Conduction and Radiation

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Lecture No. # 38

Conduction – Cylindrical and Spherical Geometries

Yeah, in today's class, first we will solve a problem concerning steady state conduction in a cylindrical geometry, namely nuclear fuel rod and then, we will go on to solving one-dimensional conduction problem in a spherical shell, which is very much used in transport of liquid nitrogen, liquid oxygen and so on.

So, in tomorrow's class, we will start with transient conduction and 2 classes, tomorrow and day after tomorrow, we will try to finish transient conduction, whatever is possible. So, please take down this problem.

Problem number 45 - a long cylindrical rod, a long cylindrical rod, long cylindrical rod of 80 millimeters radius, long cylindrical rod of 80 millimeters radius consist of a nuclear fission material; a long cylindrical rod of 80 millimeters radius consist of a nuclear fission material, within brackets, k equals 0.6 watts per meter per Kelvin, k equals 0.6 watts per meter per Kelvin, generating, generating 30000 watts per meter cube. A long cylindrical rod of 80 millimeter radius consist of a nuclear fission material, k equal to 0.6 watts per meter per Kelvin generating 30000 watts per meter cube uniformly throughout the volume, uniformly throughout the volume. The rod is encapsulated, the rod is encapsulated with a tube, the rod is encapsulated with a tube, the rod is encapsulated with a tube having an outer radius of 160 millimeter, the rod is encapsulated with a tube having an outer radius of 160 millimeters and the thermal conductivity of 4 watt per meter per Kelvin; the rod is encapsulated with a tube having an outer radius of 160 millimeters, the thermal conductivity of 4 watt per meter per Kelvin. The outer surface is surrounded by a fluid, the outer surface is surrounded by a fluid at 80 degrees, 8 0, 8 0 degree Celsius, the outer surface is surrounded by a fluid at 80 degrees Celsius and a heat transfer coefficient of, and a heat transfer coefficient of 20 watts per meter square per Kelvin, and a heat transfer coefficient of 20 watts per meter square per Kelvin.

Determine interface temperature, determine the interface temperature, determine the interface temperature and the temperature at the outer surface of the tube; determine the interface temperature and the temperature at the outer surface of the tube.

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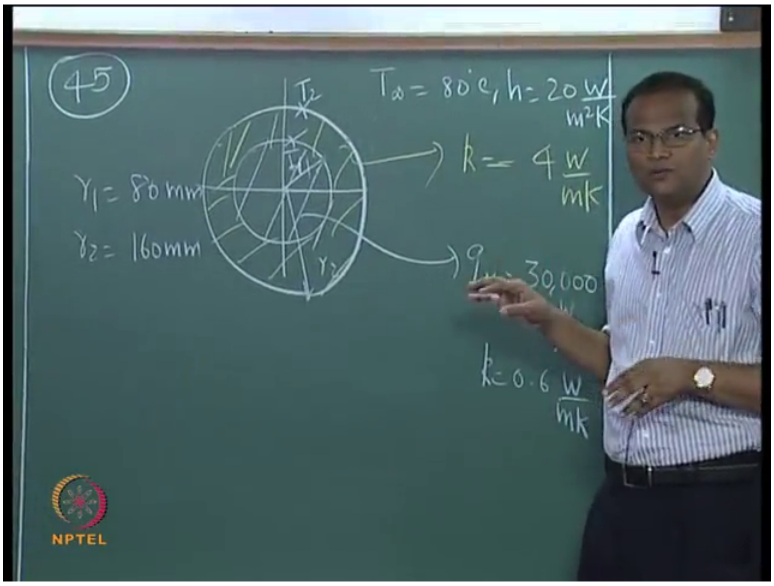

People who came late, I will draw the figure, its steady state conduction in a cylinder, from the figure you can figure out what we what we are trying to seek. So, there is a nuclear fuel rod, it is generating heat uniformly rate of 30,000 watts per meter cube. So, this material has a thermal conductivity of, you are having an outer sheath, but it is encapsulated in a tube to protect it or whatever. This material is having a k of 4 watts per meter per Kelvin, 4, did you say 4.

Now, I have given you the radius, radii. So, steady state, one-dimensional, all the heat, which, which is generated from within is transported across to the boundary, goes on to the tube and is picked up by the fluid on the outside, the fluid on the outside has got these conditions.

So, the, what we are doing is essentially called nuclear thermal hydraulics, it is very critical for a nuclear designer, I was talking about Fukushima plant and all that. So, anyway, once the criticality is reached it would, the fisher metal will continue to generate heat, but the challenge is, as a heat transfer engineer you should be able to pick up that heat, so that the temperature does not reach dangerously high levels where it will melt and lead to core meltdown and other issues.

