Conduction and Radiation

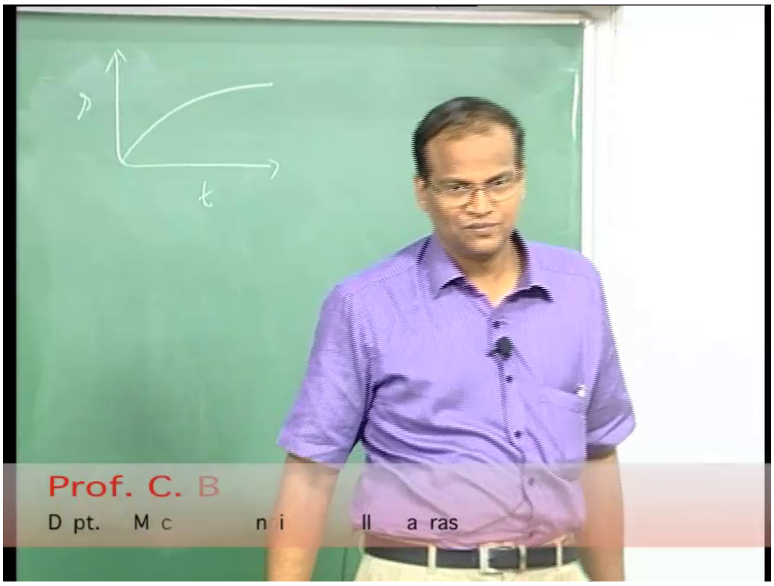
Prof. C. Balaji
Department of Mechanical Engineering
Indian Institute of Technology, Madras

Lecture No. # 46

Conduction with change of phase Contd

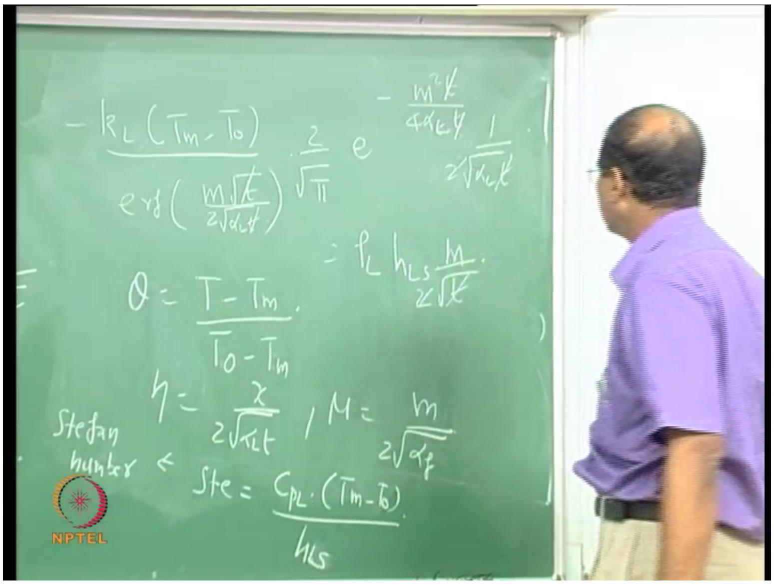
I think so towards the end of the last class, we came to an important step wherein we substituted the interface conditions then, we got a very messy result and I told you that it is known that s equal to m of root t; that is what is this s equal to? That is the melt front goes as t to the power of half that is the that is the assumption.

(Refer Slide Time: 00:32)



Now, what is the guarantee that it goes t to the power of half; I will answer that question, but before that assuming that s equal to m of root t satisfies that you can substitute it into this equation 18 and proceed with the solution.

(Refer Slide Time: 00:58)



Now, can you do this? Shall we do this minus k L; ds by dt is what? m is a constant m is a constant; it depends only on the properties m half m t to the power of minus half correct is it correct? Is it correct?

