Conduction and Radiation
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Lecture No. # 39
Transient conduction

So in today's class, we will look at some aspects of transient conduction. As you all know transient conduction is very important topic in conduction and heat transfer so in the last few classes we have been looking at the steady state temperature of systems whether it is a spherical tank storing liquid oxygen or there is a brick wall or there is a material with variable thermal conductivity or there is a nuclear fuel rod though it is generating heat is being taken away we are looking at steady state.

We are not looking at transients what happens when the nuclear fuel rod is generating heat but, enough cooling is not available what will be the temperature response of the nuclear fuel rod, these are also important. All the time we cannot expect system to be operating at steady state there will be a startup phase there will be a shutdown transients are the norm the generally you can you can see transients around, but basically if you look at the issue basically if you look at a complex power plant and all that we are interested only in the steady state operation that is fine, but there are startup and shutdown of a even for a data centre, laptop, computer, cell phone or your nuclear power plant these are all critical nuclear power plant for example, you know that the startup and shutdown are extremely critical.

So, the most complicated could be three dimensional hand steady could be a three dimensional hand steady conduction you know that we cannot solve it on the black board and all that so we have to use separation of variables and all that it will involve lot of mathematics and also if it is three dimensional unsteady and spherical or cylindrical coordinates is going to be very messy so, we will restrict our attention to Cartesian coordinates. We will start with a simple one dimensional transient conduction, but if it is one dimensional t is already a function of x I am also saying that its transient. Therefore, t is a function of time; therefore, already temperature is a function of more than one variable so temperature will be a function of two variables even in the simplest case, but the two variables will not be the spatial coordinates one will be the spatial coordinate one will be the time coordinate let us consider a simple case like that.

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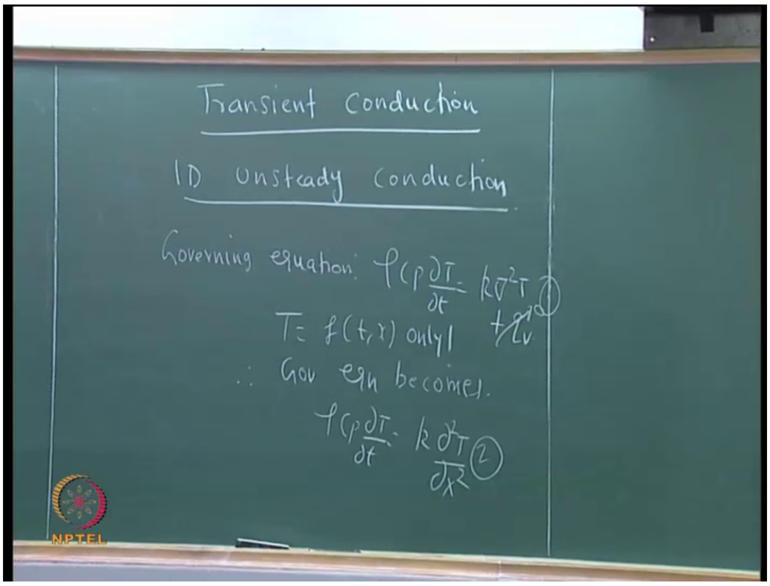

Now, so the governing equation, let us consider the situation in which q v equal to 0, I say that T is a function of t and x only. Therefore, I can say a governing equation even though T is a function only of x I am constrained to put dou square dou square d dou X square because T is a function of time also.

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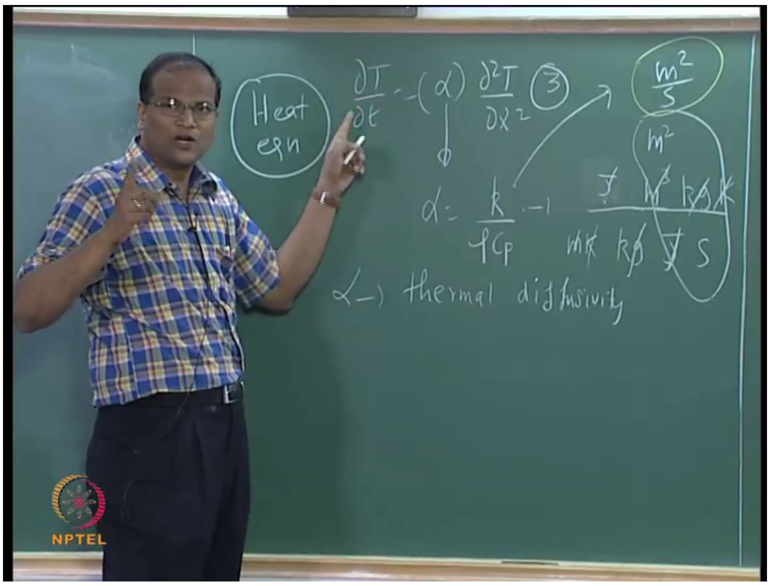

So let us call this one, we can rewrite this as so what should be units of this alpha, I wrote out the units of for k thermal conductivity rho density and C p specific heat. Deepak any problem? I will cut the k g and cut the Kelvin, so I have meter square and what is joules per second, so I have so alpha is called the thermal diffusivity this alpha makes its appearance only in transient conduction problems.

So in transient conduction problems, it is not the k which is a supreme property which is responsible for all the mischief, in steady state conduction the fin when we change from the aluminum to copper or copper to stainless steel you saw that the performance will change because m square is equal to h p by k everything will change steady state conduction thermal conductivity is king, but in unsteady it is thermal conductivity divided by rho into C p which has got the units of meter square per seconds so it gives you a rate at which the thermal disturbance propagates.

So we will look at the physical interpretation of alpha little later but, now it is sufficing to say that the alpha is a alpha is a parameter, is a quantity, is a physical, is a thermo physical property, which appears in the transient conduction equation, alpha is a property of the material, alpha for wood is different from alpha for steel and alpha for steel is different from alpha for aluminum, cooper and all that. So, if you are looking at some unsteady heat transfer and you want a design based on that then, you will chose your material appropriately mathematicians refer to this equation as the heat equation why did they call this heat equation because, this is one of the important equation they considered when they developed solutions analytical solutions by using the method of separation of variables. So mathematician used this equation they will use wave equation heat equation, so when they say heat equation they mean this.

