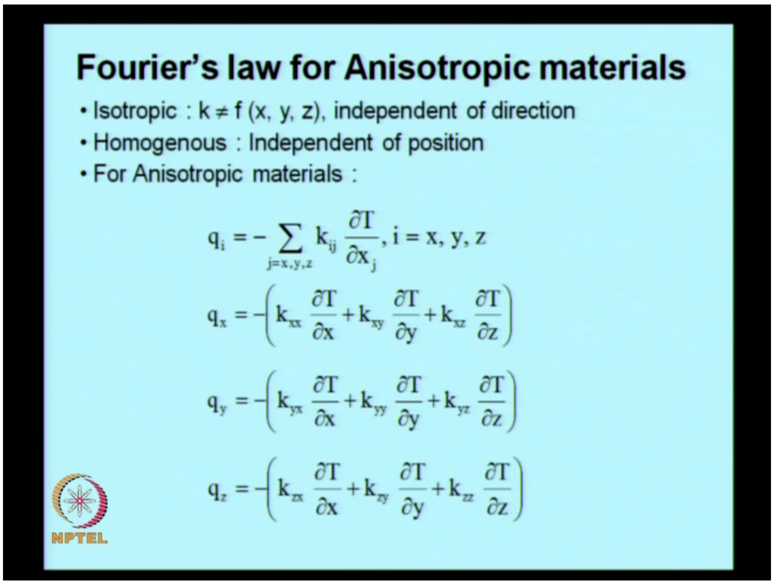

Now, there are some materials which are anisotropic, anisotropic material is one in which the thermal conductivity is independent of direction, that is thermal conductivity in x direction is the same as y is the same as z direction; if k x, k y, k z are found are dependent on the direction, it is called anisotropic material. A homogeneous material on one on the other hand, is the material in which the properties are independent of position.

So, in this case in this room for example, if you take an air sample here, measure the density and you can take an air sample there and measure the density, if the density is same in both the places, then the air is homogeneous. But thermal but isotropic is if you measuring a transport property with x direction, y direction, and z direction, and you get different values, then it is anisotropic.

Now, for anisotropic materials for anisotropic material q x, q y, q z each of these contains three components. So, here q x is k x dT by dx k x dT by d y k x dT by d z like that it follows for the other two now. So, the heat flux is related to the temperature gradient through something, this is called the thermal conductivity tensor, this is called the thermal conductivity tensor; it has got nine components, along the principle axis, main diagonal you have got k x, k y, k z, and you have got, because of symmetric k x y, k y z, k y x, k x z, k z x and so on.

So, it has got six elements, which are to be determined, other three come by symmetry right, it has to satisfy these properties k i j equal to k j i, k i i is greater than 0, k i i k j j minus k j square equal to 0 therefore, it is a positive definite matrix, it is positive definite and why does it positive definite and all that basically it comes from, what is called Onsager principle, I cannot I do not have time to do that, that is a another important concept have you studied this?

Somebody will get a doubt, sir why should be why should be like that, why should k should be greater than should be greater than 0, why should k i j equal to be k j i and so on, all these questions will be answered by Onsager's principle, it starts from the assumption that, it starts from the basic premise you cannot violate the second law of thermodynamics and so on.

When are these, where will these be important anisotropic media occur any laminated sheet any laminated sheet basically, anisotropic media, wood anisotropic transformer cores many things, where anisotropic media have to be studied. In fact, we are doing a project now for ISRO (( )) is working on that. So, ISRO wants the measurement of thermal conductivity, anisotropic thermal conductor will laminate that sheet right.

Now, they are putting it into vacuum chamber and this thing it takes a long time, so we are suggesting some inverse technique and simultaneous estimation of this property. So, it is it is not just for academic interest it is very, very important. Now the problem is the direction of heat flux is no longer normal to the isothermal surface throughout the point, and it is a lot more involved, needless to say, there will be no further consideration of anisotropic media in this course.

Now, this is the introduction to conduction. So in the next class we will start with the energy equation, where the variable will be temperature, you will relate the temperature with the properties, and you will get a differential equation, partial differential equation for temperature. And then we will take simplifying conditions and reduce the, and we will try to get as many analytical solutions as possible.