Conduction and Radiation

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Lecture No. # 08

Planck's distribution and Wien's displacement law

So, in the last class we derived the planck's black body, radiation distribution function for which more or less he was awarded the noble prize in 1918, but the noble prizes is for quantum statistics, so he got the nobel prize in 1918. What we saw yesterday was an abridged proof of his theoretical development, so I think most of the important points were coverd, in today's class we will look at the planck's distribution further, and try to see if we can extract more information from the planck's distribution

We will also look at we have been seeing only the mathematical form. If we plot the planck's distribution versus lambda, how does it look for various temperatures, can we derive additional information from this and so on? That will be the goal of today's lecture. Let us start with the planck's distribution.

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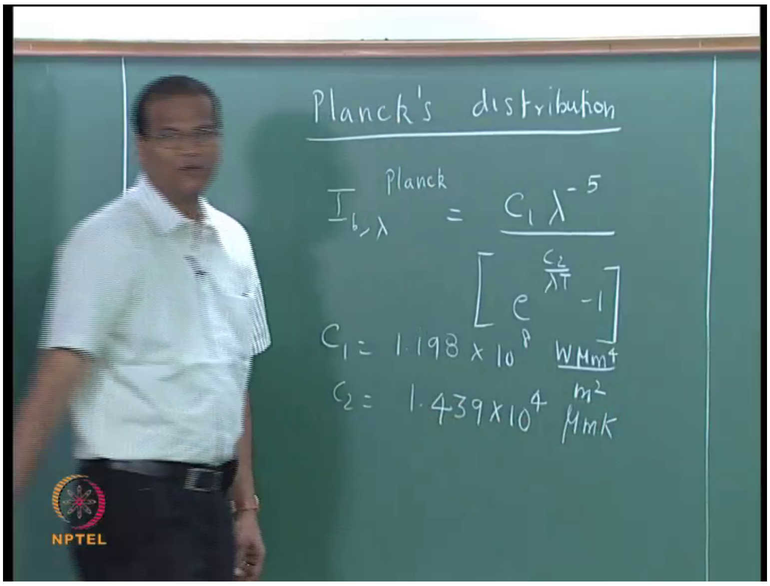

From now on we (( )) this or we will get rid of this planck, Ib lambda means it is only planck. C 1 is 1.198, is it not? 1.198. As students of radiation you must remember all this, you must remember C 1, you must remember C 2 and you must remember sigma. You do not have to remember C 1 instead even, if you know H, and boltzmann constant K you can get C is equal to 2 hc naught square C 2 is equal to H naught by K.

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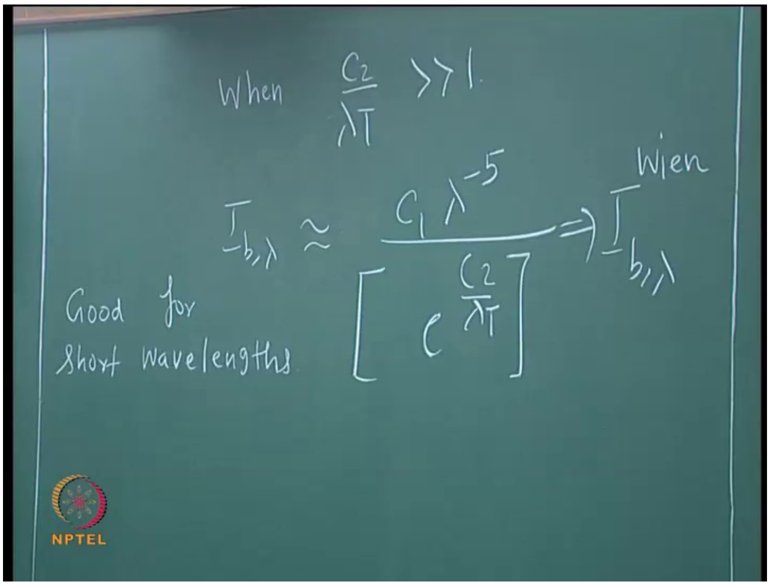

Now, when C 2 by lambda T is much much greater than 1 divided by e to the power of C 2 by lambda d is much greater than 1 therefore, E to the power C 2 by lambda d much much greater than 1, so this becomes, so what is this Ib lambda (( )) Ib lambda. So, when C 2 by lambda d is much greater than 1 it means it is valid for very short waves, very short values of lambda. Good for short wavelengths because C 2 by lambda d, when will C 2 by lambda d much much greater than 1? When lambda d is very small, so for short wavelengths is the reasonable approximate, when C 2 by lambda T and C 2 by lambda d is much less than 1, what is this now? (( )) (( )) rayleigh jeans.

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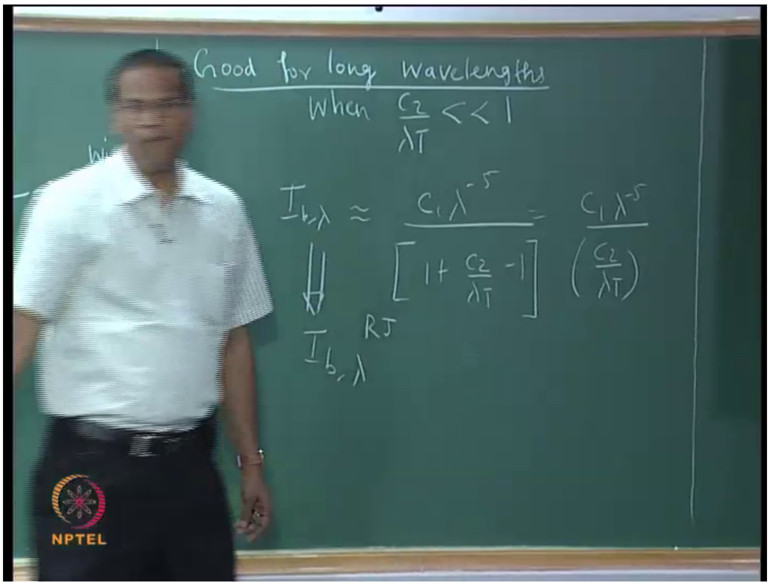

So, the wien's distribution and the rayleigh jeans distribution, are two asymptotes to planck's distribution, while planck's distribution is valid for all values of wavelengths, the wien's distribution as well as the rayleigh jeans distribution are valid for some portions of the electromagnetic spectrum. So, good for and you can do a good exercise C 2 by lambda d equal to 10, C 2 by lambda d is equal to 100, C 2 by lambda d is equal to 1000, find out Ib lambda from planck's? find Ib lambda from rayleigh jeans of wein and compute the percentage error, and you will be in a position to understand what is the penalty you pay for using an approximate expression for the 2 asymptotic limits. Is it okay?

