Conduction and Radiation
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Module No # 01
Lecture No # 04
Tutorial: Radiation pressure and radiation energy density

Good morning. So, we will continue with the one more problem on solid angle, then we will go on to radiation pressure so, you can just hold on to that you do not have to copy that now I will give you time. So, problem number four, determine the fraction of the total hemispherical emissive power within brackets you can write E in watts per meter square will make it clear.

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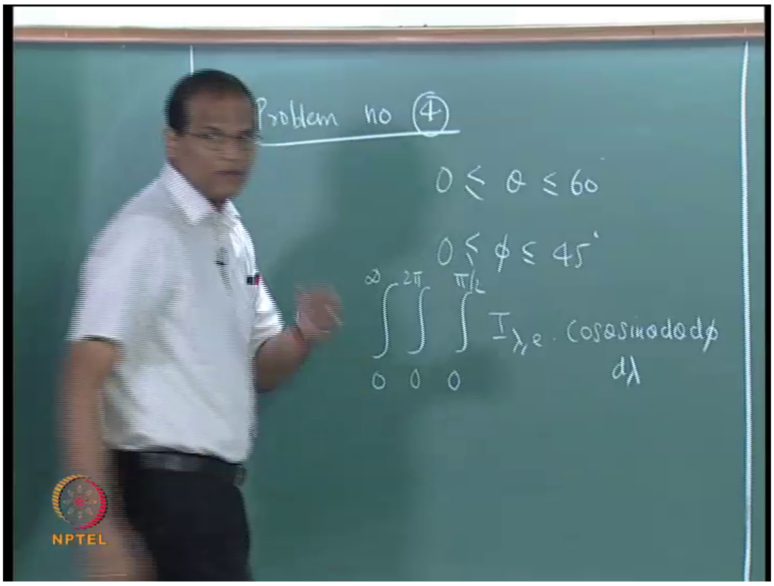

Leaving a diffuse radiator, in the directions (no voice from 01:12 to 01:24) 0 less than equal to theta less than equal to 60 degrees and 0 less than equal to phi less than equal to 45 degrees. Can you solve please go ahead.

So, I lambda E can be taken out, I can be taken out. What is this one? 45 is phi by 4. Is it Vikram? What is that cos theta sin theta sin 2 theta by 2 minus cos 2 theta by 2. How much is it?

Student: (( ))

No, tell me that into.

Student: (( ))

Into.

Student: (( ))

This is 3 by 8

Student: By 32.

So, therefore, E fraction by E 3 by 32.

0 to 60 how much of what is the fraction as opposes 0 to 90 you appreciate the logic what is that fraction?

Student: 1 by 12.

But do not do some shortcuts like 60 degrees divided by 90 degrees into 45 degrees divided by 360 degree (( )) that sine theta cos theta d theta that is because of the area. So, it is a non linear function, it is a trigonometric function, it is a function of sine theta cos theta. Therefore, if u wants to find out, what is the radiation going out in a particular direction? That is no other shortcut sine theta cos theta d theta d phi, you have to integrate. Fortunately there is a diffuse radiator, suppose a give you a problem in with I lambda e, I lambda e is equal to 3.5 cos theta into sine 5 minus 4.610 theta into cos squared fee something like that. You are sung, you will keep on all for half an hour, but whether such a surface exists is another matter.

But as I can always give some problem like that and give trouble to you. So, what I am saying is? If it is diffuse the story is very nice and if it is azimuthally independent lot of bother is solved, because 2 pi you can take out generally it is a case. But zenith angle depends usually be there particularly for solar applications are all that, that is it be a function of theta. So, during this course somewhere down the line, we will try to seep look at problems where theta, where I is also function of theta. So, that you get some exposure you get some experience in solving problems in which the directional dependence is not completely wiped out.

