Conduction and Radiation

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Lecture No. # 09

Universal blackbody function

In the last few classes, we look at the detail derivation of the rayleigh jeans. Rayleigh jeans distribution followed by the planck's distribution and the which led to the birth of quantum mechanics and then, we saw some special asymptotic limits of the planck's distribution. Which, led to the wien's distribution, as well as the Rayleigh jeans distribution. Then, the d by d lambda of the planck's distribution equating it to 0, it got lambda max T, it is 2898 micrometer Kelvin. So, we solved it for some x x plus x plus was some 4.965. We, solved it using successive substitution. So, this lambda max into T is 2898 is called the wien's displacement law, because the lambda max is equal to 2898 divided by T, as T increases the lambda max shifts to lower values.

In fact, even without that if somebody consistently made some good observations of Ib lambda for various temperatures, visually you could have located the optimum and you could join using straight line, you would have figured out that lambda max into T is 2898. However, from the planck's distribution this can be mathematically derived are you getting the point.

So, infact we saw that actually, if you replace the variable lambda by lambda T, you get the whole thing Ib lambda, becomes the function of lambda T only and all the curves, that is all the curves corresponding to the different temperatures collapse into 1 curve, that is called the universal blackbody function. I think, we stopped at this stage in between we also integrated the planck's blackbody distribution and we got the stefan boltzmann's law. We, got the value of sigma and that integral turned out to be pi to the power of 4 by 15.

Of course, we did not check it, but now you are doing your assignment and you are checking it using quadrature and all that integral x cube x cube d cube x cube d integral 0 to infinity x cube d x divided by e to the power of x minus 1. That will be pi to the power of 4 by 15 you will get the value and numeric, which is numerically very close to this pi to the power of 4 by 15.

So, all so 1 of the most important thing is, it is the quest for determining the black body radiation function. Which, led to quantum mechanics that is very very important and the best minds, it was challenged by the best minds, in physics hundread fifty years back. Now, towards end of the last class, I introduced this concept of universal blackbody function.

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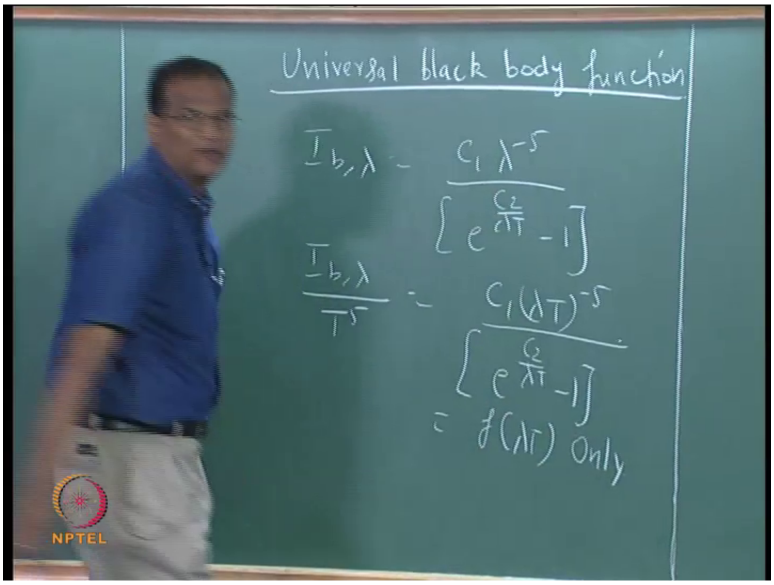

So, I b lambda then, I say I b lambda it is implicit, that it is planck. So, c 1 we divided I b lambda by sigma T to the power of 5 see there are some variants to this. You can divide Ib lambda by T to the power of 5.

Watch carefully you can divide Ib lambda by T to the power of 5 you can also divide Eb lambda by T to the power of 5. It will give the same thing except that, just scaling factor what is relationship between E and I of a blackbody, that will be pi. So, you can divide I b lambda by T to the power of 5, you can divide E b lambda by T to the power of 5, you can divide I lambda by sigma T to the power of 5 or Eb lambda by sigma T to the power of 5. So, do not get confuse. Suppose, you take different books by commode a different expressions for this.

