Conduction and Radiation

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Lecture No. # 07

Planck's Blackbody radiation distribution Function

So, in the last class we looked at the derivation of the Rayleigh-Jeans distribution. So, the Rayleigh-Jeans distribution is not totally incorrect, it predicts that correct blackbody behaviour in some parts of the spectrum or in some portions of the spectrum, only towards the ultraviolet in a very short wavelengths that i b lambda tends to infinity it is called the ultraviolet catastrophic. In today's class we will derive the correct distribution (( )) which was proposed by Planck. Of course, for proposing this distribution and developing the quantum hypothesis proposing the quantum hypothesis he got the Nobel prize in physics in 1918 at the age of 60, so Planck is 1858 to 1947.

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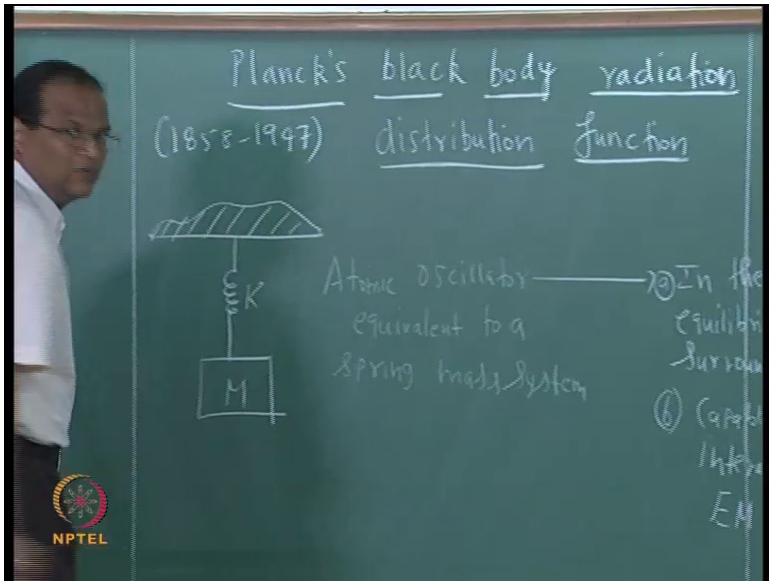

So, he got the Nobel prize at the age of 60 but, the youngest guy to get the Nobel prize in physics was Bragg, Bragg got his Nobel prize at the age of 25, Heisenberg got the Nobel prize at the age of 31, Heisenberg's PhD thesis is only twelve pages,

Student: (( ))

(( )) also 31 year's 21 but, 31 year's 5 days I think, you just go to you just go to Nobel prize in physics by age it gives all the statistics, whenever they were born the year in which they got the Nobel prize, what was the age at the time of Nobel prize, how many days they were alive after they got Nobel prize, who is the longest who was the longest survivor in Nobel (( )) bla bla bla all those things are there. So Bragg got his Nobel prize at the age of 25, 25 is you just finished your M Tech or dual degree or you were just trying to see what can be done next but, somebody all ready got Nobel prize.

So, anyway let us not worry about that we have to do our work, we should not get demoralized by such things, there are so many people who cannot solve even a quadratic equation and I hope we know how to solve.

So, now we will look at the Planck's blackbody distribution function, so the logic is more or less the same as what Rayleigh-Jeans proposed but, there is a modification in terms of the average energy, the number counting is the same as the Rayleigh-Jeans, 4 phi a cube by c cube by divided by c cube all there are 3 terms in the u nu, correct or u b nu, number of standing waves multiplied by the average energy per standing wave divided by the volume of the container, what did Planck do volume of the container is the same, number of standing waves is the same but, he had an issue with the average energy per standing wave, that he said if K T if he uses K T then is coups when you are using it at short wavelength.

Therefore, the average energy cannot be K T it has to be something, because the K T comes from the kinetic theory of gases classical physics and u b partition theorem, that is half K T for potential energy, half K T for kinetic energy, add both these are added it comes to K t.

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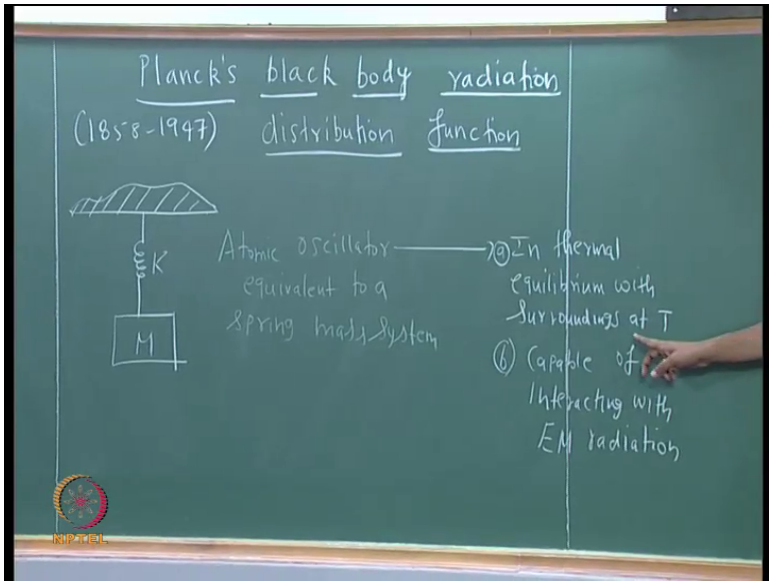

But Planck somehow figured out that the problem is with K T, so how do you overcome then he used a different approach, he used the harmonic oscillator. What is the harmonic oscillator? harmonic oscillator is a basically a spring is equivalent to a spring mass system, the spring mass shown here is the spring mass system, if the stiffness of K, and a mass of M, so he considered an atomic oscillator equivalent to a spring mass system so that he can why for a spring mass system he can get a natural frequency, and are you getting the point, natural frequency time period so he can work out some argument.

Anyway in yesterday's class also some shady argument was there no, somebody was saying so at the end he did not he got a half the he got only half the answer, so he said that it has 2 polarization plain I mean parallel and perpendicular he multiplied by 2. Now, Planck considered an atomic oscillator equivalent to a spring mass system, there are 2 important points you have to note, this spring mass system at the atomic oscillator is in thermal equilibrium which is surrounding that T, even if it is at equilibrium its surrounding T it will continue to emit radiation because some guy already told that, (( )) law this activity will seal only when T becomes 0 Kelvin, and it is capable of interacting the oscillator is capable of interacting with the electromagnetic radiation.

Now, we start from classical physics of mechanics through Boltzmann's statistics, and find out what is the energy, what is the total energy, what is the number of oscillators which are possible between 2 energy levels, what is the total energy, what is the total energy of a certain

number of oscillators, what is the total number of oscillators in a certain band? So, total energy divided by total number of oscillators will give the average energy, this average energy should be different from K T, if you get K T then you will come back to Rayleigh-Jeans distribution, so Planck did some magic there and got something which is different from K T, because of which he got E to the power of c 2 by lambda T minus 1 is the denominator.

