Conduction and Radiation
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Lecture No. # 36

Fin Heat transfer- I

So, today, we look at topic namely fin heat transfer or extended surface heat transfer, which has got lot of the applications. If you basically look at heat transfer- the study of heat transfer - essentially heat transfer is studied for three objectives; as far as I can see, there could be only three possible objectives for studying heat transfer.

The first is, you study heat transfer with the view to increase a heat transfer in certain situations; a - that is to increase heat transfer, b - to decrease heat transfer, that you want to do some insulation design, and c - could be temperature control, that is, certain equipment will work only at certain temperature. So, the idea of studying heat transfer is to achieve one of the three objectives; you augment or increase heat transfer, you decrease heat transfer and third will be to do temperature control.

But if you want to do any of these three, first you must know what is the governing equation to the problem; solve it and prove that the solution is correct. And then, start looking at parametric study or the alternative route is to do experiment; measure that temperatures and find out whether it is under control or build a prototype, and find out whether you are able to augment the heat transfer, whether you make 1 is to 20 or 1 is to 30 scaled down reactor.

Find out whether the heat of the fission, it is possible for the sodium to carry away the heat. So, you will do scaled down experiments or you do experiments on water. And then, from water, you know the Prandtl number of water, you know the thermal conductivity of water, you know the Prandtl number of sodium, and this thing using your Fundaes, Reynolds number, Prandtl number, (( )) number, Nusselt number, you see whether you can map. So, if this is the heat transfer coefficient, if this is the heat transfer rate I am getting with water, for scale down reactor, what will be the heat transfer? I will get for the full reactor with sodium something like that you, right.

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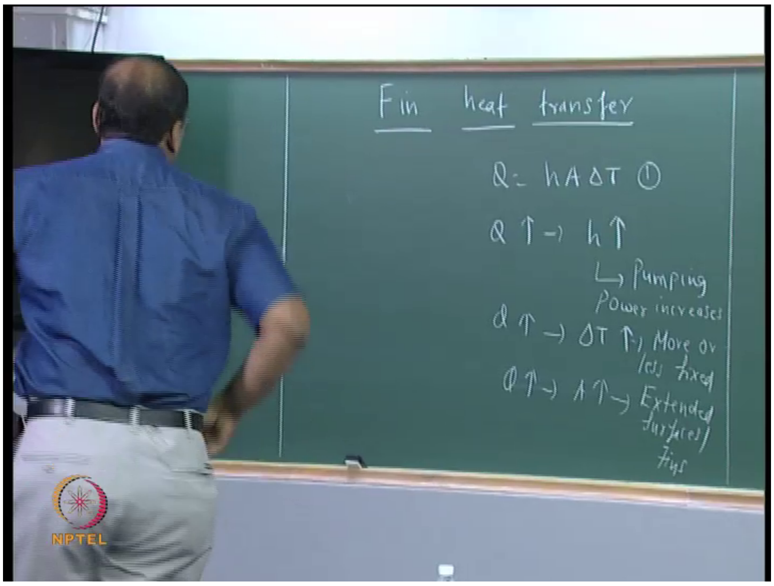

So, in the context of these three objectives, now if you want to increase heat transfer, basically in you are talking about the increasing convective heat transfer. So, the governing equation is basically the great equations that Newton's lawfully, Q is equal to…

So, if you will look at electronic equipment, the number of transistors per unit centimeter square is keeping on increasing. So, when the number of transistors is increasing, i squared r losses keep on increasing; therefore, the amount of heat which you have to dissipate keeps on increasing, but at the same time, the size of the equipment also is continuously decreasing.

People want to pack more and more with in less and less volume; therefore, there is considerable challenge in cooling these electronic devices. Therefore, there is no choice but to augment the heat transfer. Now, if you want to augment the heat transfer, there are three ways of doing this. So, Q will go up, if h goes up correct. From this equation, this is to indicate increases. So, Q increases, when h increases, but what is the difficulty in doing this? First of all h cannot definitely increase; h increases means the pumping power will also increase.

Now, Q increases, when delta T increases; are you getting the point from here? So, can you allow the computer to work at 200 degree centigrade? Not possible. In many of the situations, the delta T is fixed; delta T is not fixed, the T max is fixed; the T infinity may be changing with seasons during day time night time.

But delta T, little bit you can play; you are not completely free to change it, more or less fixed; therefore, most of the times the only option is to Q increases, when A increases; that means, you are augmenting the heat transfer by increasing the heat transfer area; so, use extended surfaces. So, the goal is to increase the heat transfer.

However, if you use the extended surfaces, the h may marginally go down, because if you put some fin and if make air flow rate, the flow path may become little more complicated; therefore, the h may go down slightly, but h into A with fin will be much higher compared to h into A without fin. So, the delta T remains fixed.

The Q with fin will in general be greater than Q without fin. If this Q with fin is less than Q without fin, there is no point in adding fin; therefore, you do not add a plastic fin or a cork fin or a bagley fin, what is the use? You will add a fin with the material of high thermal conductivity, so that there is no additional conduction resistance which is generated. You put some additional surface and it generates conduction resistance, what is the use? So, let us look at one such extended surface or fin, this is pulled out of a desktop computer; we just did it 5 minutes back.

So, this is the fan, the fan is to increase the h little bit, whatever limit is possible. So, this is this dedicated fan, this fan and the heat sink sit on the processor; there is one more fan which is on the backside of the tower, which is the general fan which is cooling all the parts; this is dedicated for the CPU, because CPU is critical, CPU is the heart of the machine.

Now, if you see this, what sort of fins are the material? Aluminum. Aluminum has got a terrific thermal conductive 205 per Watt per meter per kelvin. What process do you think they would have used to do this? What manufacturing process? They are all extruded components

Now, this is a constant area; all the fins are of constant cross section area, yes, the rectangular fins, this is a typical heat sink. So, those people will not do details calculation at what we are going to do today, these people will say so many meters, so many Watts, so many meter square. How many meter square are required for every Watt, and then, they will use a thump rule and do it.

But now, when things become smaller, and so smaller and smaller, it becomes more tricky; for example, a laptop will use a heat pipe, and then backside, the whole surface loses heat; the whole backside of the laptop is actually the fin, that is losing h a delta T to the outside. When it is hot outside, there is a conductive heat transfer taking place by natural convectors; that is an eventual heat sink, that is where the heat is transferred outside.

So, this is a simple example. So, examples of, so this is a fin system, vertical fins standing on a horizontal base; so, there are so many examples of such fins on the roadside. When you see transformers, there is an oil which is going inside; so, to cool that oil, you have transformers, you have fins - you have got fins on electrical motors, electrical generators, you have seen. So, fins are there everywhere.