So, these are basically the preliminaries, the basic grammar which is required to understand radiation properties of the black body. Now, we have set out a explore. What the die is? Yesterday we asked that question. So, it makes a question, what is I? We do not know if I is given, if it is diffuse, we can do all these things. You can find out, what is the fraction or between 2 bodies, if e is given and if it is diffuse I can put the E equal to pi I b I can do all that and find out. What is the flux? Which is coming on the second surface, if you know what is the emulsion from first surface and so on.

But suppose I know only the temperature of the first surface from the temperature. How do I get e or I? Something I do not know as it is comes from basic radiation laws which we will have to figure out. So, the first important concept in that is the concept of radiation pressure. Radiation also exerts a pressure this was something which was unknown till 1876. There was known for several this is known before also, but it was the Italian physicists a Bartoli in 1876, who first proposed a thought experiment to prove the existence of radiation pressure.

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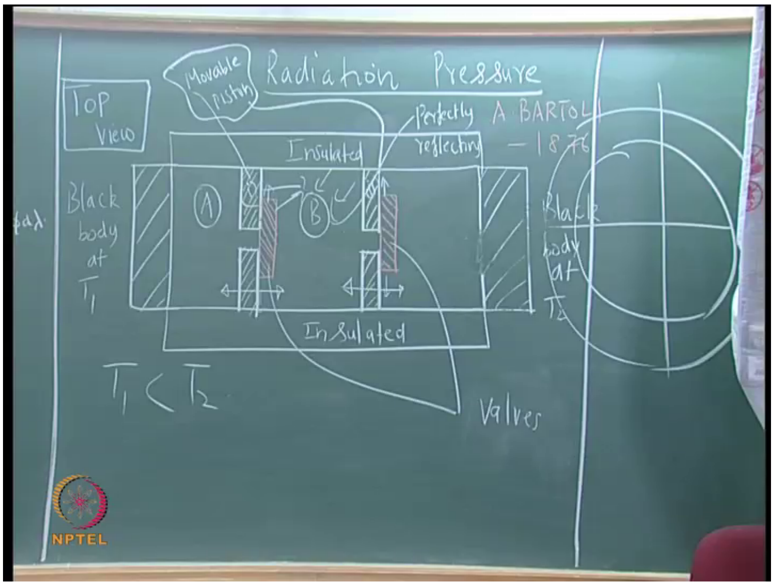

So, his apparatus is like this. So, this is A. So, this is a Bartoli is thought experiment, he is an Italian physicist this is the top view. So, please remember. So, this at top view of the Bartoli is thought experiment, it is a cylindrical chamber. So, it is a cylindrical chamber it is insulated on it is periphery so, it is like this. So, I am not allowed to go, but anyway I think you can take it. So, the side view is like this (( )) the side view will not come next to the top fusion and anyway. So, I am trying to say that it is like this if you viewed from the side it is an, it is a cylinder forget about that.

So, the will forgive me what he is listening. So, the story is like this so, you have a cylinder, you have vacuum inside not necessary vacuum, we will see it is filled with radiation corresponding to radiation energy density, which corresponds to black body at a particular temperature. Let us come to the story now this is the cylindrical chamber it is insulated on both side. So, that there is no convection or any other heat loss, it is attached to a black body on the left hand side, the black body sets a temperature T 1. It is attached to a black body at T 2 on the right hand side T 1 is less than T 2.

There are 2 movable pistons 1 and 2, which are contained in this cylindrical enclosure. this movable piston have a small gap in between there is a small gap in between which can be covered or opened by sliding valves. That means, the vales can be slid open, that is this vales can go up and down, this piston can move the pistons of frictionless and they are perfectly reflecting, all these are perfectly reflecting. he never made this experimental setup.

