Conduction and Radiation
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Lecture No. # 45
Conduction with change of phase

Good morning. In today's class, we will look at conduction with change of phase. It is very important problem from an engineering point of view, and it also involves some tricky mathematics. So, we will just see how far we can go in this.

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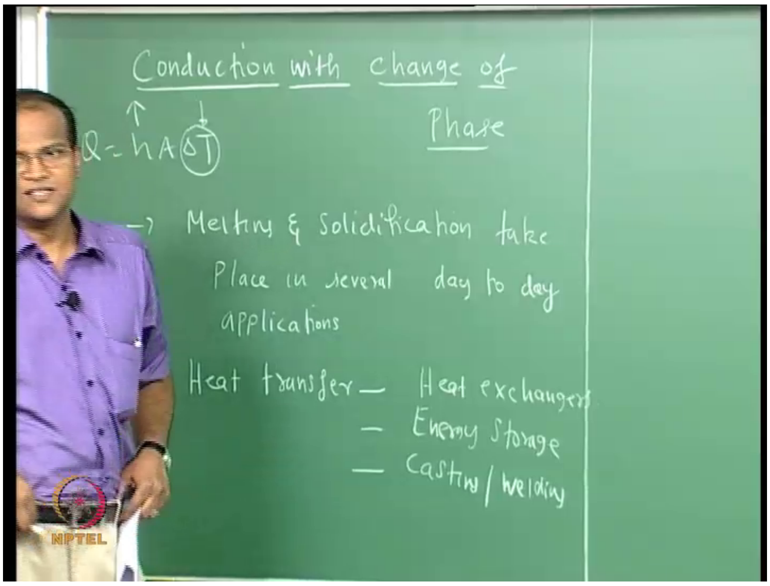

As you can very well imagine melting and solidification takes places in a variety of situations in life. Melting of ice, making ice cream, process food, dairy industries then preserving the blood in the blood bank, there are way many many applications from just ice cream making to so on. Melting and solidification and freezing they occur in lot of applications. As far as and metallurgy is full of well metallurgy is full of melting and solidification where there depending on the heating (( )) heat treatment is of course, heat treatment may not involve the change of phase. Sometimes it may involve otherwise, in metallurgy also they are faced transformation (( )) stainless steel (( )) side so on and so, they are obsessed with this melting and solidification problem.

As far as heat transfer is concerned, more than melting and solidification change of phase change of phase from liquid to vapour and vapour to liquid frequently occur in heat exchange occurs in heat exchange. So heat transfer you have heat exchanges. Most importantly, latent heat storage devices; you want to store solar energy using phase change material, using paraffin or something so the what will happen is you collect the sun's rays, you heat water; this water is circulated in a tube circulated in a in tubes inside a tank; in this tank you have got some paraffin or wax or something; the heat is transferred to this. So from solid it becomes a liquid. So it stores like this then, during the night you again send same water through this tube, but that water will be cold water, and it will get hot you will you will get hot water. So you may ask, sir what is the big deal? Why do you want to complicate things? Why do you want to use paraffin wax and all that? The simple logic is the melting heat is… The latent heat is much much more than the specific heat, and latent heat storage latent heat transfer takes place without any temperature difference. Is it iso It is isotherms. So from the second law of thermo from a thermo dynamic point of view, it is a most efficient way of transferring heat.

In fact, the goal of heat transfer is basically to reduce the delta T. In fact, we want to maximize the heat transfer performance; we want to keep on increasing h why because Q is equal to h A delta T. So if Q is fixed, you want to keep on increasing h so that delta T comes down. If you are able to accommodate the heat transfer at delta T is equal to 0 that is the best you can hope for. So this energy storage with phase change material offers that opportunity. So energy storage welding, casting all this all your foundry, business, casting and then, you pour some hot liquid into the mould then, you get the object you want; aluminum casting, dye casting all this. So melt casting and welding so, in all these applications in all these applications, you have conduction with change of phase. Therefore, it is important to know it is important to know to model conduction with change of phase up to certain level we can analytically model beyond that, you have to numerically model.

