Conduction and Radiation
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Lecture No. # 34
Conduction-1D, Steady State.

So, at last we finally, started conduction and we derive the heat conduction equation for conduction I mean the energy equation for conduction in cartesian coordinates. And we solve three types of boundary conditions as and when we come to the sphere and cylinder. I will give the governing equations for cylinder and sphere. But I deliberately and intentionally, avoided some complicated mathematics and algebra in deriving the energy equation I assume that it is a incompressible substance and all that.

So, I could have taken that it is compressible and proceeded with the whole thing and then I would have I could have written the t d s relations, t d s is equal to d s minus v d p and d s is equal to h equal to c p into t d h is equal to c p d t and then finally, I have to invoke the continuity equation then all that. But I give I avoided all that because that is not central to this course. We are not worried too much about conduction in a compressible medium and all that that is why I made it very simple.

So, that it is a form which is easily understandable and explainable to you. Now, we will quickly, review some basic problems we have still 7 or 8 hours. So, I will try to review one dimensional problems, two dimensional problems with the perspective which is not usually given in a first course. So, you will see it as it goes along we will do a face change problem and also two dimensional method of separation of variables and finite deference.

So, whatever is possible in 7, 8 hours we will have to pack so, it will be packed and it will be somewhat different from the earlier part of the course in which we will not be able to solve examples for each and every case. That pace will not be there suddenly there will be a non linier increase in pace for it is since it is only conduction. And most of the material is already known to you I hope you will be able to tolerate it. So, it will be somewhat fast. So, now let us consider the plane wall what should be the insulation on the furnace? What should be the insulation experimental setup involving a face change material? One of my student is doing how much; what should be the thickness of the cork? So, that you do not want any heat transfer from the back side. Why should be the insulation in an air conditioned room? Why should be the thickness of this thermocol false sealing? To reduce heat include air conditioner cooling load of the air conditioner so on.

So, the insulation design is a very important example, then fin system also in plane wall I use extended surfaces to increase heat transfer rate. Nuclear fuel rods, if it is a circular rod then it may not be in plane wall suppose, it has got a rectangular cross section and so on. It is not plain wall it is plain geometry. There are various applications the examples are far too many applications are far too many where one dimension steady straight is still very important. So, let us consider a simple problem. So, this is.

You can say that, this is basically occurring at the wall of a heat exchanger on which; on both sides of which there are two fluids with different temperatures. Which are affording different heat transfer coefficient, this is the common situation in conductive heat transfer. There is a solid wall, there is a fluid flowing either the fluid is moving and the solid is stationary or vice versa. You know when the air plane moves, the fluid is stationary and the solid is moving. Once there is a relative motion conductive heat transfer situation is set up. Now, T infinity 1 is greater than T infinity 2.

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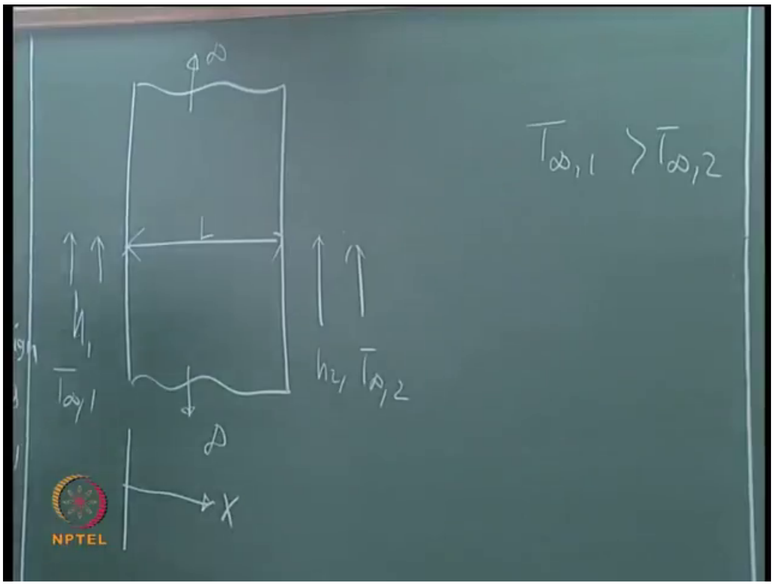

T is a function of.