So, far we have seen only for mathematical development, what is the Ib lambda look like when plotted on a graph sheet?

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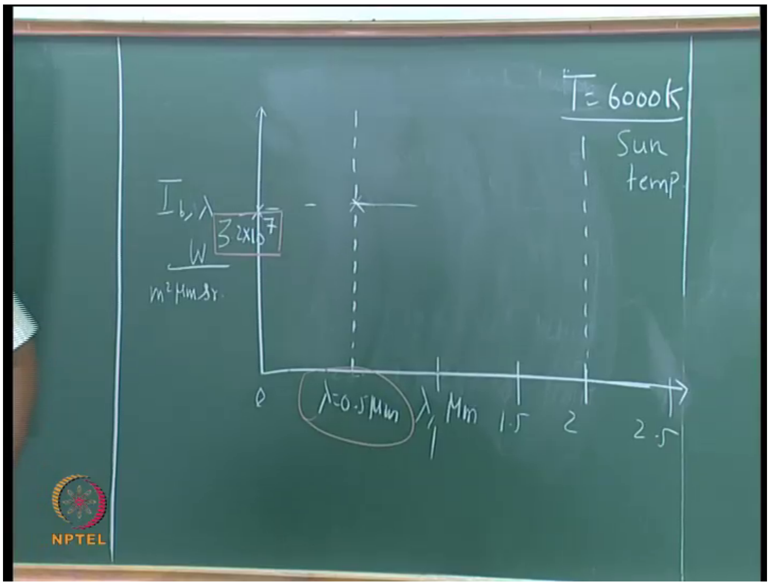

So, some books also plot Eb lambda versus lambda watts per meter square, let us take T equal to, why are you so much worried about this 6000 kelvin? (( )) Sun temperature. So, please calculate for lambda equal to 0.5 micrometer, what should be the I lambda for the sun, for the radiation from the sun? I want this values suppose, I am having this point, what is a value of the Ib lambda, please calculate? So, I will say lambda equal to 0.5 micrometer some 10 to the power of 6, 7, 8, 9 what is it? (( )) (( ))

So, it is 3.2 into 10 to the power of 7, if this is 0.5 0.5 1 1 0.5 0 0.5 1, please calculate at 2 micrometer, and somebody else should calculate at 0.1 micrometer, with 3 point we will try to generate the curve 1.6.

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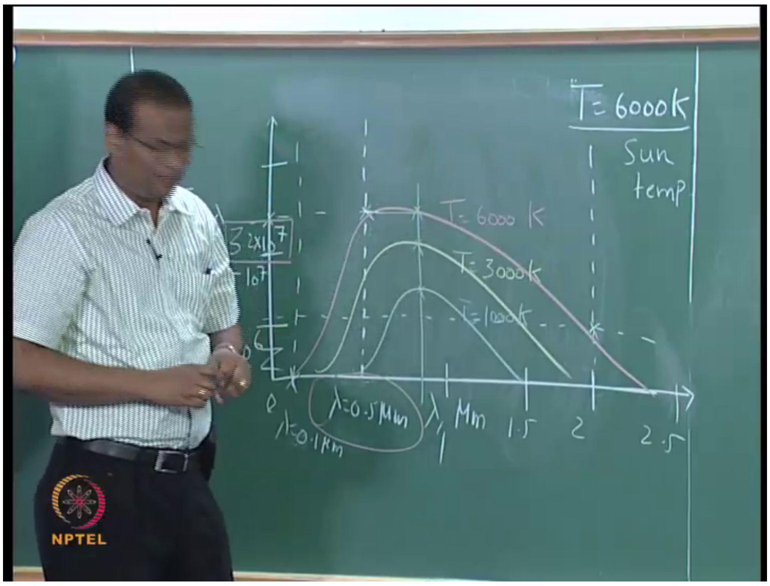

So, we will keep this, how do we start? let us keep this 10 to the power of 7, 10 to the power of 8, 10 to the power of 6, and you do not worry about this, what did you say, vikram? 1.6. Now, calculate at 0.1 micro meter, 457.2 that is all, that is 0, you can take it as 0. So, now infact it is highly is an exploded view, I mean this scale is very much stretch if you put lambda equal to 0 to 100 micrometer and all that, it will become very, the curves become sharper. Now, I can have various curves, so this let us say T equal to 3000 kelvin, and say T equal to 1000 kelvin, you can check it you can view, you can show in all the values into your calculator and then get the function out.

What are the important features of this Ib lambda? Is everybody through with this? So, what are the important features of this Ib lambda with lambda? Point number 1: Ib lambda is a continuous function of lambda, it is never discontinuous, for every value of lambda there is an Ib lambda, point number 1. So, you can see, so you can put the tittle is planck's distribution salient features, Ib planck's distribution salient features, point number 1 Ib lambda continuously varies with lambda. Number 1, Ib lambda continuously varies with lambda. Number 2, for every temperature there appears to be a peak, for every temperature there appears to be a peak for every temperature not there there is a peak for every temperature there is a peak.

Point number 3, for a given lambda Ib lambda increases the temperature, is that clear? You take any lambda, Ib lambda for 1000 kelvin is here, Ib lambda for 3000 kelvin here. Even it is also intuitively apparent from the second law of thermo dynamics, if more the temperature, higher will be the Ib lambda. So, for every value of lambda Ib lambda is Ib lambda increases the temperature. Number 4, the peak of the Ib lambda keeps shifting to the left, that is the peak of the Ib lambda keeps shifting to the left for increasing temperatures. How do we get this peak now?