So alpha alpha is a property of the medium and it changes. Now, there are several ways of attacking this, I can assume a product solution that T is a function of a solution which is a function of only x multiplied by a another solution which is a function only of T start reading dou by dou dou dou dou on dou squared everything and convert it into and then write the general solution then put the boundary conditions, and that is one way of looking at it. But, I thought I would because of the limited time because of the limited time if you get into that method of separation of variables these two classes will just go off. So, I thought we will use a different we will adopt a different tactic where we will look at we will go into the equation in depth and try to arrive at some meaningful understanding of this equations by looking at either the non dimensionalization or by looking at the scaling parameters and so on, only for a very simplified case where T is a function only of time which is called the lumped capacitance method we will get the solution and workout a problem.

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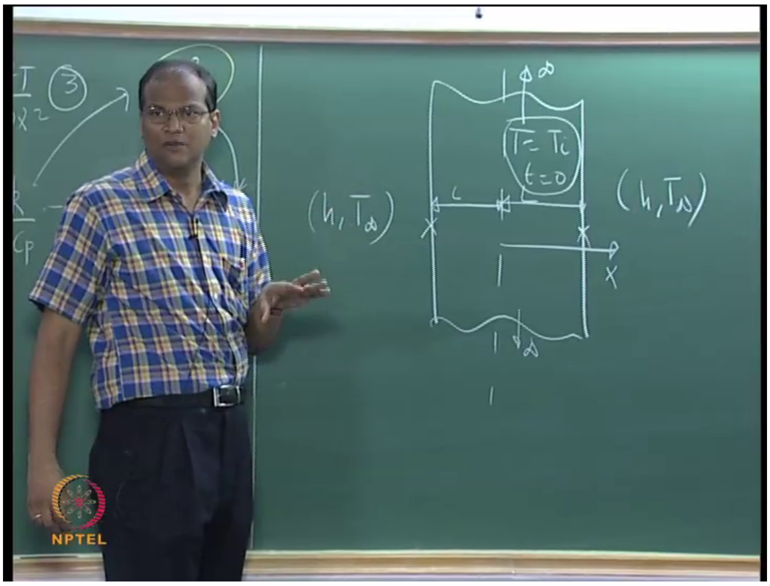

Now, let us look at some let us try to get some insides into this problem by considering a situation where I have a slab like this it is infinite in extent so this is a L and X starts from here. This slab is at a temperature of T equal to T i this slab is at a temperature of T equal to T i at time t equal to 0. Suddenly at time t equal to 0 this body is kept in is kept in a is pushed or immersed in a cold fluid which gives a heat transfer coefficient of h and there is a free stream temperature of T infinity, do not be under the impression that at x equal to L and X equal to minus l temperature is equal to T infinity, that is a separate story unless the heat transfer coefficient is infinity that would not happen. Now, there is a hot chalk piece I am putting it in cold water or cold oil the chalk piece will dissolve and I mean that is a do not do not let us not get into that now this body remains now it is undergoing cooling now as a heat transfer engineer at any at any point in the solid inside the solid if I tell the point and the time you should tell me the temperature that is a goal, this knowledge this cannot be answered with thermodynamics.

Thermodynamics does not do because it is equilibrium; it will say after everything is over it will say chalk piece will come to equilibrium with that the heat from the chalk piece will go into the water. What is a equilibrium temperature of water and chalk piece? Thermodynamics will tell you, but we have no patience we want to do heat treatment of this we want to pull this out after 20 minutes and do some other cooling and all that therefore, we need to know at a particular instant of time at a particular place what is a temperature, that is why we are doing all this.

Now, this story is clear then I have to tell that T i is greater than T infinity so this slab is cooled and you have symmetric boundary conditions on the boths on both sides now let start let us start working on the equation. This equation is first order in time and second order in X, so you require it supports two boundary conditions in x, it is first order in time it supports one condition in time the condition in time is called the initial condition because normally in these problems we will know what is a condition when you start at t equal to 0 what is the story at T greater than 0 at two points in x, you should tell the temperature or the information about the derivative of temperature or how the derivative of temperature is related to a flux or something that is there could be a convective flux or a radiative flux or a convective plus radiative flux.

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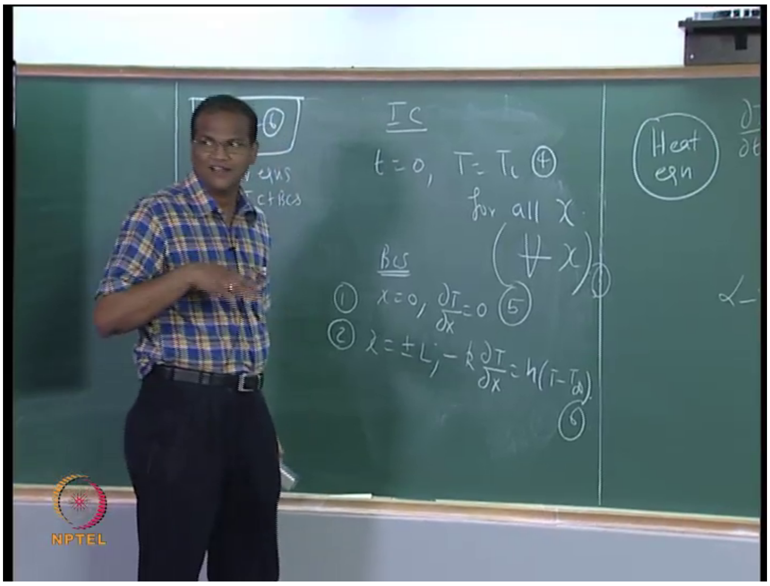

So initial condition how will mathematicians write for all x? Is it correct? Somebody will start some people may start shivering when you use the tremble but, we can as well wait for all x, fine now boundary conditions, one tell me.

Student: X equal to 0 dou T by dou X (( )).

Why why why why?

Student: Symmetric.