This is called the governing equation to the problem; the first step is to identify the governing equation to the problem, any conduction problem.

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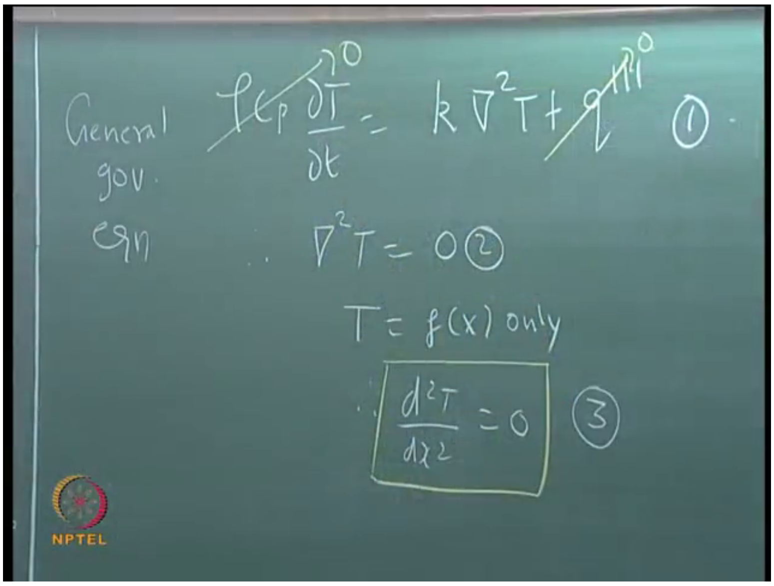

It is an ordinary differential equation for temperature second order in x therefore, it supports two boundary conditions. The two boundary conditions are available to us that x equal to zero and x equal to L, whatever heat is lost by conviction must be equal to conduction at the solid fluid interface. You can apply these two boundary conditions if the integrate, you will get two constance. We can evaluate the constance and get the temperature distribution. But you will be surprise to know, that even such a simple problem and I gave a boundary condition like that it is not as easy as you think when you do the algebra.

Because traditionally in your courses on heat transfer the teacher will say, left side is T 1 right side is T 2. Left side is T 1 and right side is T 2 is very difficult to have in practice. There is always fluid on 2 sides whose temperature alone we know. And there is a heat transfer coefficient only the heat transfer coefficient is infinity will the T 1 here, be equal to T infinity 1. And the T 2 here be equal to T infinity 2. Therefore, having a simple system T 1 and T 2 is very easy to teach, very easy to derive mathematically, but unlikely to the encounter in practice.

Now, let us take the general case, we can always take h 1 equal to infinity, h 2 equal to infinity. And reduce to the simple case which are already which are learning from high school onwards. That is q is equal to k delta t by L; I do not know when you started learning. So, that is it, but we are suppose to be heat transfer specialist. So, we will have to complicate the problem or take a more realistic case or the problem and see what happens. Now, this now the fun starts, when you do the mathematic let us do it [FL]. But getting the a and b is not going to be straight forward because I do not know the value of t aT 2 values of x.