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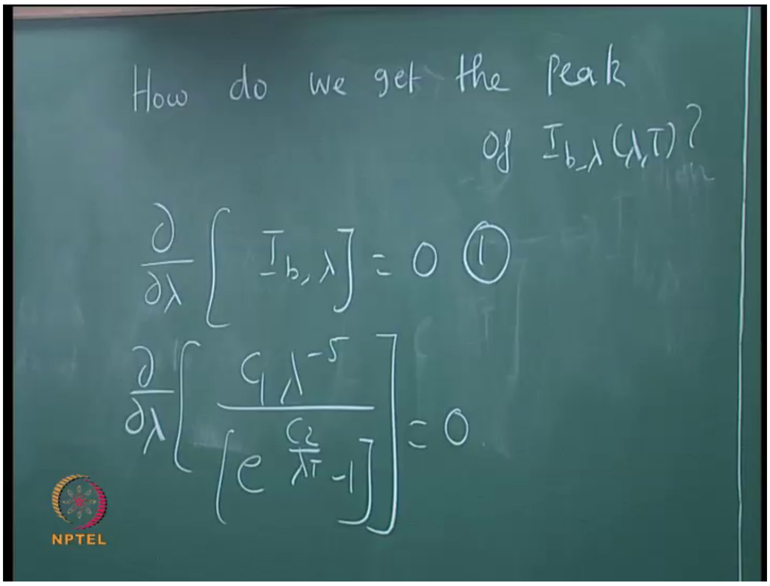

We have to get the peak now, we have do some calculus, we will do that now. You want to answer this question? Therefore, I do not want to differentiate T, because I know anyway it keeps on increasing with T. Only with lambda it increases and decreases therefore, if we put d by d lambda and make it equal to 0, there is hope that it will become state steady, I can get that value of lambda or lambda d, which will give me the maximum of Ib lambda.

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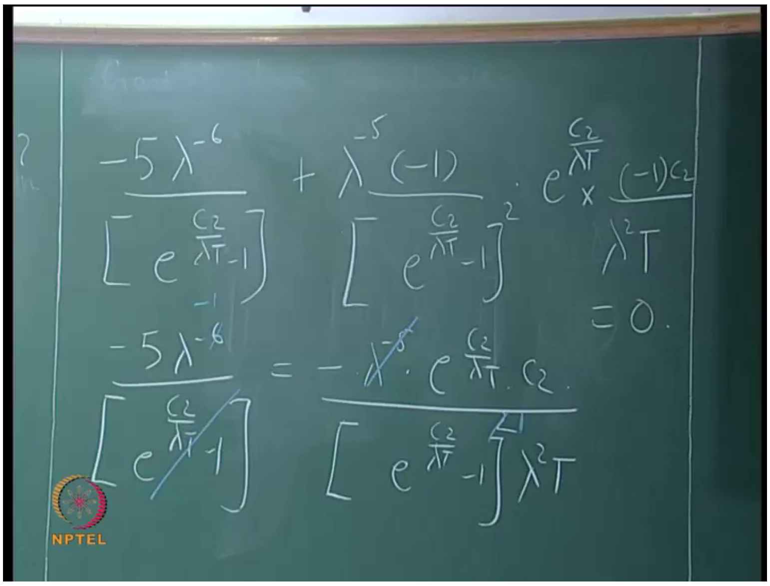

Now, let us do the… from integrating by parts are differentiating by parts, I take this as E to the power of C 2 by lambda d minus 1, I take it as a product. So, first minus 5 lambda minus 6 divided by this plus I take the lambda to the power of. Then E to the power of c 2 by lambda d minus 1 inverse will be minus 1 divided by E to the power of c 2 by lambda T minus 1 whole square multiplied by, no, into E to the power of C to the power of lambda T into minus 1 C 2 by lambda square T. Why? I am differentiating with respect to vishwas you able to see the the board? Maximizing the distance on the teacher (( )) That is your objective, I am just joking do not. You went through the optimization, isn't it? Course, minimax criterian, remember minimax criterian.

Now, this fellow has to equal to 0, I am going to work on on this since it being recorded, we will do it fresh. So, minus 5 minus, minus, minus what happens? there is a minus again, is that,

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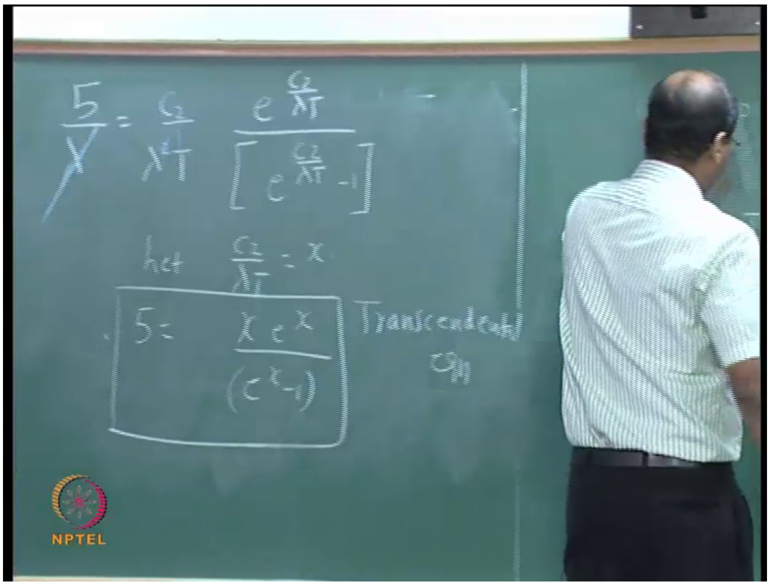

Now, we start cancelling, now minus, minus gets cancel, so 5 that is C 2 by lambda d equal to x. We need to solve for x, unfortunately it is a non-linear equation or transcendental equations, which has numerically solved. If you solve this equation, you will get x plus or x star, x star is that value of C 2 by lambda T which will make Ib lambda stationery. You can take this, we can do it, so this is called as transcendental equations. I have to numerically solve this, let me use the method of successive substitution.

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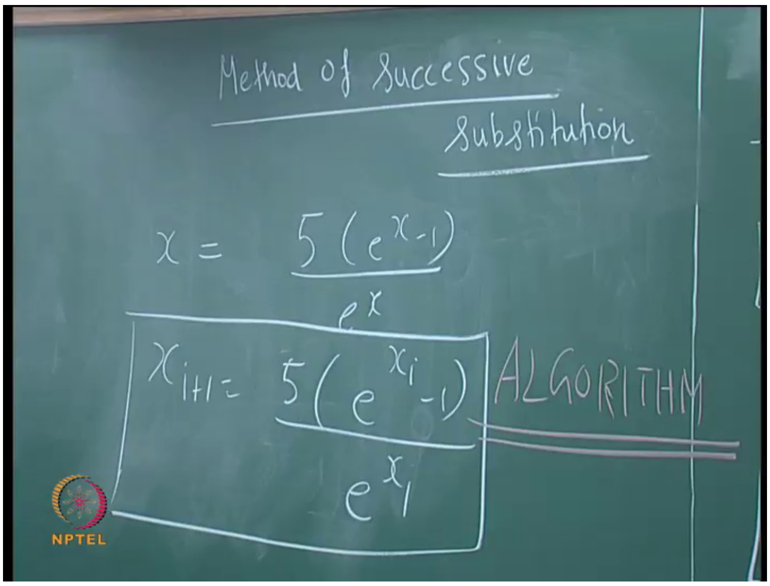

So, I will rewrite the algorithm as, or rewrite the equation as. So, I will say this is the algorithm, start with some xi get xi plus 1, treat this xi plus 1 as an xxi, keep on doing this till xi plus 1 minus xi divided by xi modules is less than equal to some expectable criterion, that is called converges criterion, it is also called the, what did we called it in earlier course? Stopping criterion, correct. let us do this.