Now, the problem is pretty straight forward. Now, if you see, I am not asking the temperature within the nuclear fuel rod, we are looking at 2 temperatures, the temperature here T 1 and the temperature here T 2, so we are interested only in the 2 temperatures. So, there is no need to solve for the conduction equation, from the center onwards. We can use the relationship, which we develop for heat conduction in a cylindrical shell, on an annular, in, in an annulus, R 1 and R 2. We can use the concept of resistance, correct. There are 2 resistances, suppose, there are 2 resistances here. So, here, the temperature is T 1, here the temperature is T 2, here the temperature is T infinity, please look at the board, it is T 1, T 2 and T infinity. We can write, we can write the relationship for Q equal to T 1 minus T infinity divided by the sum of 2 resistances.

What are the 2 resistances? There is the conduction within this sleeve or the tube plus there is convection here. When you write, that T 1 and T infinity, T infinity is known, all the other things in this resistances are known, H a is known, now this must be equal to Q. So, if you have to find the T 1, you need to know q, but q is not such a great thing, that we do not know. q is, q, q can be determined by using energy balance, after all whatever heat is generated within the volume has to be taken out by the…

So, it is a, so it is a, it is a problem where you have to intelligently use a furnace. First, get the q by simple energy balance, declare that the same q is going throughout and then equate it to q for a cylindrical shell, get the temperature T 1. Once you get the temperature T 1, the temperature T 2 can be straightaway obtained, is the problem clear? Let us work it out.

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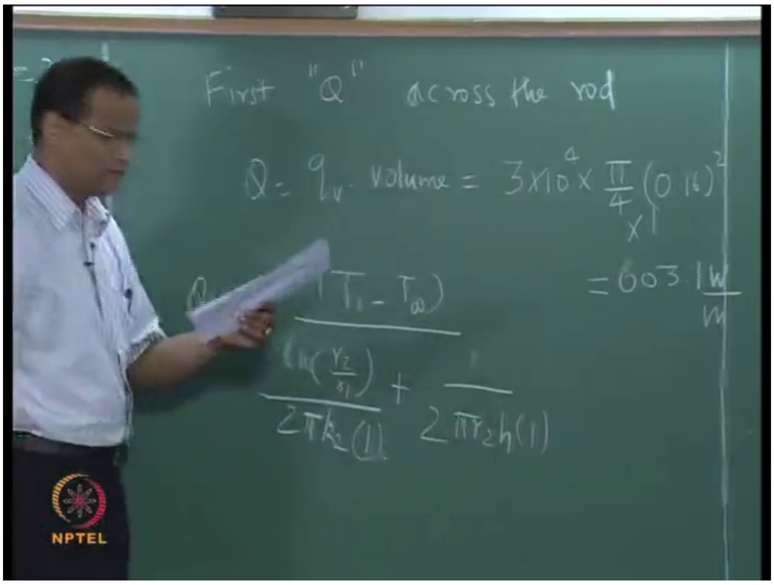

Now, first the Q across the rod is given by, I have not given the depth in the direction perpendicular to the plane of the board, what should we do about that? 1, we just take 1 meter, is this correct? 30000 watts per meter cube into pi by 4 into 0.16 because R is 8 centimeter, d is 16 centimeter, 0.13 into 1, Rohit is it clear, just give me the values, Koustav you do not have calci, start working 600 and 2.9 watts per meter, watts per, I can use watts per meter, what is that, per meter, yeah. R 1 itself is 160, do not get confused, radius, yeah, pi d square is 1, Deepak, in fact, when I worked out in the morning I also did the same thing, yeah, you have to be alert.

So, this is the same, this Q is not equal to, so I will call this k 1, k 2, 2 pi, is this correct, agree? There are only 2 resistances, conduction across the sleeve or the sheath or the tube, convection from the tube to the outside, this heat is coming from the center, it is coming from that volume.