Then the argument goes like this then the proof everything is correct finally, he gets an agreement with experiment, so if everything is correct there is a very crucial assumption he made in 1 step, that must also be correct therefore, that is that is a correct theory to explain blackbody behavior as of today, there is nobody else is able to come out with an explanation which is better than which can give better agreement to it, but somebody can challenge it tomorrow, like he is saying no theory is no theory is so (( )) that it cannot be questioned, every any theory has to be questioned, only then progress will be made but, as things stand Planck's; Planck's distribution is king.

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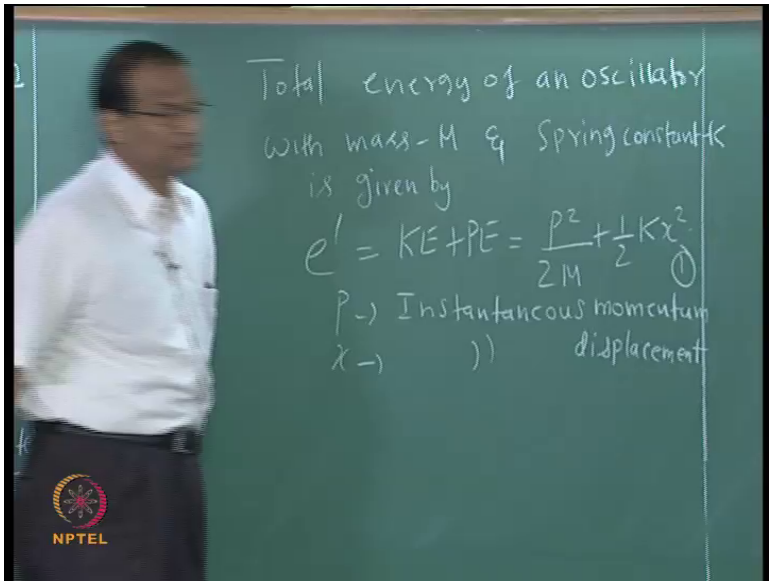

Now, total energy of again it is the not as shady as Rayleigh-Jeans but, it also mathematically involved so the follow the steps very carefully. Total energy of 1 oscillator with mass M and is given by E equal to or I do not want to use E, because E is reserved for that or electron, what is it charge of an electron something is it or mass of electron charge of electron, So let us do E dash, so E dash is K E plus P E, for the spring mass system it will be half K a squared

plus P squared by 2 M, P is the instantaneous momentum. What is x? Instantaneous displacement.

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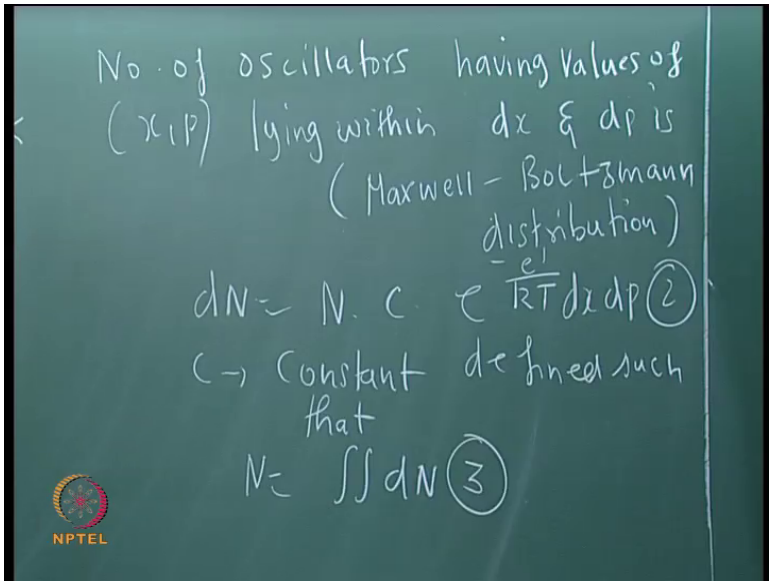

So, number of oscillators, number of oscillators having values… are you getting my point, any oscillator is characterized by its momentum and the displacement and the position, and the relationship between (( )) uncertainty will be Heisenberg's principle, del x, del b is greater than equal to h by 2 (( )) where h is again the Planck's constant. Now, the number of oscillators having values lying between this has to come from certain probability correct, this comes from what is called the Maxwell's Boltzmann's strategy, this is the pre quantum strategy, pre quantum mechanics classical so this comes from Maxwell Boltzmann's distribution.

So, Planck did not dispute all have got Rayleigh-Jeans said, he used most of his arguments he had a problem with the average energy, the number count and all that is for him, he had a problem with the energy he made the crucial change that we will see. Now, this is given by d N, n into c into E to the power of minus E dash by K T into dx dP correct, all of you know that it is a probability it follows an exponential distribution, this is what is called as a Arrhenius type distribution, what is an Arrhenius type distribution so the probability will be like this.

Now, what is c? c is not the velocity of light constant defines such that such that N is equal to double integrated dx dP, N is that is somehow if you the total number of oscillators at all the levels if the double integral is known, the left hand is known and then right hand side you can substitute it for d N integrated and pull out all the things and then except c, and then you can get the value of c, all these things will get cancelled so do not worry. Now, how will these curves of constant energy appear in the elliptic space phase? of course, it is not the full Planck's distribution I am giving the abridged version, as you know that it will take a long time it is not required for us to follow it fully, it is his life time work it is his Nobel prize winning work but, we are trying to see only the essential steps.

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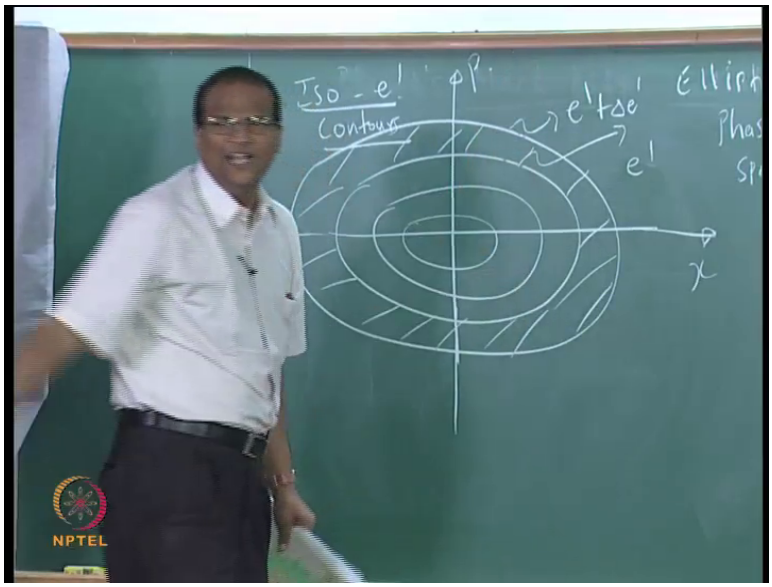

So, this is basically a elliptic in the phase space, do not get worried we would not get too much into all these physics but, this is important for the development, what are the 2 what do x and y indicate? Displacement momentum, all these are isoenergic contour, all these are isoenergic, stated in simple English there can be several combinations of momentum and displacement which can give the same energy, if you draw all this it will be an ellipse, that is all, because E is a sum of 2 things. Now, I can say that for illustration I can say this is E dash, so the next thing is E dash plus delta E dash, so the logic is very simple now, between E dash and E dash plus delta E dash if you are able to find out the energy of all the oscillators and keep it in the numerator, between E dash and E dash plus delta E dash if you find the total number of oscillators keep it in the denominator, total energy of all oscillators between 2

bands divided by the total number all oscillators between the 2 bands, it will give the average energy of any oscillator which is lying in between the 2 bands.