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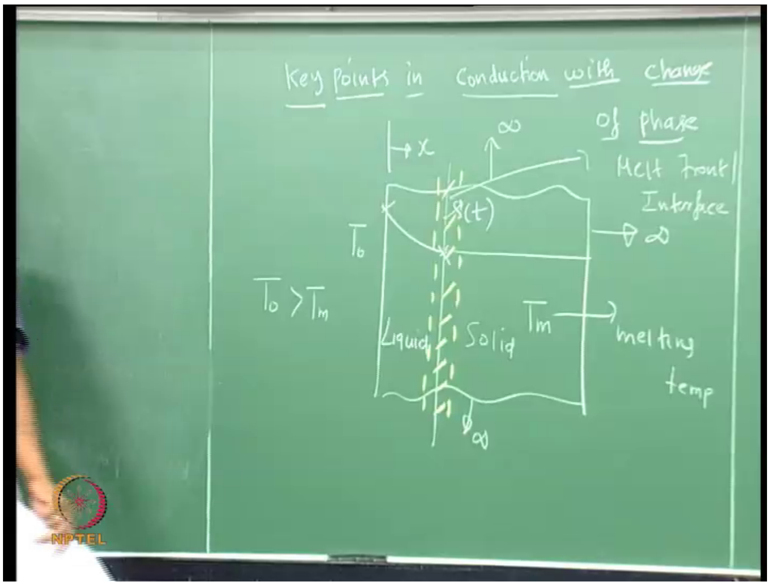

So, what are the key points? Let us consider a situation where there is a solid, the whole solid is at a temperature T m; T m is a melting temperature. Of course, it is infinite in this direction. It is also infinite in this direction that is why the therefore, it is known as the semi infinite. What is a semi infinite solid? Only the left side is specified; right side is infinite. Sir, how can you get semi infinite? Semi infinite means, the amount of the region in which this melting activity or heat transfer activity is taking place in relation to the dimension of the body is very less that is why it is semi infinite. The body is 30 centimeters thick in thickness; you are talking about the first few millimeters where this melting has started; you are you are still talking about the early regime of transient conduction so it is semi infinite.

What is semi infinite? What is a semi infiniteness there? Most of the body will not know that some disturbance took place at the left boundary. So, it will continue to sleep at the initial temperature that is semi infinite. Slowly it will get affected are you getting the point? This we have discussed earlier in this course also, where I have explained delta, the penetration, depth, how much the thermal disturbance will propagate and then, from delta can be, you can use the similarity variable x by 2 root alpha t all that we have discussed.

Now, let us say that suddenly left side you raise the temperature to T naught; so this T m everything was a previously everything was a solid; T naught is greater than T m. So and x is like this is; is the situation clear to everybody? So, where will the melting proceed? T m is the melting temperature; T naught is greater than T m; so the melting will start from left to the right. So here, we will have liquid region. Now a simplistic a simple con since was at T m, since it has not melted it will continue to be at the temperature of T m and here, it is T naught the temperature will decrease like this correct. So this will be the temperature profile (( )) by semi infinite solid when melting takes place.

What is the key engineering what is the key engineering result we are trying to seek? The key engineering or the key engineering results we are trying to seek in the problem of melting and solidification is number one, what is the position of this interface with time. How is the… What is the interfacial velocity? That is how much is the melt? How fast is the melt front moving from left to right? Second, once you have captured the melt front within the melt front, how is how is the liquid temperature changing with x for variations in sense of time. So these are the key engineering results. Therefore, the interface is not fixed the interface, if this is the interface, so the interface is a function of time. We want to know, what is this s of t is? That is the key engineering result we are trying to seek.

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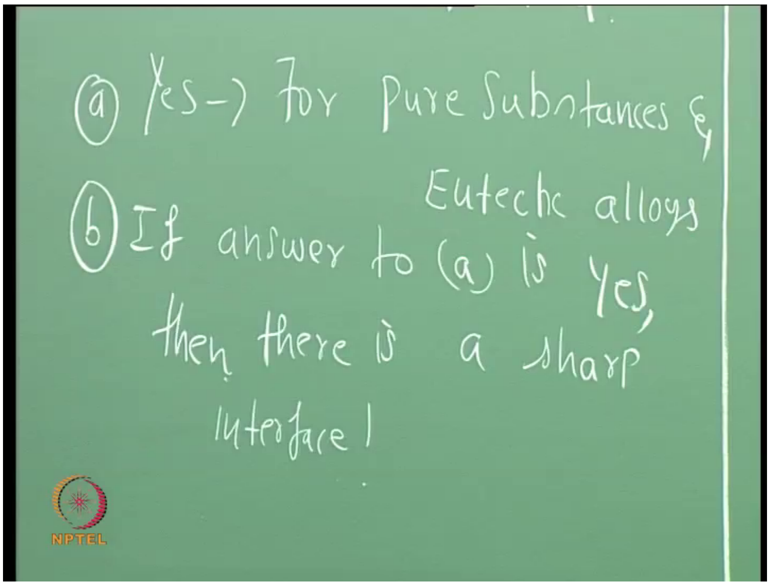