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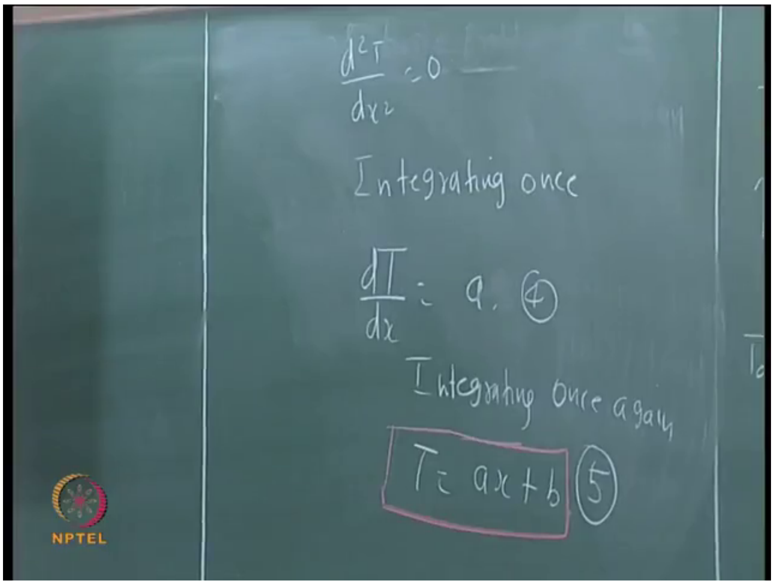

Suppose, say x equal to zero, T equal to T 1, x is equal to L, T is equal to T 2. I do not know the temperatures dT is. I know only the fluid temperatures which are bathing the wall. So, but two boundary conditions are available therefore, it should be possible for us to solve. Let us do that now, we have to it would be vary of the sign.

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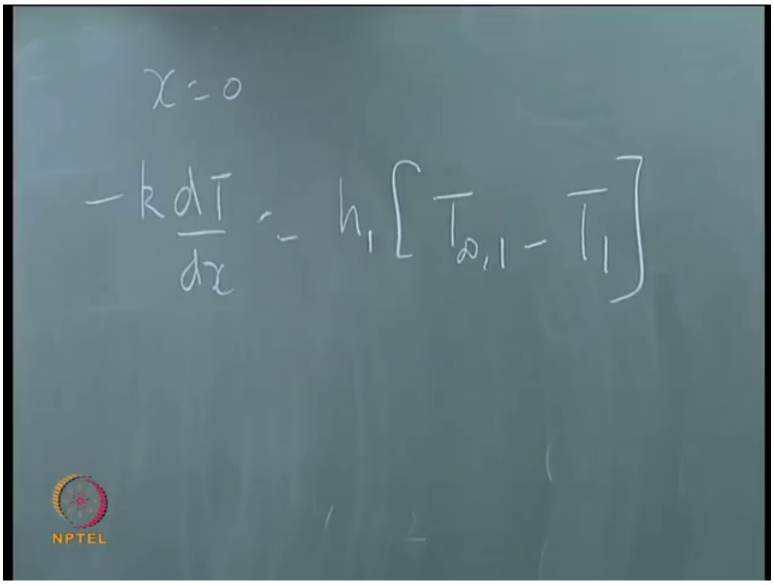

d T by d x is decreasing along x or increasing? d T by d x is positive or negative? Because we know no, the temperature should go like this therefore, d T by d x is minus it should be plus. Minus of minus is plus, right side T infinity 1 should be more than t one. Only then it is possible for heat to be transfer is not it on the second law of thermodynamics. Therefore T infinity 1 must be greater than T 1 therefore, the right side is positive. So, we are with this right now you can d t by d x therefore, minus k a

a is minus. There is a solution to the problem. That is what we propose, we hope it is all right. (( )) Where is plus T infinity 1. Why the T infinity comes here? So, we can write it as T infinity 1 minus yeah please tell me what is it in the bracket (( )) it is a correct x plus k by h 1. What are the units of k by h 1? They are in meter? What per meter by then it is correct [FL].

