Conduction and Radiation

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Lecture No. # 37

Fin heat transfer- II

So, we will continue with our discussion on fin heat transfer. In the last class, we derived the fin equation and we also saw solutions, analytical solution to the one-dimensional fin for constant area, constant cross section area. I told you, that if you have variable cross section area, it will lead to Bezel function and another type of functions, like trigonometric functions, hyperbolic function and so on, with lot more involved. But nowadays, all these fin problems can also be very, can also be easily solved using Mat lab and other things, I mean (( )) and so on.

But we are looking at analytical solution basically, because they are elegant and you can get a quick back of the envelope calculations, how many number of fins are required and all these. So, for fine tuning the results, we can use sophisticated software.

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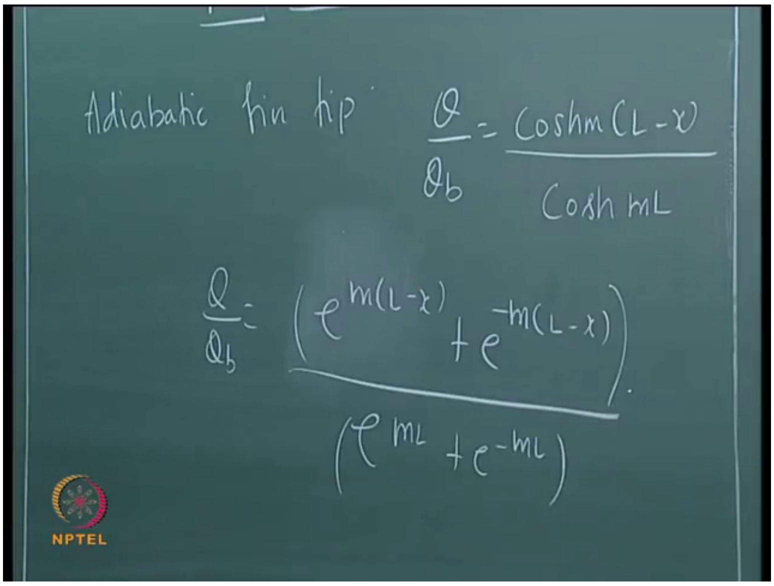

to 1:24. If you look at the right, we derived this temperature distribution in the last class.

So, correct. So, this expansion for cos hm, cos hm L minus x is the numerator and cos hm L is the denominator.

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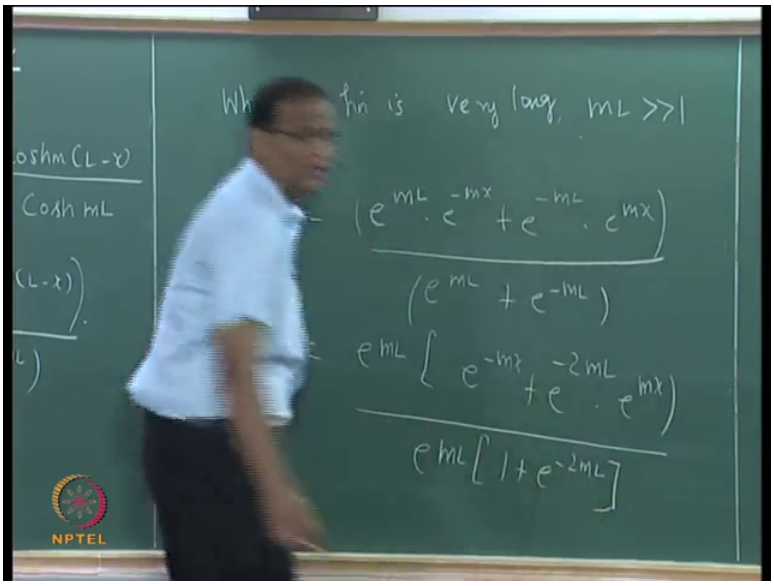

So, when the fin is very long, if we have a, if we have a case of a fin where the length is very large, then we will have, for the case of a very long fin, m L is much greater than 1. Therefore…

Student: to 03:28

Is it? So, we will call this as a long fin. So, what we are doing is, to derive the expression for the temperature distribution, as well as for the fin efficiency for the long fin, you just first consider it to be an adiabatic fin tip and start working on it. Now, it is ok for me, so I will…

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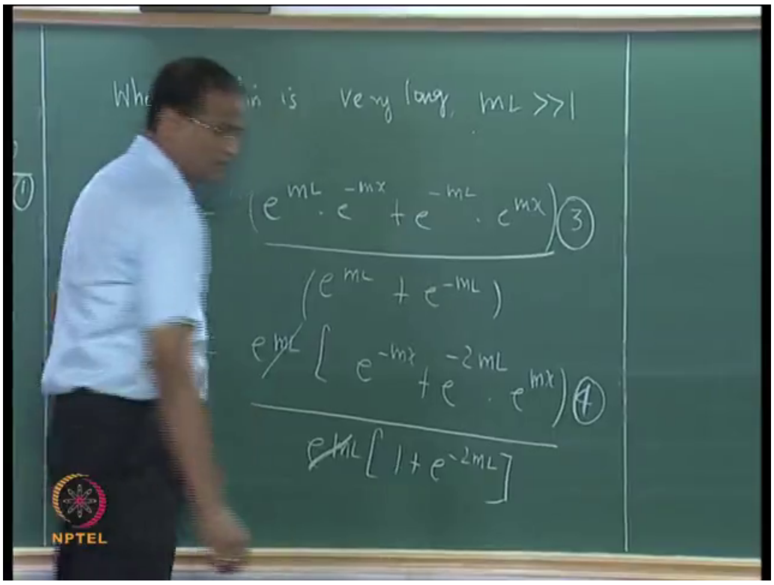

This is 1; 2. What happens when e ML is much, much greater than 1? What happens to e to the power of minus 2ML?

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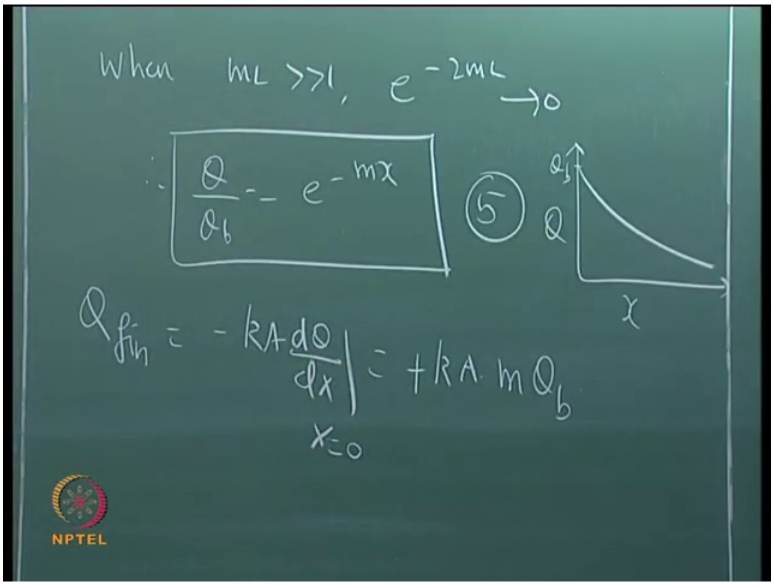

to 4:42 When ML, theta b by, theta by theta b e to the power of minus mx, it turns out to be a very simple case of exponential distribution. Even otherwise, the temperature distribution is exponential, but it has become very simple.