Now, what we do? Is there something else which is missing? e to the power of minus; now we can collect some terms. Now, we have to introduce certain dimensionless quantities; it becomes easier if we introduce those dimensionless quantities; we will introduce this now otherwise, the solution becomes tedious. Yeah so, I will introduce the following and Stefan number C pL h SL, h SL or h LS.

Student: h LS.

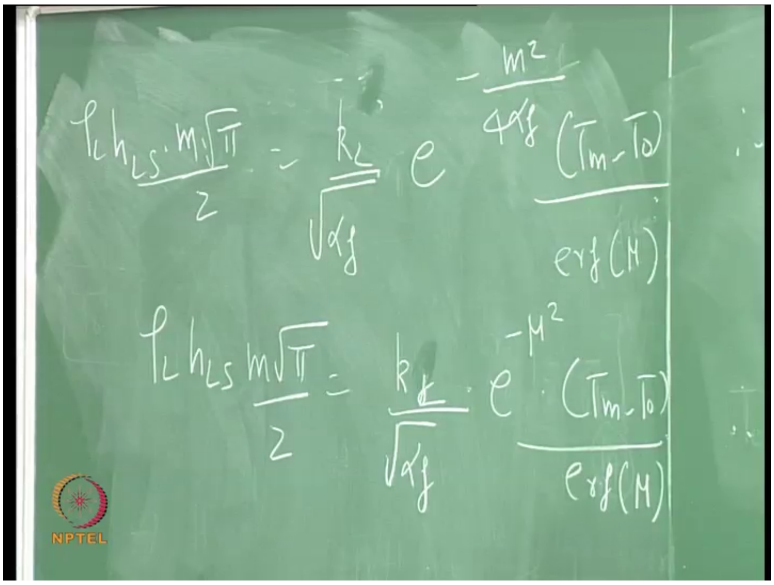
We introduced four quantities now; four dimensionless parameters theta T minus T m by T m minus T naught.

Student: T naught minus T naught.

Maybe its T naught minus T m is better. So, T minus T m by T naught minus T m is the non dimensional temperature; eta is the similarity variable non dimensional variable; mu is another non dimensional variable which depends on m and then, the Stefan number which is C pL T m minus T naught by h LS; this is the Stefan number. Please recall that this is the Stefan problem the Stefan number is very important because, the Stefan number gives you the ratio of the sensible heat addition divided by the latent heat. It is the ratio of the sensible heat to the latent heat. It is the critical link between the sensible heat addition in the problem and the latent heat addition in the problem.

Please remember that the latent heat addition takes place at the interface whereas, the sensible heat addition takes place only on the left side of the problem. In the right side or the melt front there is no sensible heat addition. With this, we will have to play with all this and then do something to that whatever we have done and hopefully, we will get the solution.

(Refer Slide Time: 05:31)



Now, we start doing let me do the right hand side so 2 by root pi is there right. So, I will have rho L m; root t is still there or root t must disappear; somewhere root t must disappear.

Student: (( ))

Did you differentiate it properly?

Student: (( ))

No no no no yeah k L T minus T naught error function into 2 by root pi, what is the differential of the error function e to the power of?

Student: (( ))

Hang on. So error function of x equal to.

Student: (( ))

The differential of this will be 2 by root pi e to the power minus eta square into that into that differential of the eta right so that term you left out. Are you getting the point? So that is why we are having trouble. So, you will have k m e to the power of minus s square by 4 alpha t into what is the different x by 2 root alpha t; if you differentiate what you get?

Student: 1 by 2 root alpha root.

With respect to x 1 by.

Student: 1 by 2 root alpha.

Now, the t is getting… Now 1 by root t is getting out did somewhere; I do not like the 1 by root 2 man is it going is it going? Yes, left hand side 1 by root t right hand side 1 by root t; the two also gets cancelled. So, what is the beauty of this when you propose when you propose that s is equal to root of m of m root t, the t disappears from this equation; yes already disappeared from this equation because, we differentiated at x equal to s. Therefore, now it is a simple equation to solve; now let us do that.