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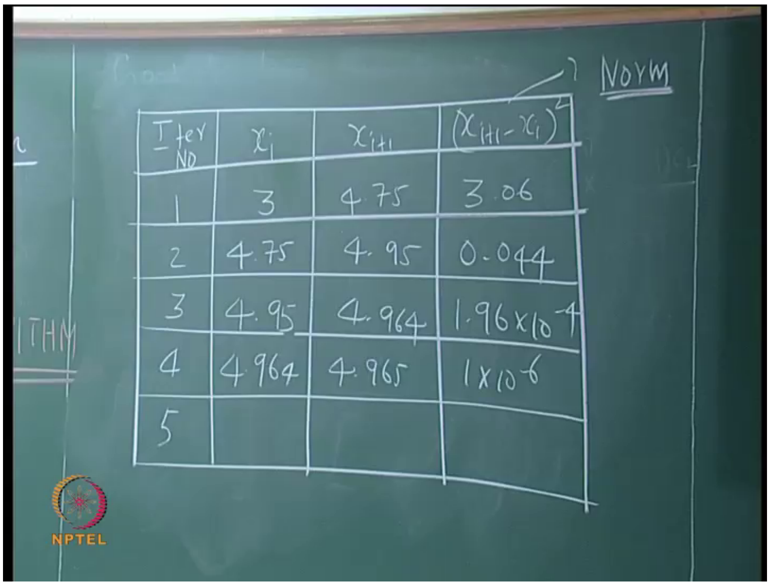

Let us start with the good start with x equal to 3, do not start with 4.8 deepak you are cheating. Start with an x equal to 3, you can start with 100 also, let us start with 3. Is everybody through with this algorithm? So, you have to put a tabular column iteration number xi xi plus 1 and error, iteration number, That is called the norm, it is called the norm square.

Norm is one important, norm is one usual criteria applied. Now, 1 3, so e to the power of 3 minus 1 divided by e to the power of 3 into 5 is the xi plus 1. So, what do you get? (( )) 4.75 error, 1.75 hole square. Samarjeet tell me the error, 1.75 whole square, 755 or 75 (( )) Fine now, use this value of 4.75 as the value of xi error. I want 3 decimal places 9 6 0. It is become silly already error 10 to the minus. Now, we will kill it 4.964 should give 4.965, 1 into 10 to the power of minus 6, stop. So, the root of this equation 5 equal to x either x by either x minus 1 is 4.965.

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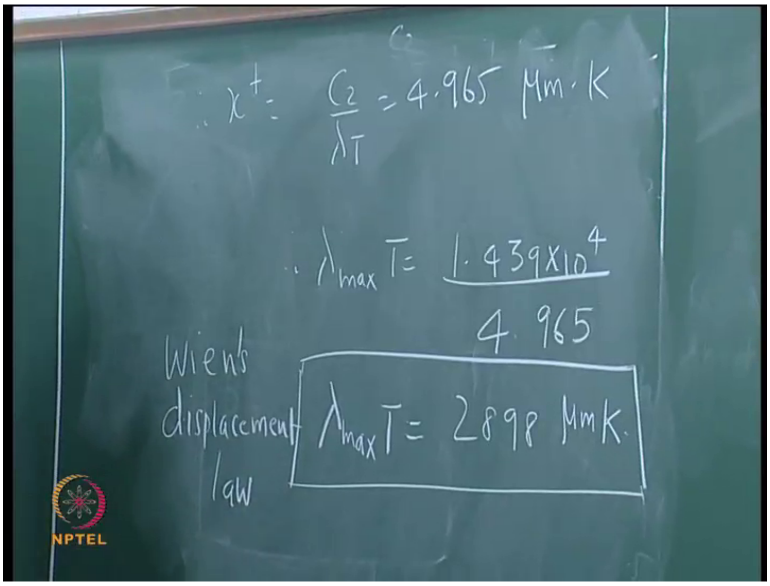

We can solve it using the newton's rawson method also by its pain full because e to the power of x by e to the power of x minus 1, but the gain is instead of 4 iteration, first hit second iteration it will hit, you will get the answer. Because, you know that it has got what type of convergence? Quadratic convergence. Therefore, x star 2898 micrometer Kelvin, this called the wien's displacement law You do not require planck's theoretical distribution to get this, from curve fitting also you can get, people could have done measurements for 5 temperatures, and then they could found why lambda max into d 2898, then what should be the Ib lambda which will ensured that lambda max T is 2898?

So, lot of people would have to try different things, only if you put E is equal to NH nu, or E dash is equal to NH nu you will get the correct you will get the correct black body behavior. So, this is the wien's displacement law, it is very profound result it is a very very profound result You thought it is simply some mathematics we differentiate something in. Now, look at the sun's temperature 6000 kelvin, so the lambda max corresponding to solar radiation is half a micrometer, because this is 3000, this is 6000, so lambda max for solar radiation is 0.5 micrometer. So, it is very important, because 0.5 lies in the visible part of the spectrum that is why you are having day light, that is why earth is so habitable. If the sun's temperature were to be some 12000 Kilometer, 24 hours you require light, because there is no sunlight, there is no photosynthesis, are you getting the point?

Lambda max is 0.5 micrometer, and 0.4 to 0.7 is the visible part of the spectrum, that is why from incandescent bulbs we went to tube light, because daylight. We want a mimic daylight, cfl, led. Because, I mean we want to have a light which is equivalent to, we want the lighting should be equalent to what we have outside, then in a normal daylight, is that clear? Now, my temperature is 37 degree centigrade, this shirt temperature is 37 degree centigrade, 310 kelvin. Suppose this green color will wear to be because of emissions, so for 300 kelvin what will be the temperature? I mean what will be the lambda? 10 micrometer is in the visible part of the spectrum, that is in a infrared.