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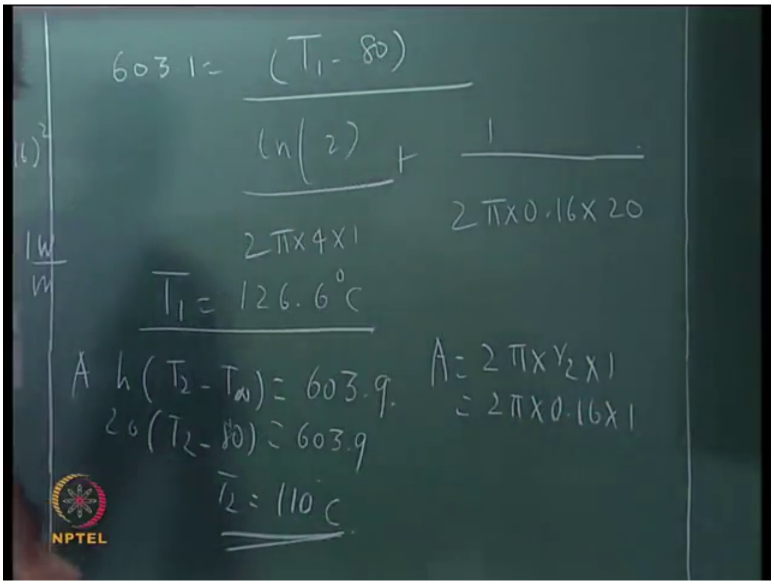

Yeah, now if you do that, R2 is 0.16, correct? So, what is T 1? 110 degrees, yeah, 126, 126 point, what did you, 126.6, just check this value, Vikram is it ok? Take it, take it, take it, 126.6. So, this is basically the, this, this, what I call as the interface temperature, temperature between the nuclear fuel rod and the, and the sheath, which surrounds it, the sheath may also be kept for (( )) reasons, starts doing the nuclear fission, suppose it disintegrates, fine. Now, we will have to get the other temperature, that is pretty straight forward, H into, 96 no, 110. So, if the problem, where the q is known, but the temperatures are unknown, Vikram is it ok. Of course, what about the area? Yeah, into, 2 pi R2 pi R.

I have already done that.

You have to solve the equation and solve individually for the inside outside, and all that will be much more difficult, much more involved, this is very intelligent way of solving it, that is using the electrical resistance network. So, the resistance network we are able to use even in a problem, where there is heat generation because from the heat generation, we are able to calculate the heat flux or the heat transfer rate and proceed further.

Now, we will add some spice to this problem. Now, all of you got these 2 temperatures, now suppose I put a rider and say, determine the centre temperature of the nuclear fuel rod, how will you do that? Determine the center that is critical, that center temperature must be far below the safe operating limit, it should be less than the creep temperature, which anyway must be much less compared to the melting temperature, is not it? What is that? You have to write the governing equation, you cannot do any tricks; you have to write the governing equation for one-dimensional conduction in a cylinder with heat generation. Now, you do not have to worry about boundary conditions because one boundary condition is T 1, which we just determined; the other boundary condition is dT by dR, must be equal to 0 at the center, that can be taken as either the maximum temperature occurs at the center or it is also called, what is known as the finiteness of the temperature, finiteness condition? Why is it called finiteness condition? That will be apparent as soon as we get the general solution to the problem.

Now, let us do problem number 46. Determine the center temperature, problem number 46, determine the center temperature; determine the center temperature of the nuclear fuel rod. Discuss, sorry, determine the center temperature of nuclear fuel rod considered in problem 45, determine the center temperature of the nuclear, nuclear fuel rod considered in problem 45.

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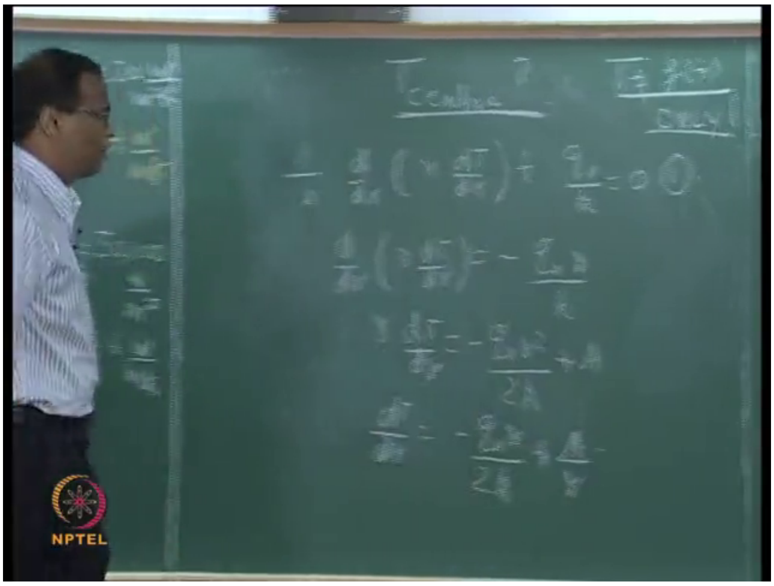

Let us do that, correct. You can get the general governing equation, remove all the unnecessary terms, unnecessary terms are rho c p dT by d dou d square T by dz square d square T by d theta square. Those 2 terms are all gone because T is a function of r only; that is the important point. Now, we have to get the solution to this. Now…

Correct, (( )), I made a mistake, yes, yeah. Now, A, B, 2 constants, 2 boundary conditions, now very simple. At the center, at the center r tends to, r is 0; if r is 0, unless A is 0 we are in trouble, therefore because r is 0, A has to be 0, this calls a finiteness of temperature or we can say, that any plane, at r equal to 0 it should be symmetric on either side of r. So, the maximum temperature will occur at r equal to 0; that is why the, the area is becoming so small, that the temperature will peak there.