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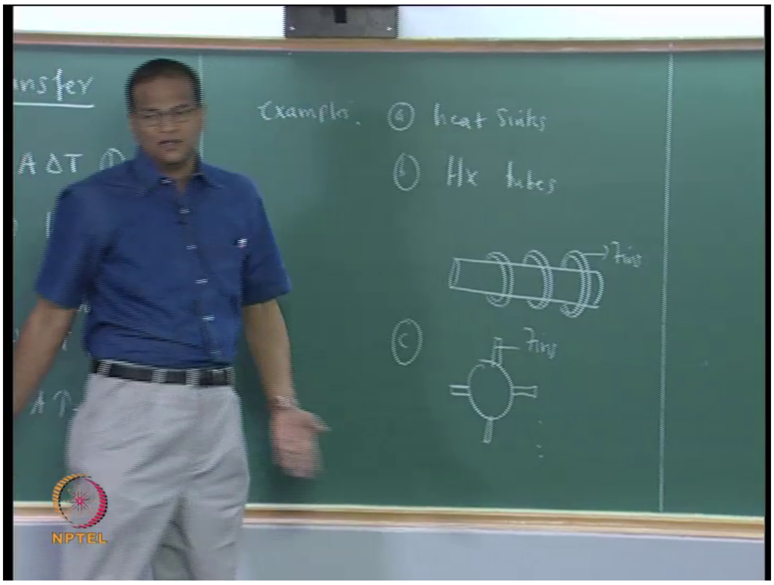

Now, we look at the heat transfer aspects of analysis of a fin. So, basically, so you can see that examples - heat exchanger tubes. So, you can have like this or you can have like this; you can have this bottle and you can have fins like this; which type of fins you will use, all that depends on you got to do an analysis, how much heat transfer augmentation you want to do on, what is your weight you can afford, extra weight you can afford and so on. There are so many things, these fins can also be used in an outer space.

In a space craft, if there is an electronic equipment in the space craft which is generating heat and you have to maintain it at a temperature, then you can still use heat sink, but the heat dissipation will not be by convection to the outside, it will be by radiation. The normal heat sink, both mode of heat transfer are available - convection and radiation.

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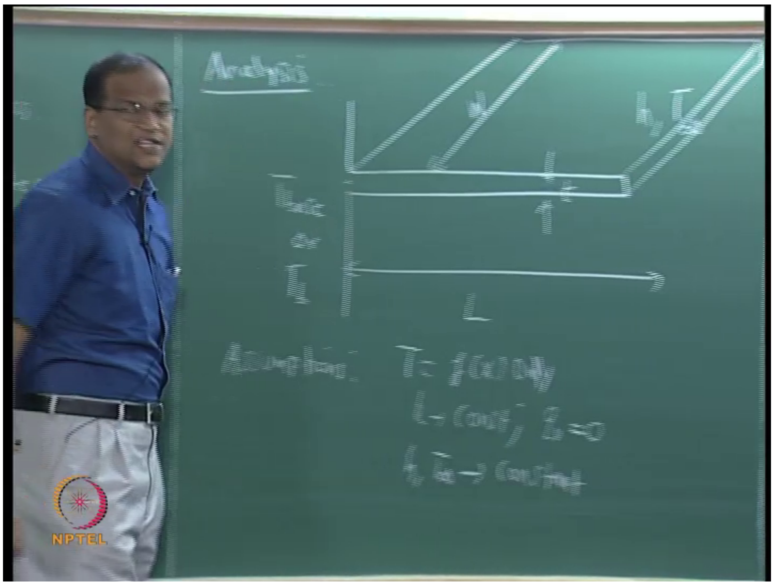

Now, let us take a simple fin. So, I am drawing something like this; I am taking one fin, right; I am taking one fin, and as far as this fin is concerned, this depth of the fin is much larger compared to the length of the fin, and then, the thickness is very small; therefore, temperature is the function only along the length of the fin.

I do not worry about temperature in the vertical direction, for temperature in the z direction. So, it is a one-dimensional steady state problem, where there is no heat generation; there is no heat generation within the fin. There is heat generation outside, but it gets on to the base, that heat is dissipated. So, I can say that, this is t, length is L. Now, the heat transfer coefficient is h and T infinity; the heat transfer coefficient can be because of, heat transfer can be because of natural convection or force convection; in this case, the heat sink, it will be because of forced convection.

So, the base temperature, that is a design variable for the CPU. You may want to restrict to 60 or 65 degree. Now, electronic equipment, 75, 80, you can go above 80, the reliability of the equipment's goes down. So, T base assuming that it is known, T infinity is known, h is known, so the width of the fin is W, the direction perpendicular to the plane of the board. Assumptions - T is a function of only, k is a constant, q v equal to 0, h and T infinity are constant; k is a constant is all right, q is a constant is all right, h is a constant ,T infinity is a constant, T is of function of x only, these three assumptions are questionable; T is a function of x only, h is a constant, T infinity is a constant, these are questionable assumptions, but let them remain questionable assumptions, but if he complicated, we cannot analyze it further. But you should know that, when you solve an actual problem, you may consider these to be variable and use advance method for solving. For today's class, we will keep them as constant; then we will proceed with the analysis.

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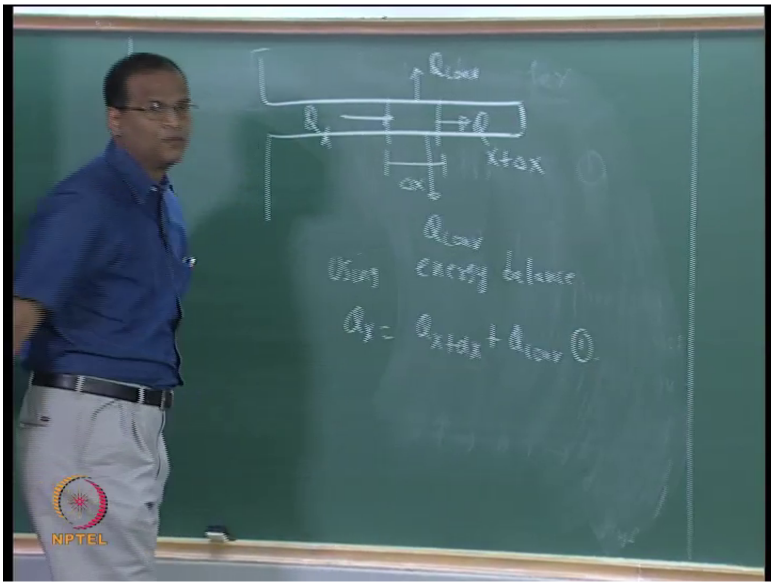

Now, we will have to take elemental; now, let us take an elemental slice; this is got a length of delta X. So, Q X is the heat presenting here, this is Q X plus delta X, and from both sides, I have Q conviction.