And then once you get that step you no more developments are required for this, go back to the Rayleigh-Jeans and instead of K T use this average energy and complete the derivation. Now, the series of the treacherous steps have to done, to get that average value of the average energy of the oscillator which is different from K T, that is where Planck introduced the hypothesis is it clear up to this stage. So, you can take curves of constant energy isoenergy contour so iso E dash contour.

Now, consider 2 energy levels E dash and E dash plus delta E dash, now we are focusing our attention on some band, the story is about this band, it could be any E dash and any E dash with delta E dash we can take 1 such delta E dash, and now try to find out.

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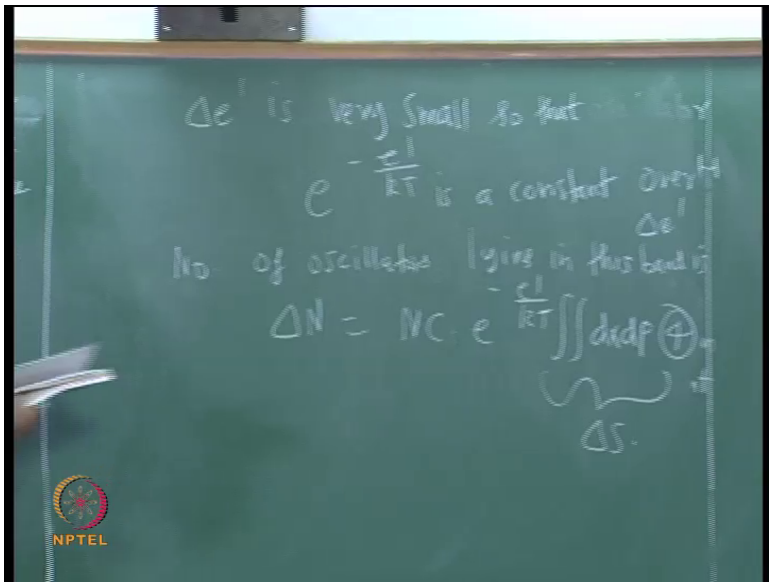

Now, let us say delta E dash is very small, so that E to the power of minus E dash by K T is a constant, where it is a constant where over delta E dash, is a constant. So, now if you count the number of oscillators, we count the number of oscillator lying in this band E dash to E dash plus delta E dash that is given by… So, d N is differential, delta N means I am doing some integration, so n c E to the power of minus E dash by K T double integral dx dP, what is the equation number 4 very good, 4 so far it is very clear isn't it so you have to double integral with respect to dx dp.

What is this dx dP? it is the area of the area of the slice of the ellipse, if you take a slice of the cassata ice cream, there will be several 1 vanilla ice cream will be there, one coffee ice will be there, orange ice cream then 1 cake slice also will be there, let us assume that this delta E dash is like that cake slice in the cassata slice, have you tried it yet let us not get distracted we are doing something very serious. So, what is this; this is basically that area of that cassata slice whatever, area between the 2 ellipses you will write a story, area between the 2 ellipses given by the hatched theory on the black board.

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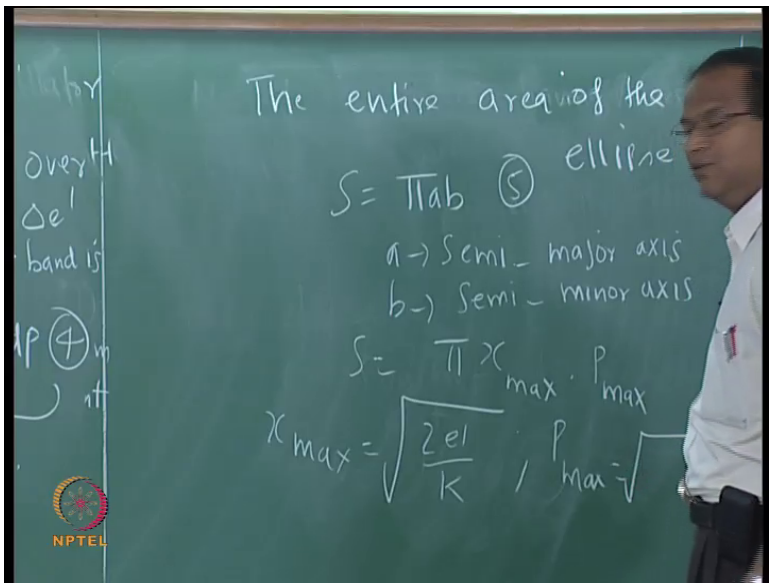

What is the entire area what is the entire area of the ellipse, s is given by phi a b where a is a semi major axis and b is a semi minor axis, if you want you can put an equation number for this also it is not required. So, s will be equal to pi in this case in the elliptic phase space s is s will be equal to pi into x max into P max, from the equation of ellipse s square by a square plus y square y square by b square is equal to 1, can you get a expression for x max and P max. You know the equation of the ellipse isn't it x square by a square plus y square by b square is equal to 1, and can you get the expressions for x max and y max, anyway it is part of the assignment yeah root very good 2 x max 2 E dash by K, P max yeah 2 M E dash.

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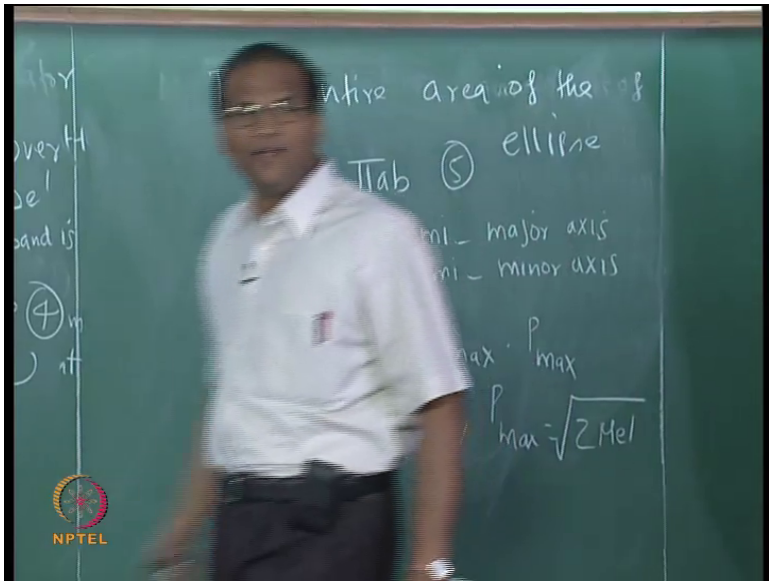

I do not have time to derive this, do not worry people who are not able to get this take this as true, this is 1 of the questions in the assignment which I am going to give tomorrow, it is just the equation of ellipse, so x max and this are given like this, Now you can get the area s.