So, rho L h SL m by m root 2 by pi is equal to k L or k f 4 alpha f correct 4 alpha f into Root pi by 2 root pi by 2; m root pi by 2. Correct because, no no no lets 2 by root pi I am taking it to this side, it will be root pi by 2 correct. What is that right side will be equal to the power of m squared by 4 what is that? So yeah if there are some mistakes you tell me we will fix all that. Any mistake?

Student: Minus s square.

Where?

Student: (( ))

No no no there is a minus on both sides no; forget the minus. There is a root alpha so the two is still there two is there or two is there.

Student: (( ))

Let it be there. Now what

Student: (( ))

No no when we differentiated I got this.

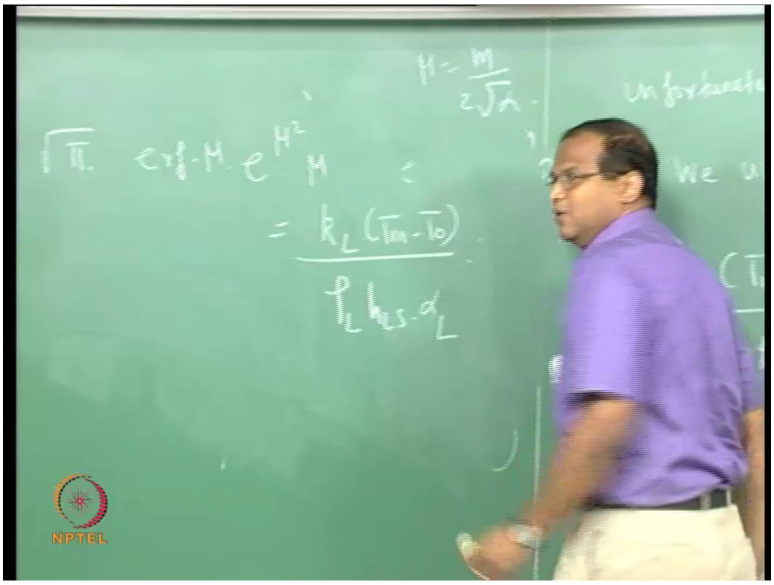
Student: That is in exponent no sir into.

This is into.

Student: (( ))

No no where did we put alpha f. Now, I have to introduce the Stefan number I have to introduce the Stefan number.

(Refer Slide Time: 11:32)



So, what I can do is the root pi error function of mu into e to the power of mu square is it correct? root pi mu e to the power of; this e to the power of minus mu square I am taking it to the left hand side, and mu is equal to k f rho L T m minus T naught.

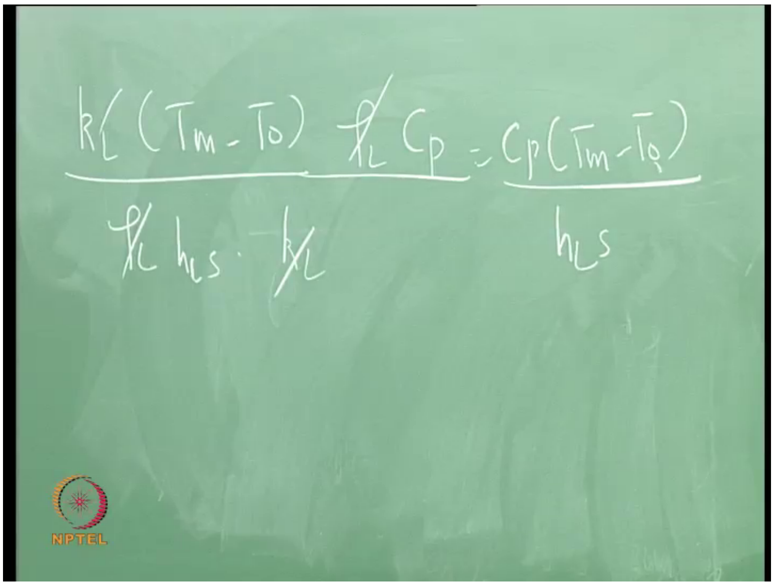
Student: (( ))

What is mu? m by.