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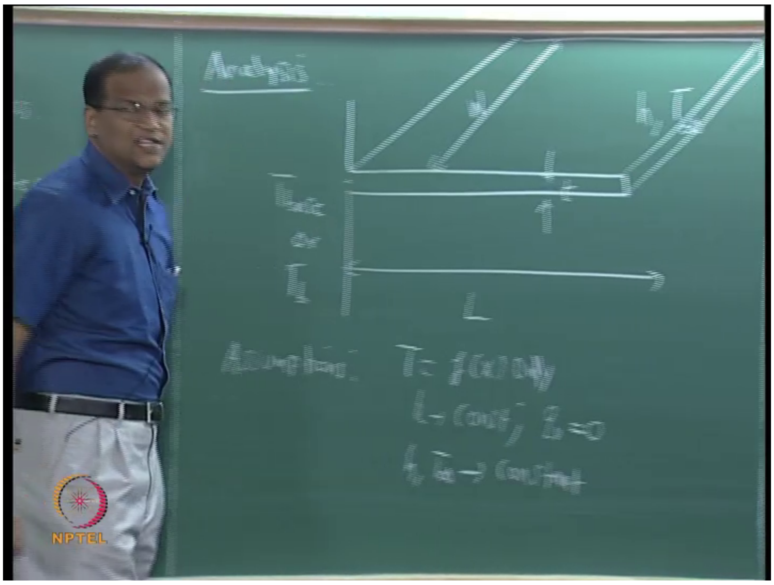

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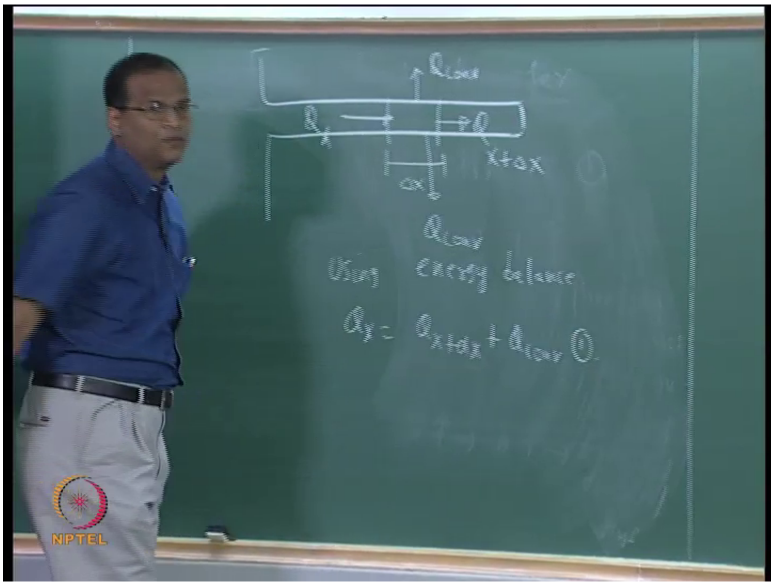

If you take any fin here from bottom side, and bottom and top, there will be heat transfer. Now, if you consider radiation, we are in trouble; you have to consider view factors and all that; let us omit radiation for the time being, it becomes more complicated if you want to. Now, using energy balance, we are not solving for an unsteady fin; steady state conditions exist using energy balance.

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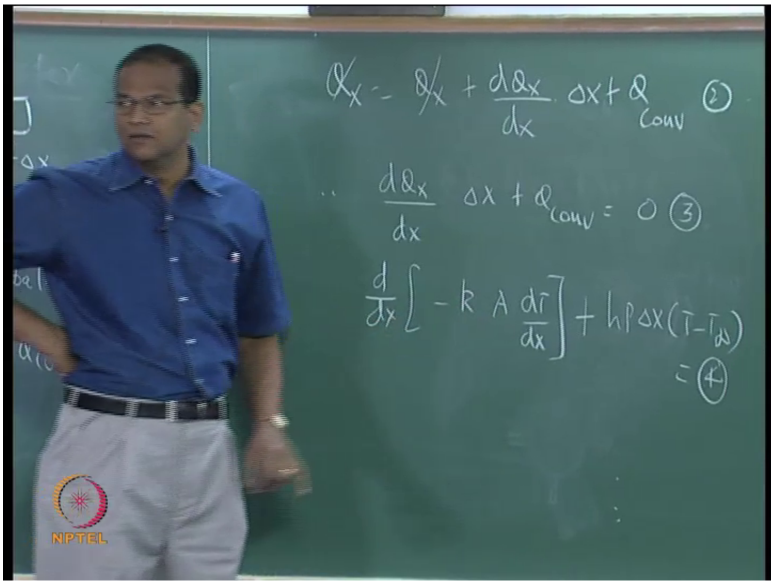

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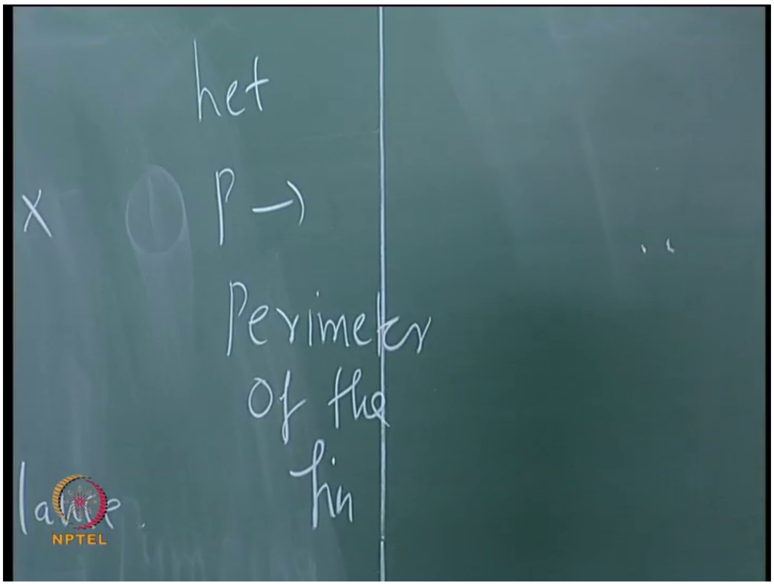

Whatever is entering must be equal to whatever is leaving; therefore… any problem? What happen ? So, let P be the perimeter of the fin; now, dQ x is, is that correct? The elemental area of the slice is P delta X; watch out A is a cross sectional area - W into T; the Fourier's law is always for the cross sectional area, right; Newton's laws of cooling is always for the surface area.