So, it will be into 10 to the power of minus 13 into 10 to the power of minus 6 7, whatever depending on, how it was divided, but essentially the fundal is, it is divided by T to the power of 5. All the other things are constants. When, you divided by T to the power of 5, all the curves collapse.

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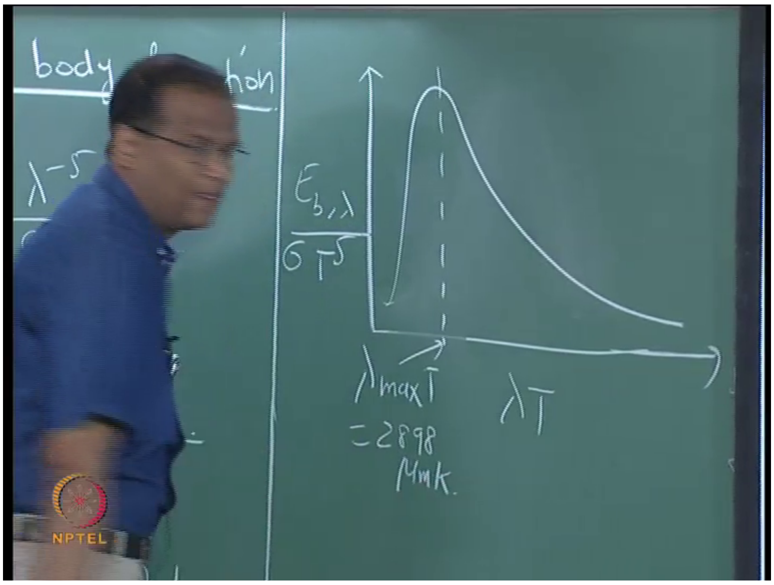

So, you have so let me say now, it is Eb lambda by sigma T to the power of 5. I am allowed to do this. I am allowed to do this. So, if it is now it is not be like this. That will be the shape, What will this be? 2898.

Now, all the time we are not just interested in finding out, what is the total area and the curve from lambda is equal to 0, to lambda equal to infinity. Some time, we are interested in finding out how much of energy is in the visible part of the spectrum is absorbed. How much of the energy is absorbed in the infrared part of the spectrum or I have a satellite orbiting let it be put in 0 stationary orbit, that will be height at 36000 kilometer.

It is stationary with respect to the earth it is always looking at india. I have an instrument an infrared imager or sounder, which captures the radiation coming from the earth surface. Which is altered by the clouds rains and all that in the atmosphere. Now, this instrument it can be a multi frequency or multispectral instrument. That is, it is capable of capturing radiation in different frequencies, those frequencies are chosen based on electromagnetic spectrum, where the atmosphere absorbs; where the atmosphere is transparent; so on.

So, a particular instrument may have frequency response in various channels. In each channel, you cannot have a direct delta function, you getting the point it cannot have 100 are you getting the point around the particular lambda is equal to 4.5 micrometer it is not that, at 4.4 or 4.6, the radiation which is captured by that instrument will be 0.

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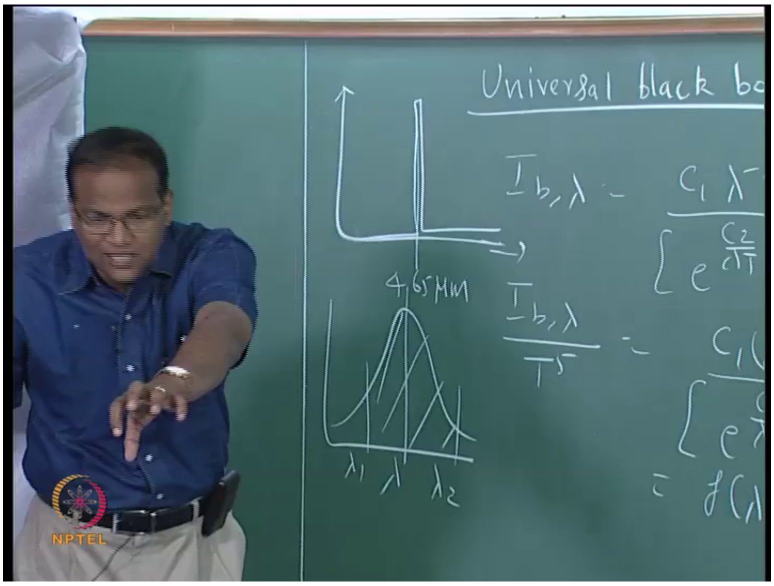