It is symmetric and you expect the temperature should be maximum at the center, why? Hot chalk piece is put in cold water; the center of the chalk piece for this coldness to penetrate the center it will take a longtime first the outer surface will get disturbed then, the layers will start getting disturbed progressively now so x equal to 0 dT by dX is equal to 0 what is the second boundary.

Convective boundary.

Yeah yeah yes yes what is that?

Student: X equal to l (( )).

X equal to l or.

Student: Plus minus l.

Plus minus l correct.

Student: (( )).

So this is 3, this is 4. Now, 3 to e 3 to 6 is called the governing equation plus initial condition plus boundary condition after you give 3 to 6, mathematician is enough, heat transfer engineer is not required so it is just a for them it is partial differential equation they will try to solve right. So once you write one initial condition two boundary conditions and the governing equation we say it is mathematically closed, c l o s e d, that is it satisfies that closure it requires all the information you you you have all the information required to solve the problem now if we I am still trying to dodge by not solving so we are trying to get some physical interpretation.

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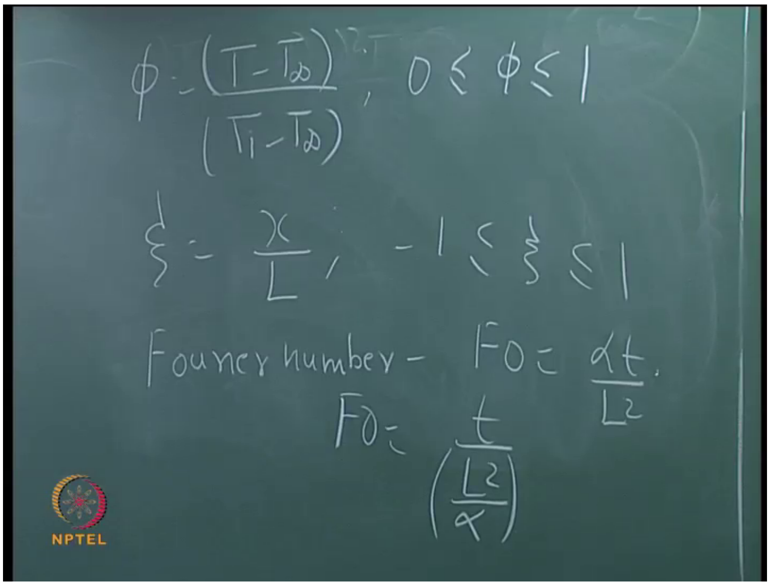

Now, let us try to do some non dimensionalization or normalization of the equations, so I define a new dimensionless temperature or dimensionless temperature excess which I call it as T minus T infinity by T i minus T what is the behind this what will be the minimum and maximum values of this phi.

Student: 0 (( )).

Correct. So, 0 less than equal to so when I write a computer program if all the temperatures are varying between 0 and one it is very good for me double precision maximum information I can retain otherwise if I have 300 points or something some 2 or 3 digits I am already losing fine. Now, xi equal to x by L so what are the values of xi in this problem correct x we have done this we have done, now I defined what is called the I define what is called the Fourier number is a product of the thermal diffusivity multiplied by time divided by L square when I write it like that alpha is meter square per second, time is second so meter square by meter square Fourier number is dimensionless so it is yet another dimensionless number in heat transfer.

So Joseph Fourier worked on it so we want to on so we have put this as Fourier number what is this Fourier number Fourier number is nothing, but the dimensionless time so time is that, so L square by alpha is the timescale for the problem L square by alpha is a timescale for the problem. Therefore, if you divide t by L square by alpha you will get something which is dimensionless, but which is indicative of the temperature because L and alpha are already fixed for the chalk piece you know what is the alpha you know the dimension so only the t is changing so it gives you information on the time. Now with this we will with this we will now normalize or non dimensionalize the governing equations.

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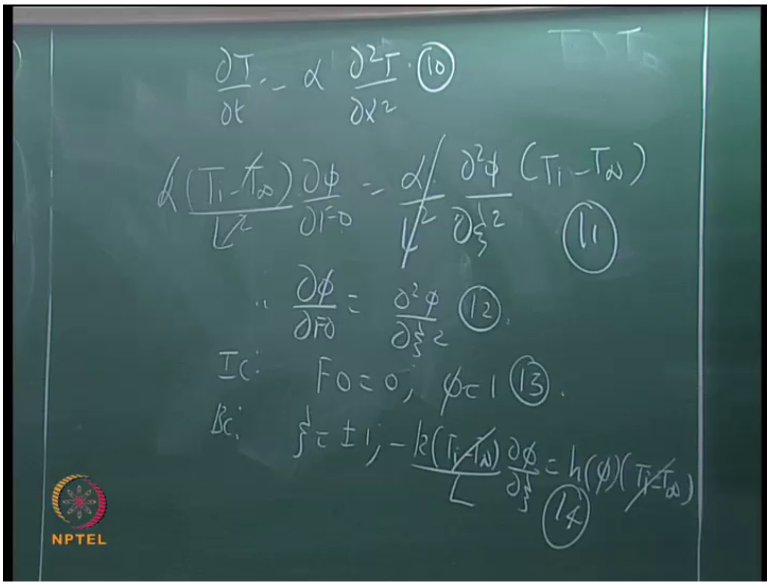

So the governing equation is basically from T i have to go to phi, so this will be just is that correct yeah please check whether it is d of T and d of T minus T infinity is the same because d of T infinity is 0 when you are taking the differential you can subtract add or whatever so long as the other one is a constant so what does what does this equation become now? So, this is 7, 8, 9; this is again the same thing 10. Therefore, the governing equation has become like this what about initial condition time equal to 0 means Fourier number equal to 0 phi equal to.

Student: 0 phi equal to 1.

1 phi equal to 1 now boundary condition is it so you can knockoff these fellows, what under the (( )) yeah yeah we will do that so this one is xi equal to plus or minus 1 yeah Vikram, what is that xi equal to 0 so is that correct, now this h L by k is called the Biot number.