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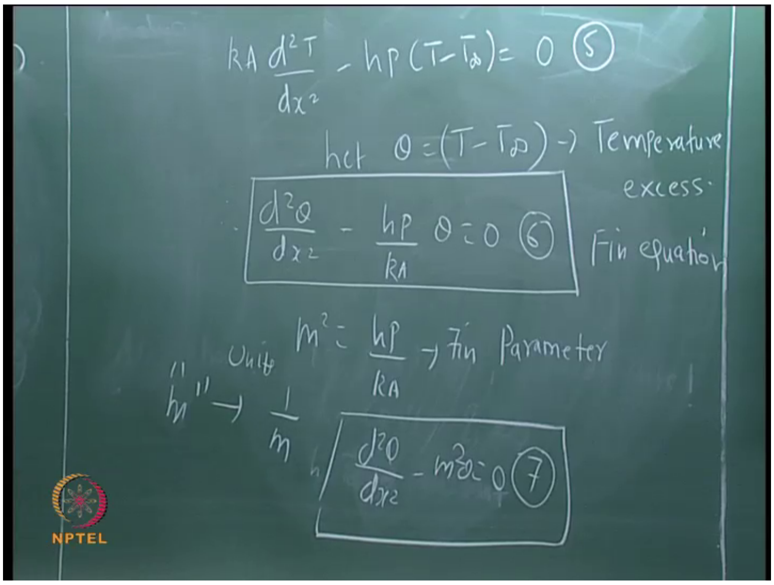

So, did we say that k is a constant ? Yes, A is a constant? May not be constant, but for our consideration, we are taking a rectangle fin of a constant cross section area, but suppose I have a fin like this or I have a fin like this, then I am in trouble, but that will lead to Bezel function - modified Bezel function - first time, second k and k naught i naught k 1, but I used to teach all these before, but now it is not required; you can do it on the computer itself.

Let us not worry about the variable area - constant area; therefore, you are looking only at constant area. So, let us we can adjust the minus, yes, it is there; now, let what is theta? Temperature excess; therefore, equation number 6 is known as the fin equation. It is an ordinarily differential equation; it is a second order equation, it is suppose two boundary conditions.

The two boundary conditions are pretty clear to us, at x equal to 0, the temperature at the base is known; at x equal to l, that is, the tip of the fin, several conditions are possible; you can have a convection condition, you can have a radiation condition, you can have a convection plus radiation condition or you can have a very simple condition, that you ignore the heat transfer of the tip and say that the fin tip is adiabatic or insulated.

But now, this hP by kA is called the fin parameter of the square of the fin parameter, because the heat transfer coefficient, perimeter, thermal conductivity and area of cross section of the fin, all of these are available before we start the analysis; therefore, you can put, you can calculate hP by kA and take the root of that, that will be m - the fin parameter - that gives you the horoscope of the fin; fin with low m, fin with medium m, fin with high m, that will give you an idea what will be the efficiency of ultimately the efficiency of the fin.

Now, I am using one more term - efficiency of the fin. I will define it at an appropriate time. So, this is kelvin per meter square, this is kelvin; therefore, m square will have 1 by meter square m will have the units of…

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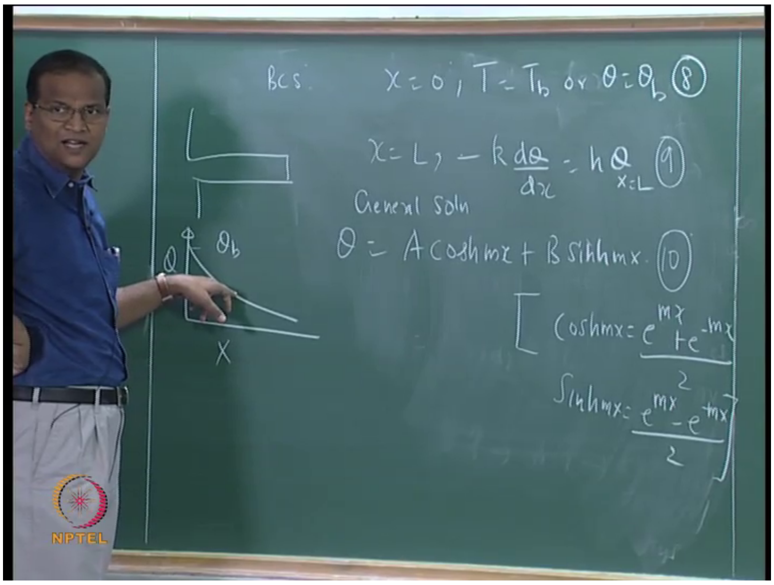

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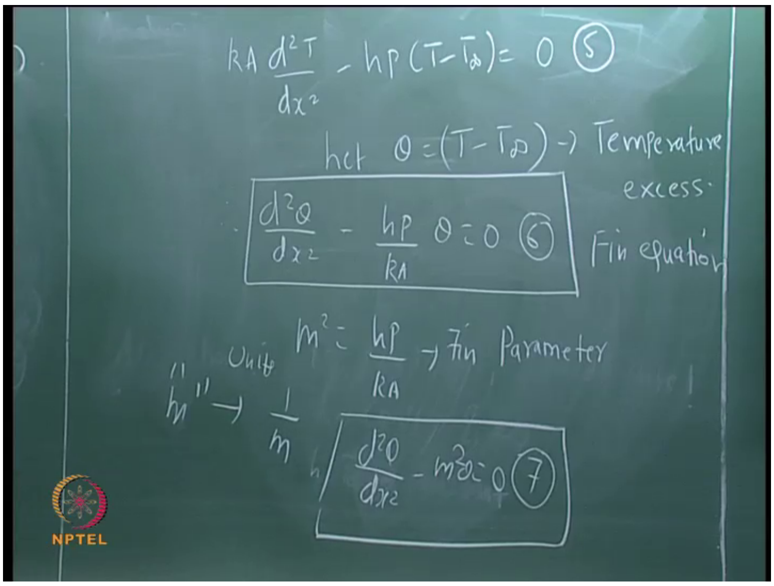

It looks like a silly thing, but this m is a fin parameter, this is the meter. So, now, boundary conditions, did I write that d square theta by? This will be equation number 9. Your fin equation given by equation number 7 along with the two boundary conditions 8 and 9, close the problem mathematically; it is not possible for us to write general solution to the problem and apply the two boundary conditions, and find out the solution; let us do it now.

So, the general solution to 7 is given by, so square… you can write the solution as a e to the power of mx plus b e to the power of minus mx, but for strategic reasons, I want to keep the cos h; you can start the solution with a e to the power of mx plus b e to the power of minus mx, and then you can work out, but the algebra is more involved for this boundary conditions.

Now, the challenge is to come out with the constants a and b, and then find out k d theta by dx at x equal to 0, find out the heat transfer from the base and what will be the maximum possible heat transfer from the fin; therefore, actual heat transfer from the fin divided by the maximum possible heat transfer gives you the efficiency. Then, what is a heat transfer with fin divided by what is heat transfer without fin, that is called effectiveness.

Once you have all these numbers, therefore, an actual fin for the heat sink, which we pulled out of the desktop computer; you can actually find out what will be the thing; I can straight away, say, have 98 percent efficiency, you cannot do any more; this is already aluminum, it is having a terrific thermal conductivity. It is so thin and then this will be more or less isothermal. So, the key point is, if the fin is more or less isothermal, you are going to get good efficiency. The challenge is to make it isothermal, do not make it 1 kilometer long.