What are the key points? First, whether T m is a single temperature or there is a region; there is a there is a range of temperatures about which the melting takes place. That means, it starts at 37 and ends at 39 then, we are in trouble. Then, you would not have such a sharp interface, you will have some mushy region, half melt, half liquid, 25 percent melt, 75 percent liquid like that then, you have to model that mushy zone also. Obviously, you can very well imagine that you have if that you have a situation where T m is not specified at one point, there is a small range for T m then, it is much more difficult.

So, whether T m is a single temperature? Yes, for pure substances and eutectic alloy. If answer to a is yes then, we have a sharp interface correct. There is a sharp interface left side is liquid.

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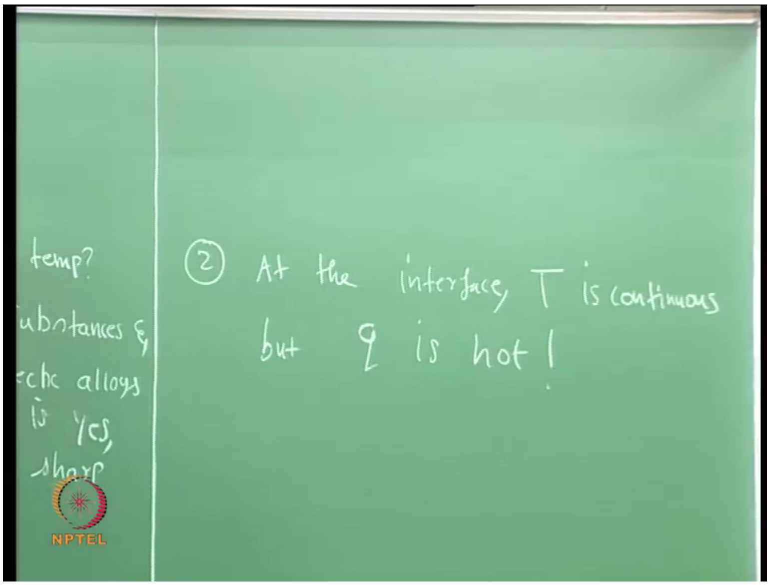

Right side is solid. That is the issue number one. Obviously in this course, I am going to consider only a pure substance or eutectic alloy which has got a sharp interface only one melting temperature. Then so, this is also called the interface or the melt front, freeze front or melt front at the interface temperature is continuous correct. Whether we are approaching slightly from the left side or slightly from the right side, the temperature will be T m, but heat steps will not be continuous. This is the major difference between the normal conduction which you study and this, and conduction with change of phase.

Because, lot of people will think, sir what is the big deal? If there is a melting which takes place, there is an additional resistance there is an additional resistance; it can be treated like an additional resistance because, there is finite amount of heat involved in melting and then, the solid will become liquid then, you can continue with the process why cannot you add that melting model that melting phenomena, the contact resistance and have a resistance network and proceed. You cannot model melting in solidification with simple formula with simple approaches which you learned in conduction without change of phase basically because, minus k dt by dx from left side is not equal to minus k dt by dx on the right side; minus k dt by dx on the left side is equal to minus k dt by dx on the right side plus, whatever heat is generated because of the whatever heat transfer or energy transfer takes place because of the phase change that we will model. That is the first thing you have to learn. That is developing of the interface condition that is called Stefan condition Stefan found this out; how do you model this. So, is this situation clear?

So, the key differences are the temperature is continuous heat flux is not, and T m need not be a single temperature, there should be a range then, you have a mushy zone that means, you have some region where the melting starts some region where the melting ends. That is much more difficult to solve; it can be modeled using software or you can write your own code, but if we if you do not worry about that, consider a single temperature which is for pure substances eutectic alloy, you can attack the problem analytically.

Now, I have to we have to get this interface condition that is very very critical. Shall I erase this? Let us consider this itself. Now, let us consider the interface; consider a thin slice control volume around this interface, we are going to study the story around this control volume. So, there are several ways in which you can consider this. Now you can assume we can understand, you can use a control volume, you can use a Lagrangian approach or an (( )) approach right; to figure out what? To figure out the state of affair happening within the control volume.