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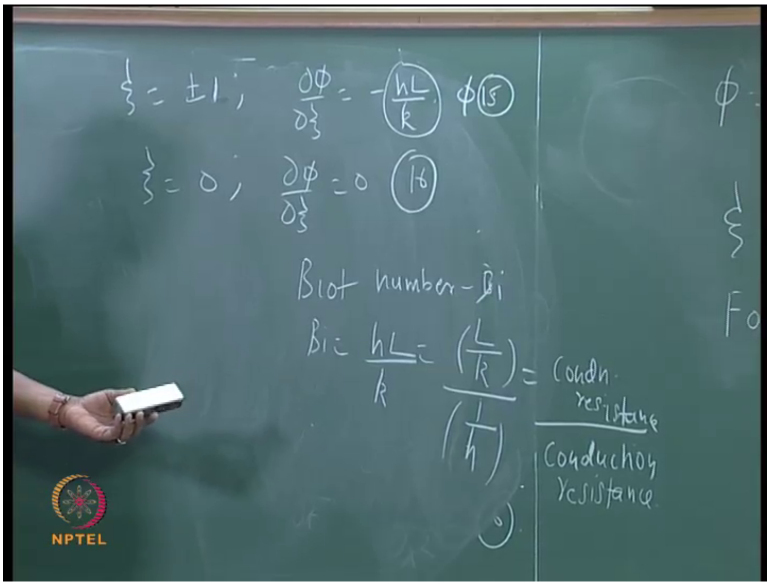

So this h L by k, why am I writing like this? When we solve the one dimensional steady state conduction equation for the wall, what was L by k?

Student: (( )).

Resistance 1 by h.

Student: (( )).

Resistance l by k is the resistance for.

Student: Conduction.

Conduction 1 by h is the resistance for.

Conduction therefore, h L by k is actually the conduction resistance by convection resistance now, we will take an we will take a detour for a few minutes and then we will come back yeah yeah yeah yeah.

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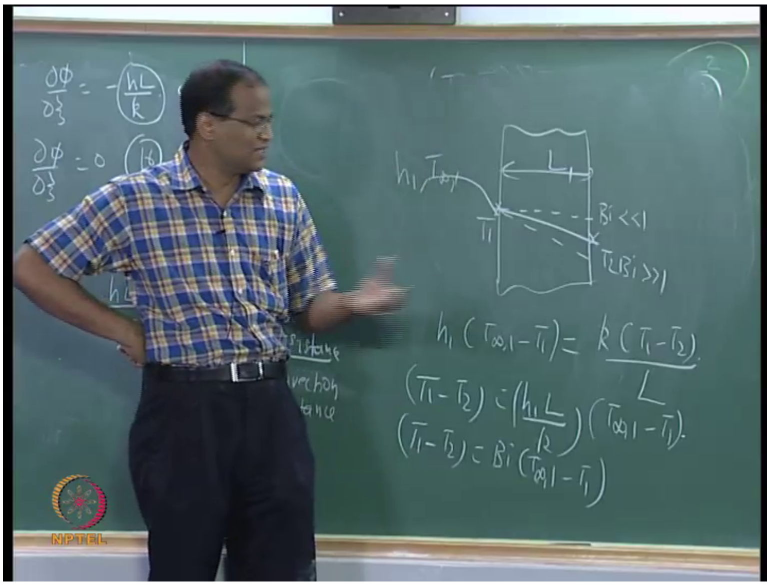

Now, we will take a short detour for a few minutes to get this concept better let us take this let us take this example where we have h 1 T infinity this T 1 and this T 2 the some other solid with L 1 so what do we do here so we said h 1 into, is it correct? Now correct, you must put a title for this you will not you should not get confused with the transient conduction I am doing something else here, so T 1 minus T 2 therefore, if the Biot number is very small which could mean that the thermal conductivity of the wall is very high and the Biot number is very small. Therefore, the right hand side is negligibly small therefore, T 1 minus T 2 will be very gentle, if the thermal conductivity is very poor to accomplish the same heat transfer rate is very poor the Biot number is very high.

Therefore, T 1 minus T 2 will be substantial therefore, so the Biot number lets us decide whether there are temperature gradiance within the solid if the Biot number is very very small. Therefore, we are at liberty to assume that the whole solid is at one temperature which is called the lumped formulation or lumped capacitance method, granted no no heat transfer will take place without a temperature gradient but, for a finite flux q, q is equal to k a dT by dX the k is so high that for a finite q dT by dX is negligibly small in relation to what? In relation to the temperature drop which is required for accomplishing the convective heat transfer from the stream of the fluid to the solid.

So compared to T infinity 1 minus T 1 T 1 minus T 2 is substantially small therefore, in relation to this we need not worry about this therefore, if you have a situation where Biot number is much much less than one then this system is dominated completely by the surface resistance of the convection resistance which happens in the interface between the solid and the fluid. On the other hand, if you have a situation where you have a plastic or an extremely poor conductor then the given that you have a situation where there is a conduction and convection coupling or conjugate heat transfer which is taking place, this the controlling resistance will be the large conduction resistance or the poor conductivity of the solid under question. Therefore, you cannot ignore the gradience within the medium is that, therefore, Biot number less than 0.1 T.

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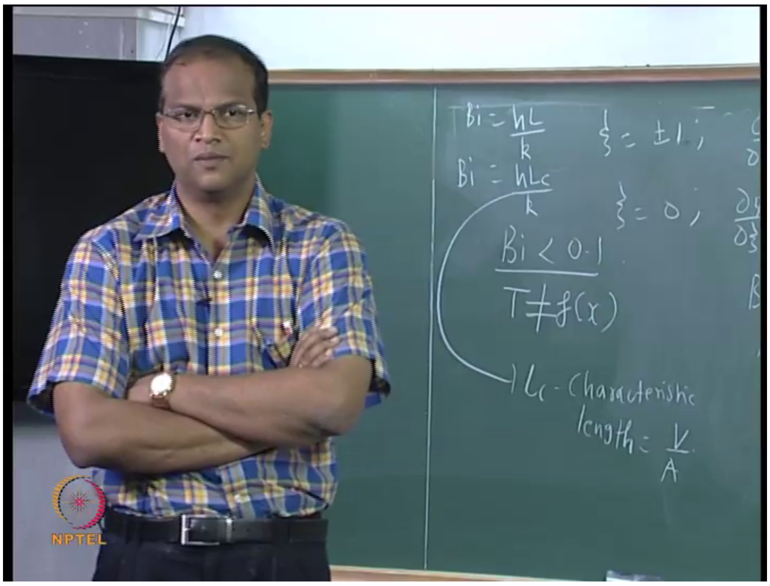

So, if I give you a transient conduction, if you encounter a transient conduction problem the first step is to calculate the Biot number if the Biot number is less than 0.1, you can go ahead and convert it into a problem where there is only one temperature within the whole solid now this Biot number a h L by k in this case h L was clear otherwise you have what is called h L c by k, L c is the characteristic length equal to V by A. I hope you get L; I do not I hope you do not get 2 L in this problem if you do V by A.