Generally, it is laps something like (( )) suppose this is 4.65 micrometer, there is no device on the earth, which is an engineer will make, which will have a frequency response like this, that is a response of the instrument. So, generally it is laps something like this, you getting the point. Around this lambda or nu, that will be d nu associated with this. Therefore, there will be a small band, there will be a small range of frequency or lambda over, which this instrument will responds. So, we have o capture from lambda 1, which is left of this. So, you have to capture from lambda 1 to lambda 2. Therefore, we are interested. So, this area under the curve, will be the total energy radiation energy, which is captured by any instrument. Therefore, now we are interested in, what is the energy; which is captured in some portions of the spectrum.

I am trying to establish,the need for doing that. Therefore, we are also interested in knowing what is the fraction of the energy; what is the energy, which is absorbed transmitted or reflected or whatever between lambda 1 and lambda 2. Therefore, at will I

(( )) I will say from this lambda 1 to this lambda 2, it is a blackbody. Can you tell me, how does sigma T to the power of 4; how much fraction is there? Therefore, we are interested in finding out this 1.

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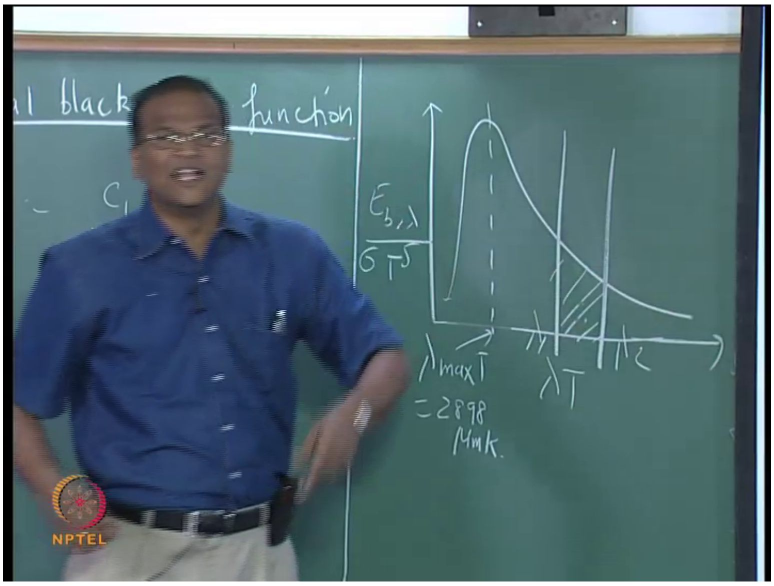

So, this is lambda 1 and this is lambda 2. We are finding out, we want to know what is this shaded area from 0 to infinity? The shaded area sigma T to the power of 4, if you take E v out of that, if I know the fraction, I can multiply by sigma T to the power of 4, from a body at temperature T and I can get, so many watts per meter square. In this wavelength band therefore, I need to work on this fraction. So, this called f function.

So, this f function is f of lambda 1 to or what is the fraction of energy emitted by the blackbody, between the 2 limits lambda 1 and lambda 2. I am not putting T 1 and T 2. Watch,because it is for a body at temperature T. You can put T 1 and T 2 with understanding that, T 1 equal to T 2 equal to T.