What does it means? This is green because it reflects, it absorbs all the other wavelengths. So, from wien’s displacement law we can understand for all practical purposes, color is basically based on reflection rather than emission. If we want to see really emission temperature based on a emission, you have to take iron rod heat it in the smithy, heat it red hot, then whatever temperature getting is the emission temperature, otherwise this color of the chair is red, because it is because it is the reflecting red, are you getting the point? So, color can be because of reflection, can also be because of emission. Is that, then the fundamental question why is the sky blue in color, raman, scattering all that release all those questions. So, the wien's displacement law is very powerful law.

So, where you want your, suppose you would selectively absorb some radiation, where do you want, what should be the characteristics of a plate and all that? That should be based on the temperature of the source, then Ib lambda corresponding to the source, you ensure that you want to capture the maximum amount of radiation, in that portion of the spectrum where its maximum also is there, are you getting the point? What is a point in having I absorptivity in the wrong part of the spectrum? You cannot be a good solar designer unless you are stud in electromagnetic radiation, and you are good in radiation loss.

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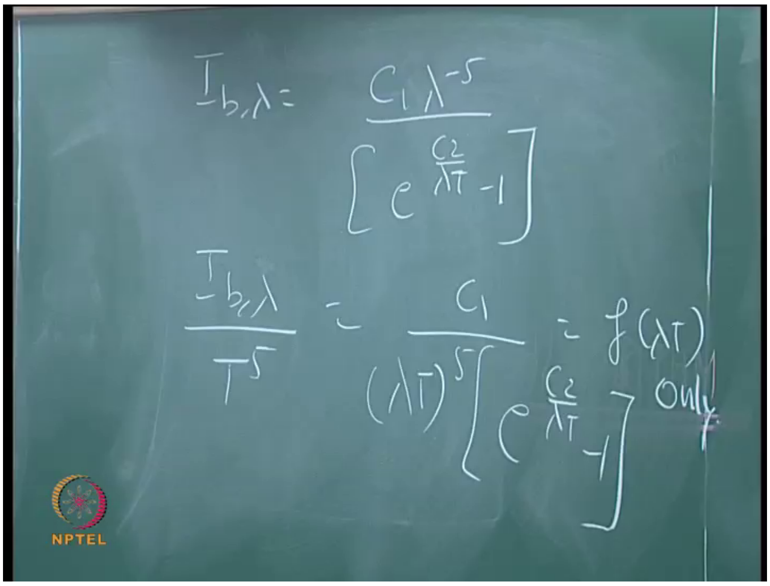

Now, let us look at something is even more interesting, Ib lambda. I did not do anything except to write the plank's distribution, but I would like to do something now. I want to look at this, I just wanted to take Ib lambda divided by T to the power of 5 and see what is the story. By innocuously dividing by a simple quantity like T to the power of 5, right hand side I am getting very profound result. What is that result? The right hand side is a function only of lambda T only.

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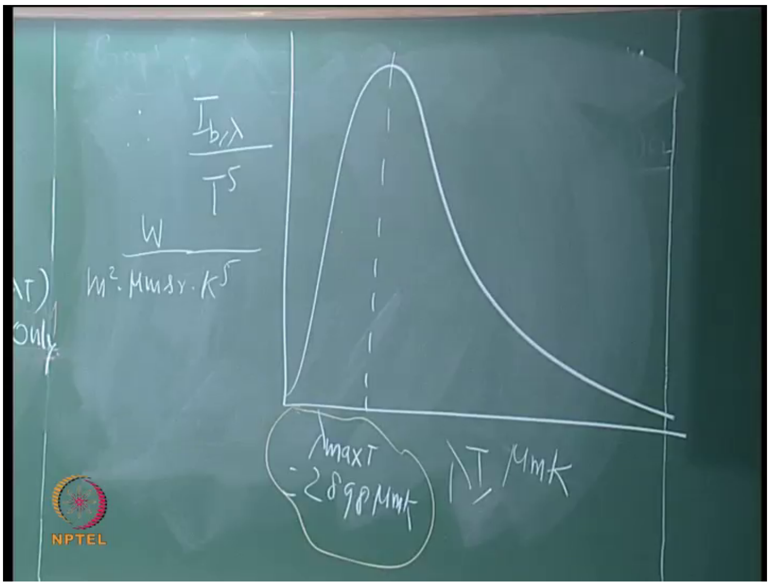

Therefore, I get only 1 curve, lambda and T I fused, I merged. So, the units are very difficult for this watts ceredian (( )) this fellow. Now, there is an acid test of whether you understood the developments in today's class. This corresponds to (( )) very good, 2898 micrometer Kelvin, that is the story. This is max (( )) lambda T (( )) I am taking, if you take lambda T product body is at 500 kelvin, it is emitting radiation at 0.4 micrometer, 0.4 into 500 or 0.4 into 2000, and finding out Ib lambda by T to the power of 5. All the curves merge into 1 curve, you do whatever you want. You have the equation go to excel and keep doing it finally, everything will fuse into one curve, that is called universal black body distribution function black body distribution function, this called the universal. See in that in that the maximum occurs that lambda max C 2898, which is basically the result wien's displacement law.

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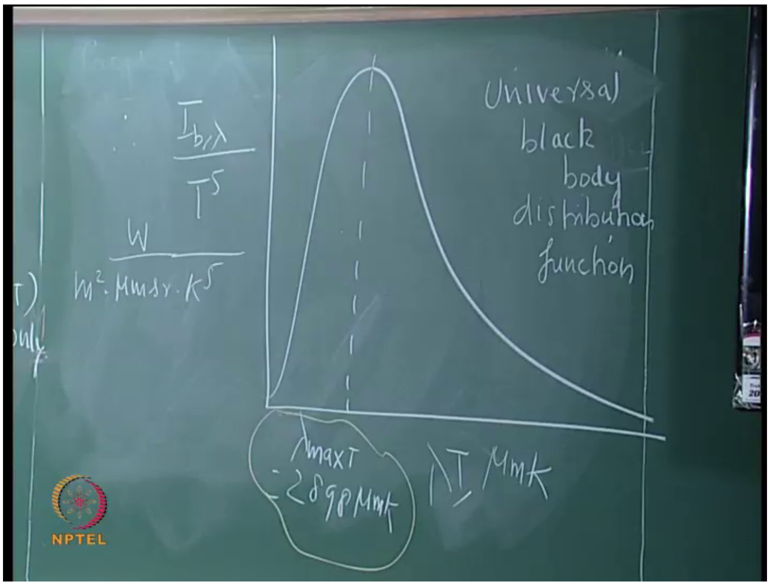

If you get the area into the curve and treat it to be unity, then between two particular lambda 1 to lambda 2 and you can find what is the fraction which is emitted in a particular band, that is very important for a solar collector what is a for example, you want to design a solar collector, what is the percentage of radiation which is absorbed? or for example, emitted in individual part of the spectrum. So, you have to find out these fractions? So, blackbody radiation functions, these are called F function charts, what is a fraction which is absorbed? In next class i will distribute the charts you can solve some problem. So, if Ib lambda by is divided by T to the power of 5, then this curve gets displaced the curve gets displaced such that only 1 curve emerges, which is the universal blackbody distribution function. Therefore, some people argue that because certain curves get displaced, this should be the wien's displacement law and not this. This is actually is the wien's displacement law, all the curves are getting displaced and they become 1 curve.