Student: m by 2 root alpha.

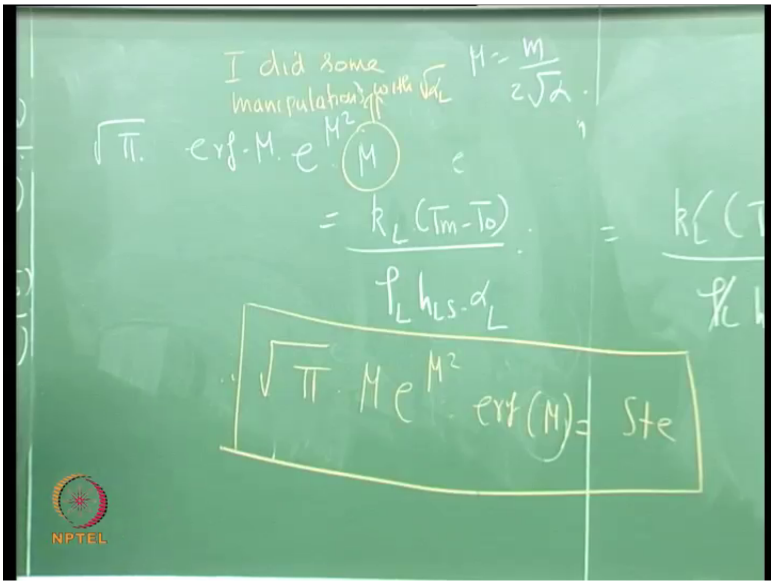
m by 2 root alpha m by 2 root alpha; if I root of alpha by root of alpha if I substitute denominator and numerator, this m root alpha by 2 that becomes, mu root alpha into root alpha becomes alpha that alpha L remains here. No mistake no golmaal everything is correct k some. So Stefan see, how much trouble he is giving all these are in paper man this is paper.

(Refer Slide Time: 13:43)



Now, I have to bring Stefan number, so easy for me equal to k L T m minus T naught divided by rho L h LS alpha is k L by rho C p. Finally, we solved the Stefan problem.

(Refer Slide Time: 14:12)



Therefore, given the Stefan number, you have to solve it to find out the mu. What is mu? m by. From mu m by root 2 alpha; alpha f is alpha L is known to you. So, if you find out the mu from that equation you know the alpha you will get the m; what is m itself? m is s by root t; if you know the time you will get the s. Finally, what is this goal of all this to get s. It is such a complicated non-linear equation. Therefore, this equation can be better used if you say, I want to get this much; I want to have this much s; you decide what is the s you for a particular time; s is equal to m root t; you decide the s you decide the t, you will get the m from there you will get the mu. For that mu, you find out what Stefan number you have to use. From that Stefan number, you can find out what could be the T naught for you to get that melt front. How much heating must be applied at the left side; this is the better way to use the equation instead of correct instead of finding.

Suppose, you decide at the end of 15 minutes 12 up to 12 millimeter, I want the melting peaceful so, you decide mu everything you decide. Stefan number C p is not under your control; h LS is not under your control; T m is not under your control, but T naught is under your control; T naught minus T m. Did we make a mistake? Throughout, I am writing T naught minus T m or.

Student: (( ))

How?