Now, but we will have to answer the question, how long is long? We will try to answer that question. Now, we can work out the…, agree? e to the power of minus mx, derivative is, m, minus m, e to the power of minus mx, substitute x equal to 0. So, this minus and minus will get cancelled, so we will get plus… k is units of watts per meter per Kelvin; A is meter square and m is 1 per meter. So, there is a 1 per meter in k, there is 1 per meter in m, there is a meter square in A. So, the meter square meter square get cancelled, the whole expression is in watts, correct.

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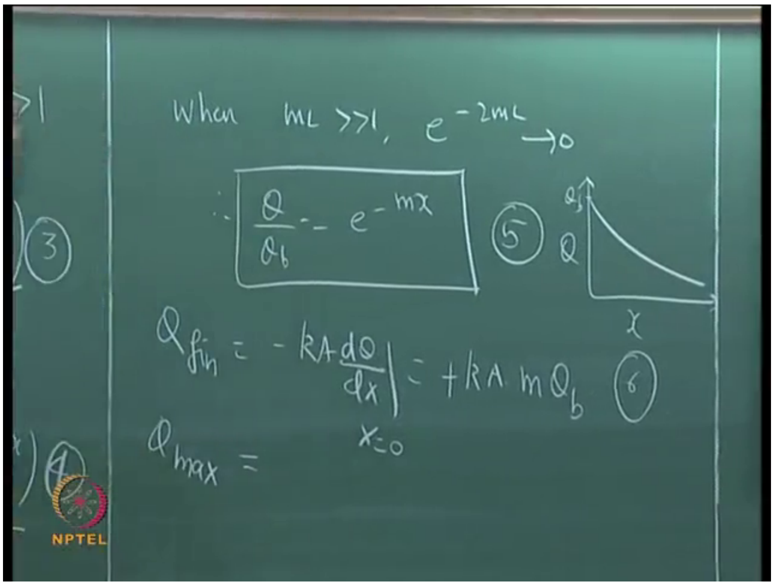

So, what is a Q max? The maximum heat transfer possible, the maximum heat transfer, which is possible from the fin occurs, happens in the whole fin, is at the base temperature. That is, t equal to t b throughout or theta equal to theta b throughout. Please remember, theta is a temperature excess t minus t infinity. So, this will this will be h PL to 7:40

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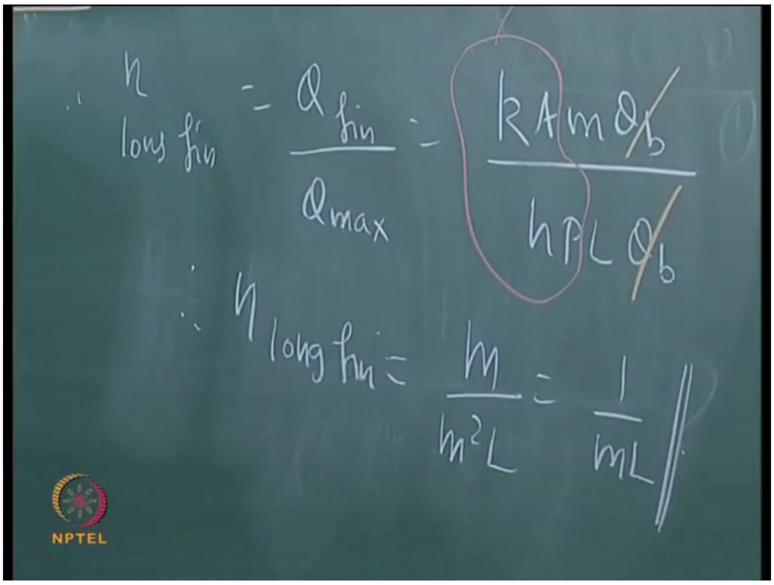

Therefore… What is this? 1 by, 1 by m square to 8:08

As expected, deficiency of the fin will decrease as the length of the fin increases because theta is 1 by mL. We should not use long fins, long fin itself is an approximation, but we should use less number of short fins instead of small number of long fins because towards the end, the temperature difference will be, the temperature difference with respect to the ambient will become lower and lower consequent upon the temperature distribution being like this.

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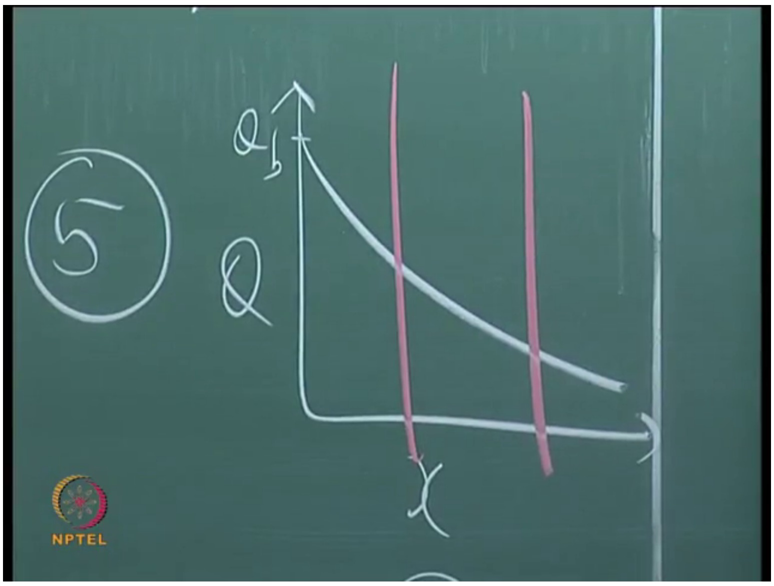

So, if we divide the fin into 3 parts, if we take a 10 centimeter long fin, divide into 3 parts: 3.3, 3.3, 3.3 centimeters, the first 3.3 centimeters will give terrific heat transfer rate because theta is very high; the second 3.3 will give ok; the last 3.3 will give very poor heat transfer rate. Keep on increasing the fin length, the last portions of the fin are totally useless because they will be at only a few degree above the ambient temperature, no point. Instead, you can cut them down and crowd the base with more fins; oh, I forgot, that may locally affect the heat transfer coefficient, but you can put some fan, something increases velocity and do something. That is what I showed you, the heat sink, with a fan very closely packed fins.