There are two things which are taking place. There is a transfer of mass and there is a transfer of energy. So you have model this transfer of mass. Now, we can assume that the interface and the control volume surrounding it are stationary. One possibility is, to assume that the interface and the control volume surrounding it are stationary, but material is just coming from left to right or material is coming from right to left for example, material is coming as solid from the right entering the control volume as if as if it is some process, as if it is some machine; material is coming from left getting melted exiting as liquid. At a particular instant of time, since the control volume is not expanding in size are contracting it is a fixed area. Now, what can you say about the mass balance across the control volume.

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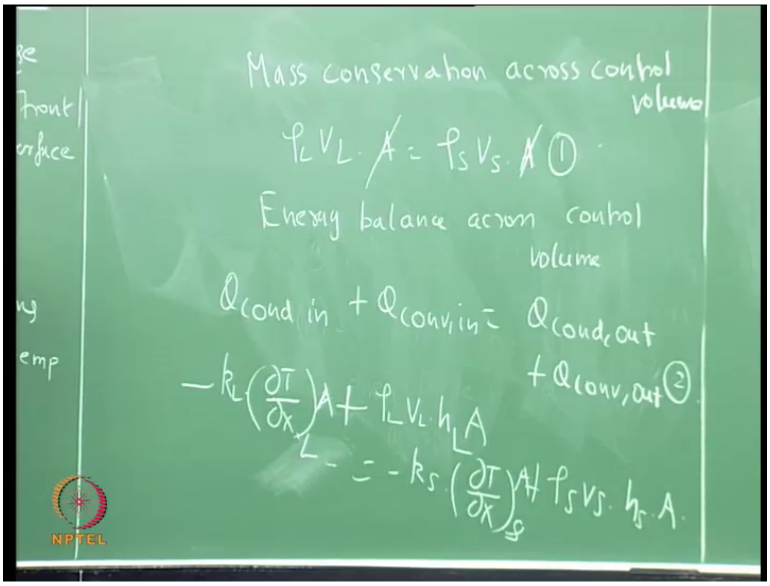

Mass conservation; you can consider the area of cross section to be A; so rho L rho S; A is the same then velocity; what is this velocity sir? What is this velocity? This is that velocity which is actually the velocity of the front.

Now, what can you say about this? Now rho L V L into A is equal to rho S into V S into A. So, what is the funda here? If rho L is different from rho S rho L is different from rho S in a particular instance of time 5 centimeters of solid thickness may translate to 7 centimeters of liquid depending on the density that is what I am trying to say. If both the solid and liquid phase are of the same density, it will cancelled does not matter, but when it melts the density could be changing, but the mass has to be conserved. So we should make the fatal assumption that the velocity at the liquid side must be equal to the velocity of the solid side, but rho L V L equal to rho S V S. Is that clear?

Of course, many materials rho L is equal to rho S, and we will not get we will not get off, and we do not worry that is another story. Now rho L V L is equal to rho S V S, so that is the first that is point number one. Now energy balance energy balance from if you consider from left from left from the left it enters as liquid sorry from the right it enters as solid and goes out as liquid. That is the melting problem or from the left it can enter as L and go out as solid that is the solidification problem.

So, when it is coming there is a flux the conductive heat flux which is coming because of a temperature difference. There could also be a conductive heat flux here I am just saying that it is a T m. Initially, it need not be a T m, it can be at some other temperature then, there could temperature gradient within the right side also. I can I started the lecture to make it clear. So there could be conductive heat fluxes on both sides. Then, this liquid is coming with an enthalpy of h L, this liquid is this solid is coming with an enthalpy of h S. Therefore, the energy balance demands that whatever is conducted in, plus whatever enthalpy is brought in must be equal to whatever was conducted out, plus whatever enthalpy is taken out correct; so these two are critical.

Now, how can you do this? Q, what is Q conduct minus k A? dou T by dou x into A you want to take or A A gets cancelled or you will cancel plus rho is you put it is as rho L rho L. That is convection; it treated as convection enthalpy no that is enthalpy liquid enthalpy. So, this is equal to minus k solid dou T by dou x, this is liquid and solid. See sometimes, I may use small s you be consistent; L is liquid; s is solid plus A. I am coming to it h s, you do not be do not jump to conclusion. Left side is L; right side is s. Now, I am saying that h L minus h s is h f j I am coming to that because, when it is coming it is coming with a liquid left side. Across the control volume only this transformation takes place. I cannot straight away put h f g when I derive; what is the fundamental principle? How do you how do you get that equation if you ask. Suppose, I straight away write minus k L dT by dx is equal to minus k s dT by dx plus h f g; you will ask me, sir how? I am explaining now that h f g is coming.