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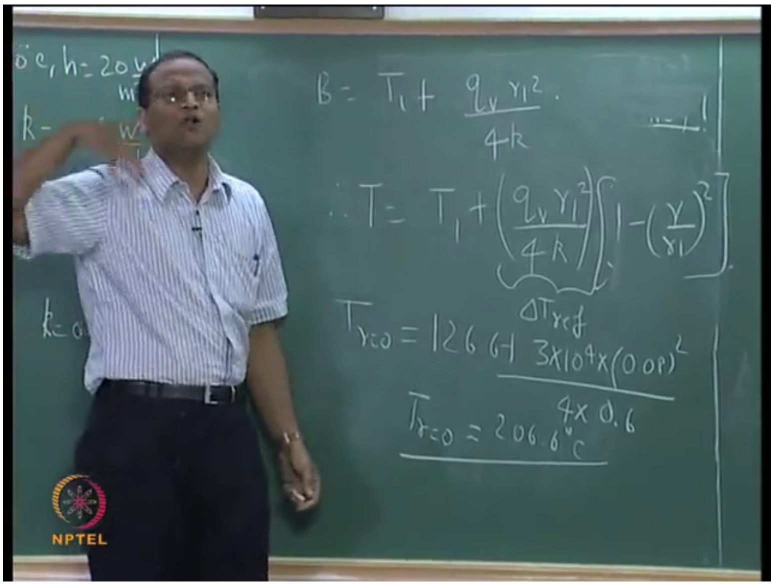

So, dT by dr is equal to 0, therefore from equation 4, therefore what about B, at r equal to r 1, T equal to, T equal to T1. So, now, I will do…

So, T equal to T1, what is B? Yeah, did I complicate it further or its ok, by 4 k here, just check? I can call it as A, I can call it as an equivalent temperature difference, correct, because this q v r square, look at this term please, q v r square by 4 k will have the units of Kelvin, that is the temperature rise, which is caused by the heat generation. We can also call it as a dimensionless heat generation parameter.

It is not dimensionless, but it is Kelvin, Kelvin. You can make it as a dimensionless heat generation parameter by dividing by T infinity or something, it, it, it is a temperature difference, you can divide it by T infinity and make it dimensionless heat generation parameter. So, call it (( )), whatever. Now, we will work out the temperature T, center at T r equal to 0 equal to 126.6 plus… r 1 (( )) point naught 8, correct, 0.6. How much was it? So, it is safe.

So, if you recall, this T 1, please look at the board, this T 1 was decided by the ability of the outside heat transfer coefficient to take away the heat, which is generated by the nuclear reaction, but this T depends on how much of heat is carried away by convection, but how much of, but how much temperature rise is caused because of the finiteness of the thermal conductivity of the nuclear heat generating material.

If the k is very high you do not have very big issues, the controlling the systems will only be, most of T will be contributed by T 1, if k is very small most of T will be contributed by the 2nd term. So, there is a conduction convection coupling already happening in, in the, this form, correct, but in the nuclear fuel material, k will not be under your control, r also we cannot make big, may be 20, this itself is too much. Normally, they will have 20 or 30 and q v also, physics people will tell what is q v. When one is criticality and all that, therefore, ask thermal engineers. We can control T 1, how can I control T 1 by circulating that fluids and this thing, then you can put some sleeve. We do not want to put too much of sleeve on at because if you are, if you are putting a sleeve, it is an impediment to the transfer of heat, but that sleeve also gives you protection, structural protection. So, there is a trade-off. So, there is a multiple objective optimization framework there. So, these are the issues. So, you have to be and you cannot, you do not have the luxury of repeating experiments in sodium and all that one is to one scale model.