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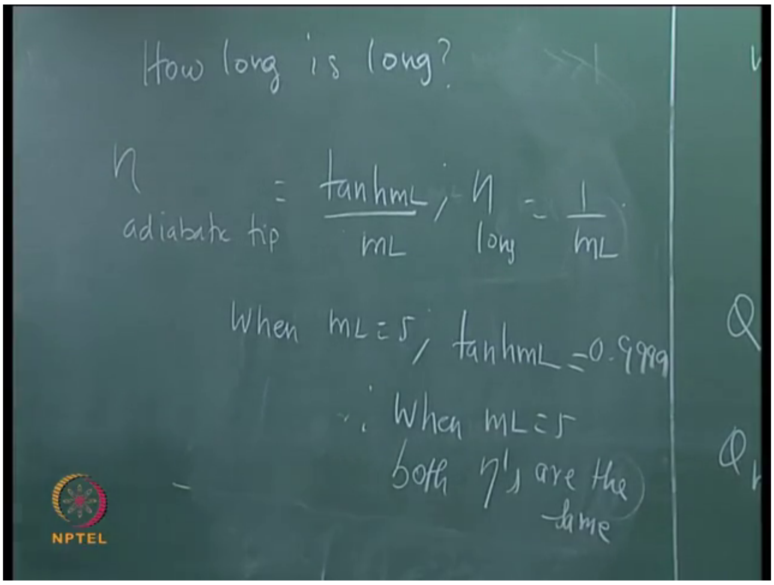

Now, how long is long, that question has to be answered

Student: to 10:25

Yeah, can you tell me, when mL equal to 5? 0.9999, right? The 1st step in technology is to calculate the m of the fin multiplied by L, get m L. The mL is 5 or above, straightaway use the long fin, release, there is no need to worry whether it will be adiabatic fin tip. It is so long, that the tip of the fin will be just few degrees above, above the ambient and the tip, tip area is also less; the area is less, the theta is also less, why would it lose so much of heat.

Let us solve a problem, problem number 44; problem number 44. A long circular aluminum rod, problem number 44, a long circular aluminum rod, k equal to 205 watt per meter per Kelvin; a long circular aluminum rod k equal to 205 watt per meter per Kelvin is attached at one end, is attached at one end to a heated wall. Long circular aluminum rod k equal to 205 watt per meter per Kelvin is attached at one end to a heated wall, full stop. The fin transfers heat by convection; the fin transfers heat by convection to a fluid, full stop. The fin transfers heat by convection to a fluid, to a cold fluid, full stop. a, if the diameter of the rod is tripled; a, if the diameter of the rod is tripled, how much will the heat transfer rate change. If the diameter of the rod is tripled, how much will the heat transfer rate change, question mark. b, if a copper rod is used, if a copper rod, within brackets, k equal to 390, 390 watt per meter per Kelvin, if a copper rod, bracket k equal to 390 watt per meter per Kelvin, close bracket, is used instead of aluminum; if a copper rod is used in the place of aluminum, then how much will the heat transfer rate change, then how much will the heat transfer rate change?

This is a simple problem, which gives you a practical aspect of what is an effect of the change in the geometry on the heat transfer performance of a fin, part a. Part b, what is a change in the material used in fins? I have already told you, a long, you have to watch, you have to look out for the buzz words, a long copper rod, a long aluminum rod, means it is a long fin and it is attached to the base, you do not have to write the governing equation for, governing energy equation for conduction and cylindrical coordinate to solve and all that, simple; simple problem.

At this point I would like to, I would like to caution you when we solve the one-dimensional problems. If you, I do not know how I expected this question from somebody, when we solve this fin equation, we already spent 1, 1 full hour, full hour trying to take a control volume, identifying what is the energy term inside this thing, moving velocity and all that and got the energy equation. And then, we started looking at slab without conduction, without heat generation, with heat generation and we applied boundary condition and solved the problems. Why would we have to derive the governing equation separately for a fin, why cannot we use governing equation, which derives the generic governing equation, nobody asks this question.

Yeah, there, the, the problem is, what we are doing is you can still solve the fin equation using the concept of general, using the general energy equation, but you have to say, that t is a function of x and y and then apply the boundary condition, apply the boundary condition separately in each of the wall. But here, we took minus k dt by dx is entering, minus k dt by is leaving and then, in other direction we are just lumping it by a convective heat transfer process directly, we are supplying the rate. Therefore, by energy balance, which used to come as boundary condition, at the boundary convection is equal to conduction that is now integrated because we are lumping.

If on the other hand, if you want to use t, if want to solve the fin problem as t is a function of x and y, then go ahead and use the original governing equation, are you getting the point? So, the general governing equation cannot, can, cannot be used to solve the one-dimensional fin equation. To derive the one-dimensional fin equation you have to do it separately because we are lumping the heat transfer across the wide direction, because we are saying, that the thickness is so small, that there is no temperature distribution in this. If there is no temperature distribution in this, but there is a convection, which is taking place, how to factor in this, h p d, h p d x into theta; we, we put it into the equation, is it ok?

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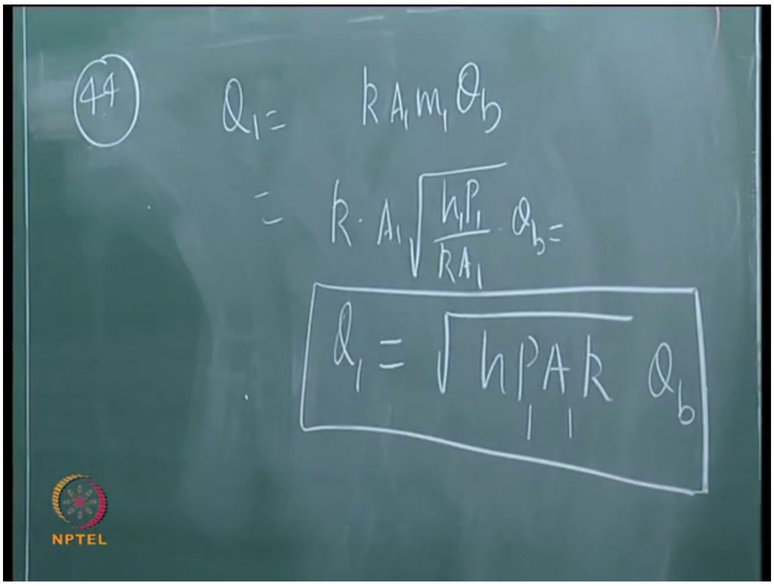

Student: to 17:03; 17:04 to 17:16; 17:22 to 17:50

So, correct, in the 1st case, we are changing only the m Q1 is k. There is a, there is a mnemonic here; Q is root of hPAK theta b, so you can remember like that. Q is equal to root of hPAK theta b, I would, ok.