Student: (( ))

So it should be T naught minus T m; somehow you adjust that so otherwise, T m minus T naught will be negative. So this solves this Stefan problem. What is the Neumann problem? In the Neumann problem, the T i is less than T m. Initially, the body is at a temperature lower than the

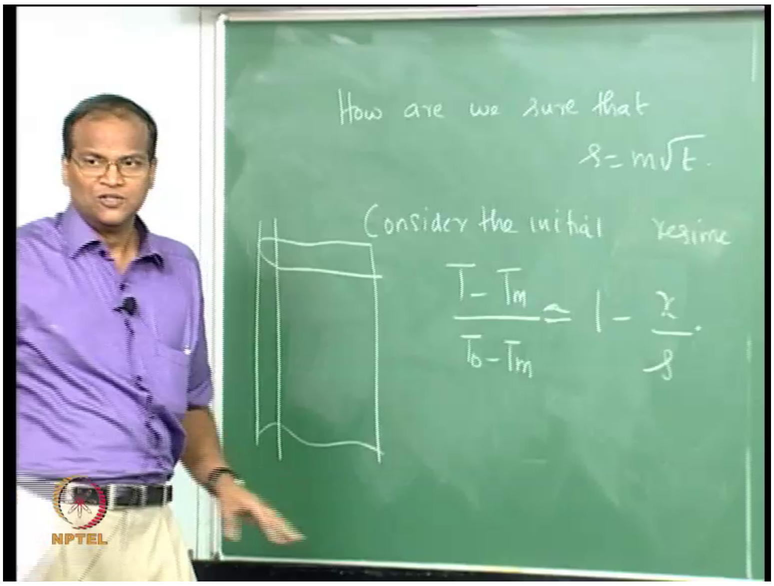
Student: (( ))

Everywhere it is at T i; there also will be sensible heat and you will have to consider you will have to consider conduction in both the liquid phase and the solid phase. So there is something called the sub cooling parameter. So the Neumann problem is more general than the Stefan problem; it is more difficult to solve. The whole body is not at T m when you start either of the temperature lower than; it has to be heated to that and then it will start melting.

Student: (( ))

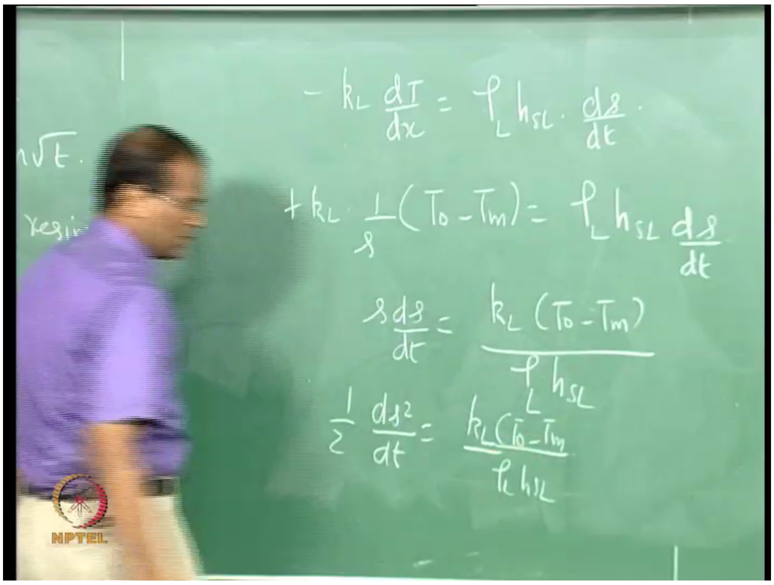
Yeah so you will have to consider conduction both in the liquid phase as well as in the solid. That is the Neumann problem.

(Refer Slide Time: 18:05)