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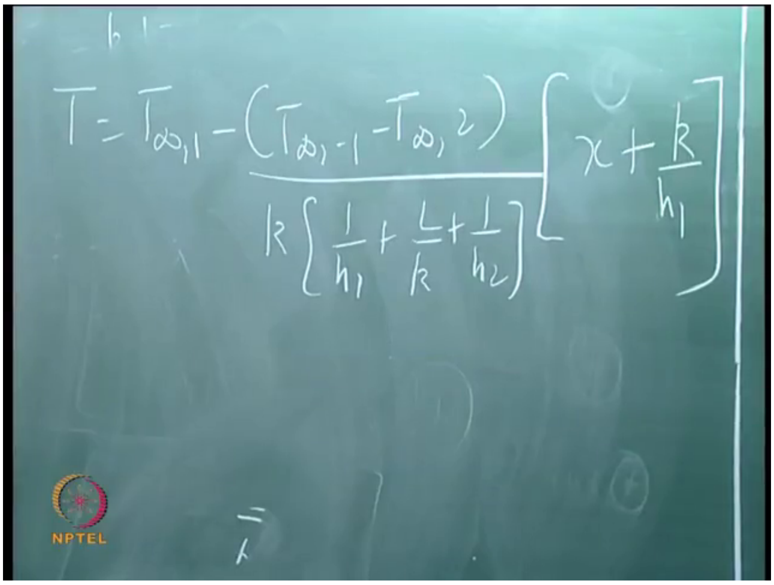

The expression may be complicated in involve, but is it linear in x? Yes of course, it is linear then x therefore, the temperature distribution is linear. If the temperature is distribution is linear d t by d x will be a constant. q is equal to k d t by d x and k is a constant thermal conductivity. So, linear temperature distribution will give rise to a constant heat flux. A contradicting variation in temperature will lead to a linearly varying heat flux. A constant heat flux does not always assure a constant heat transfer rate, if the area itself is changing for example, a cylinder or a sphere.

So, d t by d r may be constant, but a may come a may be two pie r d r then we are in trouble. So, you have to be careful. So, you should not work with heat fluxes always particularly for cylinder or sphere problem. You can work with flux for those problems where the area is a constant like a plane wall now. So, this is the now, I told you I just took a plain wall now, we have been working it is eighteen minuets, we have been working for the last thirty two minutes. So, you thought that it is a very simple problem, but this is the general case h 1 and h 2 being infinity are the most specific cases now, we can substitute if.

Not nine, (( )) fifteen. So, these are also asymptotically, correct because on this will be actually T 1 this is a linear temperature profile. People working in heat transfer we call it as a l t p linear temperature.

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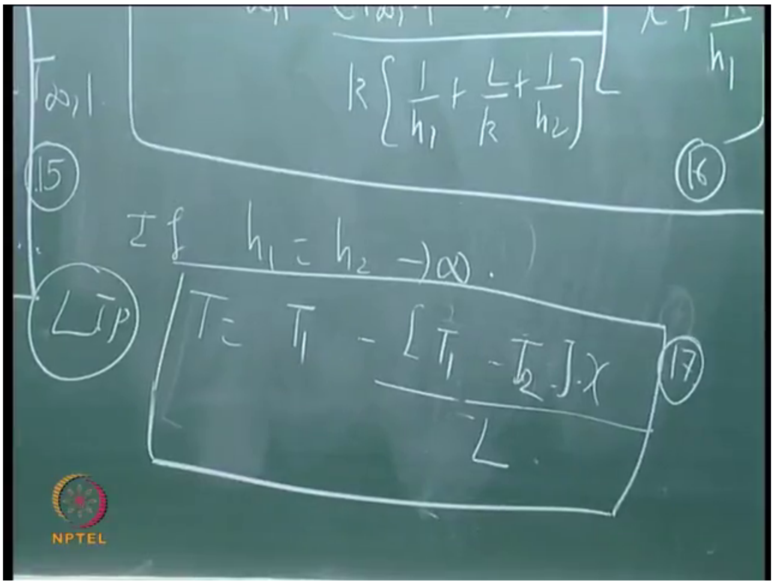

Profile at x equal to zero, t equal to T 1 at x equal to l t equal to T 2 for this special case right q is equal to k delta t by L. Now what was your q? Got this right? Correct? Now.