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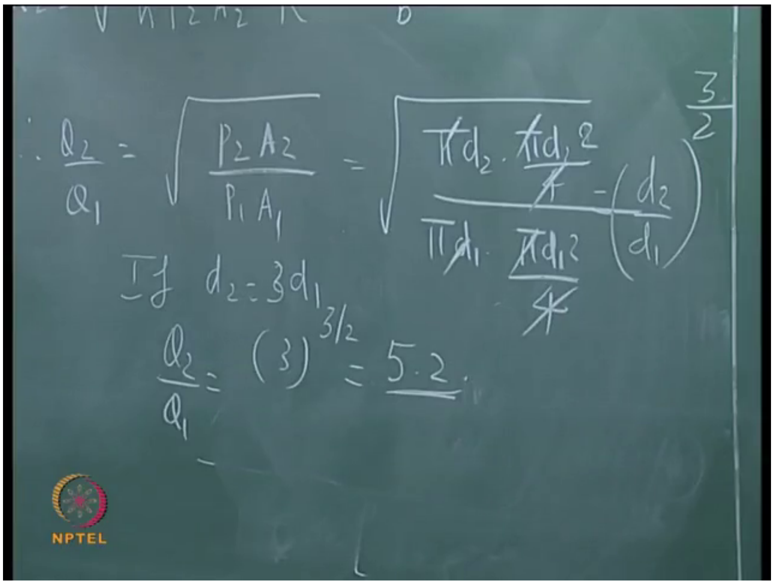

to 18:24; 18:26 to 19:09; Now, Q2… Therefore… correct. So, the diameter is doubled, oh, tripled, tripled (( )), 5 point, very impressive. But suppose, we are trying to do something like this in a satellite, which is placed in the PSLV, you want to do some heat sink design and all that, (( )) what happened, is it not clear? The heat transfer rate is increased by 5 times, very good, but the weight has increased by 9 times. (( )) 100s of fins, it is not commensurate with this. So, just because if hard, very good, the things have, the fins have become thicker and become heavy. So, there is a penalty associated every time you try to do something, that there is some other penalty. Now, part b, so the weight has increased 9 times because the diameter is tripled. So it goes, diameter square, weight into 3 into 3, 9 times.

Student: (( ))

You want Q to be constant; what I am saying is, suppose you want to increase the Q itself; what I am, what I am saying is, if you want to Q, increase the Q itself, I am saying that this just because you tripled this 3, this 5.2 times is not coming for free, that is a point I want to emphasize.

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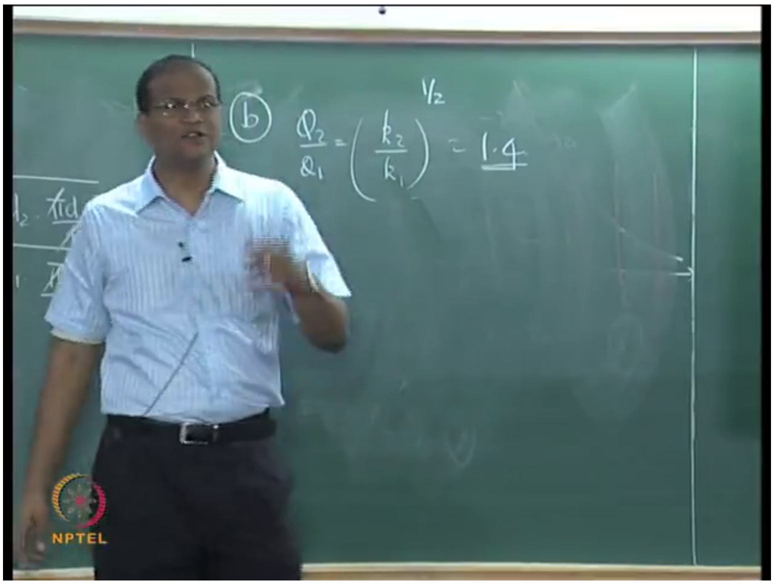

b is… to 21:56. All other things are the same. How much is it? 1 point… Again, what is the density of copper? 8099 kilogram per meter cube. What is the density of aluminum? 2000 (( )).

So, if you replace aluminum by copper, there are several dangers, weight is increased. So, copper is very expensive. Now, you can see, this sort of exercise gives you an idea of the basic heat transfer processes, how each of these parameters influences the heat transfer rate.

So, we, this gives you a good idea of the role of various parameters. I can also use this, I can also conduct some experiments, where I have long fin, I measure the temperature at 2 places, then I know the theta, theta by theta b e to the power of minus m x.

Now, I can solve it as an inverse problem and get the value of m, from the value of m I can get the value of thermal conductivity. So, you can do some additional, I do not normally do a problem like this, I do not have time now, that is, you have a fin, you have a fin setup and measure some temperatures, then you keep on guessing the value of thermal conductivity, it will give you theta minus, it will give a theta versus x like this for various values of k. There will be only 1 value of k, which gives a curve, which is closest to the experiment and that curve should be the best representation of the thermal conductivity. So, that is a basically, what is called an inverse problem.

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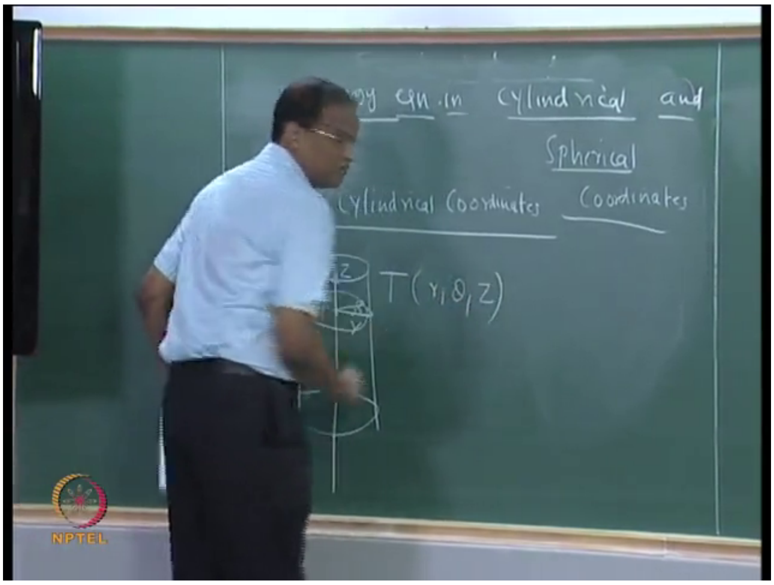

Now, we will go to… to 24:06. We have to look at the energy equation in cylindrical and spherical coordinates, I do not have time to derive that, but it will, they, they can be derived in a manner analogous to how, how it was derived for the Cartesian coordinate.

to 24:50. So, if you have something like this, this sort of geometry occurs basically in double pipe heat exchanges, that is, concentric tube heat exchanges, nuclear fuel rods, they are mostly like this. So, the temperature could be a function of the radius r, the temperature can change along the axial length, axial distance z, then there is this coordinate. So, this we call theta, this angle. So, t of r, it is called r theta z coordinate, theta r z, mostly, unless you have a spherical, a circumferentially varying heat flux, theta will not be a function of or t will not be a function of this angle theta unless you have special heat flux, sinusoidal varign heat flux or some non-uniform heat flux. Generally, it will be uniformed along the azimuth, but temperature will vary, usually across the radius it will vary and often times it will vary with z also.