So, sometimes nuclear heat transfer is also called, you can just juggle the letters and call it as unclear heat transfer. If somebody wants to write a program to solve this conduction in sphere or cylinder, it will be a challenge to determine the temperature at the center in a finite volume method, only if you write your own code (( )). Now, before winding up our discussion on steady state conduction in one-dimension, we look at a spherical shell which is a, which is also got lot of practical application. So, let us look at…

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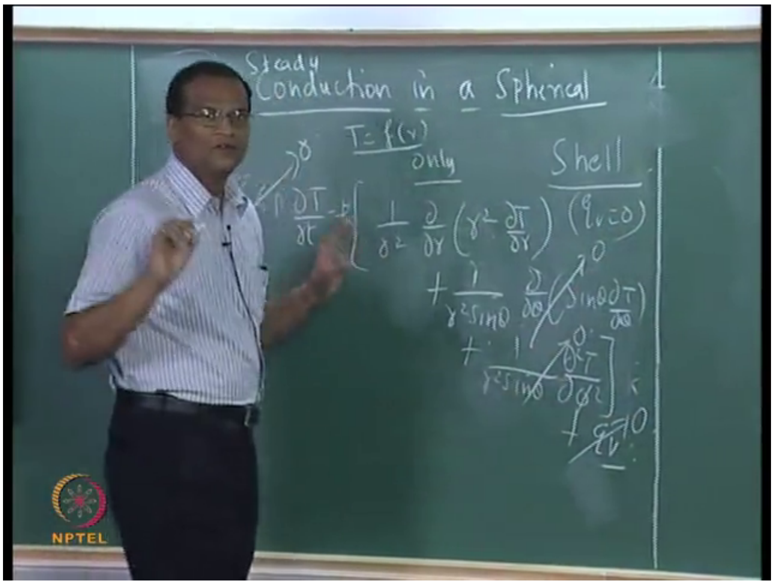

So, what is the equation? Theta by plus 1 by r square sin theta, is it (( )), dou by dou theta of sine theta, then plus r square sine theta dou square T by dou phi by k. So, T is a function of r only. I will put a k here, oh, I have a k there, is it, then we will take this off. What did we write? No, no, is dou by dou r of r square dou d by dou r, k is here, is correct, T is a function of r only. So, this gets knocked off, 1 d steady conduction, now I do not have q v. So, it is very simple.

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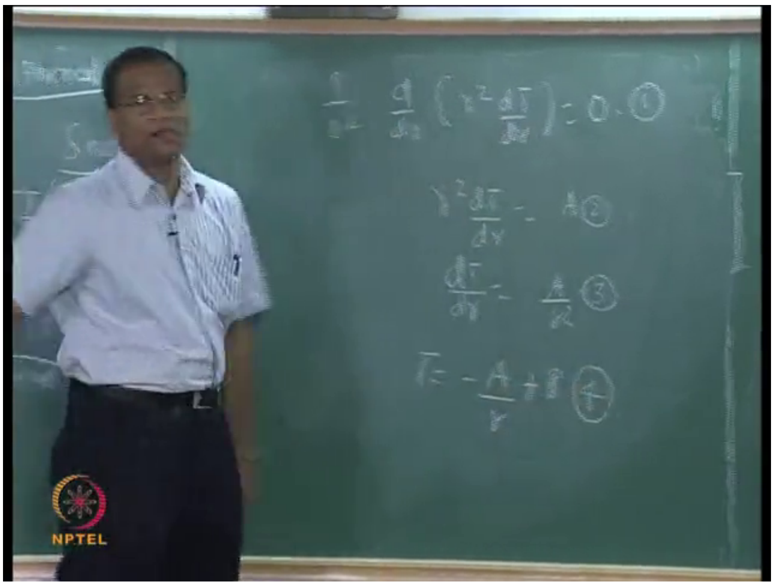

So, again, I will start with 1, it is correct? T, T equal to minus A by r plus B. So, you get the lon of r for the cylinder, we get 1 by r for the sphere, now let us substitute.

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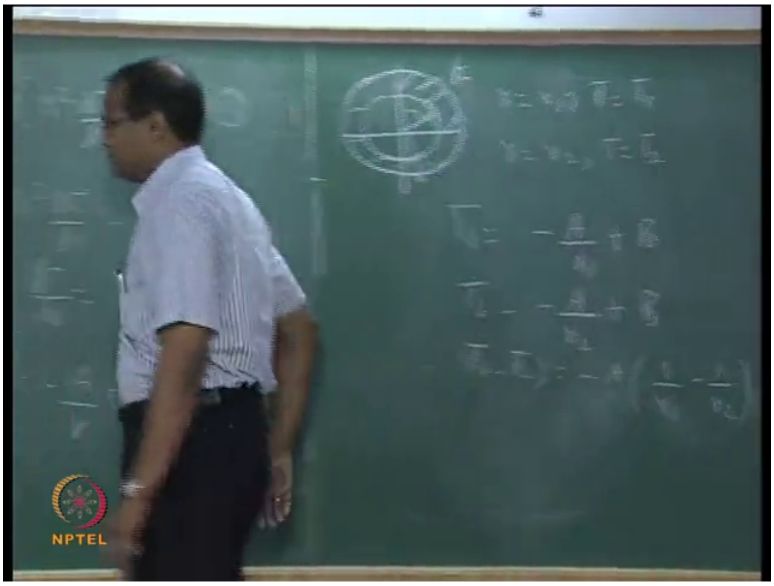