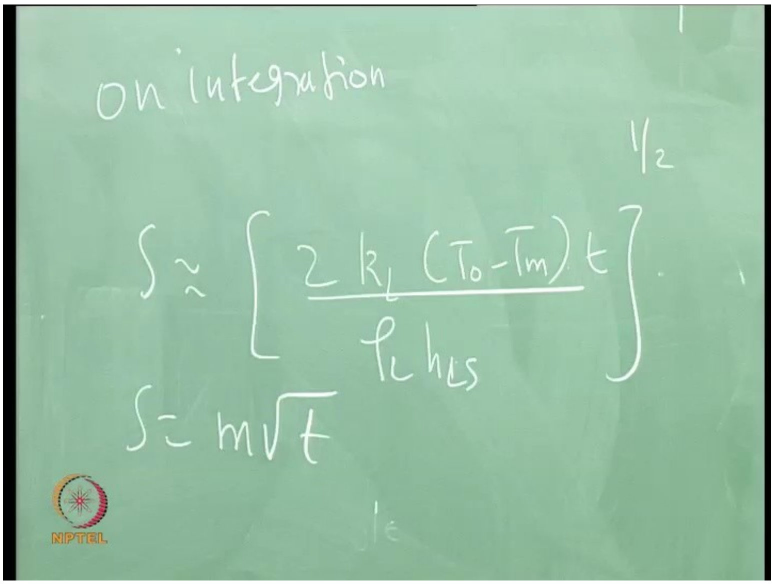
Now, you have to answer, how are you sure that s equal to m root alpha t. So, consider the initial portion of cooling; that is you are looking at the early stages. So the melter come only up to this phase. So you can say that, can we say like this? I cannot put equal, I can say approximate. I am assuming a linear temperature profile for liquid portion of the interval. Am I allowed to do that? This is correct at x equal to 0, T equal to T naught because this will be 1; at x equal to s, this is 0 T equal to T m; so it satisfies.

(Refer Slide Time: 19:56)



Now, with this approximate temperature profile please differentiate the interface condition. So if you do this, what do you get? Minus 1 by s correct. Therefore, s ds by dt is equal to k L rho s; what is it? Is it rho L? Yes, no no do not do not give a blank look. So dT by dx I am differentiating from this temperature profile. Then now, this is nothing, but half of dx squared by dt. I can take this here then, I can integrate it is a ordinary differential equation in t. I can integrate.

(Refer Slide Time: 21:47)



Now, k L t k L rho L h L are all properties of the medium and T naught minus T m is the initial temperature to excess the temperature exceeds with respect to the boundary. Therefore, s is indeed m root t. So, in the initial portion of the regime when you are saying that, the temperature profile is approximately linear. This is called the scaling analysis. You are approximately getting a scale you are approximately getting a scale for this instead of boundary layer thickness we call it as a melt thickness. So, you have to get s equal to m of root t from scaling analysis. Now, it is fundamentally correct s equal to s equal to m of root t. Using this hypothesis you proceed further, you will get a relation you will finally get a solution where you can solve for mu. It is a highly complicated in mu. Therefore given the mu, you can calculate work out the Stefan number and then find out how much heating is required.

But, the Stefan heat the Stefan problem is infinitesimally slow heating because, the body is initially at a the Stefan problem and Neumann problem are different because, the body is initially at a temperature which is equal to T m everywhere it is T m. But, in the Neumann problem, you can have a temperature which is different from T m. These these these are the basic differences. Therefore, you have to say 1 by alpha dou t by dou tau is equal to dou square t by dou x square both for the solid and the liquid you have to consider for the Neumann problem.

Now we assume that, the melt front is always like this; if this is the x direction, the melt front is proceeding vertically. This is far from the truth in many of the applications, where there may be natural convection; there may be the two dimensional epics and so on. The melt front may be at an angle to the x so, that is… So invariably melting will be two dimensional. So, this x will be a function of x and y apart so, the x will x will not be a s; s itself is a function of time that is one thing, and the melt front will also be two dimensional.

So, once you want to model those cases; you want to consider a sub cooling super heating; you want to consider mushy zone and all that, the analytical solutions will break down. You have to take the course in numerical methods; you can write your code. And so, the important thing is the capturing of this melt front. That is why we spent considerable effort now to get s; capturing this melt front as a function of time, that is the key engineering result. With that, we come to an end in melting and solidification. These are the basic in ideas involved. So, it is tractable analytically up to a certain level of complexity beyond that you have take recourse to numerical methods.

Thank you.