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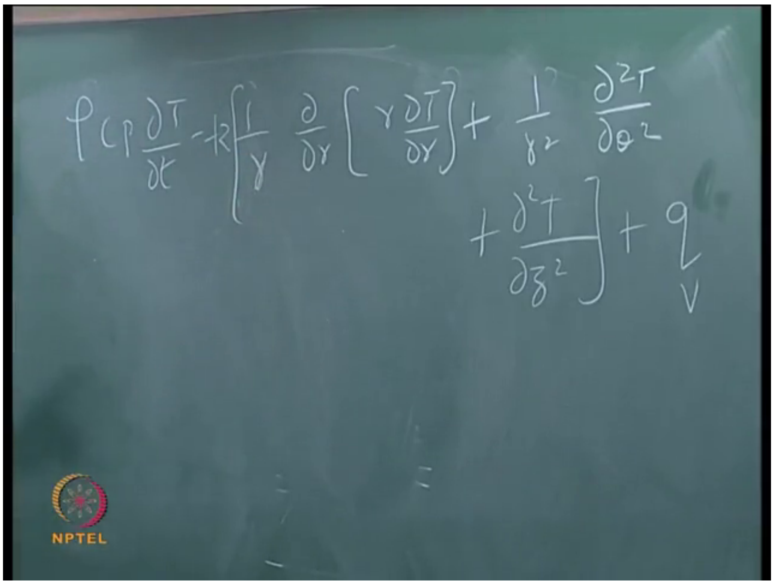

So, the governing equation for this will be rho Cp. to 26:42. This is for, uniform thermal, uniform thermal conductivity K. So, please note the essential differences between this equation and Cartesian coordinate equation; have you written this down? The essential differences are the left hand side is same as the right hand side, you got 1 by r dou by dou r of r dt by dr, we used to get d square t by dx square there.

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But why is it 1 by r is coming is basically, if you take a section r, if you section, if you take a section r plus dr, there we put minus small q at x is equal to minus small q at x plus dx, here you cannot put that, it will (( )), you have to put minus capital Qr equal to because capital Qr is small q r into 2 pi r d r 2 pi 2 pi r plus d r. So, that is why the dou by dou r of r dt by dr is coming. The area continuously changes, it is not a constant, heat flux across the radius constant. There may be constant heat transfer rate, but heat flux is not. Then, theta is dimensionless there; therefore d square, d square theta by d theta square has to be multiplied by 1 by r square because it is not dimensionally consistent; dimensionally consistent, then d square t by d z square.

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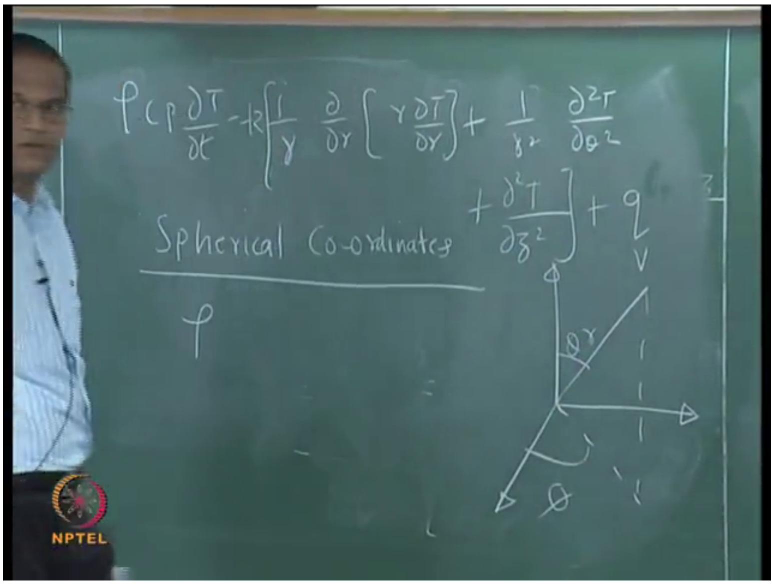

Now, for the spherical coordinates, we saw this spherical coordinates in our discussion on radiation. So, there is r.

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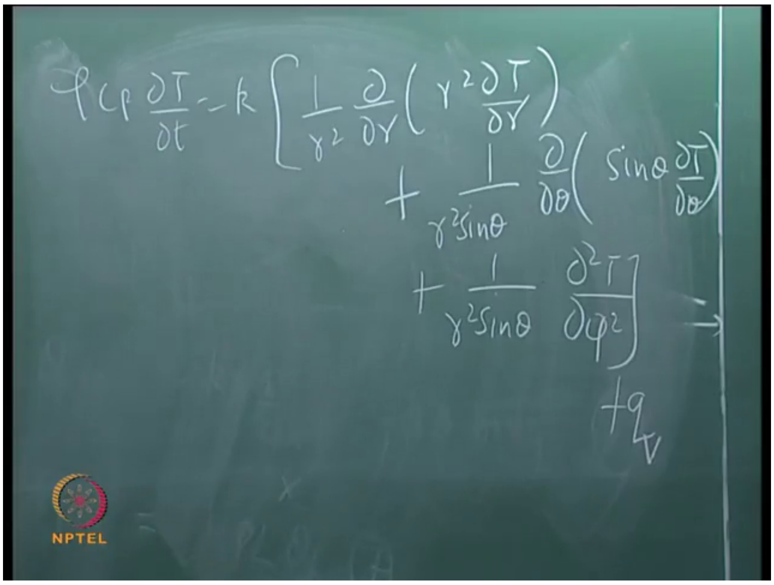

So instead of 1 by r, what do you expect for spherical coordinate? 1 by r square plus 1 by r square sin theta to 29:41. So, I have asked the derivation of this equation as a question to examination 8, 10 years back, do not worry, nowadays I do not do, this thing can we derived, but it is very painful, takes a long time, just a conduction course, we can do it now.

Please keep this, suppose in the exam I ask questions on simplified forms of this, you start from this equation and if it is unsteady, remove the left hand side, no qv, remove the qv term on the right hand side. But if it is only one-dimensional, remove the functional variation with respect to, here in this case theta and z and proceed.

Let us get some analytical solutions for simple cases.