So, if it is a sphere 4 by 3 pi r cubed divided by 4 pi r squared L c will be r by 3, so take the radius calculate r by 3 get the thermal conductivity of the sphere heat transfer coefficient will be given to you calculate h L by k if it is greater than 0.1 then lumped capacitance will not work, if less than 0.1 then you can use the lumped capacitance what is this lumped capacitance we will we will talk about it a little later.

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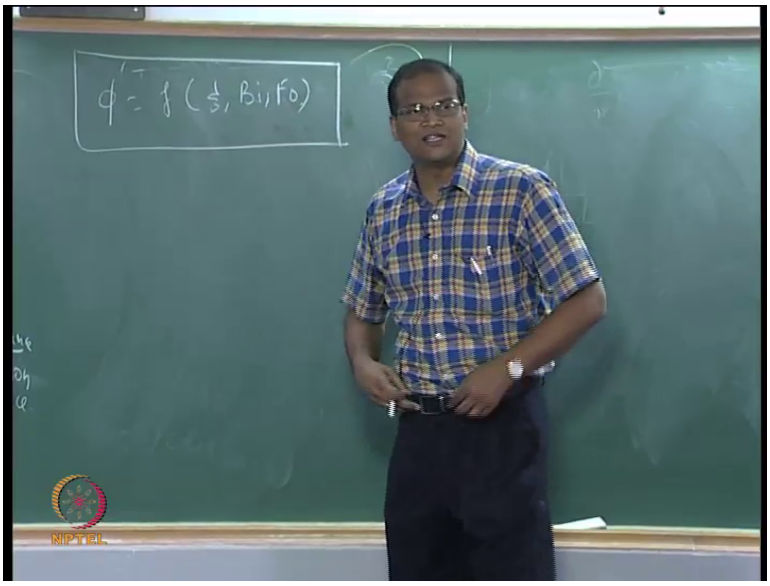

So this is to explain the concept of Biot number what it means is what it now means is you have a xi you have a Biot number you have a Fourier number. Therefore, p is the function of xi Biot number so this is the result we got by using by doing a normalization or non dimensionalization of the governing equations. This is considered as standard practice in fluid mechanics and heat transfer there is yet another way of getting these three parameters, what is that method?

Student: (( )).

You can use the Buckingham pi theorem you can use the Buckingham pi theorem and get the same thing however it is easier to do it with non dimensionalization also. The temperature dimensionless temperature will be function of the dimensionalized distance that is the way it should be because this gives the conduction convection coupling this gives the time and this gives the space. So, temperature is a function of the spatial coordinate the time coordinate and the coupling between the conduction and convection then these equations that we mathematically solved and all that, but now let us look at the physical interpretation of this what is actually happening in the transient conduction .

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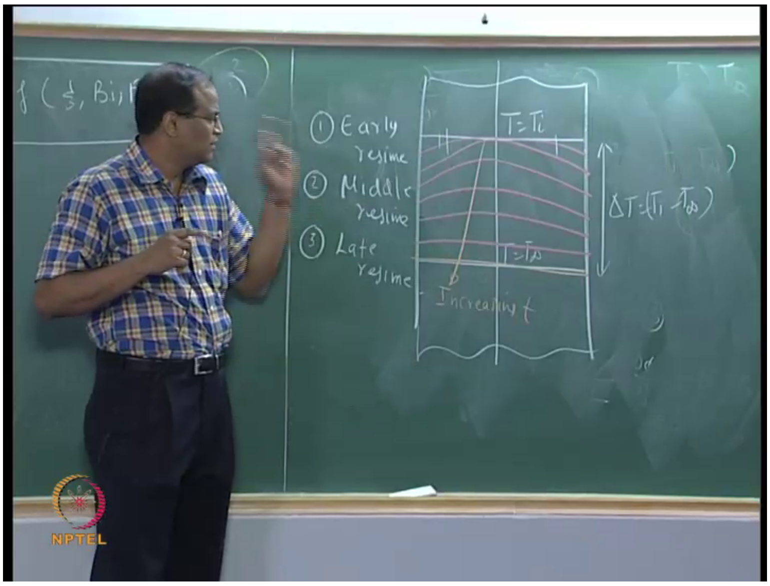

So this is the slab I am looking at the temperature levels initially T equal to T I everywhere, correct? After everything after everything goes after everything is completed what will be the temperature everywhere?

T equal to (( )).

T equal to T infinity therefore, this delta T, T i minus T infinity is the maximum temperature difference in the problem all the temperatures have to lie between T i and T infinity, agree?

Now initially, you have T equal to T i then what will happen is after some time the system will behave like this, after sometime this, correct? So what is this? There are three regimes there are three regimes for the conduction transient conduction there is an initial regime where there is a portion of this slab which does not know that there is a disturbance which has taken place at x equal to plus L and x equal to minus L. So, that is the early regime, there is a middle regime in which at all points in the domain the effect of the disturbance at the wall is felt that is this temperature is coming doun this temperature is coming doun of course. The temperatures are coming doun at different rates depending upon the governing equation what the equation tells you that is called the middle regime.

Then we have a late regime, watch the late regime in the late regime the temperatures have already become very small. All the temperatures are tending to approach T infinity therefore, the gradients within the medium are also very small this is what corresponds to Biot number much less than 0.1, so that is the late regime case.