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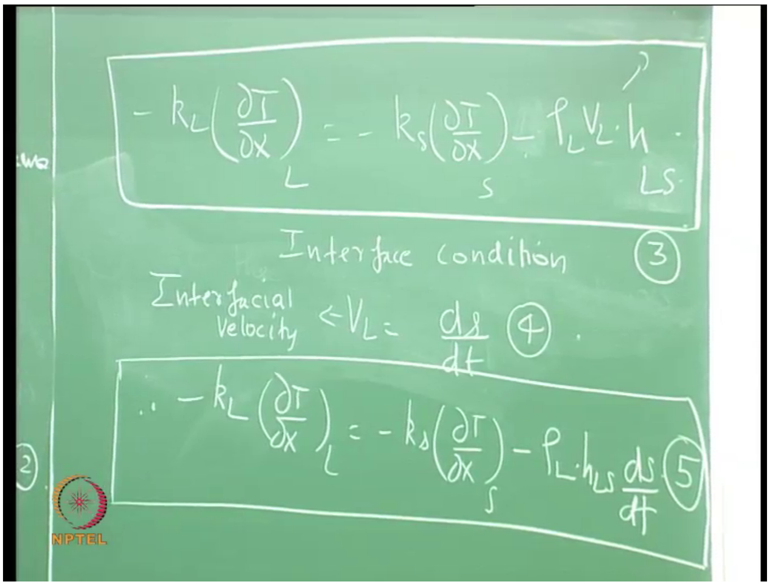

Now, you can do what you are saying; so minus k L of the liquid is equal to… So usually the solid has more than enthalpy than the liquid or h L minus h s what do you do?

Student: h s minus h L.

h s minus h L. That is what you want to do. Now, what is the beauty? Rho s v s is equal to rho L V L. So, this is that is your h f g; we do not think h f g because, h f g is for water to lea. I have put h s minus h L; is that correct? I knocked off the A; is it correct? This is called the interface condition very important. Now are you confident that h SL is positive are you confident that h SL is positive.

Student: (( ))

That is why I am also no no there is nothing wrong in what we have written

Student: (( ))

That is why is it correct? Or did I make many mistakes? Plus, now h L minus.

Student: (( ))

Is it correct?

Student: (( ))

That is why I do not want to get a negative value; will that h LS be positive or

It will be negative.

Yeah because, that is no no no h f g means h f g means fluid to

Student: (( ))

h g minus h f gas to so liquid to solid.

Student: Liquid will have more enthalpy.

Yeah, liquid will have more enthalpy that is why if you bring it like this, it is h s minus h L we call it as h LS, we put the minus sign now. It sets it to rest. This is called the interface condition. Now, you have to solve the conduction equation with this interface condition. That is the added thing so the first half an hour what we did was, we derived this interface condition without this interface condition you cannot proceed.

Now, what about the interfacial velocity? V L can be considered to be ds by dt where s is the location of the interface where s is the location of the interface. Therefore, so this is the interfacial velocity. So apart from the time which enters the unsteady conduction, the time enters the interface condition. Now this condition we have to use; so equation 5 is valid for both melting and solidification. You want you can write equation 5 is valid for both melting and solidification.

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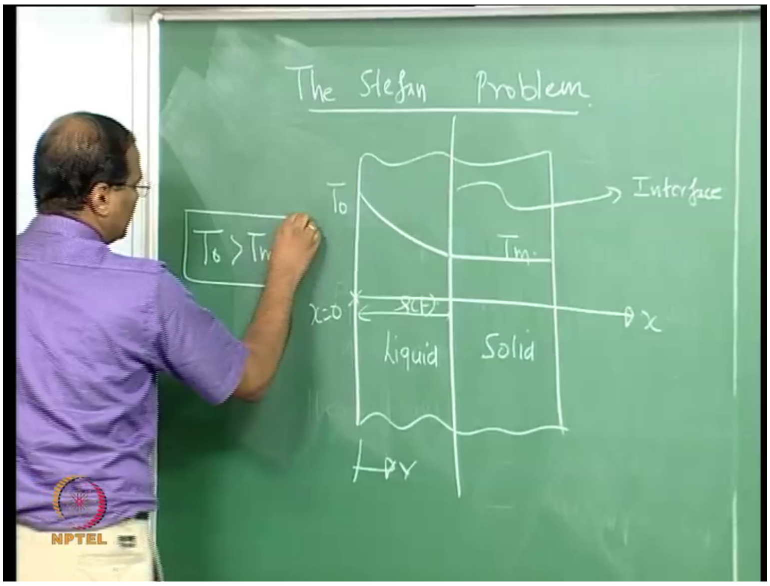

Now, let us consider the Stefan's problem. What we considered little while ago is Stefan problem heat conduction equation which we are aware of. It supports two boundary conditions in x and one initial condition.