If you make it 1 kilometer long, the tip of the fin will be at the temperature as outside; therefore, the last portions of the fin will be very poor in transferring the heat. So, it is always advantages to have more number of short fins other than small number of long fins. So, that will become clear; once we get the temperature distribution, how the temperature distribution looks like.

So, theta A cos hmx plus, but your common sense tells you that temperature should go like this, cos hmx in fact is a good candidate for this, because though it is a composite of e to the power mx plus e to the power minus mx.

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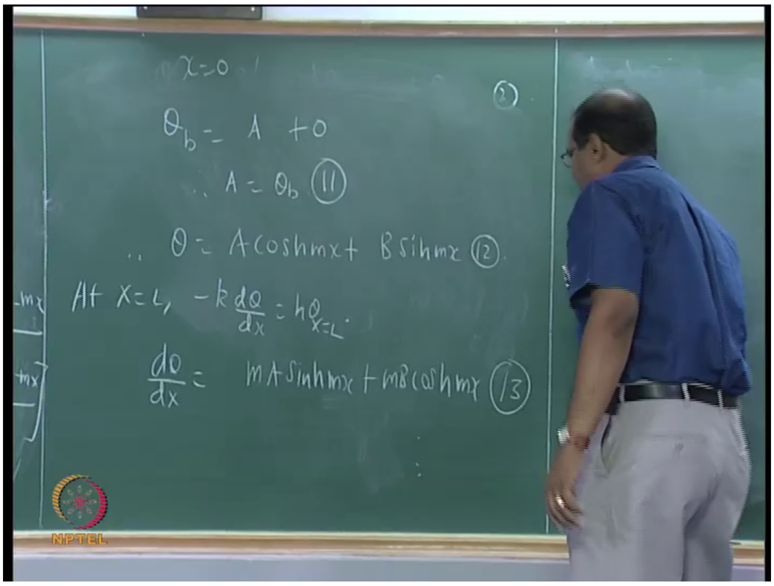

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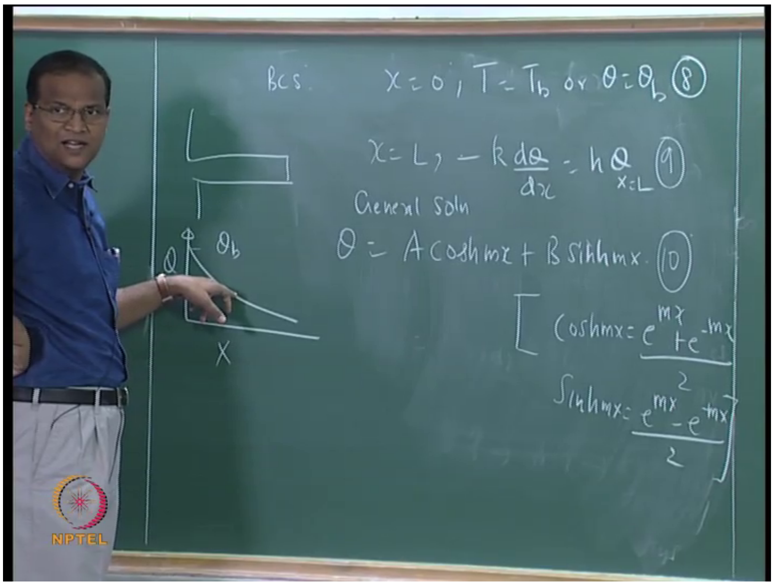

Now, x equal to 0, is it correct? sin h of 0 is 0, cos h of 0 is 1; therefore, now you have to apply the second boundary condition, is it correct? d by dx of cos h will be e to the power of m x minus e to the power of m will be there. So, it will be m sin m sin hmx. This will be m e to the power of mx plus m e to the power of minus mx divided by 2, m you take out, it will be sin h will be m cos hmx, because there is a minus sign; you will be tempted to believe that it will be minus cos hmx, it is not, it is cos hmx.

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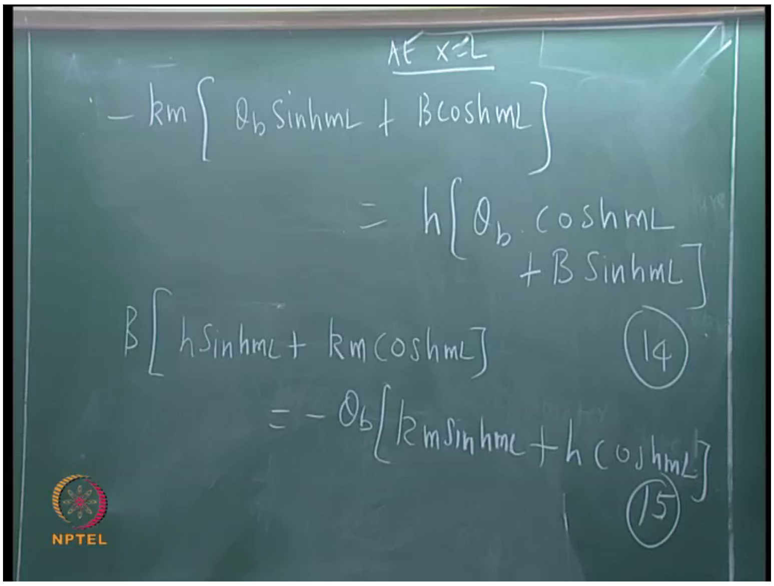

Now, therefore, why A? theta b. Therefore, minus km; now, where I am applying this? At X equal to L. What was that? Is it fine? Collect the terms with B; this km, it is theta b?

No.

Is it do you have h theta b?

No sir.

No.

km theta b?

No.

km cos hmL.

Very good.

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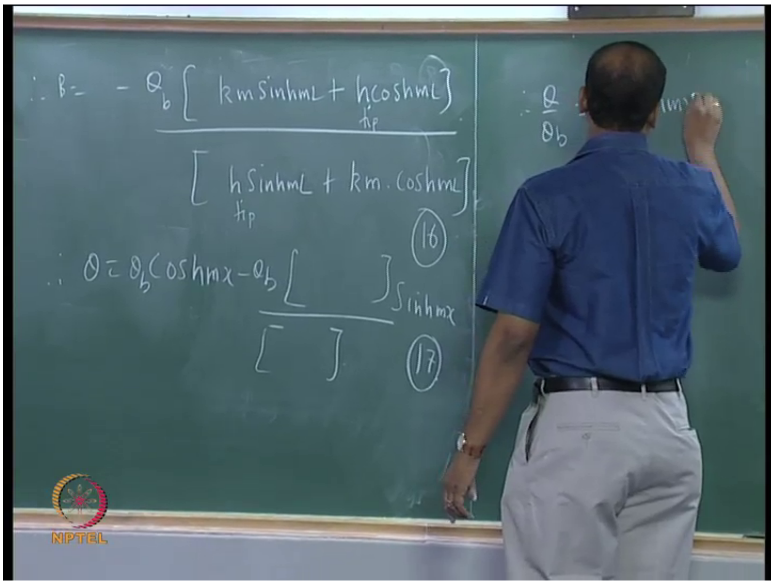