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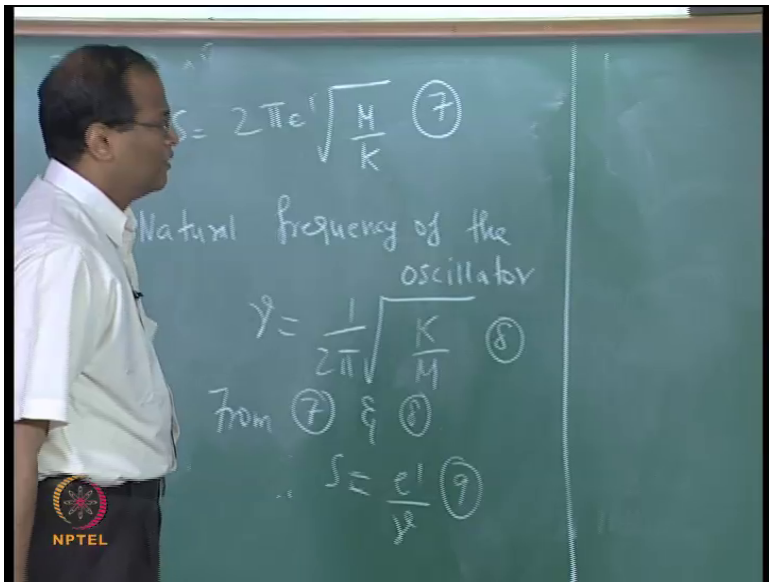

Therefore, s is 2 pi E dash root M by K, so it is an important step 6, 7, what is the natural frequency of oscillation on the atomic oscillator with a spring constant of K and a mass of m, so the natural frequency nu will be 1 by 2 pi, 1 by 2 pi root of M by K, or K by M, K by M, K

by M, yeah from 7 and 8 from 7 and 8 s equal to E dash by nu very good, from 7 and 8 you can just substitute for this so K by M root of K by M will be 2 pi nu, root of M by K will be 1 by 2 pi nu, substitute 1 by 2 pi nu, 2 pi 2 pi will get cancelled so s will be E dash by nu.

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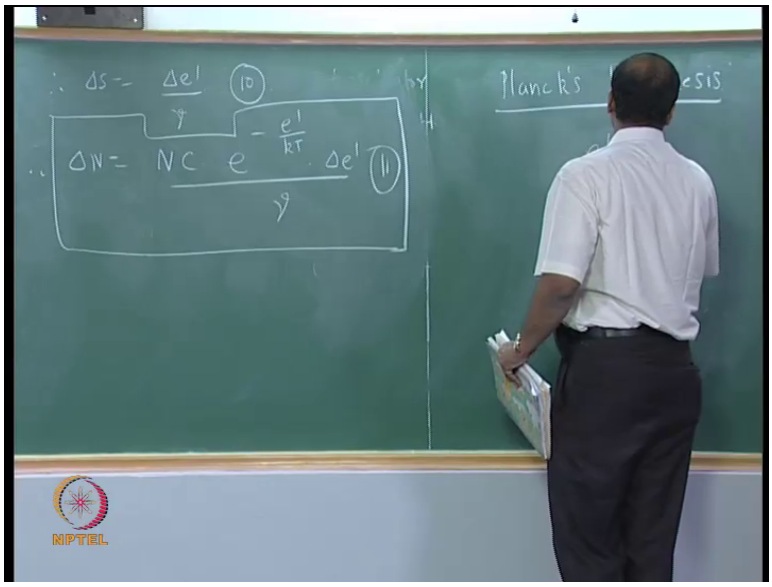

Therefore delta s will be… so getting back to our expression for delta N therefore delta n will be N c E to the power of minus E dash by K T, yeah this is very important intermediate result we have, please correct me if I made some mistake, I think its fine. Now, comes the Planck's hypothesis, which he proposed, now consider this elliptic phase space is divided into increments, and into equal bands of area h, so Planck's proposal is a very important step he numbered these rings he put lot of rings elliptical rings, he numbered these rings N equal to 0, 1, 2, 3, 4, 5, 6 and he proposed that the energy of an oscillator located at the inner boundary of a particular ring can be put as E dash is equal to N h nu, E dash equal to s nu, what is s? What is that area between the 2 bands, now that area is replacing as N h, if h becomes 0 we are in trouble it will reduce to kt.

But do not ask me how we got this idea and this thing may whether God came in his dream we don't k now but, he proposed this, and it finally it when you do that you get the correct blackbody behavior, so this is the Planck's hypothesis, now comes the crucial step.

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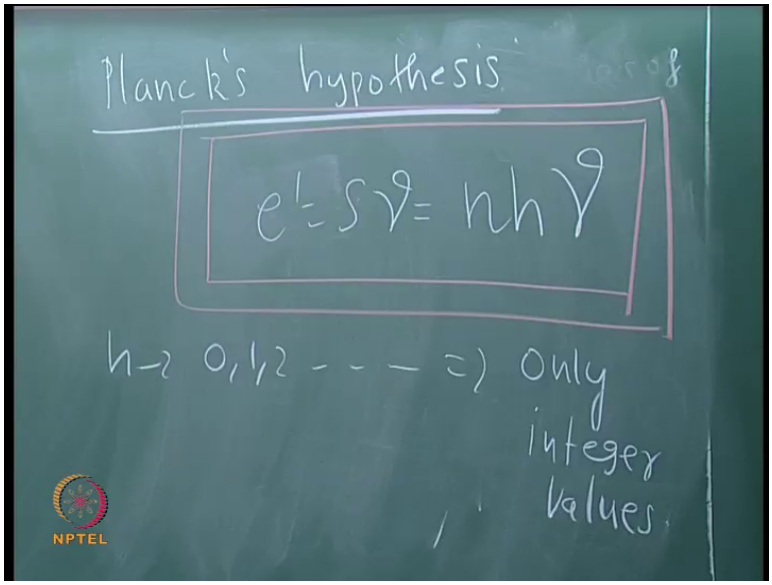

So, s equal to E dash equal to there is no rational basis of this there is no rational basis for this it cannot be established in mathematical principle, he just proposed that you can divide the whole thing into rings N, n is a number N can take only integer values.