This, I will say that is F from lambda 1 T to lambda 2 T is equal to F of 0 to lambda T minus F of 0 to lambda 1 T. If, I have a way by which I can calculate this area first and subtracted to this area then, I have this if, I have a formula or if, I have a table, if I have a look up chart, which will give me F of 0 to lambda, for any value of the lambda that problem is solved between 0.4 to 0.7. I will take lambda 1 is 0.4, lambda 2 is 0.7, you tell me the temperature, I will multiply lambda 1 into T lambda 2 into T. If, I have a first column lambda 1 T and the second column if, I have this fraction then, I have I essentially solve the problem. Mathematically this is so.

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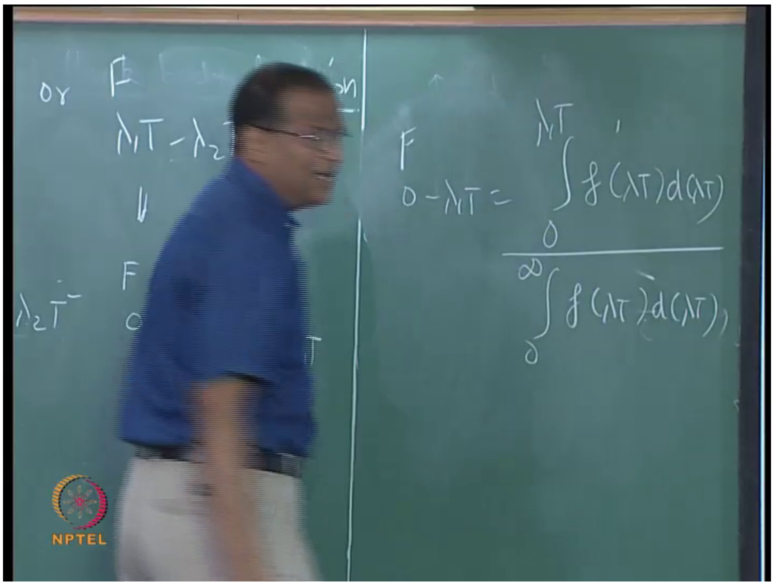

F of 0 to lambda 1 T. So, this fraction is the total energy in the band 0 to lambda 1 T divided by the total energy in the band 0 to infinity. So, F is a dimensionless number which varies between (( ))

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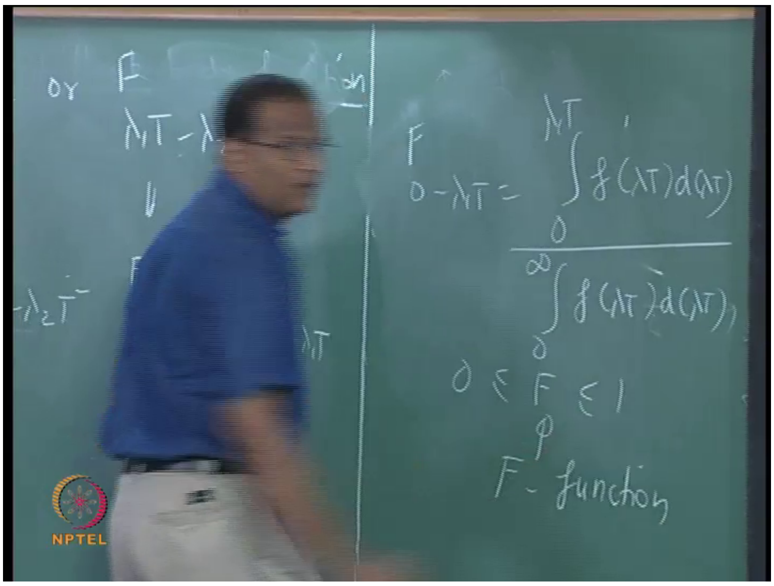

So, 0 less than equal to we call it as a F function (( )) lambda T, because it is universal. I can make 1 table. Otherwise, I have to do for different temperatures lambda is not good.