It equals both are minus now plus h; tell me, if I made any mistake. It is all right, theta b? It is should be fine; therefore, b is…

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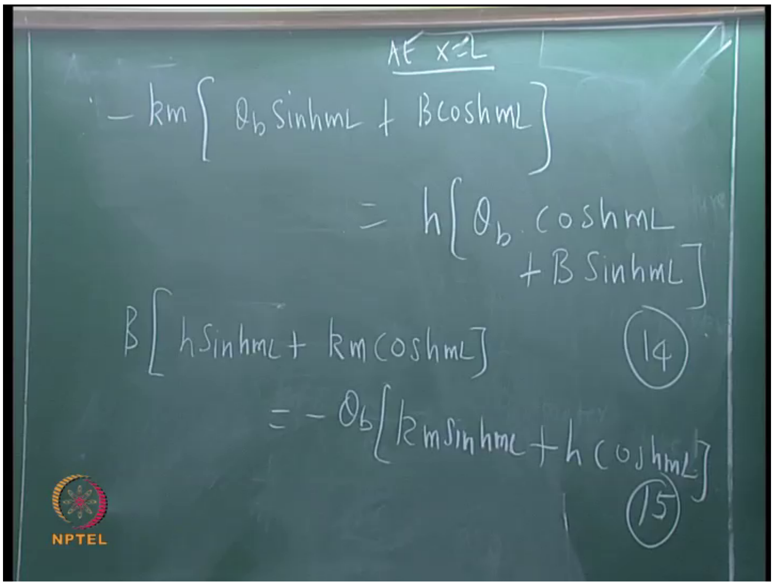

No, theta b now, we have to be careful; this is the boundary condition the tip; watch here, the boundary condition the tip minus k d theta by dx is h theta and x equal to L; there is no need that the h for the fin which came in fin parameter root hP by kA should be the same as this. So, I will make it, I will be more cautious and call this as h tip, correct.

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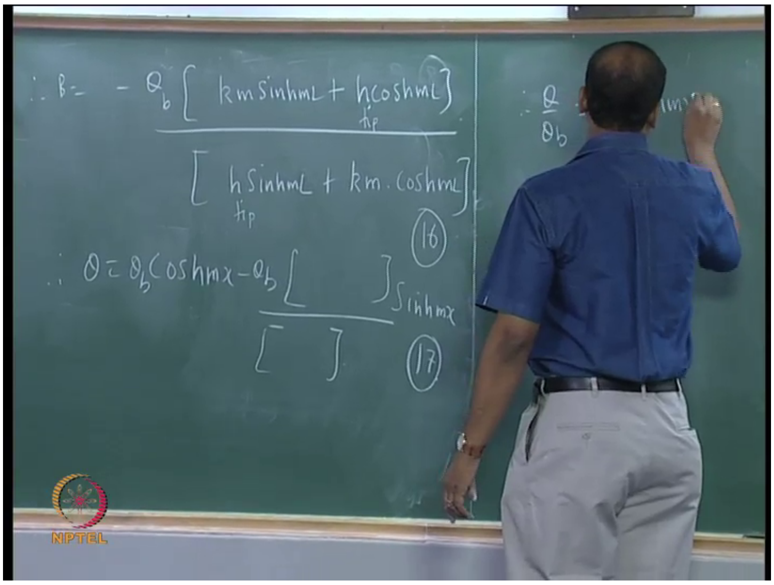

Because the tip P transfer coefficient could be different, because it may be a combined convection radiation or because this is basically along this; it is heat transfer from horizontal surface, but at the tip, it is heat transfer from a vertical surface; there could be small difference.

The tip is a vertical surface, is not it? So, the heat transfer, how you calculate a convective heat transfer from the surface which is vertical; this will become very critical, if it is only natural convection of a surface place like this and a surface place like this will give you give rise to different heat transfer coefficient.

My famous example of this; Limca bottle if placed in the fridge like this or this, which will give maximum heat transfer coefficient? Which will give maximum cooling? Why? Because of the growth of the boundary layer. If the boundary layer develops like this, the boundary layer grows only up to this high. So, the average boundary layer thickness will be much smaller will displace like this; if it is place like this, the boundary layer will develop like this.

So, the average boundary layer thickness will be more. Boundary layer thickness is the resistance to the flow of heat; thicker the boundary layer, lower the heat transfer rate; therefore, the average boundary layer thickness for vertically placed Limca bottle is more than the average boundary layer thickness in a horizontally placed Limca bottle; therefore, all these fellows bakery fellows will put there.

That is also for compacting; they want the porosity to be less, that is one more thing, but this will cool better, like that, no, because Deepak did not like; when I say horizontal this, now you understand. When you are blowing air and all that, if it is force convection, it would not matter much. But still vertical plate, horizontal plate, different correlations are available. So, in order to be clinically correct, we will have this. Now, 16, these obtain; therefore, Abhinandan, what is the problem? Today, you miss anything.

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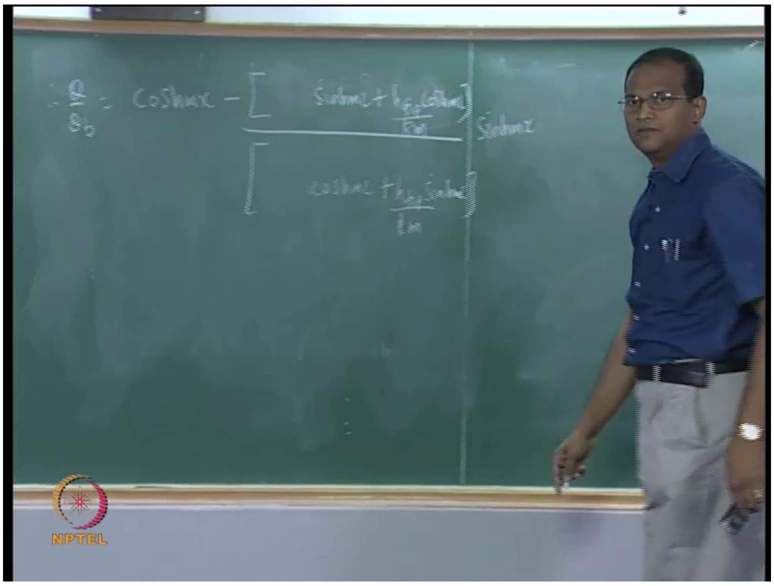

Therefore, is it correct? Thank god, theta b is on both the terms. So, I can bring theta b to the left hand side; therefore, let us cross multiply.