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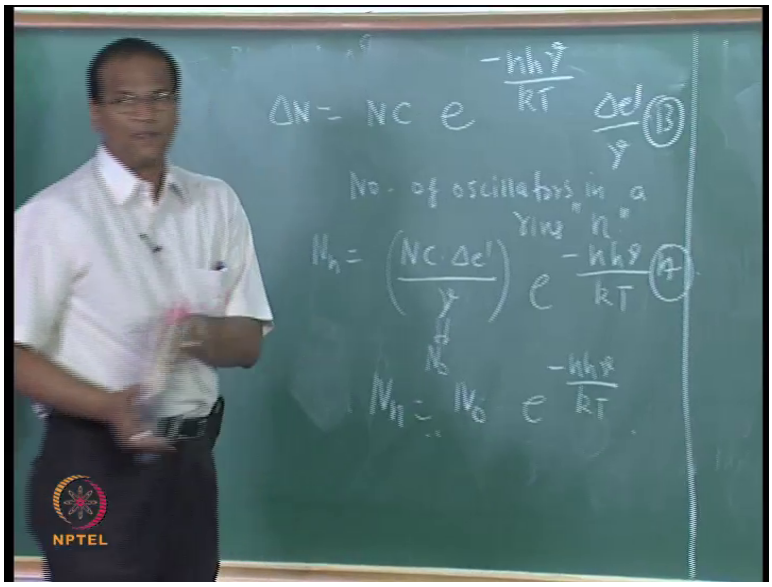

Now, if this is the story, so delta N becomes with this Planck's hypothesis N minus N h nu, N h nu, by K T is there into delta E dash by nu let him let this fellow be there. Therefore number of oscillators number of oscillators in a ring N is given by N l is the same as this, N c

into yeah please so I have put this as 12, so call this 13, that N delta E dash and nu, N delta E dash and nu are basically you can replace it by some N naught is a constant into we got an expression like this.

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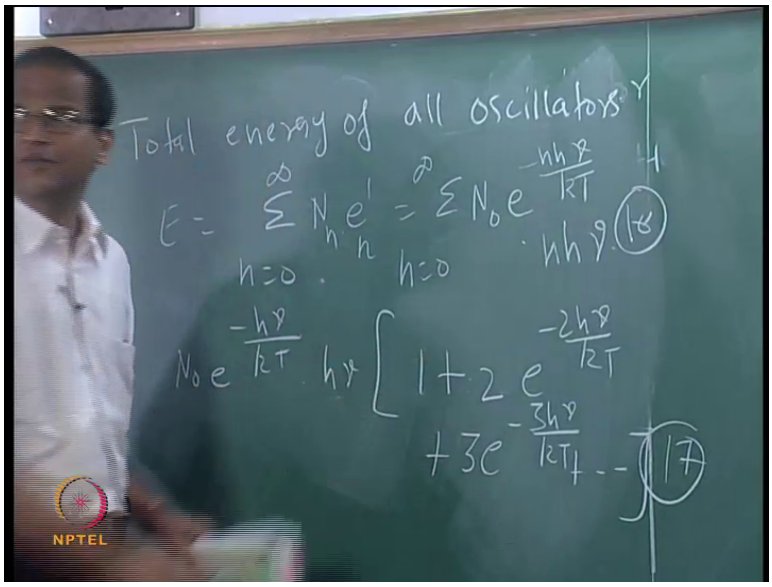

Now, what is the total energy of all oscillators, what could be the minimum and maximum value of N, n minimum 0, maximum infinity, so you have to add up for all values of N, total energy we call it as a capital E is equal to N is equal to 0 to infinity, N n multiplied by correct, E dash corresponding to a particular value of N and we have already have the expression for this, E dash is given by N h nu.

So, if we expand this N naught is there, so can we take N naught into h nu into E to the power of minus h nu by K T outside this summation, so 16 so can I take N naught E to the power of minus h nu by K T into h nu into 1 plus correct, 1 plus 2 into E to the power of plus 3 into E to the power of… did I make a mistake I think it is fine, yeah it is all fine.

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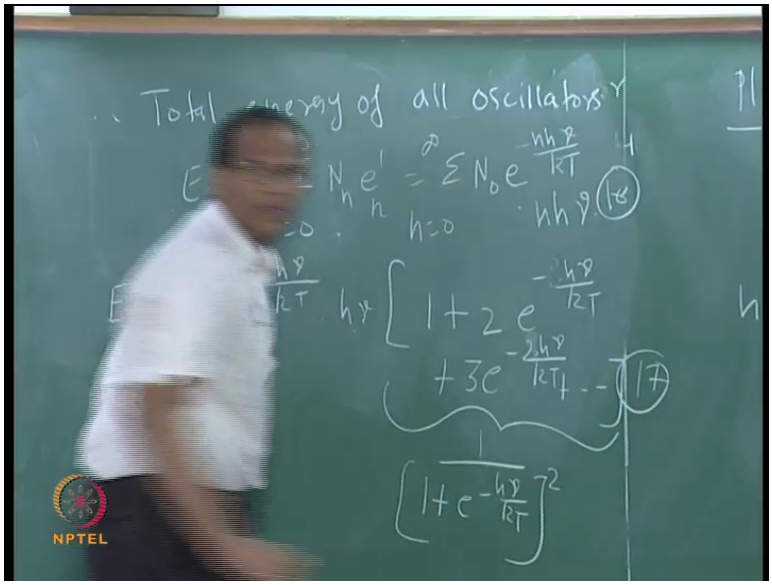

What is the sum of the series?

Student: (( ))

yeah sorry this is E to the power of 1 mistaken, is it correct, what is the sum of the series? yeah this is 1 by you can check 1 minus I do not think so, let me check, yeah it cannot be 1 plus yeah it has to be minus, is it correct, we can expand and check it, which 1 no, square will be there, you have doubt no then we have to waste 2 minutes.

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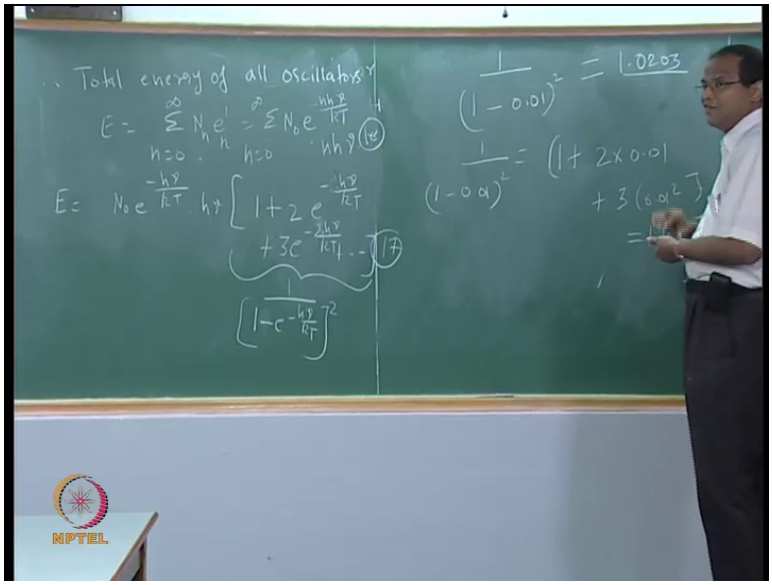

So, please check please tell me the value please tell me the value, how much is this 0.99 whole square 1.0203, Now you want to expand that no, what I am claiming that it is 1 plus 2 times point naught 1, that is 3 times yeah what is this Pradeep?