Let us take a spherical shell k, r 1, r 2. Let us say, let us say, take a spherical shell of thickness r 2 minus r 1, at r equal to r 1 temperature is T 1, at r equal to r 2 temperature equal T 2; even if you do not know directly we can get it indirectly, exactly in the same way as we did for problem 45. When do you think such problems will arise?

So, you started operating a power plant, you got so much of nuclear fuel waste, you want to dispose the fuel waste. So, you will put it into a spherical mass, then you put stainless steel all that and put canister, then you will bury a, deep bury deep down and put it inside. Now, we will have to find out, what will be the temperature in the outside?

As far as we are concerned as thermal engineers, we want to find out temperature outside and all that, physics people will be interested in what is the radiation level. They will take a meter and find out what is happening over time, how the radiation. But as far as we are concerned, what will be the temperature on the outside, or I am taking a spherical shell, I am, for example, I want liquid nitrogen or liquid oxygen for my PSLV launch and my liquid propulsion systems are LPSE and the fuel, I am making the fuel in Mahindragiri or Trivandrum. So, I am, I am taking a truck and transporting the fuel from Mahindragiri to Shreeharikota, I am putting it in a spherical container, what will happen is you would have temperatures, liquid nitrogen is 90 Kelvin, outside is blazing, now today it is very cool and it, may be, it is 40 degree centigrade. Now, this terrific heat leak inside because of which this oxygen will get converted to vapor. So, then, we will need, slowly the oxygen, some liquid will be lost. So, you will have to find out, suppose this journey takes so many hours, at the end of the day how much is, how much of liquid oxygen will be lost? So, you turn around and say, let us say 2 kgs per day is alone allowed, if 2 kg per day is to be allowed, what should be the thickness of the insulation? That is engineering, so this is a very practical problem. Now, so what we started out with very, with some mathematics, we can apply to some hard core engineering problem.

So, let us, I will give you the examples. Now, you take liquid oxygen and spherical tank and do that. Now, T 1 minus A by r 1 plus B, therefore…, correct. So, what is A?

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This is also ok, but 1 by r 2 minus 1 by r 1 will be negative, this will be negative. No, I did something there, I did something there Vikram, I took care of the minus sign, I did not make a mistake, right. Now, B, so T 1… by r 1, correct.

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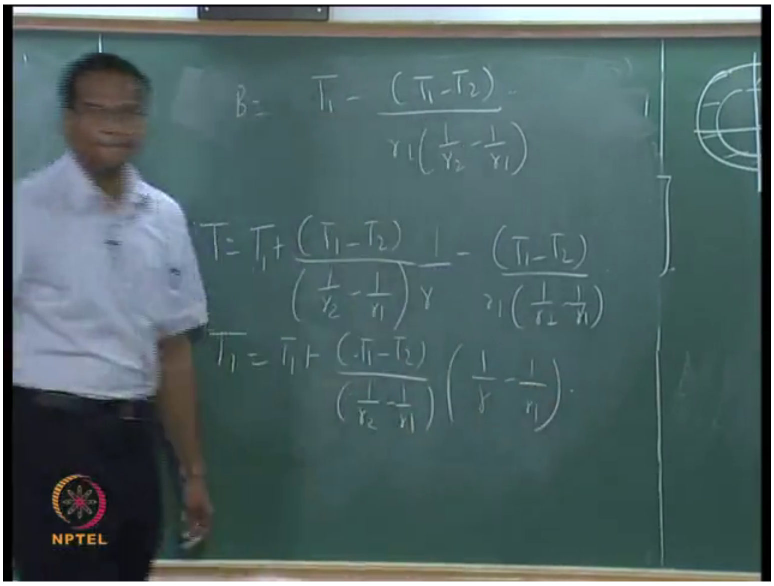

This is correct? Therefore… Some, which is minus, which is plus?

Student: Sir, B will be T 1 plus.

So, ok, then B will be plus, very good. So, anything is plus, then we are in trouble, this is it? May be, I made more mistakes, is it ok? Yeah, r equal to R 1, T equal to T 1, (( )), but R equal to R 2 yeah. So, this is 1 by R 1 minus, minus 1 minus 1, fine, feels good. Now, what is Q?