So simplifications are possible in regime one and regime three simplifications are possible in regime one because all the solid is not taking part in the heat transfer activity. In regime three, the complete solid is taking place is a taking part in the heat transfer activity, but the gradients of temperatures are very small because the body itself is sluggishly approaching towards T infinity. Therefore, you do not worry about the temperature gradients within the body. Therefore, simplifications are possible to the governing equation in regimes one and regime three, but in regime two you have to solve the full transient conduction equation.

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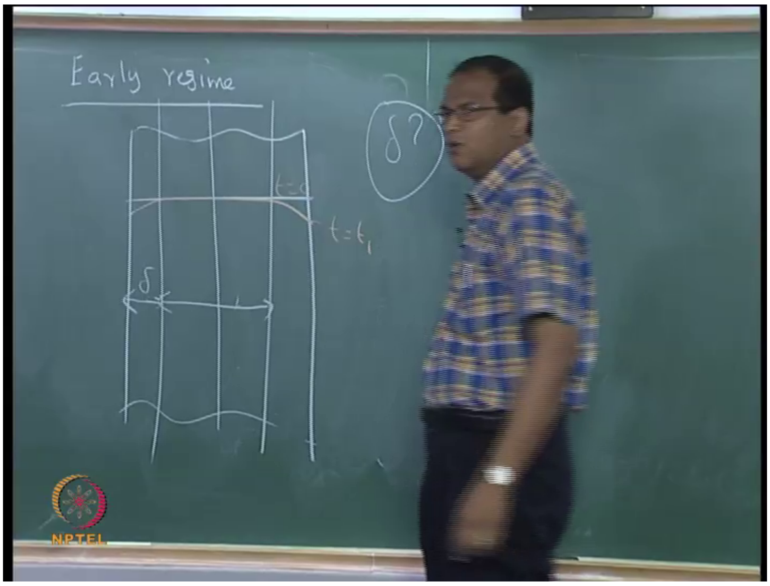

Let us look at the early regime now so after sometime the temperature is this is t equal to 0 this is t equal to t 1 what about this region? This region which have enclosed by the two arrows is completely ignorant of the presence of a disturbance that is some cooling has taken place at x equal to plus or minus l this region this is the region where the effect of the disturbance is felt.

Therefore, at a particular time t is equal to t 1 if you try to find out this region where the thermal disturbance is felt and this thickness if we call it as delta that is the penetration depth that is the depth of penetration of the thermal disturbance from the surface how does this penetration depth go how does it scale as are there some. Can we know more about this delta?

So for this we have to do what is called scaling analysis, what is this scaling analysis? The governing equation is replaced by approximate terms.

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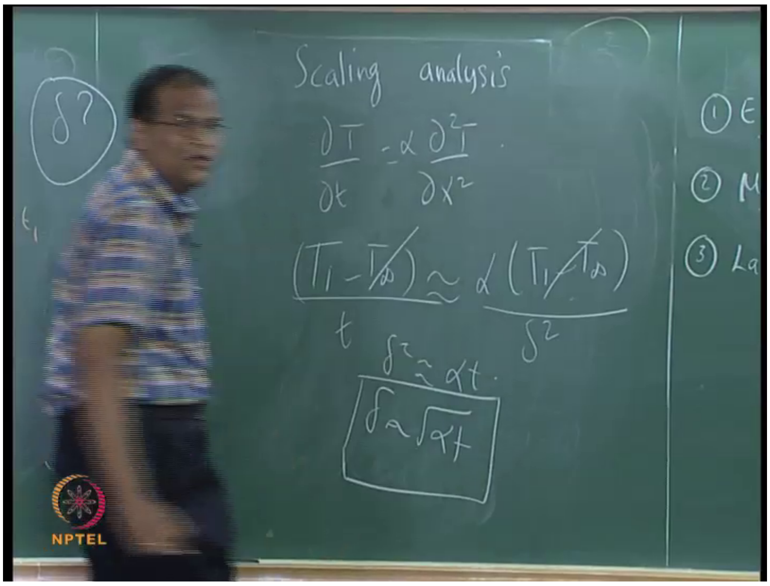

So, I say what is a good scale for delta T in the problem maximum delta T what is T I minus let us say we are talking about a time T compare to time T equal to 0 therefore, delta T is T minus 0 it is T. Now, I am doing an approximate this thing therefore, I will put approximately goes equals alpha so in scaling analysis. I am trying to replace the governing equation by an approximate relationship which does not involve the derivative terns so that I get a quick understanding of the various terms involved delta x.

Now, what is the delta x is the length scale what is the length scale? Now, for the penetration delta delta square is that what is what is it what is it that I am trying to do? I am trying to get an estimate of delta, so please remember that if once I equation is given you do not have to go to the computer straight away, so much of work is possible before going to the computer.

Now, if I do this very interesting, delta square goes as delta goes as root of alpha T so, the penetration depth goes as root of alpha into time. It is intuitively apparent that as the time increases the penetration depth will increase. As the alpha increases that is why if some or is boiling and there is a ladle you touch the stainless steel spoon immediately within a few seconds you will feel that, but if you put a wooden spoon it will take a long time for you to (( )) there it is not the k that is a alpha whether at the place where you have touch that disturbance has propagated that is from the burner it has come to the or it has come to the ladle, and I can modulate the ladle I can modulate as the one dimensional fin Daniel Dennis book, heat transfer book if people are interested you can do that.

So with that handle you can modulate as a one dimensional annular fin or some fin or something then you can find out exactly transient fin at what time you will you will feel hot is it so this is so delta goes as root of alpha T between two materials if you want to find out the penetration depth delta 2 by delta 1 for the same time will be root of alpha 2 by alpha 1 we can work out problems. On this all that is separate story, but most important thing I because I am because I got delta as root of alpha t.