If, you do not like any of these, you can just use the planck's distribution find out lambda 1 lambda 2. You can do, you can work with that also, this is to make it easier that is all, because that is why I introduced the universal black body function first. The question is why are we hanging around with that T, my point is if I multiply lambda then, T then this is only 1 curve. I cannot just 1 compact table with which I can figure out.

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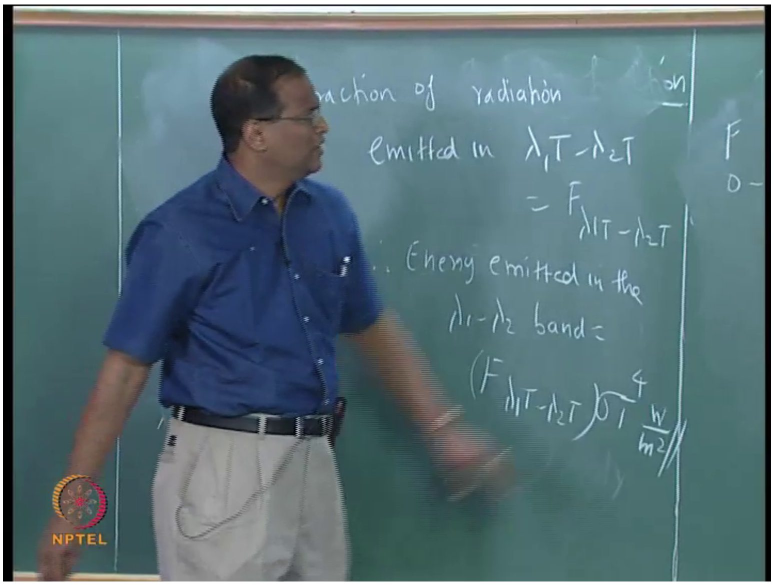

Therefore, the fraction of energy will be we cannot say fraction of energy emitted no is given by F of it does not matter. Whether, F of lambda d is taken care of I lambda E lambda, all it was numerator and denominator all the constants will get canceled. Multiplied by sigma T to the power of 4 is a key point.

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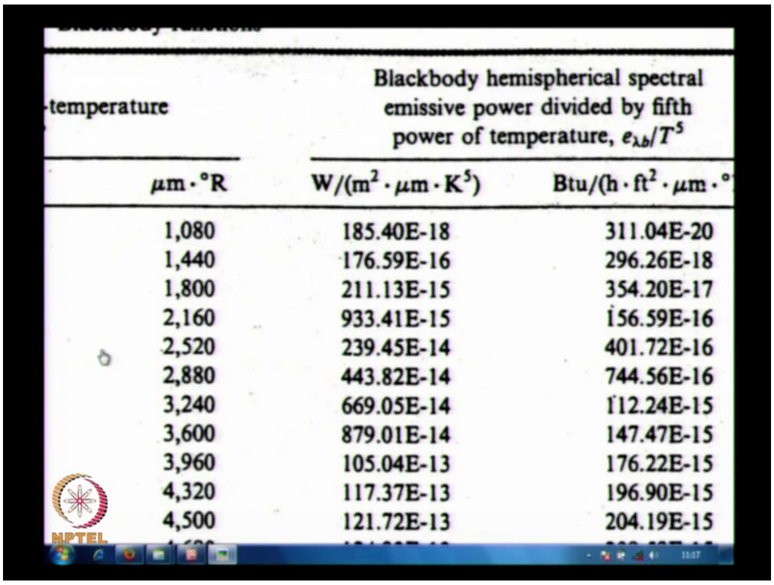

temperature	Blackbody hemispherical spectral emissive power divided by fifth power of temperature, $e_{\lambda b}/T^5$	
$\mu m \cdot {}^\circ R$	$W/(m^2 \cdot \mu m \cdot K^5)$	$Btu/(h \cdot ft^2 \cdot \mu m \cdot {}^\circ$
1,080	185.40E-18	311.04E-20
1,440	176.59E-16	296.26E-18
1,800	211.13E-15	354.20E-17
2,160	933.41E-15	156.59E-16
2,520	239.45E-14	401.72E-16
2,880	443.82E-14	744.56E-16
3,240	669.05E-14	112.24E-15
3,600	879.01E-14	147.47E-15
3,960	105.04E-13	176.22E-15
4,320	117.37E-13	196.90E-15
4,500	121.72E-13	204.19E-15