Student: Same 1.02

Same we are doing something like this low level our level Planck is doing at his level that is all. But no problem you can get a doubt I also get that doubt, I may instead of 1 it whether it could be 2, whether instead of plus it should be minus, when you get that doubt like this I will always do 1 by 1 plus point naught 1, or 1 plus suppose you got what is the expansion for log x, if you forget calculator is available you can just do it.

If you do not want to get confused for the rest of your life the basic goal is very simple, you have to remember very few things, by remembering very few things you should be able to derive many things, then that means your fundas are intact, if you try to remember too many things that is where the problem is. If you just know d by d x of sin x you can get all the other things isn't it, and d by d x you can all the other things integral this thing and nu, so you have to remember few things then those you should never get doubt about these things and you have expand all these things, now everybody is convinced. Therefore one by 1 minus x whole square is this, this is not a part of the Planck's distribution fine.

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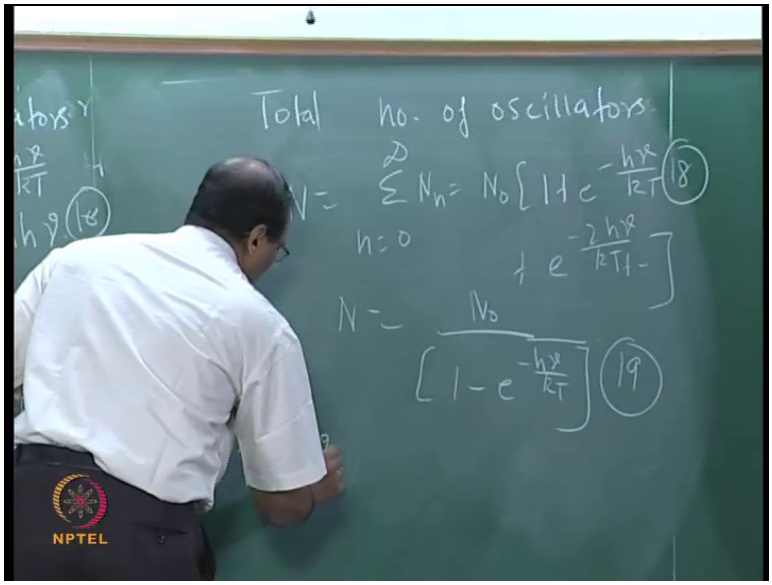

Now, we got the total energy of all the oscillators, what is the total number of oscillators? N n there is something called N n isn't it yeah that is a notation, so what is this now, N naught into 1 plus E to the power of minus 2 (( )) it goes like this, is this correct, so what is the sum of the series, N naught is equal to N equal to. So, we got the total energy of all the oscillators, we got the total number of oscillators, so the average energy for oscillator is basically 17 divided by 19.

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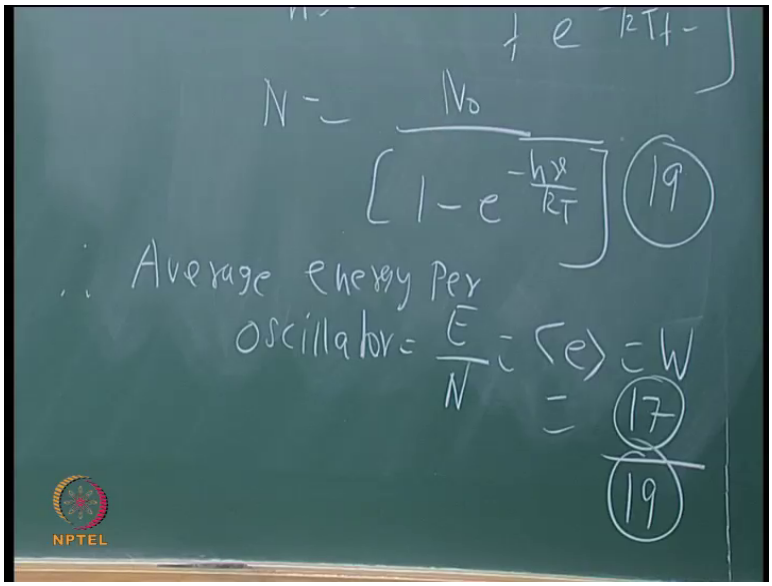

Therefore, I do not know what is the symbol Planck used w; w, that is 17 by 19 yeah tell me what is it?

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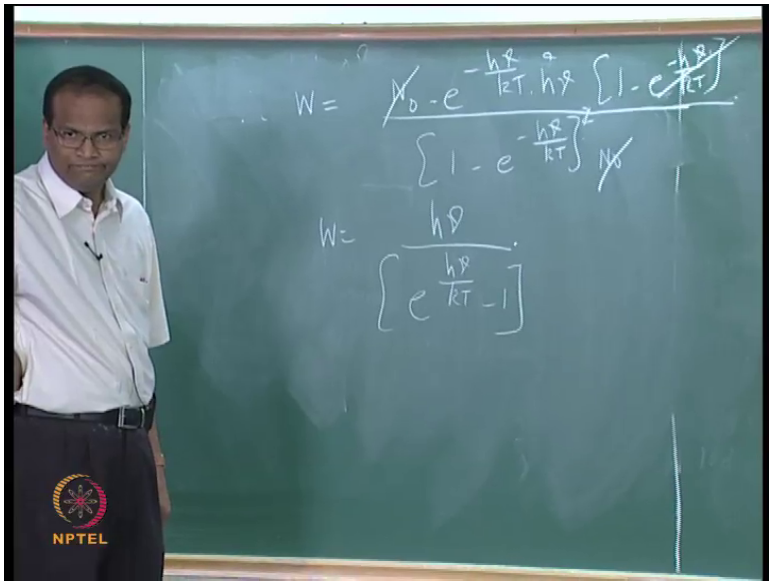

W equal to N naught; N naught this is a very important step, he got the answer already, he is already solved the problem. Now, Planck wanted to check what will happen when h by h nu by K T is very small.