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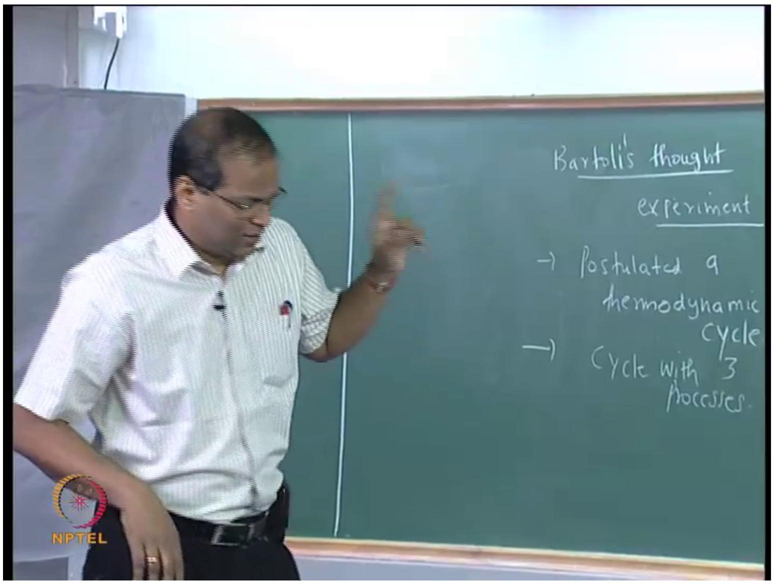

But he proposed it is thermodynamically correct so, we accept it as truth. So, he postulated a cycle thermodynamic cycle. So, cycle… now let us consider same this four this coordinate system and dA1 will haunt you throughout the semester. r theta dA it is emitting area.

I lambda e into. What is that story? D Q lambda theta phi is equal to I lambda e (( )) d A cos theta d omega (( )) so, d lambda there is nothing new. No that we can worry later radiant power passing through we can it is a radiation energy density corresponding to that (( )) if you want you can take it like that, but radiant power on it radiant power is coming onto it. Now, what is the net momentum flux passing through dA. What is a net momentum flux?

What are the units of the radian power? Watts no problem right. What the units of this? Area is d A speed is the speed which radiation propagates, what is unit of the (( )) meter cube per second. I have power in the Newton in the numerator (( )) do not what will be the unit? Watt per meter squared meta second. What is the watt second? Joules per meter cube. Thermodynamically joule represents what (( )) energy divided by meter cube can represent (( )) energy density. How can I arrange joule in terms of Newton, what is this? Newton per meter square is what? Pressure .And who proposes radiation as a pressure? Bartoli.

So, all these who called croup their trying to get the expression for radiation pressure do not worry about everything is correct, we can be verified by experiment. Because you are a getting expose to this entire concept for the first time it is difficult for you to understand, but it is all correct. So, the net momentum flux can be figured out by taking the radiant power and dividing it by area into the speed. Can you do that now? So, can I say d sine of 2, you are convinced it is got the units of joules per meter cube or Newton per meter square. So, if you proceed along this direction we should get an expression for radiation pressure that is the goal of this exercise.

The title is radiation pressure for this subsection. So, by doing this mathematical derivation by going through these steps we should be in a position to get an expression for a radiation pressure. What are we trying to do in this? (( )) or at least relate I to pressure see it is something like I is the only thing I know, but with I am not able to proceed further through my fundamental geometry like solid geometry and definitions of spherical coordinates system. If you give I can find so, many things. What is how much will the second surface it is see? What is orientation? How much will be the fraction of energy in a particular direction? All those things I can find out.

But given a temperature what is I that question is not answered that is more fundamental. So, many people so many physicists the last 150 years 1850 from 1800 to or 1800 to 1920 they were trying to find some expression for I. Some thought by just using plain thermodynamics they can get. some people thought by using classical physics that is kinetic energy kinetic theory of gas and all that they can get. But none of these worked out and eventually could new hypothesis called quantum hypothesis has to be figured out. Now, we are seeing the first part there are three part of the story a b c, a is based on thermodynamics where, b is based on classical theory and c is the actual quantum hypothesis.

As far as, a is concerned you can get to a position where you can say I is proportional to the power of something that is all. But how various with lambda cannot be obtained from it is beyond the scope of thermodynamics, but to put things in perspective we have to proceed from thermodynamics. So, basically we are trying to get an expression for I and if I know I can do all these calculations. But now, I have no way to I have to start somewhere, suppose thermodynamics I feel the thermodynamics is the tool with which everything can be solved. If I am able to relate I to energy density I to the pressure and all that then I will assume radiation to be an ideal gas.