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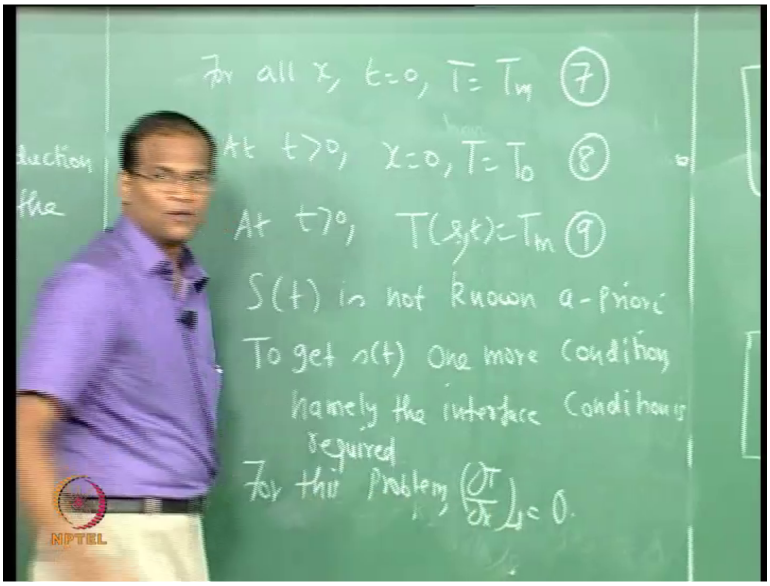

So, the initial condition is for all x, t equal to 0; T is equal to T m. So one initial condition and one boundary condition are given; what is the third boundary condition? t s is like this spatial coordinate; t is the time again. At any time at T of s t, T is equal to T m. Therefore, the solution can be worked out only up to s; the solution can be worked out only up to x equal to s. Now, the difficulty the difficulty that arises in the problem is s is not known for us, because s is a function of time. Therefore, s has to be obtained as a part of the solution. In fact, how s changes with time is the key engineering result, which we are trying seek, because from the engineering point of view, how fast or how slowly the melt proceeds is an information which we are seeking.

So therefore, we want to know, we do not know what this we do not know what this s is right now. But now, one thing is very sure is in order to find s, one more condition is required; that one more condition comes from the interface condition. Yes, agreed. Now in that interface condition, what is dT by dX of the solid? Is there any temperature gradient in the solid? No, because the whole solid is at T m. Therefore, the interface condition can be simplified for the Stefan problem.

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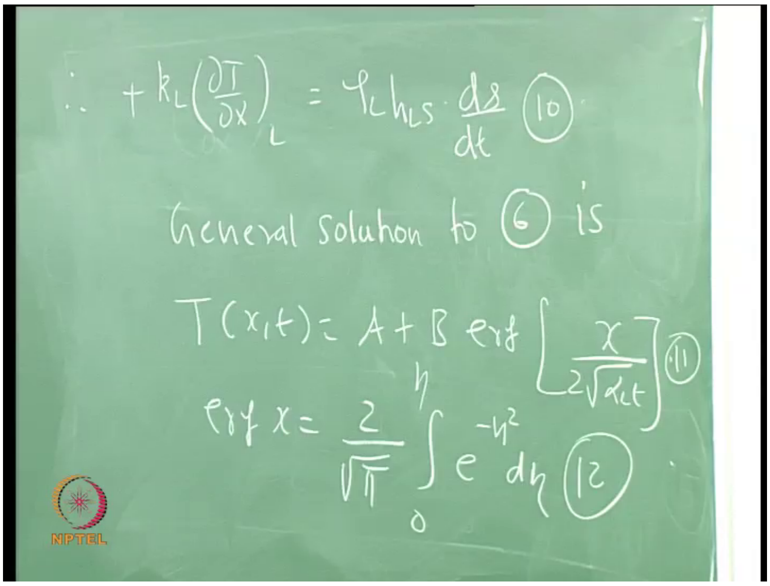

Now, for this problem because there are no temperature gradients within the solid; the solid continues to be at T m. Therefore, minus k L

Student: (( ))

No rho minus or plus.