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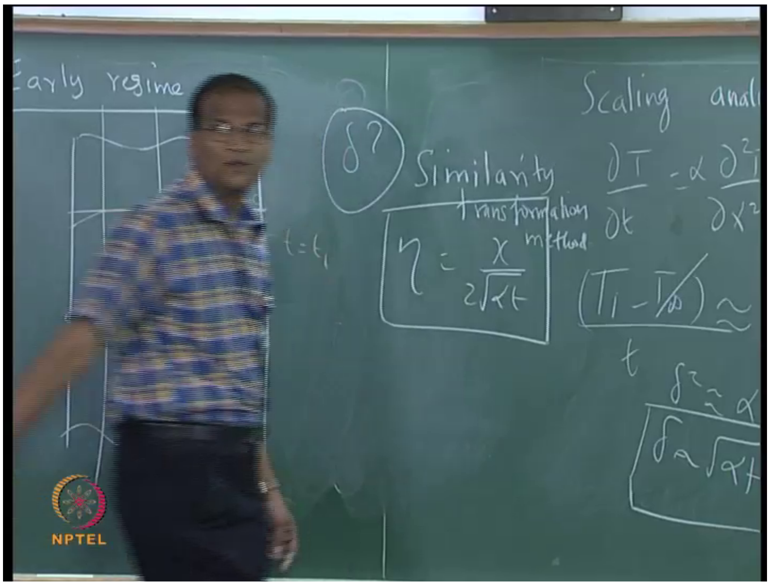

Suppose I introduce what is called I know from previous experience you may ask me sir why did you choose 2? I can choose it as a and then I can find out a that is as formal procedure. Now I can introduce an eta which is x by 2 root alpha t because root of alpha t comes from a scaling analysis, then this eta; eta is basically dimensionless correct by the introduction of eta it is possible for me to convert the original partial differential equation into an ordinary differential equation and I proceed.

So if you use this for the early regime this is called the similarity transformation method minus.

Why correct minus?

Student: And there is a alpha (( )) the last equation.

What what what?

Student: (( )).

No no no why alpha should some here and finite?

Student: Just differential (( )).

Alpha is coming in the original therefore, yeah Pavan. Now the alpha will come d square T by, so what do you get dT by T eta is whether 1 by 2 alpha t you can be taken out you can just write it as d square T by d eta square hand side correct.

Now you can write is not it?

Student: (( )).

What you want to do that I will have to proceed to the next proceed to the next topic I do not have time to finish this off, but what lapse in is I know for sure that the PD is converted to OD. So now, let us leave the mathematics let us get back to the physics T equal to 0 or T equal to infinity when T equal to infinity eta tends to correct T equal to infinity is mathematically as far as this thing is concerned T equal to infinity corresponds to eta equal to 0 x equal to 0 also corresponds to eta equal to 0.

So by the combination of x by 2 root alpha t when eta equal to 0 or eta is equal to infinity you can keep the T constant and see what is happening with respect to x or you can keep x constant and see what is happening with respect to T therefore, by the combination of x by by by a very intelligent combination of x and T we can put x by 2 root alpha T such that we get one parameter eta called the similarity parameter, similarity variable, which will tell the whole story of the transient conduction in this law you will get d square T by it will result in what is called the error function the solution will result in what is called the error function complimentary error function and so on. I do not have time to discuss that.

Now, we will go to the late regime early regime,. so it is possible to do some simplification where you can introduce a similarity transformation and then why are we doing all these struggle because our belief is solving ODE simpler compared to solving PDE. So, if that belief is true then it is worth doing this otherwise you go to mat lab and solve it is your PDE solver and for the early regime. It is not required for the late regime.

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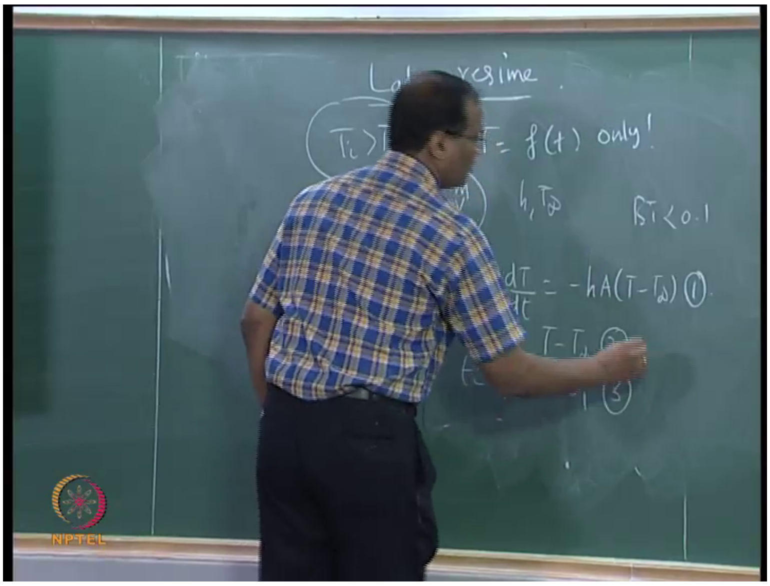

So late regime so consider an object which has a mass m volume c p it is h n T infinity it is at a temperature T and it is cooling initially T i it is lumped because Biot number is much much Biot number is less than point one because of which I can write which is the governing equation for the problem I can define a theta. Since we are lumping, it we cannot start from the original governing equation you have to get the governing equation from first principles, I already told you the same logic we use for getting the fin equation.

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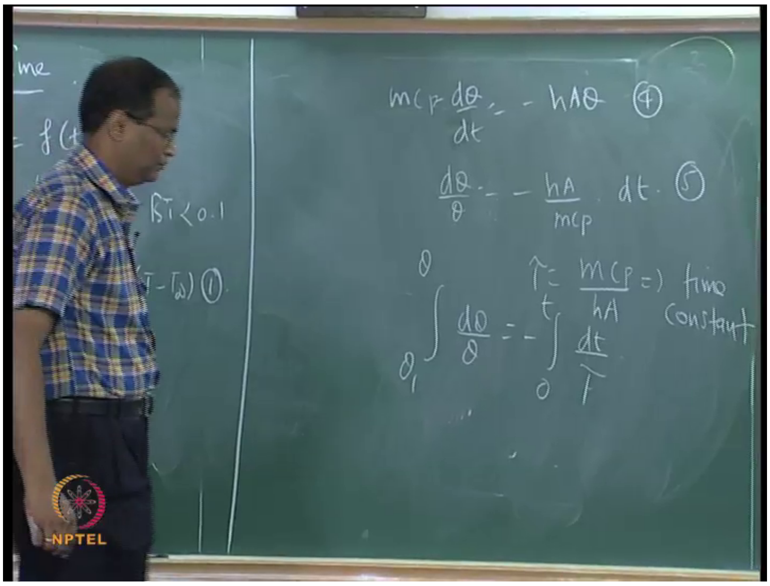

So we can say, what are the units of m c p by h A?