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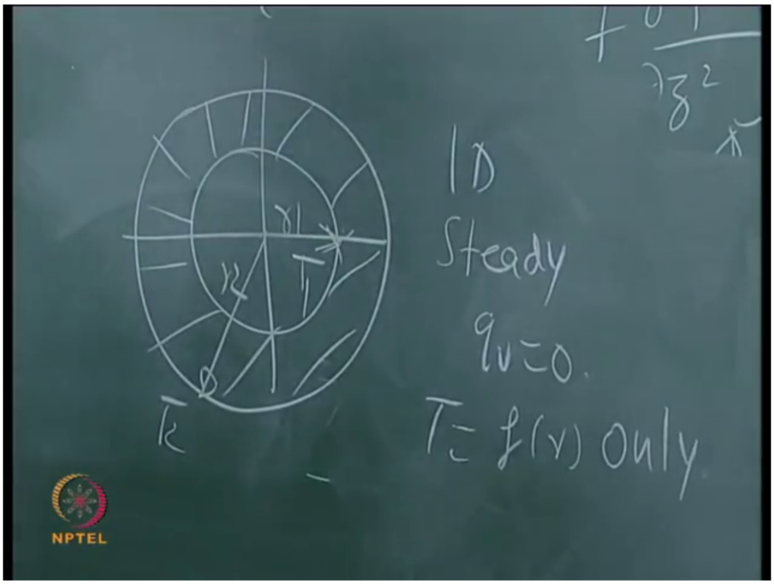

So, let us consider a double pipe heat exchanger, we are not worried about temperature as function of z or theta, temperature is a function of r only. So, I have a solid material here, so it is an annulus; it is an annulus. The temperature here is T1 at r1, this is r 2, it is T2. This, why it is T1 and T2 is because basically, because some fluids may be changed in face. The correct description would be to give a heat transfer h1 and t infinity 1 and h2 and t infinity 2, but if I give you that it becomes very messy, but we will incorporate that into our analysis by using our resistance concept. But first, we will solve the simplified form of the governing equation; is the situation clear? So, it is 1 d steady q e equal to 0, t is a function of r only.

There is a pipe, which is like this, I am taking the sectional view, it is the sectional view of the pipe. So, that is, this is a thickness of the pipe material or tube material.

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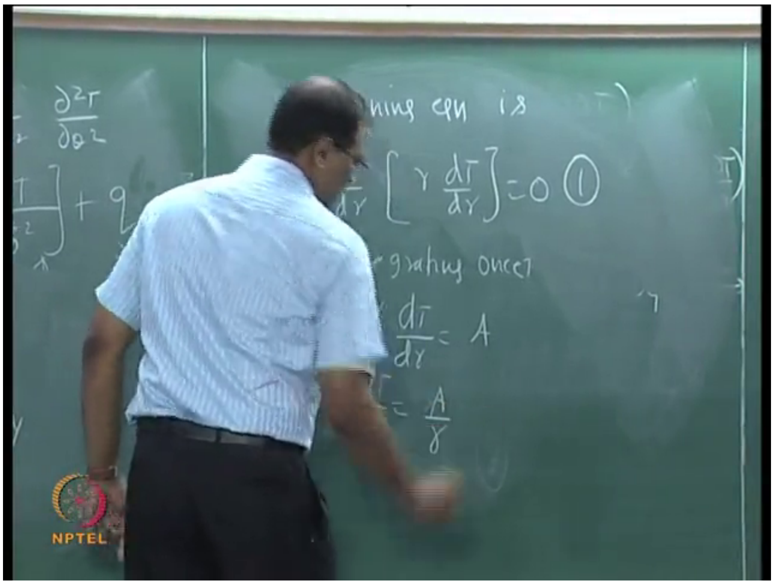

Now, please reduce the governing equation will be, the governing equation is from this dou by dou r of r dt by dr equal to 0, that is it. I can make it d also, no need for dou, is everybody through with this? The left side unsteady term 0, right side, right side d square theta by d theta square 0 d square t by d s square 0 q v equal to 0 because everything else is 0, dou by dou r can be changed, d into d by dr. Now, integrating once… to 33:23

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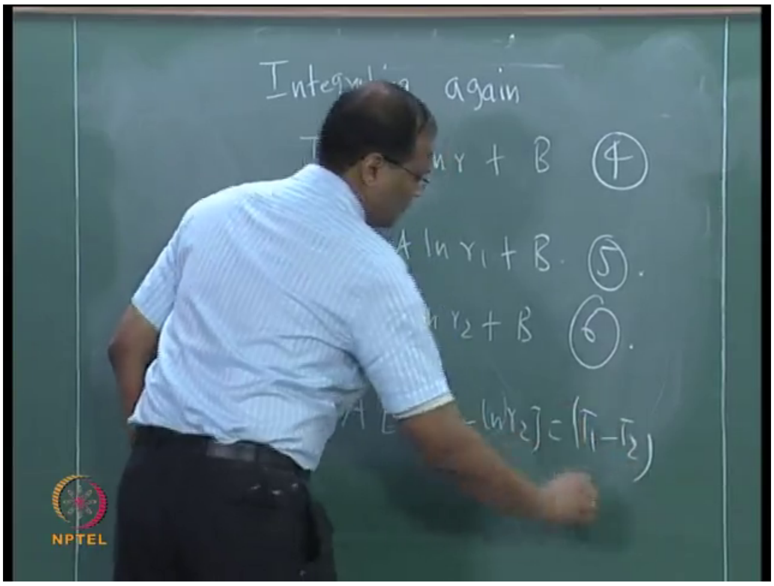

Now, integrating again, t equal to A log r plus b, substitute the 2 boundary conditions at r equal to r 1, t equal to T1; at r equal to r 2, T equal to T 2, that is specified to us.

Subtracting 6 from 5, the B will get cancelled, you can straight away evaluate A. Is it ok? log of r1 minus log of r2 is log of r1 by r2.

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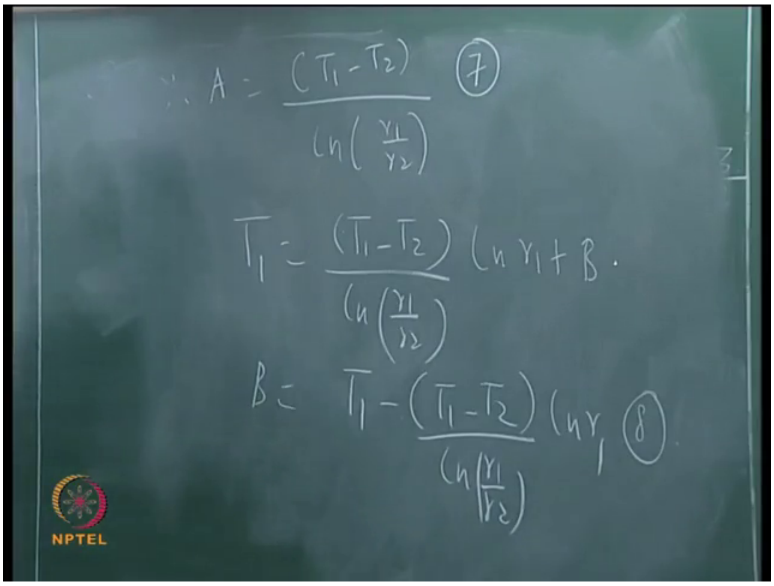

Student: to 35:47

Is it correct, Anirban, is it correct?