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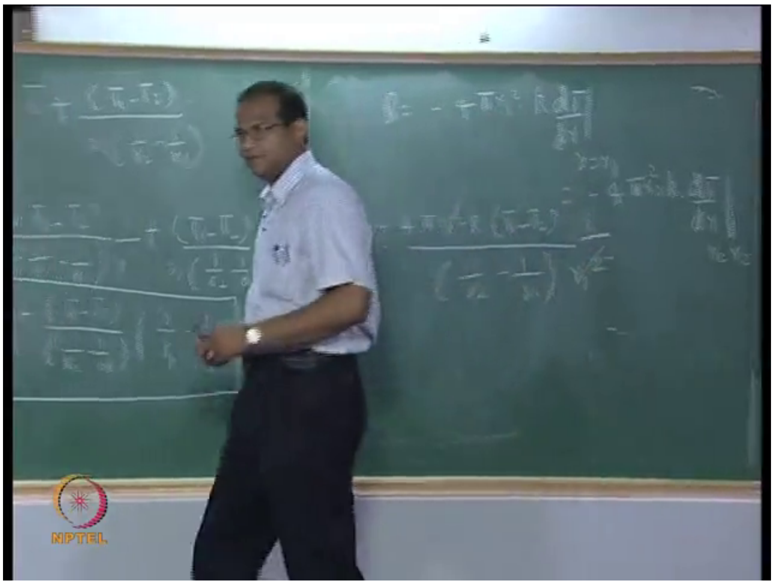

minus 4 pi r 1 square k dT by dr at r equal to r 1 is also equal to minus 4 pi r 2 square k dT by dr at r 2, same thing, which is going, yeah, it is fine now, yeah, r 2 square. Shrikant is it ok, following?

Now, let us take k dT by dr at r equal to r 1. So, q equal, what is dT by dr? Let us keep this, so this term will get knocked off. So, it is, minus 1 by, minus 1 by 1 by 1 minus 1 by r square, there is already a minus 1 here, this is plus, plus 1 by r; r 1 r 1 gets cancelled, 1 by r 2 minus 1 by r 1 is minus r 2 is greater than r. Therefore, the denominator is negative, I already have a minus sign, so (( )). I have no problem, therefore…,

So, what is this, what is this? What? Resistance, shell resistance, conduction resistance, therefore, this is… Now, if you have a situation where there is a spherical shell and r h 1 T infinity 1 k r 1 r 2 h 2 T infinity 2, this Q will be the same as…, correct.

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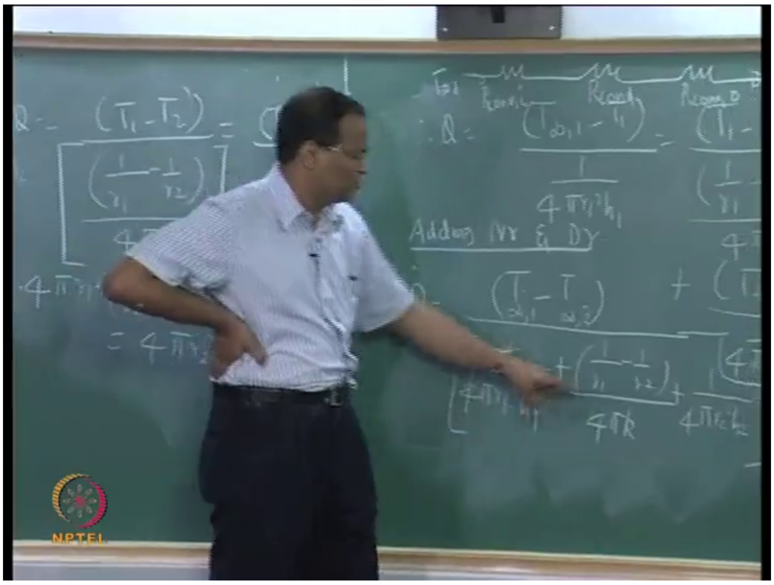

So, you can add the numerator separately and the denominator separately and using the, dividendo, dividendo-componendo rule, if you add the numerator separately denominator separately, the resultant, the expression will also be equal to Q. Therefore, adding n r and d r, numerator denominator…

You can draw this like this, T infinity 1. So, r convection inside, r conduction, r convection outside. Now, you can keep on adding slits of varying radii, r 2, r 3, r 4 with different materials k 1, k 2, k 3. You can have a composite spherical shell, which is made of several materials and then, which has got a convective heat transfer coefficient on the inside, but also a convective heat transfer coefficient on the outside.