If I assume radiation to be an ideal gas then I use my Maxwell relation t d s relations and I can get some. That is the approach people have taken that is what we are trying to conical now whatever those. So, many grate physicals their lifetimes work, we are doing in 20 minutes 30 minutes 50 minutes. Now, so my goal is left hand side is what are the units Newton per meter square pressure, right hand side I am getting d Q, but already in the expression 1 d Q is written in terms of I. So, if a complete this exercise I will say that radiation pressure is some function of I, if I know that I will keep it.

Then, what is radiation energy density? As a function of I will keep it, I will say that radiation energy density is like any other energy density or internal energy. Now, if I have expression for both u and p then I will go to the expression t d s is du plus p d v. And then I will use my all thermodynamic, I will manipulate and then tried to get something so, that is a goal. So, first we have to get the expression or I can I need not say all this suppose I say p is equal to 4 pi I b by 3 or something then you will you all be flummoxed, you would not know how this expressions have come.

Therefore, this becomes innavetiple, these derivations are innavetiplethis, is expression number 2. What is a component normal to the radiation? Component normal to d A right that is very important. Is not it? What is a component which is normal to d A? Because it is going in this direction. Is it not? Or it is coming in the direction. What is the direction, what is the component normal to d A? There will be a cos theta.

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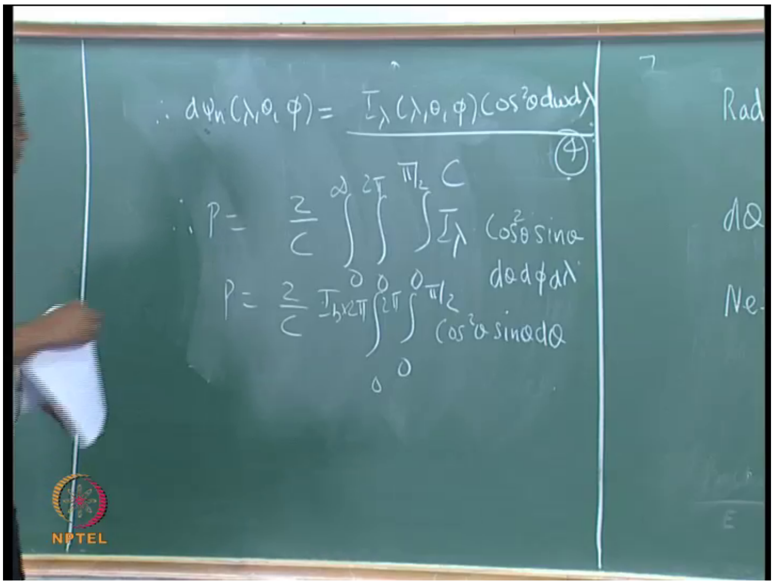

Therefore, d syn cos theta. d A here cancelled because I divided radian power divided by C into dA. So, did he follow this argument so, if you have to get the net pressure this is the elemental rate of momentum flux. This has to be this has undergo 3 integrations 1 integration respect to theta, 1 integration respect to phi and 1 integration respect to lambda. But if you do that this answer is only half correct, because I am saying that it is a perfect reflector whatever is coming and whatever is going out. If you want to find out what is the net momentum change incoming minus outgoing, incoming is x outgoing is minus x so, it is x minus x it will be equal to 2 x, that is given by this.

Now, it is for a black body the P, yes sure cos square. Now, this will be given by P by 2 C and we can do the, for the black body we can do the I b 0 to 2 pi again do the 2 pi also cos square theta. So, people who are not able to I have done several things between 4 and 5 I have done several things. First I have gotten greed of integrations respect to lambda by saying that I lambda d lambda integral it is just replaced by I b, I b is not a function of theta b because we are assuming everything in black body so, that is pulled out point number 2. Second I already did my integration with respect to d 5 because I is anyway independent I multiplied 2 pi. So, I done an integration respect to pi I have done integration respect to lambda.