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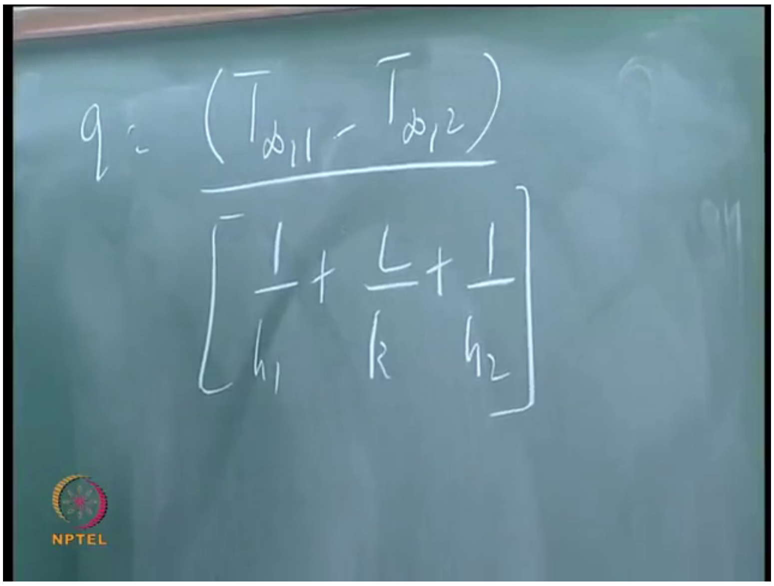

Let us look at the physical interpretation of this. Or let us see there is another way of getting the heat flux, which we got by solving the original governing equation. So, I will take the same problem of course, T 1 and T 2 I do not know after all it is a same q therefore, fine. I will use the same thing now. This was also be equal to what is this? Componendo dividendo rule therefore, quick and smart way of solving it, but it is not always possible to do it. There should be no heat generation and q must be constant and all. But if you are not using this method, you can straight away get the heat transfer rate. But to be want to find out, the interfacial temperatures or temperature anywhere, what you have to do is using this you will get q.

And this q will be equal to T infinity 1, minus T 1 divided by 1 by h 1 you will get T 1. Then from that you will get T 1 similarly, you can use this formula to get T 2 and you will have to proceed.

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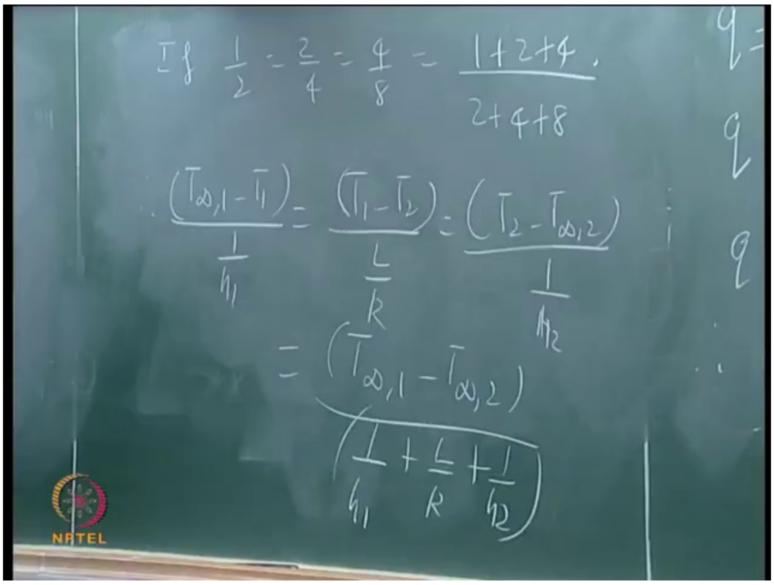

So, what it essentially means, is I have a temperature difference, global temperature difference T infinity 1 minus T infinity 2 between the two fluids. There are some resistances to the flow of the heat. The resistance is arise as the consequence of the finiteness of the heat transfer coefficient on the outside. The finiteness of the heat transfer coefficient on the inside. And the finiteness of the thermal conductivity of the solid.