Now, so the first 1 is 600 for example, if you look at 600 and micrometer Kelvin. You will get 1 8 5 into 10 to the power of minus 18. So, but this is basically, so this is Eb lambda divided by sigma T to the power of 5.

So, if you see this value keeps going up, with lambda T. So, 2898 it becomes maximum where it is 2898. It becomes maximum that, corresponds to the wien's displacement law. Now, it goes up and so this is that, so this is watts per meter square, per micrometer kelvin to the power of 5. The last column, gives the F of 0 to lambda T.That is the important thing, so you can see that now, I am not able to show everything on the (( )) so, what should be the maximum so when it is maximum it is a no it will reach F. The F fraction will reach 0.99. You will stop with the 0.99998.

So, these every comprehensive, you can use it in your advantage, you can use this table intelligently (( )) infact get watts per meter square for the whole thing and you can do lot of things with this table. But it is very useful to solve problems, where I ask you to find out, what is the total energy emitted in a particular wavelength (( )) for example, solar energy, considering the outer surface of the sun to be a blackbody at 6000 kelvin. Standard question will be, what is the fraction of the total energy emitted by the sun, which is in the visible part of the spectrum? That is a very standard question, like that vocalized-nose for different bodies. Bodies, which are emitting radiation, which are at different temperatures. You get an idea or the fraction of the radiation, which is emitted in a particular wavelength band, near infrared, far infrared, ultraviolet and so on this is.

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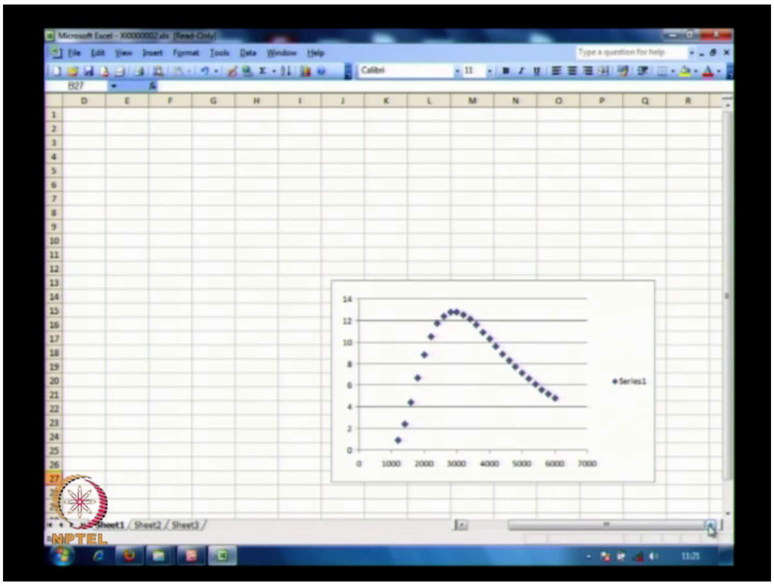

Now, I will do some work in excel. Now, I can just put close now is this clear now. The first column is lambda into T corresponding to 1200. There was some 0.99 like, it was 90 9 or something in the table correct corresponding to 1200 0.9 is 9 3 (( )) so, 1400 is 2.39 and then, 1800 have done all that up to 6000. I have gone, what is 6200 if you want, I can add 6400. How much what you get for 6400 (( )).

Now, you see this is the universal blackbody function, with lambda max are you getting the point, the maximum is about 2898 and corresponds to this. So, I am able to plot this, so from that table you can re-plot and confirm that you will get universal blackbody distribution function and the fraction is also given.