Student: (( )).

It has got the units of time so it is basically it gives an idea what is the scale for time in this problem so it is called the tau the time constant now we can integrate, now is it no dT by tau correct?

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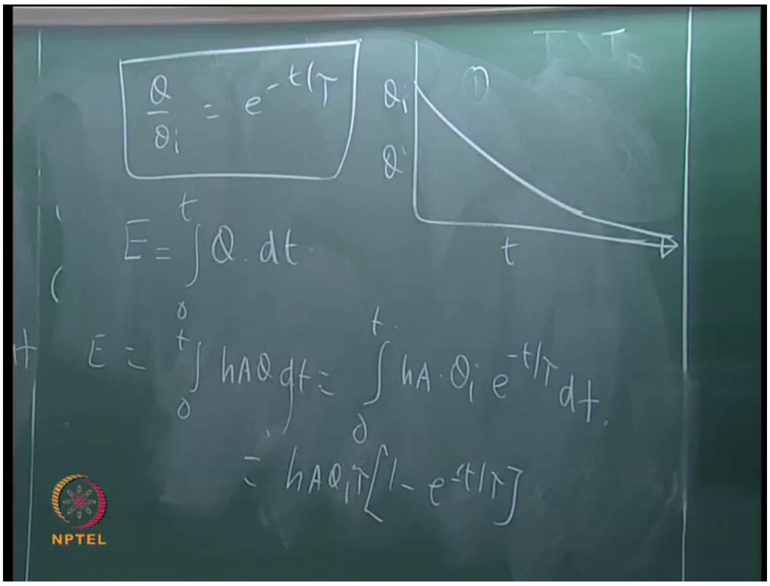

So, we get… So for the lump capacitance system for cooling so the temperature distribution is exponential.

So this theta will asymptotically tend to 0 as time T tends to infinity. So, if you want to do a heat treatment you are having a stainless steel bearing or bowl barring or whatever, then if you know the initial temperature at which it is heated and you want to know and you want to pull it out at the end of you want to pull it out when the temperature reaches 400 Kelvin if you in or 500 Kelvin, if you know that the bowl barring can be assumed to be lumped that is specially lump then you know all the properties of the stainless steel thermal conductivity c p h will be given you first calculate the time constant left side theta by theta I, you will know then you will find out the total time. So in your actual process, when that time is gone you will pull out the ball bearing and continue your heat treatment process, so you are able to get handle on this so this will work only the Biot number is less than point one the first step is to check whether the Biot number is less than point.

Now, what is the total energy which is transferred? Will be? What is the total heat which is lost from the body in a time? In a time T, so that will be q into d T so this will be in joules or kilo joules.

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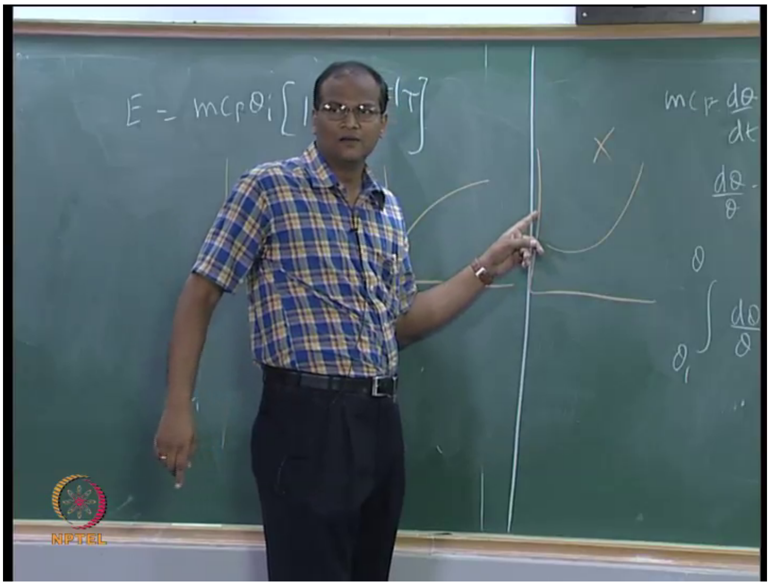

So this will be can I, so this will be this will be, please look at this expression theta please look at this expression that can be easily derived theta is initial temperature access that is T minus T T i minus T infinity m C p theta i, i was the original enthalpy access of the body with respect to the surroundings at time T equal to 0 e to the power of minus 0 is 1 so there is no energy which has been transferred. When sufficient time elapse such that the T approaches infinity e to the power of minus T by tau will become 0 this will become 1. So energy trans will be m C p theta I so all the initial temperature access the body enthalpy access is dissipated to the surroundings. So e takes on a value between 0 and m C p theta i, so from the e itself you can find out what will be the average temperature during the of the body during the cooling process? The average temperature is the temperature itself but, this average temperature is an important concept if (( )) also the temperature is changing.

Now, I can complicate the problem by adding a heat generation term it is heating it is heating the body is heating; for example, this was the case when you are with your immersion heater problem in a immersion heater problem are you getting the point m C p dT by d tau equal to h A theta that is minus plus q which is the electrical input. So there will be a phase where the heating will take place, then there will be steady state then when you switch off the heater cooling will take place.

So, you have to appropriately model the governing equation. Is it clear? So whenever you have a cooling curve it will be like this whenever you have a heating curve; it will be like this do not ever make a curve like this; this is dy by dx d square by d q everything is positive this is called an exponential curve exponential growth rate is unsustainable no engineering system can even the growth of bacteria and all that which we say that it depends on the current population, there is a finiteness in a amount of nutrients which is available so that will stabilize after sometime. So, as an engineer you should not that fatal mistake a to the power of we should not casually sketch a curve, like that some people like me catch fine.