Please take down this problem, problem number 47. A spherical tank for storing liquid oxygen, a spherical tank for storing liquid oxygen, a spherical tank for storing liquid oxygen on a space shuttle, problem 47; a spherical tank for storing liquid oxygen on a space shuttle is to be made from stainless steel, is to be made from stainless steel of 80 centimeter outer diameter, 8 0 centimeter outer diameter and 5 millimeter wall thickness. Liquid oxygen is contained inside the thickness of the wall is 5 millimeter, it is a stainless steel shell, boiling point of liquid oxygen is 90 Kelvin, boiling point of liquid oxygen is 90 Kelvin, latent heat of fusion of liquid oxygen, you want to call it fusion or you call it vaporization, the latent heat of vaporization of liquid oxygen is 213 kilo joules per kilo gram; the latent heat vaporization of liquid oxygen 213 kilo joules per kilo gram, is it, yeah. The tank is to be installed in a large compartment, the tank is to be installed in a large compartment whose temperature is, the tank is to be installed in a large compartment whose temperature is maintained at 240 Kelvin, 240 Kelvin. Design a thermal insulation system, determine, design a thermal insulation system, that will maintain oxygen losses, design a thermal insulation system, that will maintain oxygen losses due to boiling below 1 kilogram per day; design a thermal insulation system, that will maintain oxygen losses due to boiling below 1 kilogram per day, k of s is 15 watts per meter per Kelvin, k s is, k of s is 15 watts per meter per Kelvin of a foam based insulation, of a foam based insulation with k equal to 0.05, of a foam based insulation with k equal to 0.05 watts per meter per Kelvin, a foam based insulation with k equal to 0.05 watts per meter per Kelvin may be used.

So, this problem is adapted from (( )), because we are on air, this from (( )), just you can use the final formula right away. So, 1st get the Q, heat transfer loses 1 k g, 1 k g per day. How much will be the Q? First calculate the Q, then you use this formula, use this formula to h 1, h 2 these are not there, but you will have 2 1 by r 1 minus 1 r 2 by 4 pi k 1 plus 1 by r 2 minus 1 by r 3 divided by 4 pi k 2; k 1 is a thermal conductor of stainless steel, k 2 is a thermal conductor of the foam material, all the things are unknown except r 3. So, you will design a, you will design the insulation thickness, ok.

Now let us complete this. So, consider everything is well mixed in inside and outside or the resistance is associated with them are so small compared to the conduction resistance, in all our question paper we will say, make reasonable assumptions with justification, if nothing else is gained what do we do, you cannot say, that data is not there I cannot solve the problem.

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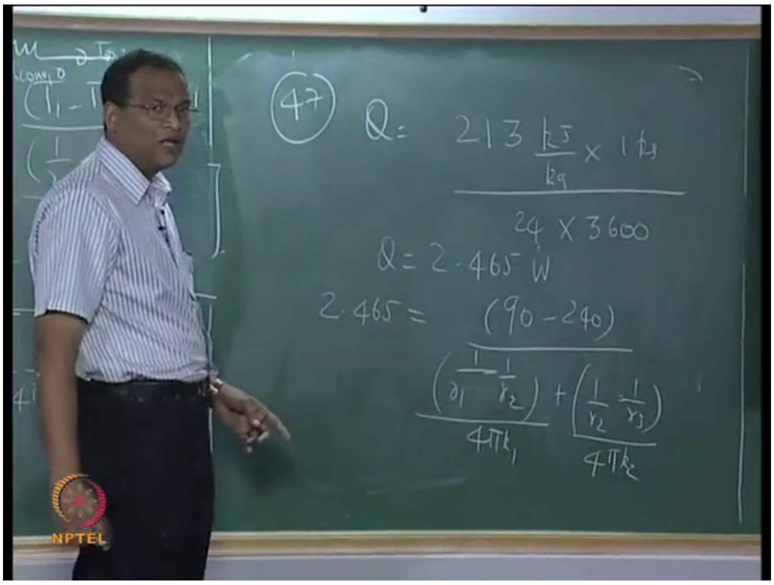

So, 47, so what will be the Q? So, 213 kilo joule per kg into 1 kg in 24 hours, please remember, it is in kilo joules in kg. So, therefore, Q will give you in kilo-watts. So, Q will be how much? 2.465 kilo-watt, why? 2.465.

Student: Watts Sir, (( ))

Watts o.k. So, this will be, I hope the values are meaning meaningful. Now, let us write the first, yeah, now tell me Vikram, what is the problem? This one, yeah, 2.465 too small, very small, then 213 kilo joule per kg should be all right, is not it, it look like reasonable, everything looks reasonable in this problem. So, may be insulation is not required, I do not know.

We have just completed.