Student: to 37:04

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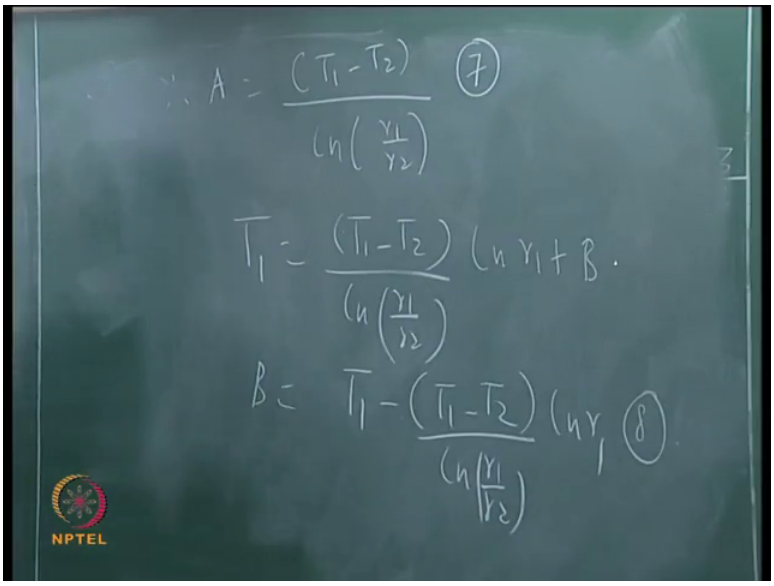

Therefore, yeah it is easy to see, but r is equal to r 1, what is lon of r by r 1? (( )) where? (( )), ok. That r is equal to r1, lon of 1 is 0, so T equal to T1, r is equal to r2, this will become r2 by r1, this will be minus 1, so t2 minus t1 plus t2, so the equation is correct. So, we check it for asymptotic correctness. So, we got the temperature distribution. We are interested in heat transfer rate.

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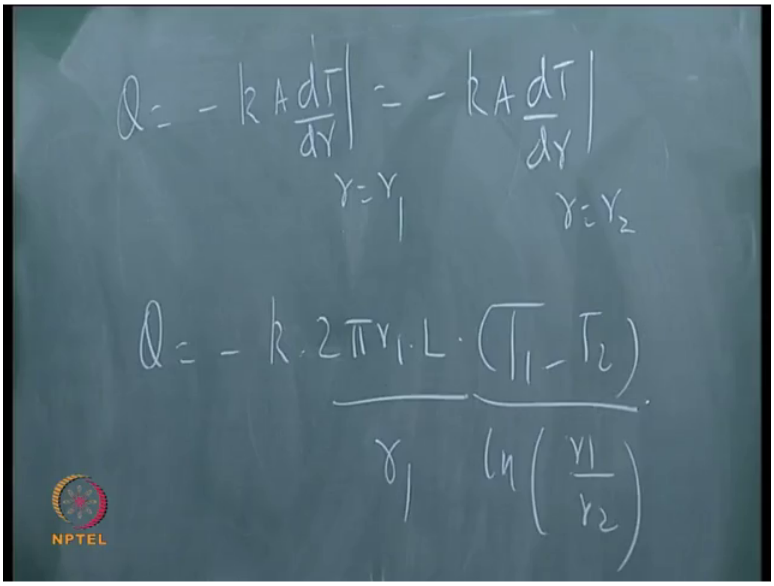

Student: to 38:12

This has to be correct, that is how, that is how we wrote the equation d by dr of r dt by dr is equal to 0, so this is the case. What is dt by dr? A by r; A by r dt by dr at r1 is A by r1; A by r1. What is A? T1 minus T2, that is it, by r1. I can take care of the minus sign by writing the denominator lon of r2 by r1.

Student: to 39:43

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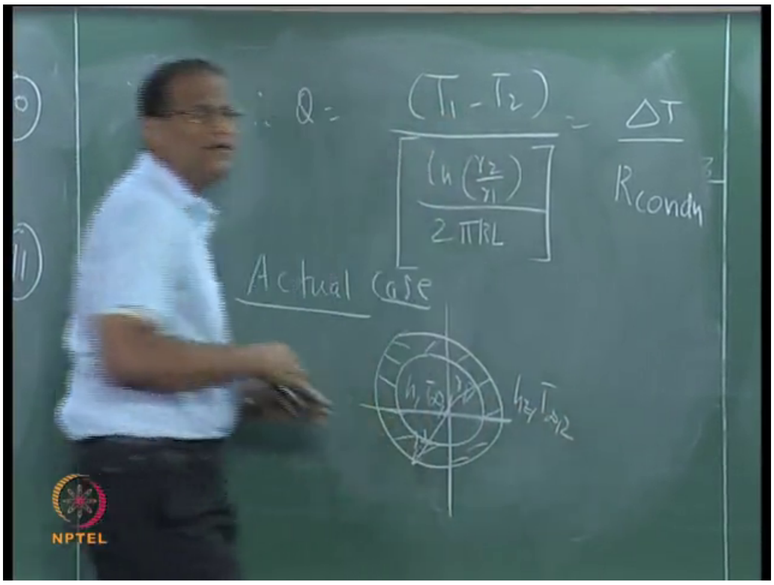

When I say Q equal to delta t by something, what is that delta t by something? Resistance; that is the resistance. What is the only, only resistance, which is occurring in this problem? This is a conduction resistance because of finite conduction across the thickness of the tube; that resistance will vanish in the event of thermal conductivity of the material tending to infinity.

So, this is, therefore, the conduction resistance involves a log of ratio of the radii. So, the natural log occurs naturally in the case of conduction in cylindrical coordinates. This did not come in the case of conduction in Cartesian coordinates; it is basically the, the only source. The only culprit for this is the changing area 2 pi r dr, that led to A by r, which led to A lon r, which may also lead to problems. When you are solving for conduction within a solid, then what will happen to the temperature at the center? So, you have to, you have to take care of the boundary condition at the center.

You may say, dt by dr 0 at the center if it is a solid cylinder; dt by dr does not mean it is insulated, sometime what we do is at the center, we apply what is called the finiteness of temperature condition, the temperature has to be finite. If there, if time permits, we will solve the problems where we take a solid cylinder.

Now, this is pretty straight forward, now we go to the actual case, what is actual case? h1 t infinity 1 r 1 r 2 h 2 t infinity 2. So, I do not know the boundary, I do not know the temperatures T 1 and T 2 at r 1 and r 2 respectively, rather I know the temperature of the fluids, which are flowing in the inside and the outside and the heat transfer coefficient. This is possible in a double pipe heat exchanger where cold fluid is flowing inside and hot fluid is flowing outside, you are trying to accomplish heat transfer putting the 2 fluids. Why cannot you mix the fluids? We do not want to do that, one could be oil, one could, one could be sodium, it happens, sodium to water heat exchanger or something.