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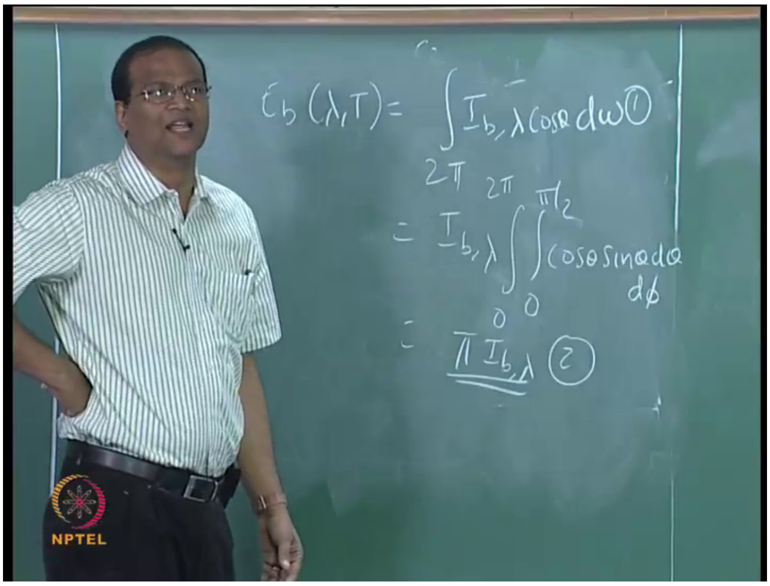

What happens, if the planck's distribution integrate? So, you integrate over the hemisphere, d lambda sorry correct? particular lambda only. No, I am still having…. blackbody diffuse into cos theta, correct? This we can replace it…. I could have simply written like this, I just writing it so that you are able to recollect what we did in an earlier class.

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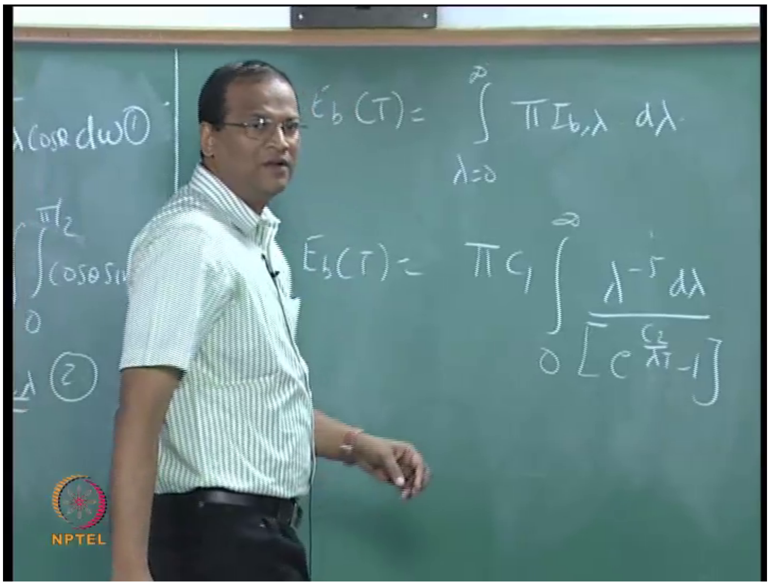

Now, Eb of T correct? Is that dcT integrate this, so you can or you are already able to see that it is not goanna be easy, let us try, if we can do something with this. So, already we introduced an x, x was C 2 by lambda d, and wants to introduce the x it is easy for the get the wien's displacement law therefore, common sense demands or common sense at least tells us it hints that we can try at a x equal to C 2 by lambda d, and C whether we can proceed a little further. Let, so first what will happen to the limits of integration? We are doing it in a particular temperature, we are integrating only with respect to the lambda, all this is for a particular temperature, blackbody at particular temperature. What will happened to the limits? This is 0 to infinity, infinity to 0.

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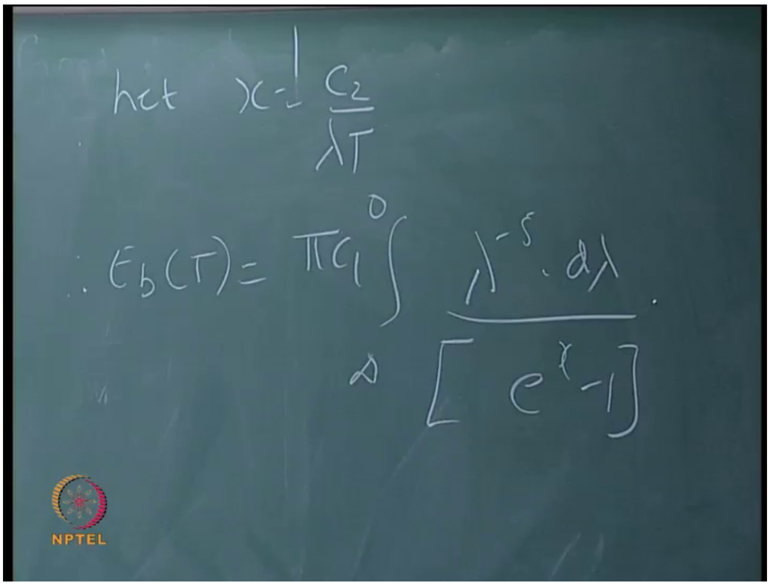

So therefore, I am not really doing justice, i will write the simple things first, much more work is required, we will do that.