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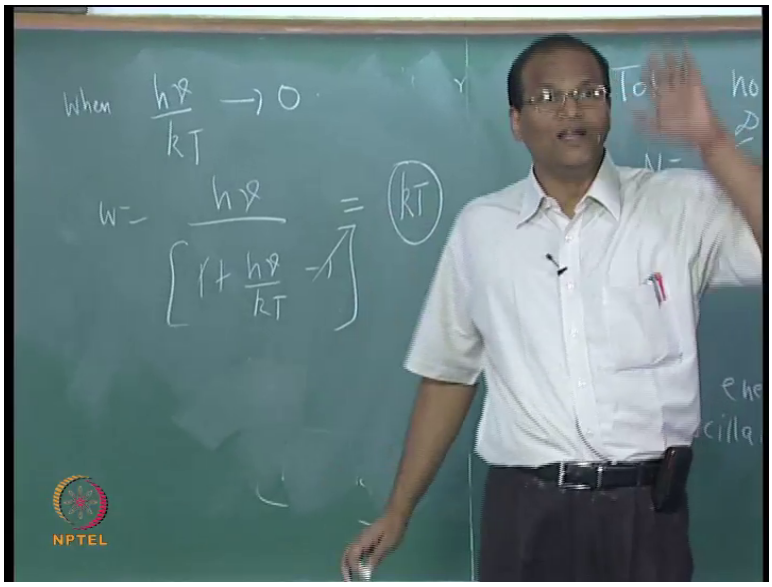

So, for small E to the power of h can be expanded as 1 plus K T, w equal to K T is used in which distribution Rayleigh-Jeans, so there is consistency as well as Planck was forced to come to the conclusion that h cannot be 0, h has to be small but, finite, so he introduced a fundamental physical constant in nature, he introduced a fundamental physical constant in physics it was either too unknown, and when he matched with the experiment he found out to be 6.627 into 10 to minus 34 joule second, and in honor of him we call it as a Planck's constant, so this is this first part of the story.

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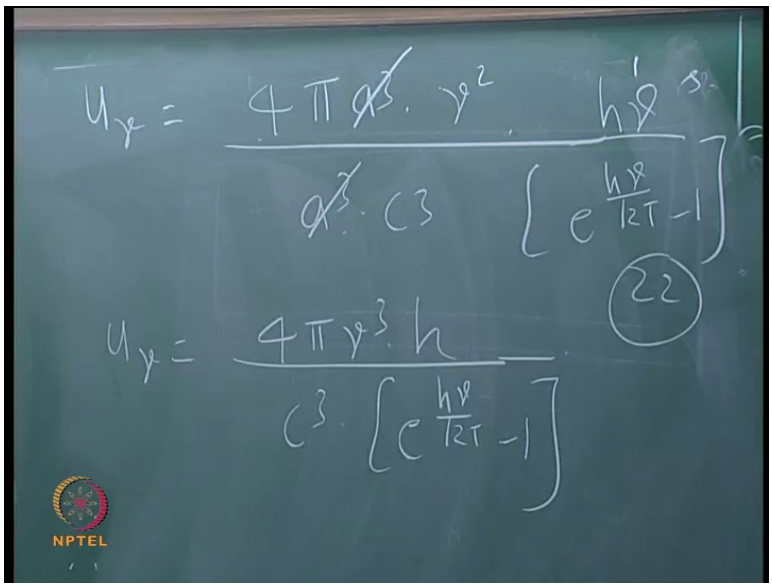

Now, we have to complete the derivation, so nu equals 4 pi a cube what was that 4 pi a cube.

Student: Nu squared.

Nu squared is it correct, please let me know if made any mistake, is it correct, so what is that we have to give some numbers, what is the number for this 20, good 20, 21, 22 can you develop on this, so u nu is given by 4 pi is nu cube, 4 phi nu cube h it is all slippery please let me know if I am making some mistakes.

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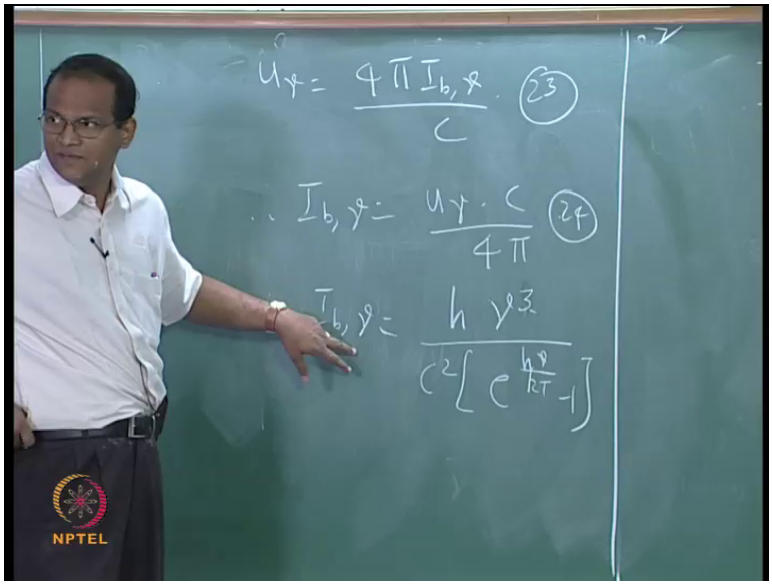

Now, u nu is it correct, what therefore, i b nu 23, so tell me the expression for i b nu equal to 4 pi goes off h nu squared h nu squared by c squared.

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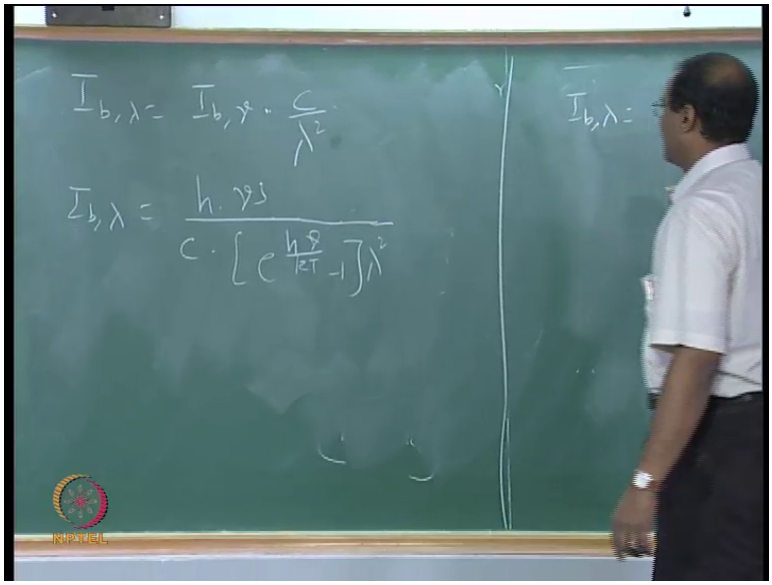

Now, i b lambda equal to i b nu into… no where is the problem?

Student: Nu cube

nu does not get cancelled. No

Is this correct now tell me let us I will hold on for thirty seconds is this correct.