The only thing this fellow is still now I have to remove this right I have to do this integrate respect to theta, can you do that?

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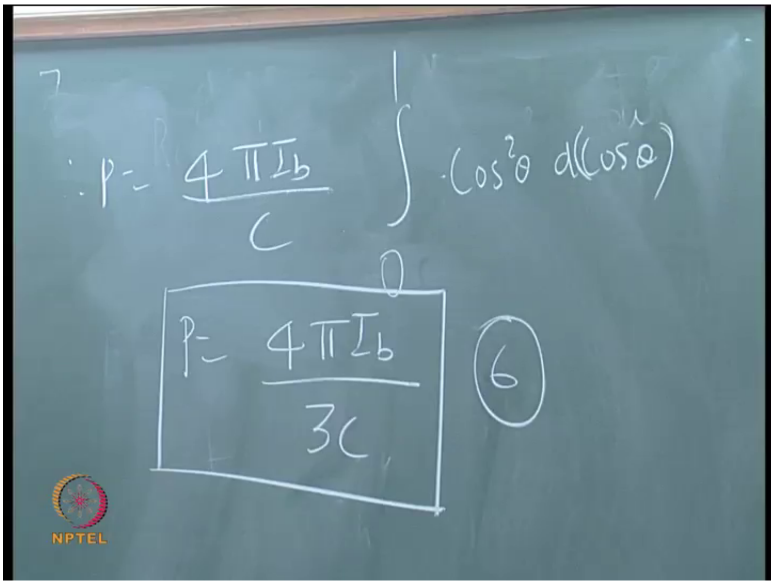

The first 1 d A cos theta we already learned that will come here no there is no problem, what I have written it will be a cos theta component we cannot divided by cos theta. When do you divided by cos theta? (( )) Taking the component of the flux you already divided by d A in the first place you divided by d A vikram and finally, you got an expression for flux. The flux has the flux you can resolve the flux and have a component in whatever direction you want now you are having the component in theta phi direction.

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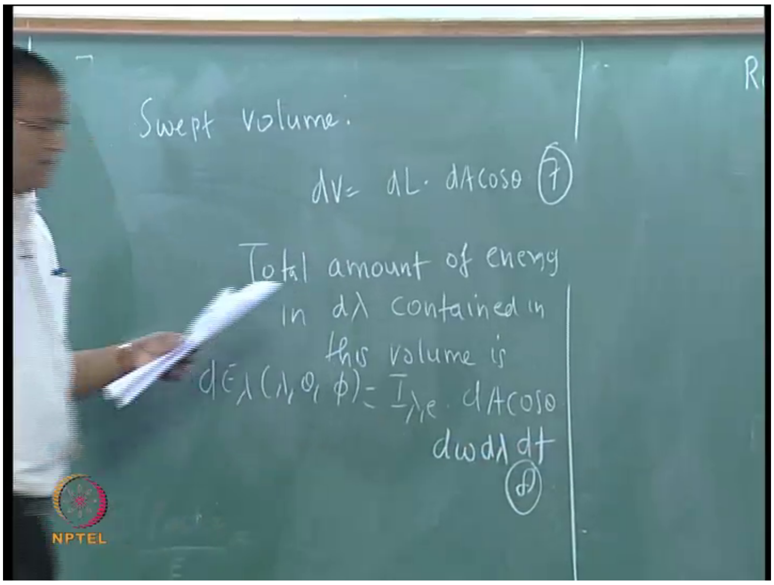

Now, at the (( )) the next classes is it not?. So, swept volume d v equal to d L so, I think there is a small change. So, you should draw the swept volume based on what based on d A cos theta not d A.