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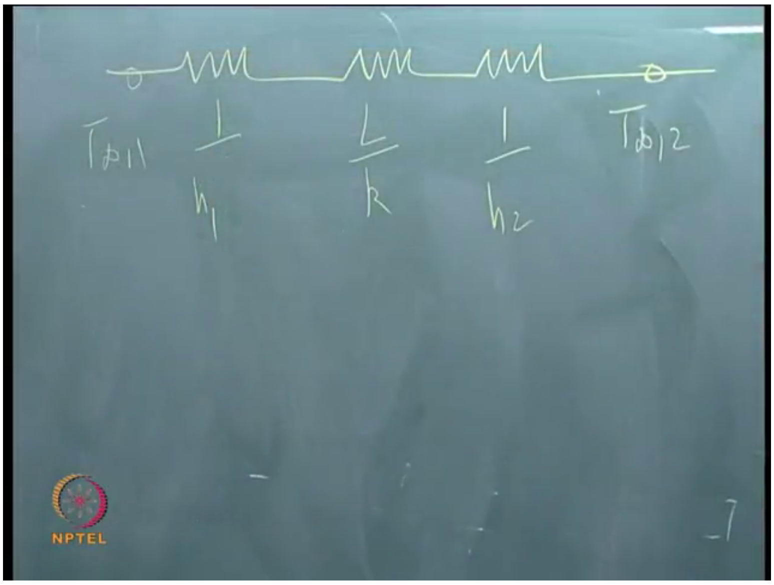

So, 1 by h 1 can be considered to be the convective resistance on the outside. 1 by h T 2 is a convective resistance on the inside and l b k is a conduction resistance. If h 1 and h 2 are both equal to infinity, then the 2 convective resistances will evaporate. Straight away you will get a problem, you will say q equal to k delta t by L. So, this is what is called as the actually, there is a. what is a? It is a area of cross section, the direction perpendicular to the plain of the board. So, this is called the convective resistance.

So, the governing equation would be, what is this? So, it is one d what? So, this fellow zero because its steady state, this fellow also zero no heat generated.

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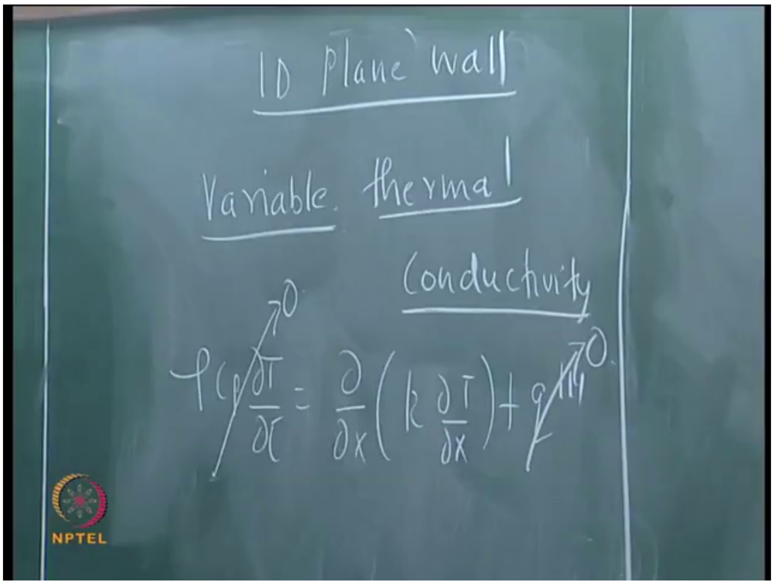

And since, it is only d T is a function only of x, I do not have to put dou. So, we will again start some numbers. So, it is a wall like this. So, it is an infinite in extent T 1 T 2, but unfortunately k is a function of x. Far should be k a function of temperature itself, which makes it a non-linear problem. We will consider both the cases k could be the function of x; that means, the material is an isotropic or homogenous or whatever. Now, k is a function of t it is a non linier thermal conductivity. That is see you are trying to get the temperature the temperature is dependent on the thermal conductivity. But the stupid thermal conductivity is itself is dependent on the temperature.