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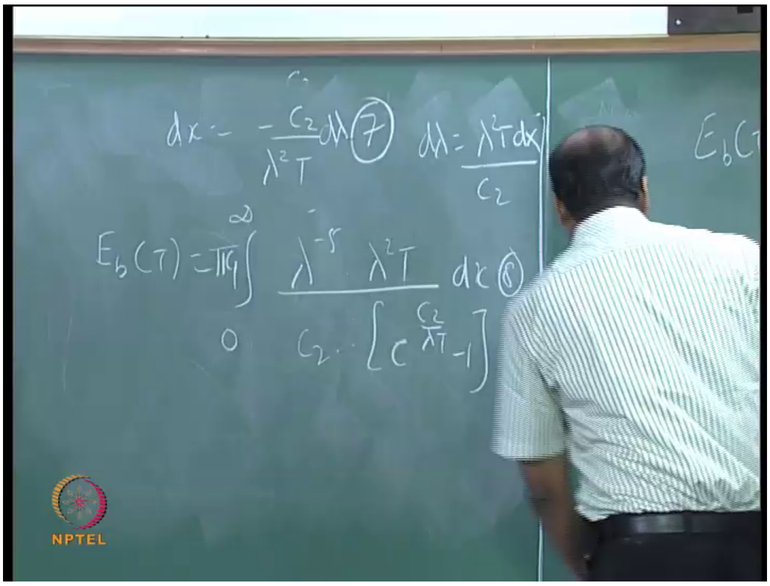

Dx, what are the….will you give some numbers? Why dont you give numbers, what this 3? We will give the same number 3, it is easier. What happened now? Or minus comes again. I can change it, I can get rid of the minus, I can make it again 0 to infinity no problem is it not?

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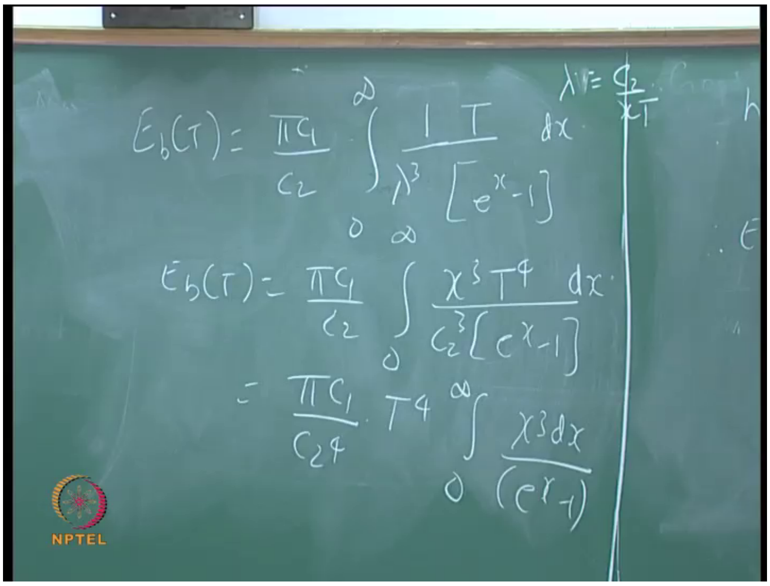

Lambda to the power of minus 5 C 2 by lambda square T e to the power of C 2 by lambda T minus 1 dx. So, Eb of T is now some pi 1 pi was there, pi C 1 is there, so pi C 1 C 2 by 0 to infinity, I made a mistake (( )) here dx into (( )) so d lambda here wait correct? So, lambda square T by C 2 then minus C 2 got canceled.

So, it is all slippery so have to be careful, so pi C 1 by C 2, where is C 1 getting 1 more T, there is no d here? Now, lambda cube, what I get here? lambda q (( )) lambda to the power of minus 3 1 by lambda q T C 2 is there have taken out, that is it. Why did not we do this? e to the power of x minus 1 dx, again put again lambda equal to, correct? lambda equal to C 2 by xT therefore, x 2 x cube divided by. Now, you have to integrate x cube dx minus e to the power of minus 1, it is not easy to integrate. Some special formulas are derived for that, but we now know the result. Please note that ihave taken T out of the integral because the integration is with respect to x or lambda.

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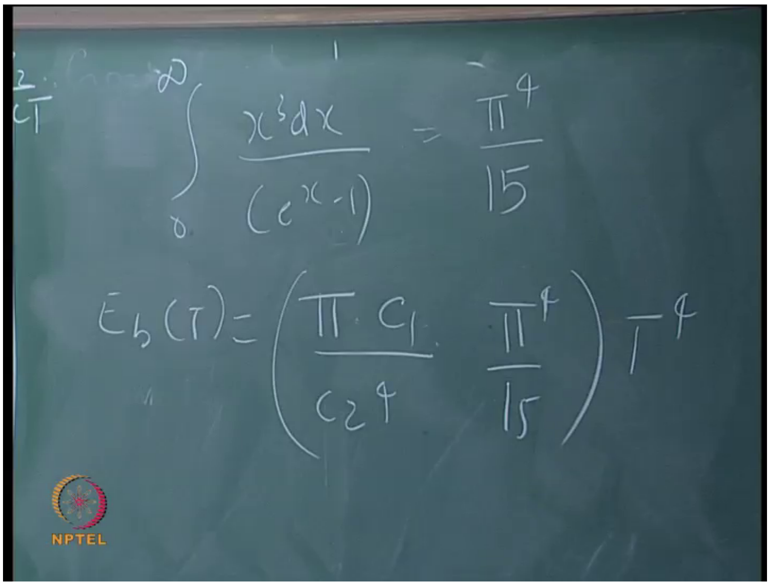

Now, this is where people are integrated this, and they found that this equal to pi to the power of 4 by 15. If you do not believe this the assignment is coming, so numerical integrate using (( )) whatever. Now, we are home Eb of T equal to, now we put in all the values know you put in other values So, this is that ac by 4 from thermodynamics, we should get 5.67 on 10 to the power of minus 8 that means we are home, if we do not get that then we have made some fatal mistakes somewhere.

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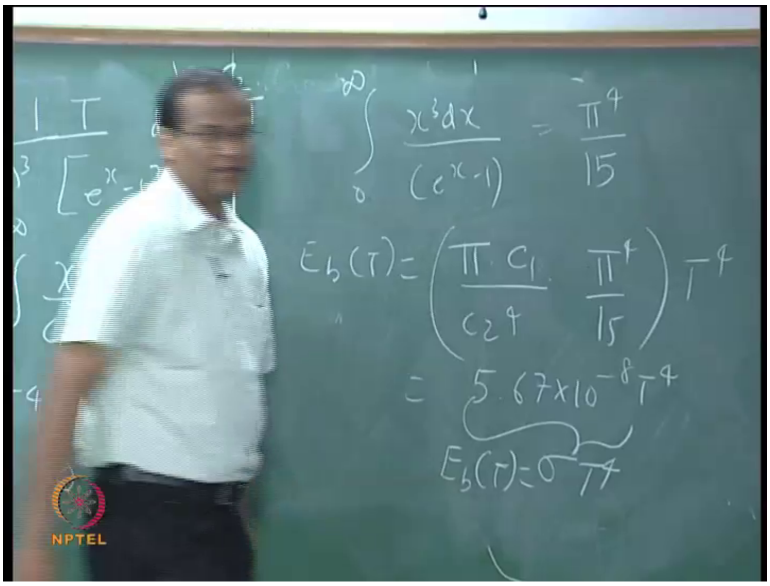

Deepak is checking (( )) 5.67 Sampath okay. You can give that concession to me 5., so this is 5.67 10 to the power of minus 8.