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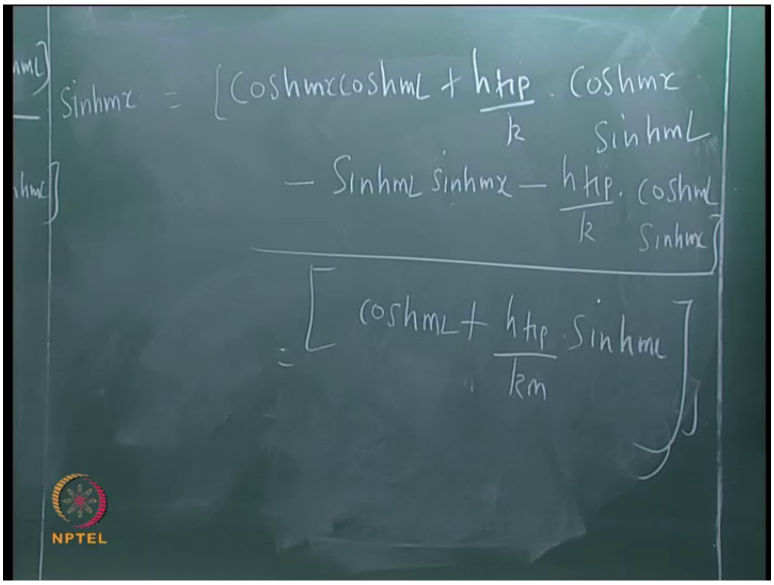

I can take the… of course, without that, I am trouble. Now, any objection with find? I remove the km from both the numerator and the denominator. Now, I can cross multiply.

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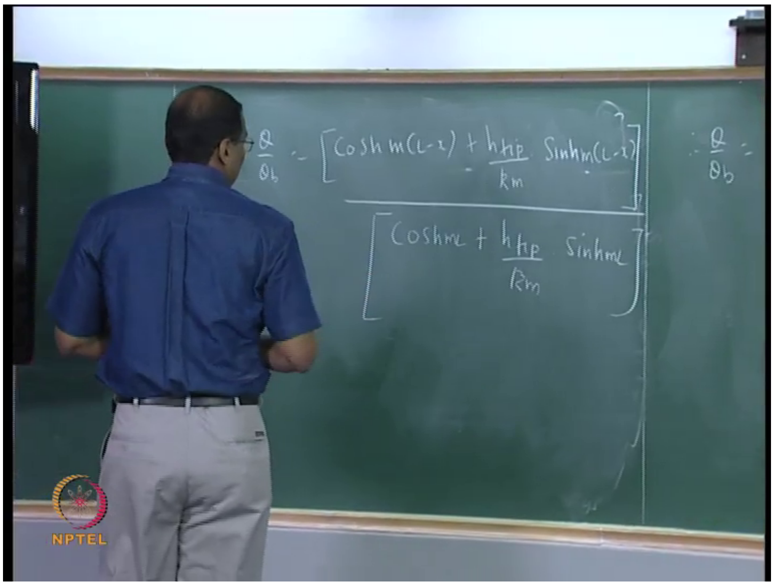

Basically you have to collect the term h tip by k and the terms without that, what is cos h a cos h b minus sin h a sin h b? cos h of a minus b, correct; cos h a cos h b minus sin h a sin, you can do it, take it out; therefore, cos hmx sin hmL minus cos hmL sin hmx is sin h of m into L minus x.

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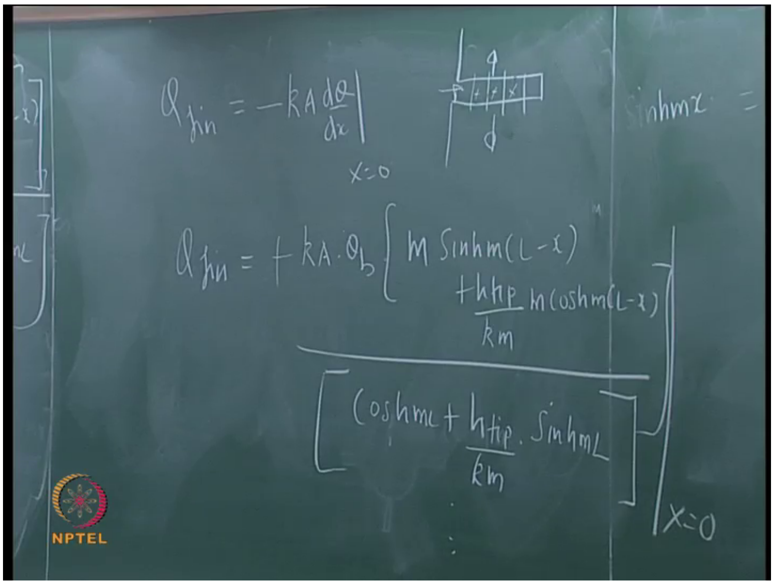

This material must be new to you; no undergraduate text book on heat transfer has given this. They will always consider the case of an adiabatic fin tip, and then, they will use a correction and all, because it is not really complicated, we can derive it. Most US books prefer not to do this; they will take this simple adiabatic fin case, fine; sin hmL cos hmL, that is what you get. Now, what is the heat transfer from the fin? Minus kA d theta by dx

X equal to 0.

And X equals to 0; please remember, whatever heat is coming from the base, this is only convicted out; this is what we wrote mathematically as d square theta by dx square is minus m square theta equal to 0; that is the formal mathematical statement of the energy balance. Therefore, if you find out what is the heat transfer which is entering the base, that is the heat which is convicted out. Alternatively, I can find out what is the theta at every station X, and then, I can divide the whole thing into several stations, and find out hA delta d for each of these sections and numerically add up, that will also be the same. What polarav, you did not follow.

But this is analytical formula is available, we will do this; both should be equal with the numerical limits. Now, so this Q fin must be equal to minus kA theta b; now, this will be cos hmL minus x plus h minus m plus m. So, this minus, so this will be m, and because there is a minus L minus x; there will be a minus which comes again.

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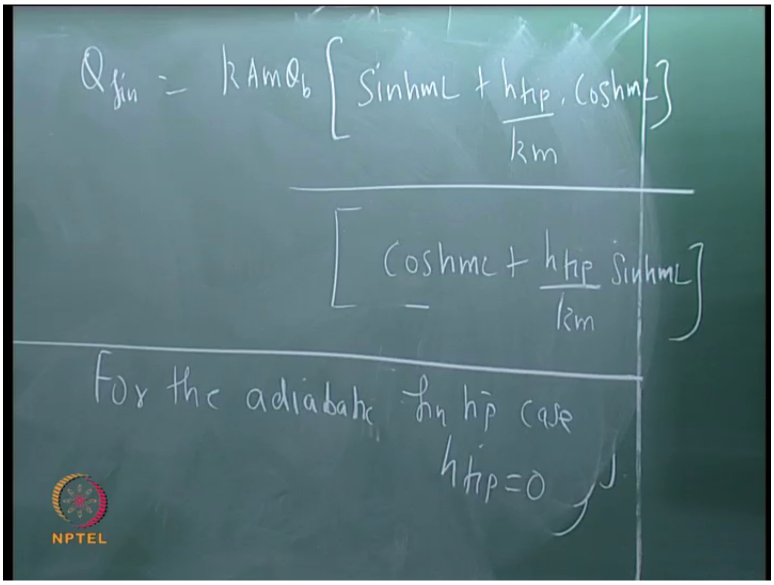

Now, this you have to substitute at X equal to 0, is that correct? What is the answer? You are getting, is it very complicated? So, Q fin kAm theta b sin hmL plus.