Student: (( ))

Plus.

Student: (( ))

That is it. No, we have not solved it, but.

Student: (( ))

It is not so easy now we will see. That therein, lies some fundas are there. Now, we have correctly formulated the problem. Now, there is no nothing is there in the problem whatever physics is there, whatever mathematics is there we have incorporated. Now, we have to seek a general solution to the problem with the boundary condition, initial condition and the interspatial condition. We have to find the solution to the Stefan problem.

Now, this is this conduction in semi infinite solid. Do not take it up with the melting front and all that. As far as the left side is concerned, it is basically conduction in a semi infinite solid that is the initial portion. For the initial portion, I have told you that we can introduce a similarity variable, though I may not have worked out the full similarity solution in the class. In the course, I have already told you that x by 2 root alpha t can break down this can convert this p d into o d, and you can get an error solution in the form of an error function.

So, the general solution to 6 the general solution to 6 is given by T of x comma t; I do not have time to derive that here, but it can be shown that, once you have eta equal to x by root alpha t, you can substitute for eta here and then, this will convert into o d and then in just in another 10 or 15 minutes you can show that, the p d is converted to o d and the solution for that o d is given by the error function.

Now, the error function of x is given by 2 by root pi; do not worry that right hand side that integral, the variable under integral sign, it can be anything; it can be alpha, beta, gamma; it is a dummy it is a dummy variable because you are substituting 0 to eta. Now so, this is 11. Now slowly one by one, you have to get the constant A B as well as s, we will do that slowly. What is the error function of 0? the error function of 0? Integral 0 to 0 error function of 0, 0 at x equal to 0, what is the condition? t equal to?

Student: (( ))

Error function of 0 is 0. Therefore, this term vanishes therefore, A must be equal to T naught.

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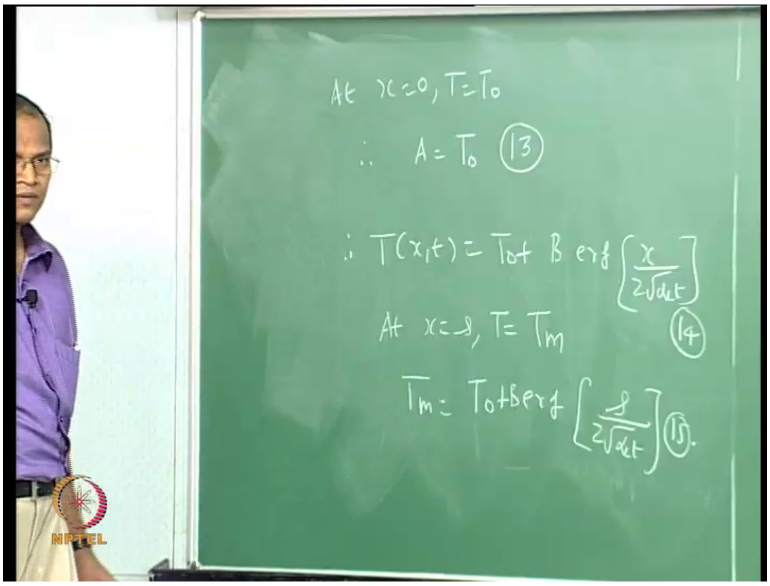

At x equal to s, what is T?

Student: T m.

Very good. At x equal to s, T equal to T m because, that is the place where the that is the melt front for the interface.

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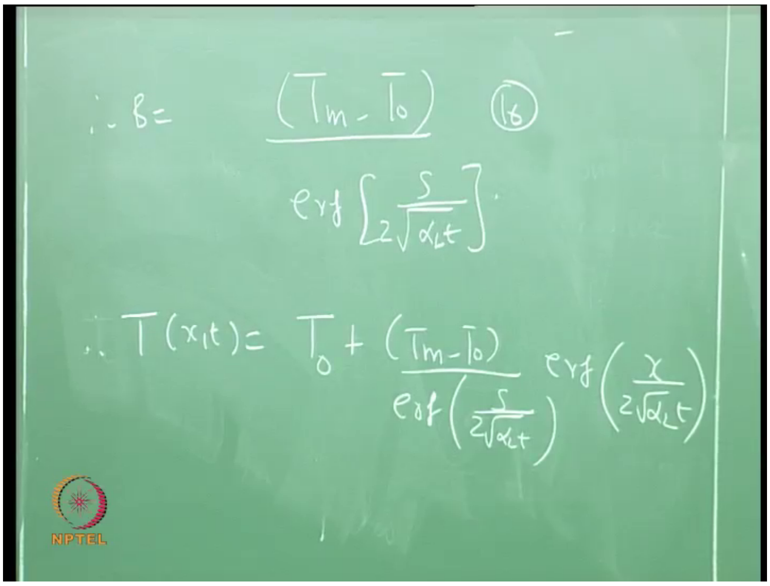