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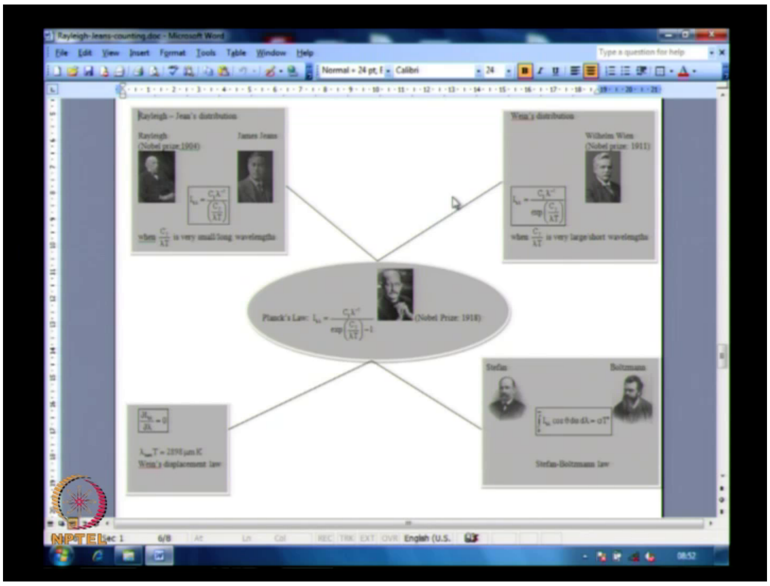

What I am trying to tell you is, this is like the sun of the solar system, black planck's blackbody distribution. I put the planck's law, his picture and the year he got nobel prize. Now, to the limit lambda C 2 by lambda d is very small rayleigh and jeans, when C 2 by lambda d is very large wien's law, when you differentiate you get the wien's displacement law, when integrate you get the stefan boltzmann's law. So, one picture says it all, how many classes we spend to derive all this? Now, but it is… I thought about it yesterday, so I gave this framework my phg student put all these picture together, put the picture and I will make copies and give it to you.

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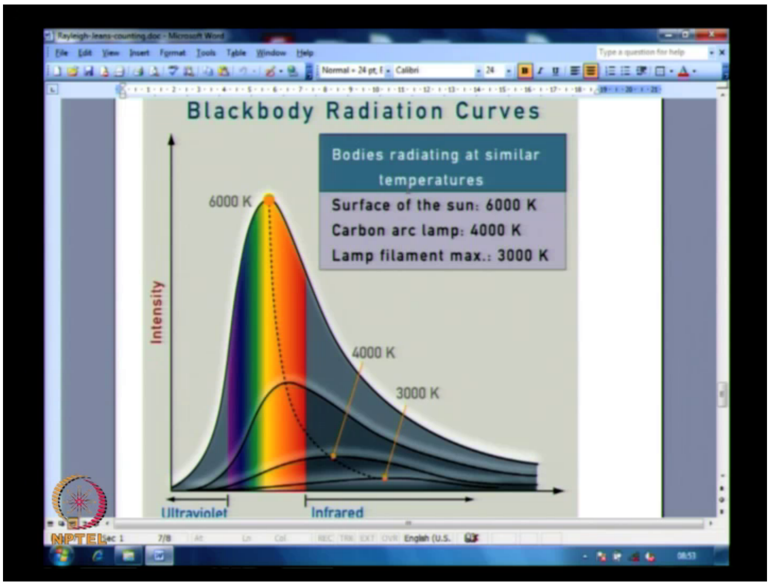

These are all from internet, so make it more colorful blackbody radiation, so that the 3 surfaces given: surface temperature of sun, carbon arc lamp and lamp filament. You can see the visible part of the spectrum is here Ib lambda versus lambda looks like this.

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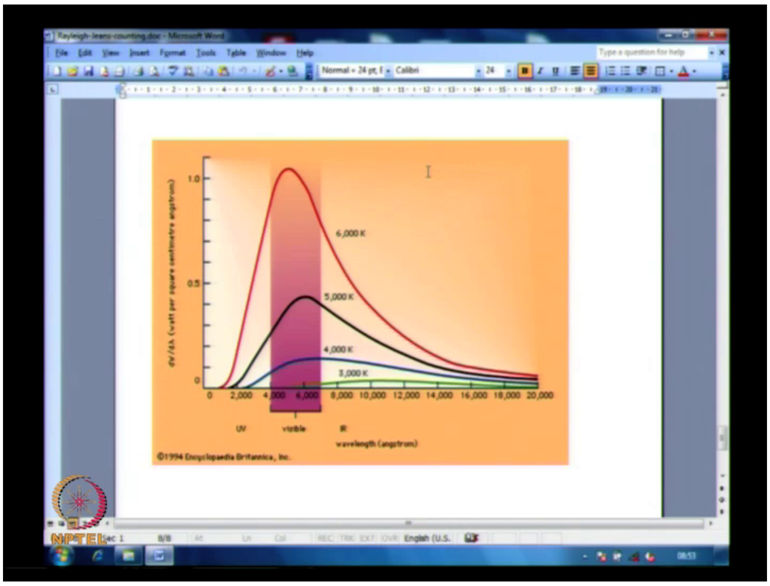

It is also of interestive physicists, astronomers and all these people are interested in this. And you can see that the Ib lambda non-linearly increases with temperature for 3000, 4000 obviously, they are looking an celestial bodies, they are having high temperatures, we are looking at 300, 373 maximum 1200, 1500 that is the engineering temperature. We are not worried about 6000 is a limit for us, after that we are not interested, but if they are looking at stellar dynamics, they are looking at the radiation from other stars, they want to find out inter planet distance, they are proposing some new theories and astrophysics, people like Hawlking and all those people will be interested.