Huh

Fine now i b nu is equal to i b lambda i b nu into

Student: (( ))

Lambda square by c or c by lambda squared, c by lambda square, sure h nu cube by c correct, lambda square I am just writing what you are telling me, I hope in the next 5 minutes we will get the Planck's distribution fine.

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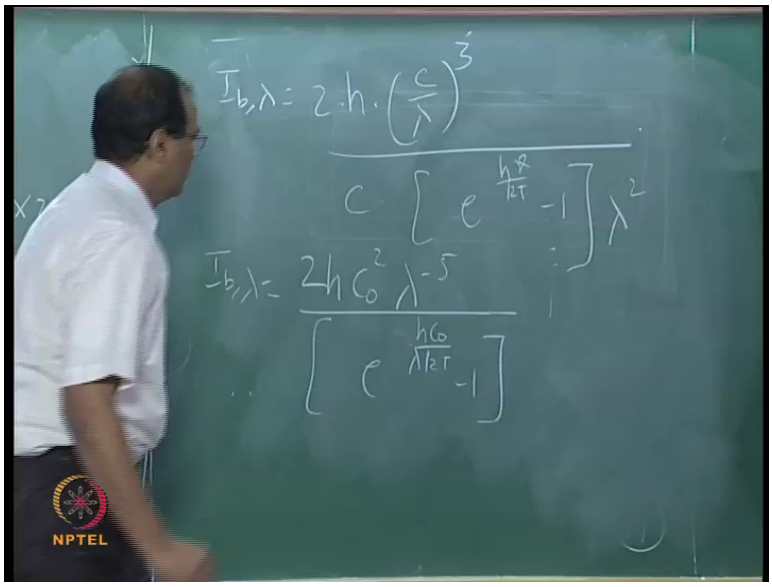

So, i b lambda h, we multiplied by 2 somewhere we left that, to take care of the 2 polarization, see you are letting me down, some where it is multiplied by 2, so u nu is no that is here u nu was

Student: 2 into

Into 2 I can 2 h c naught squared, where h c can be… that is it.

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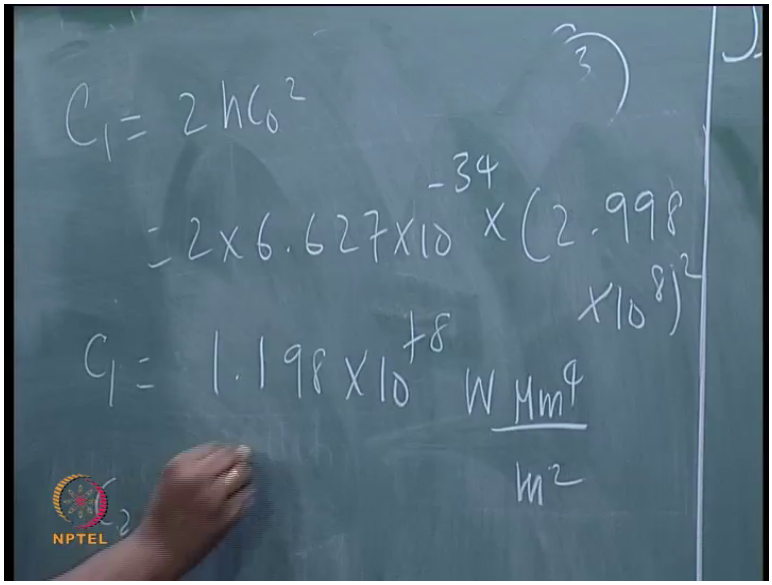

Now, c 1 and c 2 we can work out the values, yeah tell me the value, 1.198 no 10 to the power of.

Student: Minus16.

Minus 16 joules what is it joules per joules second no. Now instead of making it in I gave you the unit when I derived the Rayleigh-Jeans first, I can make it as please tell if this is correct, is this the correct unit for this, I did 10 to the power of meter to micrometer 10 to the power of 6 into 4 10 to the power of 24, 24 and minus 16 gave me plus 8 therefore, c 2 is equal to h c naught by K 14.

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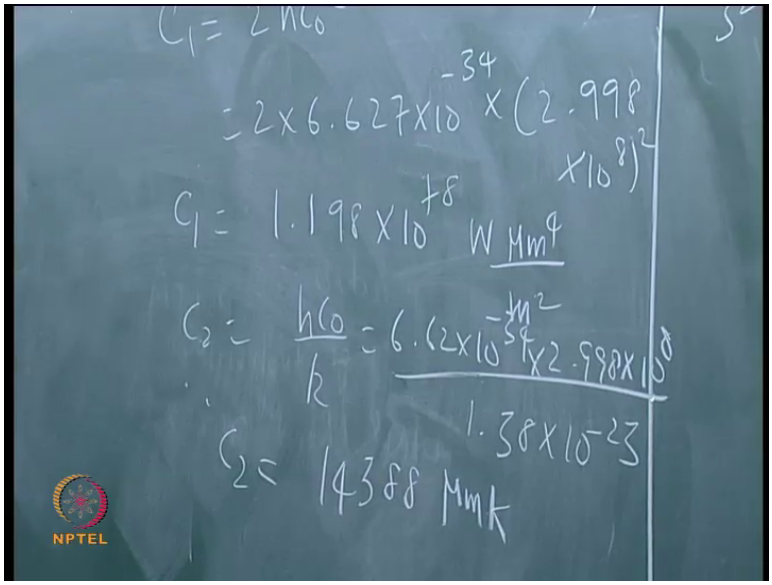

So, c 1 is called the first Rayleigh-Jeans constant, c 2 is called the second Rayleigh-Jeans constant, so what is so great about this, he also did something he multiplied by 2 polarization and all that, he got an expression for i b lambda which when plotted gives you exactly the same thing as what people measured for all wavelengths and for all temperatures therefore, his result must be correct, his result if his result has to be correct all the steps which he has done in the result must also be correct, all the steps which he has taken that all but 1 of the steps were exactly the same as what Rayleigh-Jeans has done that was based on the Maxwell Boltzmann's statistics.

There was 1 crucial departure from the Maxwell-Boltzmann's strategies when he proposed that the energy can be divided into that bands and s E dash will be N h nu, therefore, e dash is equal to N h nu must be correct, because only if you do that only if you make that assumption you get an expression which agrees with experiment therefore, this theory must be correct, therefore, the hypothesis that the energy transfer can take place only in discrete multiples of h nu is correct, this is the beginning of quantum mechanics. So, after 19 naught 1 he proposed, after that he spent so many years in trying to figure this out so in 1918 finally, they gave him the Nobel prize.

In tomorrow's class we will see what happens is there a maximum for this distribution for a particular temperature, what happens please remember that a triple integral the integral with respect to lambda cos theta, sin theta, d theta, d phi all that must lead to sigma d to the power

of 4. In tomorrow's class we will derive this Stefan-Boltzmann's law and the Veins displacement law and that will complete basic blackbody radiation physics. And then there is something called Band emission that is between two wavelengths what is the fraction of the energy which is emitted that we will have to see.