So, that goes into a spin. So, that is why it is a non-linear. Now this is so, this is the governing equation how do we proceed?

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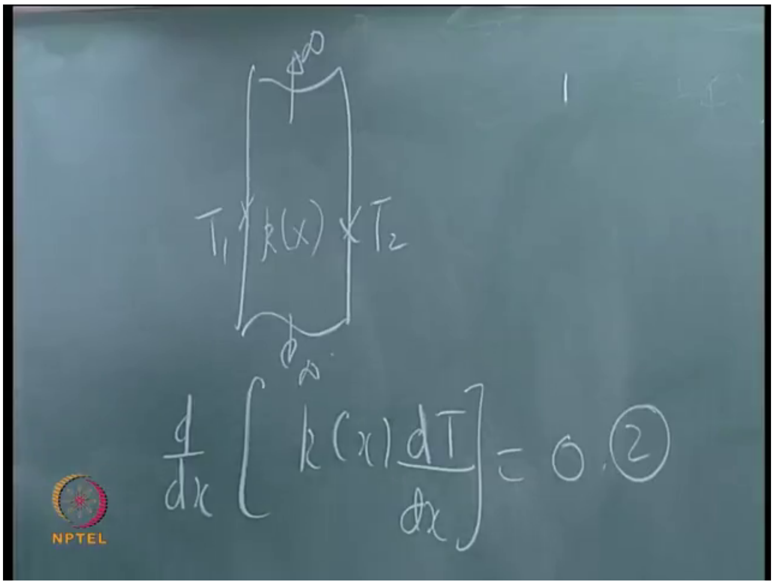

k x d t by d x what did they do? Integrated ones. Now, what we do now? What is a after all? Minus of the q lets keep it like this now, let us not get into the argument now, we just proceed with derivation. But you should remember that what can I do now? watch right.

Student: (( ))

k x should come on the right correct. No if you do this, you will mess up. We should not do like this what do now? d t is a d x by k x it is more like it right now, I start from t to any t. So, I start from zero to x You integrate between T 1 to T 2. So, you integrate zero to L straight way to get a. what is this?

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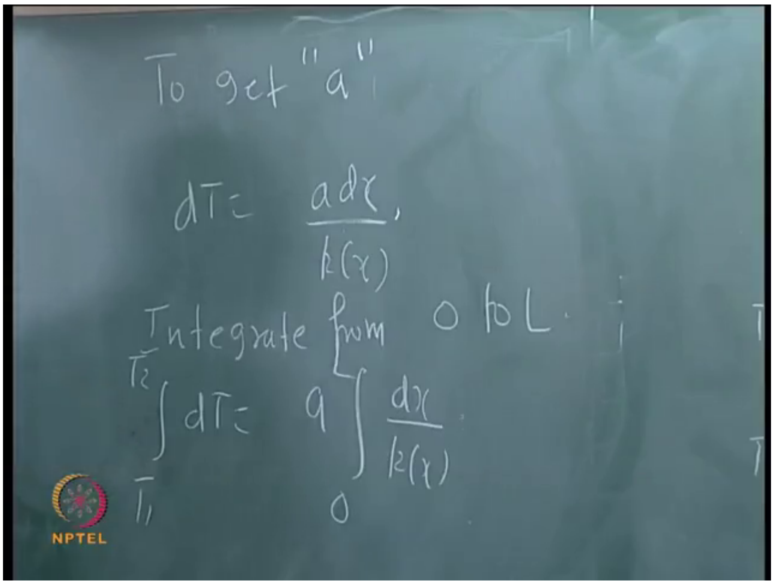

T 2 minus T 1 therefore, a is right zero to L.