Now, T m error function of tell me exactly error function of.

Student: s by.

s by correct. Therefore, B equal to T m minus T naught. Therefore, agreed. Yeah T m is known; T naught is known; alpha L is known; time is known. If you give the position of x, it is possible for you to calculate the error function of x. You can solve the problem provided you know what s is, but you do not know what s is. So the problem is not over, you are able to understand that.

This solution is valid only up to s; you can work out the temperature profile only up to s. Beyond s, the whole soul the body is at a temperature of T m, but that s is itself is changing with time; the unfortunately the solution has s. I do not know what the s is. But, I do not have to lose hope because, my interface condition is there. So that interface condition, if we substitute it by general solution because, k dT by dx is there at the interface. So, if you substitute k dT by dx, the only unknown is this is s; left side also I will get s; right side also I will get some function of s. I will get an ordinary differential equation in s. If I solve the ordinary differential equation in s, I can go home. So much for the simple Stefan problem. Now, let us substitute the interface condition and stop, we will work out the solution in the next class. Now we will just is this clear?

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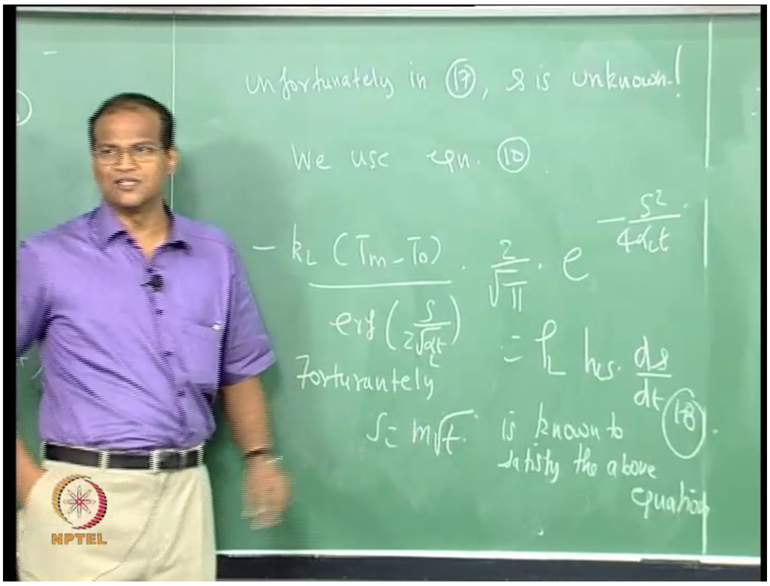

That is our problem. Therefore, what you have to do is, we use equation 10 yeah please start integrate differentiating minus k L; what is the integral of error for differential of the error function? The differentiate term goes; so you have 2 by root pi eta square you are applying at x is equal to s. So, it is now threatening as some error function of s is there; some e to the power A square is there; dx by dt is there; where is the hope? I want to get s as a function of time.

If you solve equation 18, you will get s as a function of time. So dependant the dependence of s on x is gone because, we have substituted, we have worked out interface conduction at x equal to s. Fortunately, it is known; fortunately s is equal to m of t is known to satisfy the above equation, where m is a parameter which depends only on the properties of this, properties of the medium under question. So, you can substitute s equal to m of t there and then, resultant equation will be put in terms of m and you solve for m and then, once you solve for m then, you will this for m into root t m into root t is please note this satisfies this.

So, if you substitute s equal to m equal to root t the whole thing can be solved, and you can find out the solution and that completes, we will work out the solution in the next class. So, how do you we get this? That means, what it means is, the s goes as t to the power of half; that is intuitive just as the boundary layers, it grows as k to the power of half. Will see from an integral analysis, we will see how that is actually true and then, we will work this out and get the value of it solve it, and we will complete the solution in the next class.