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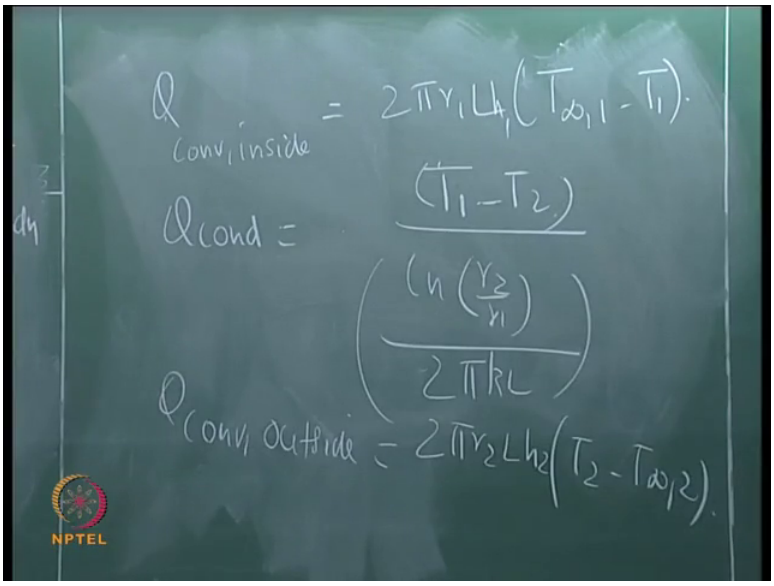

Now, what is a heat transfer rate? What is the L I am keeping, talking some L? L is the length in the direction perpendicular to the plane of the board. So, yes, ok.

Student: to 43:39

So, now, we use the dividendo-componendo rule and say, it is a same Q, which is coming outside from inside outside, it is the same Q. Therefore…

Student: to 44:32

Using the dividendo-componendo rule, we can add the numerators and denominators separately; still the resulting expression will be equal to the Q, which is occurring across the Q. Therefore... to 45:10

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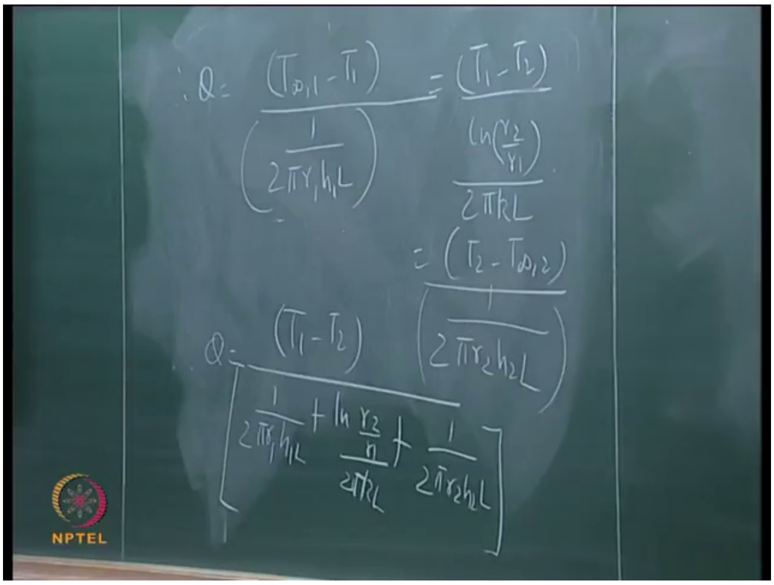

So, the 3 terms in the denominator are, the 1st term represents the convective resistance on the inside, the 2nd term represents the conduction resistance across the thickness of the tube and the 3rd term represents the convection resistance on the outside. The 1st and 3rd terms are also called as the, yeah, the 1st and 3rd terms are also called surface resistances. So, using the concept of electrical analogy…

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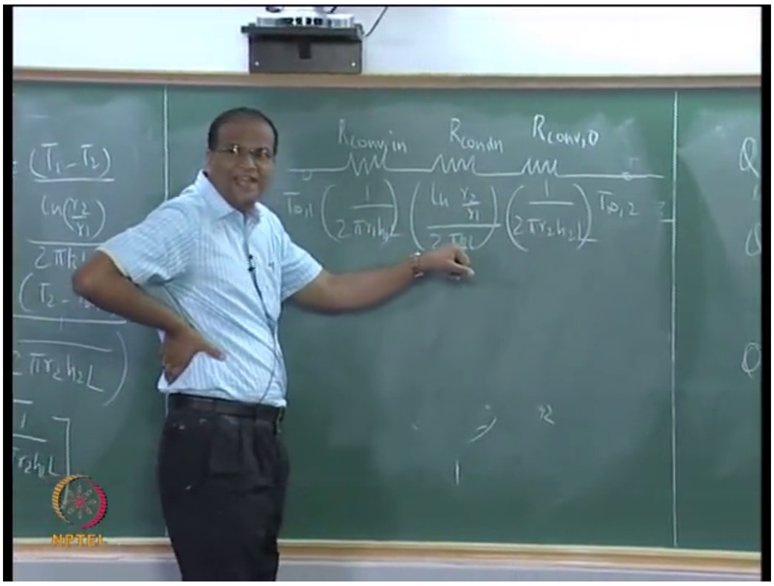

So, we can say, so your potential difference T infinity 1 to T infinity 2…. to 46:33. In the event of h 1 equal to h 2 equal to infinity, it will reduce the simple expression, which we derived some few minutes ago.

Now, we can have a composite wall, where apart from the inside and outside surface resistances we can pack this with insulation. Therefore, you can have a, you can have terms like lon of r 2 by r 1 2 pi k A into l plus lon of r 3 by r 2 into 2 pi k B into l, that is you have got. Suppose, you are looking at an insulation design, you are using 2, 3 types of insulation, there is a metallic wall, then you put glass, then you put something else, what is the idea behind this? In all these industry establishments, when you are carrying hot fluid, the outside temperature cannot exceed 65 degrees because that is a temperature at which people will get hurt. So, the outside temperature should not be more than 65, all these open end because somebody will go and touch it. So, you will have to keep on insulating and also, you do not want, by the time it leaves this boiler room and goes to the turbine room, you do not want it to get cool, you want to, you want to expect the full enthalpy of the steam.

So, when you come to the insulation design all these things will come. There is also a point of critical; you can also work out what is called the critical insulation thickness, which you, may, must have done in your undergraduate heat transfer course. So, we can use the dividendo-componendo rule and I have got the results like this in an exactly analogous equation fashion. You can derive basic relation analytical solutions for one-dimensional heat transfer across its spherical shell.

Why spherical shell is so important? Because you want to carry cryogenic fluids, you want to carry nuclear, nuclear fuel waste and all that, then if you want to put several layers, there can be k a, k b, k c. So, if you are not able to solve, we can solve problems in the exam also, the class. So, there are lots of opportunities for me to ask question on conduction in the exam. Now, we will stop here.