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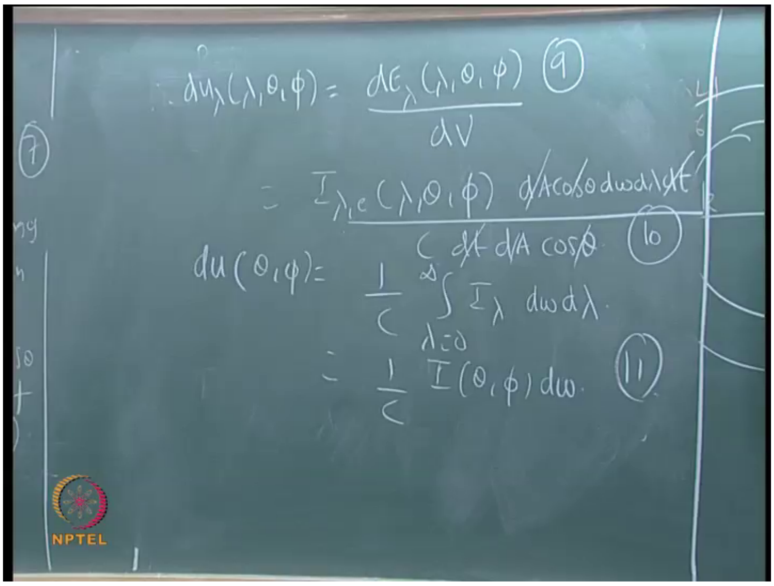

So, this swept volume is d L d A cos theta now total amount of energy. Therefore, which one, there is no problem this is what will be the x what be the units for this in brackets watts multiplied by time joules. What are the units of e? Joules. So, numerator is joules denominator is meter cube. Now, can we do this? d L c, d t, d t d A, d A cos theta, cos theta for the black body.

So, 11 12 so, the major progress we achieved in this class is we got an expression for radiation pressure in terms of the intensity. We got an expression for the radian energy density in terms of intensity and finally we got a very crucial relationship that the radiation pressure is basically with the radiation energy density divided by 3. So, now I will give you a sneak peak of what we are going to do in next class. The radiant energy the total internal energy U is equal to the internal energy per kilogram or internal energy per volume into volume capital U small u into v. Therefore, when you take the U d of capital U it is U d v plus v d u that is there.

Now, t d s is basically equal to t u plus p d v that comes from thermodynamics. In the du, u is written as a product of a small u into v and p can also be written in terms of u and p can also be write in terms of u now is equal to u by 3. Now, I will say that radiation is assumed to be a gas which is having a temperature having a corresponding to a volume v. And therefore, all the thermodynamic laws are applicable and if I had the t d s relationship and then t d s must be equal to exact differential because inter phase of property. Therefore, dough by dough y suppose you have an expression d z equal to m d x plus n d y, dough m by dough y is equal to dough n by dough x.

Therefore, I can take dough by dough by dough t of the first term equal to dough v of the second depending on how it comes and then finally, I will get internal energy is nothing but sigma into t to the power of 4 that will come out. But what that I is as a function of as a function of lambda specifically between 4.5 micro meter and 4.6 micrometer. What is I lambda? You cannot thermodynamics cannot answer. But for the complete wavelength interval lambda is equal to 0 to infinity all the angels are taken into account you tell me the temperature I can tell you the blackbody hemispherical emissive power, that is for engineering applications.

But suppose you are interested in solar energy in this thing you are doing some applications and you are you want to have selective absorbers and selective reflectors this knowledge is no good, it is not adequate. So, the next class first 20 30 minutes you will I will just give you the final result Stefan Boltzmanns law I will proceed with that. But what is that I? For that quantum mechanical consideration, have to be brought in. So, the next 3 classes we will work out the quantum much how plank and others derived this.

So, we are getting holidays in the subsequent week. And when is inauguration 5 o clock right on twenty fifth. So, to cut a long story short next Tuesday we will have a class 5 o clock, that is what I told you suppose I have 1 on twenty fifth you will say inauguration is there u are going home eighteen shall we have the class 5 to 6 reserved lab cannot go beyond five (( )) no, but you say classes there it is an official slot you come 5 10 minutes late I will allow you can come through this door. Because there is no other it is now heterogeneous class.