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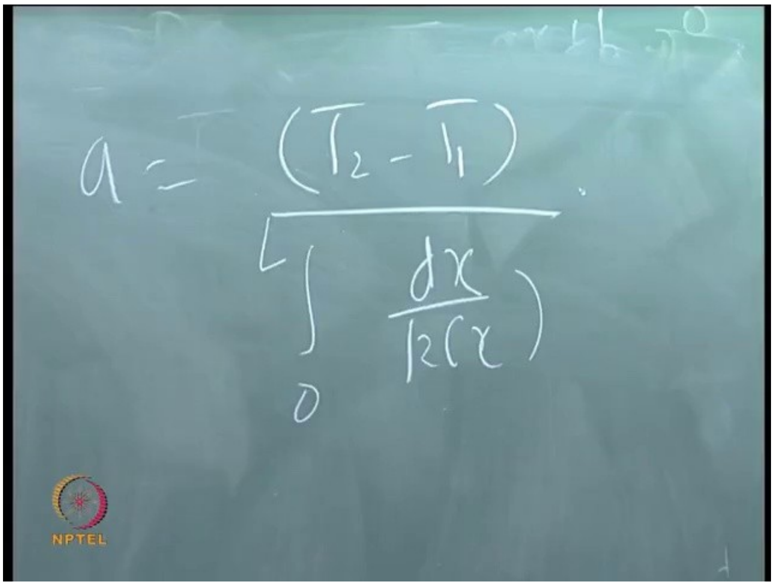

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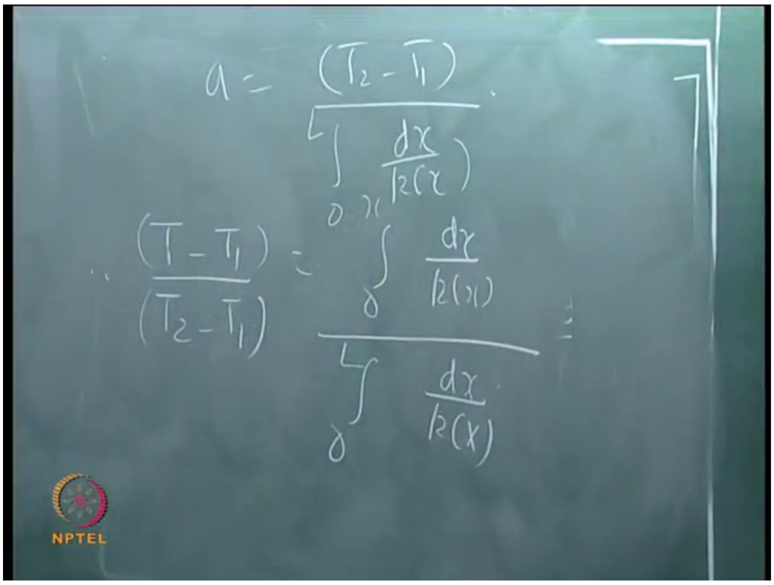

Again each and every stage you should be able to figure out for yourself whether, you are proceeding along the right track. A good way to check whether, you are proceeding along the right track is, to look at what this expression will become if k of x is constant. If k of x is constant, it can be pulled out both the nominator and denominator right side will be just x by L. Which is the linear temperature distribution model, for the constant thermal conductivity case therefore, it is correct what you have done is correct.

Suppose, I tell you somebody gives you k x equal to a plus b x. There a and b are given, it can be straight away integrated, you can get the temperature distribution. a minus of a is anyway minus of a here, is anyway the heat transfer rate. So, we will stop here, and tomorrows class we will look at the case k as function of t. We will try to solve some numerical examples and then go to the heat generation constant. How to take away the heat generation is a topical because, this heat generation takes place in nuclear reactors, how is the heat taken away by the coolant. So, I stop here.