So, as the algebra gets tougher and almost about to consume you, I will give you some relief. For most practical cases, we can assume that the tip is adiabatic; you can substitute h tip equal to 0. So, the adiabatic fin does not mean, it will be absolute 0; in relation to the h from the other surfaces, it is negligible, you do not have to consider.

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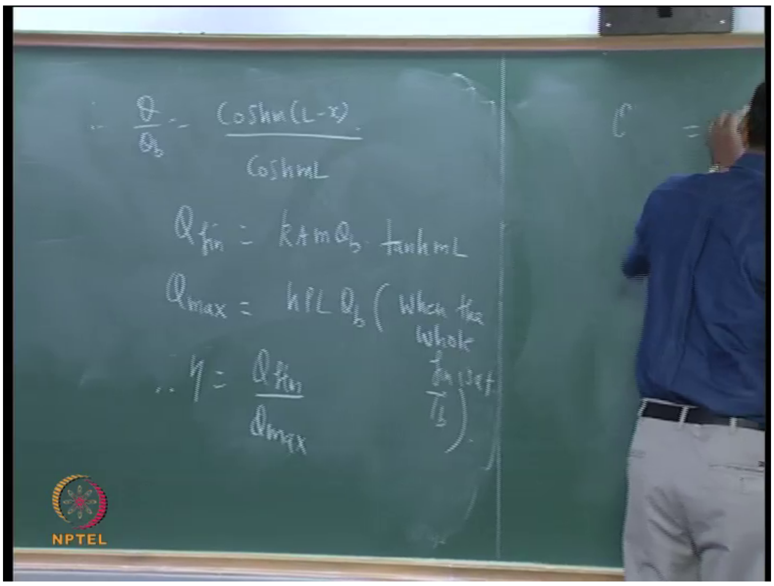

What will this become? Therefore, you have to give some numbers, is that k? What is the Q maximum from the fin? When will the maximum heat transfer take place from the fin? If the fin is isothermal and is at a temperature equal to T base theta b; therefore, the surface area corresponding to that will be perimeter into length.

So, if you use the Newton's law of cooling hA delta T h into P into L into theta b, that is the maximum, that is the denominator; numerator is Q actual from the fin. If you divide Q fin by the Q max, it will give you a performance metric like a CGPA of the fin, whether the fin is working all; maximum value is 1, minimum value is 0.

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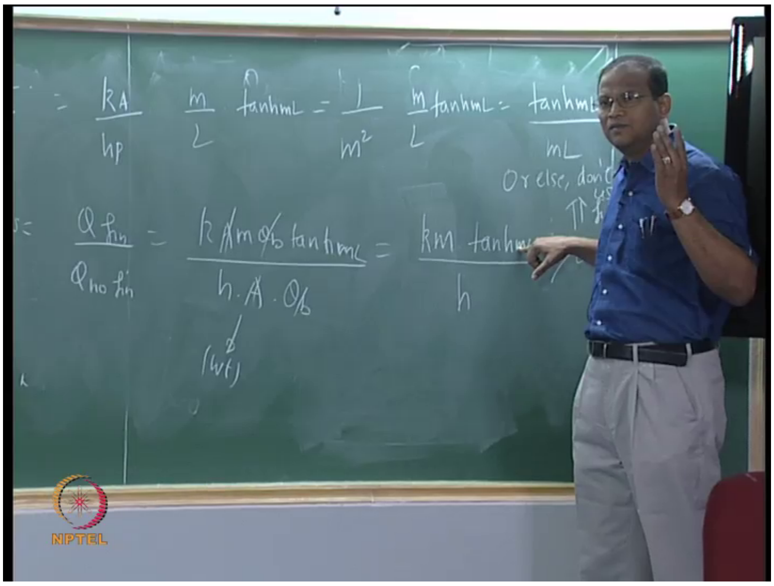

Let us do that; Q max equal to when whole fin, no, therefore, the fin efficiency, you are at, why? Is it ok? The efficiency of an adiabatic fin with a adiabatic fin tip is tan hmL by mL. Now, something called effectiveness; so, look at this, now you got surface area, that is, front side, top side, bottom side of each of these fins and so many fins are there. If you find out the efficiency and the total heat transfer from each fin, you can get the heat transfer from all the fins.

Now, if the fins are not there, this base surface will be there. The base surface will transfer heat equal to hA delta T. Now, you have to find out what is the heat transfer from the fin divided by heat transfer without the fin, and if it is not greater than 2 or something, do not add fin - do not add a plastic fin - and screw up the heat transfer.

So, this is the effectiveness. So, this is the performance metric. Now, they should be greater than 1; it is not an efficiency parameter. Now, this will be equal to denominator; please tell me the denominator,

Student: (( )) H into A.

What is A?

Student: (( ))

W t correct into

Student: Number of fins

No number of fins; only one fin; for every fin, theta b. So, this is with the this will be… a also goes. So, km and hmL divided by k is Watts per meter per kelvin, this will be per meter. So, Watts per meter squire per kelvin divided by this. So, this must be greater than 2 or else, do not use fins. You can quickly see that, what is tan h of 2; no, tan hmL tan h of 2.

Student: 0.96

0.96. tan h of 3? 0.98 or 99?

Student: 0.99.

Therefore, tan hmL is not a big parameter, once m crosses certain value; therefore, if the fin has to be effective, please look here the fin has to be made of a material whose thermal conductivity is high.

The fin should necessarily be having a metal would which has a reasonable value of m, but length should not be very large; it should have and there should be low heat transfer coefficient for you to justify. If there is a surface already with a high heat transfer coefficient, you are very energetic, you put a lot of fins with the poor thermal conductivity, you are blocking up the whole thing. You should know where to put the fin; the fin should always be put on a surface on where the heat transfer coefficient is lower. If there is a tube water is flowing inside, air is flowing outside, you do not do an optimization and put the fin on the inside; already terrific heat transfer coefficient available.

You should know where to put your money as I say; so, the fin should be in the outside. The fin should always be on that side where the heat transfer coefficient is weak, because that is the controlling resistance for the overall heat transfer.