Now, we solve a problem which will all our ideas about I b lambda wien's displacement law and F function everything. So, please take down this problem number 7 8 9 it is only 6, what is the problem number it is not 1 5 5.

Problem number 5 consider a blackbody problem, number 5 consider a blackbody at a temperature of 6000 kelvin determine the following, consider a blackbody problem number 5 consider a blackbody at a temperature of 6000 kelvin determine the following consider a blackbody problem number 5 consider a blackbody at a temperature of 6000 kelvin determine the following, A Ib lambda at 0.4 micrometer, B Ib lambda at point 0 1 micrometer, A Ib lambda at 0.4 micrometer B Ib lambda at point 0 1 micrometer C Ib lambda 10 micrometer, C Ib lambda 10 micrometer D total hemispherical emissive power D total hemispherical emissive power D total hemispherical emissive power E Ib lambda corresponding to lambda max, E Ib lambda corresponding to lambda max F ratio of Ib lambda at lambda max F ratio of Ib lambda at lambda max to Ib lambda 10 micrometer, F ratio of Ib lambda at lambda max to Ib lambda 10 micrometer G fraction of the radiation in the visible part of the spectrum G fraction of radiation in the visible part of the spectrum

Now, so if you have to complete all the parts it will take 10 minutes, but it will reinforce all the concepts, all the formulas which we have derived. So, I will quickly go through the problem again.

Consider a blackbody at a temperature of 6000 kelvin. Determine Ib lambda 0.4 micrometer point naught 1 micrometer, 10 micrometer determine Eb, determine Ib lambda corresponding to lambda max, Ib lambda lambda max divided by Ib lambda 10 micrometer, that is part F and part G, fraction of radiation in visible part of the spectrum.

If u do not have calculator, you can still manage with their function chart, I have given or take out, your calculator and do it is that, this will give you Eb lambda. So, you divided by pi even then, I think sigma T to the power of 5 you will require the calculator.

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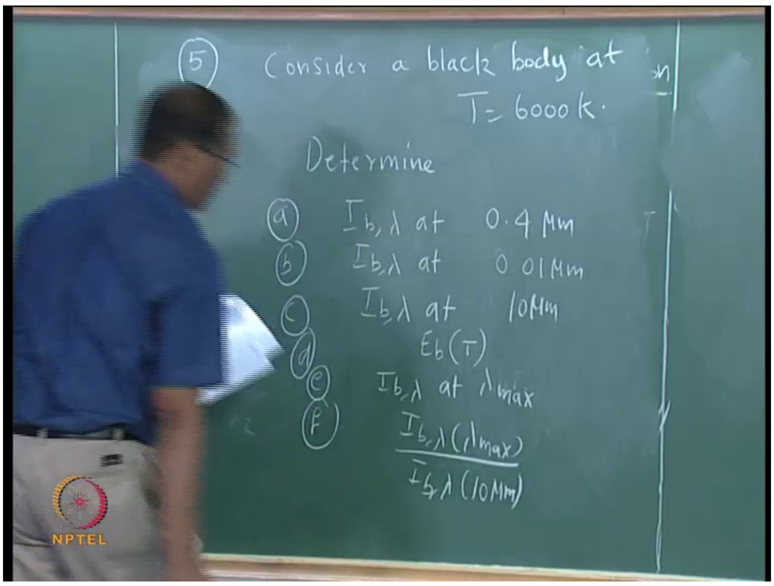

For the sake of completeness, I will write it on the board. You just proceed, I am writing the question again, these are problem's statements for the sake of completeness and for the sake of nptel I have written it.

Now, part by part we can solve this. So, we will get a feel for this C 1 C 2 and all that, so what I will normally do is? I will first calculate lambda T and I will calculate C 2 by lambda T and get e to the power of C 2 by lambda T minus 1 and keep it. Otherwise, when I do in you have there is a minus 1, all that there is a chance of making mistake. So, I will proceed from the denominator maybe, you have better way of doing. So, this way I will do it, I just calculate lambda T first divide C 2 by lambda T and have that ratio.

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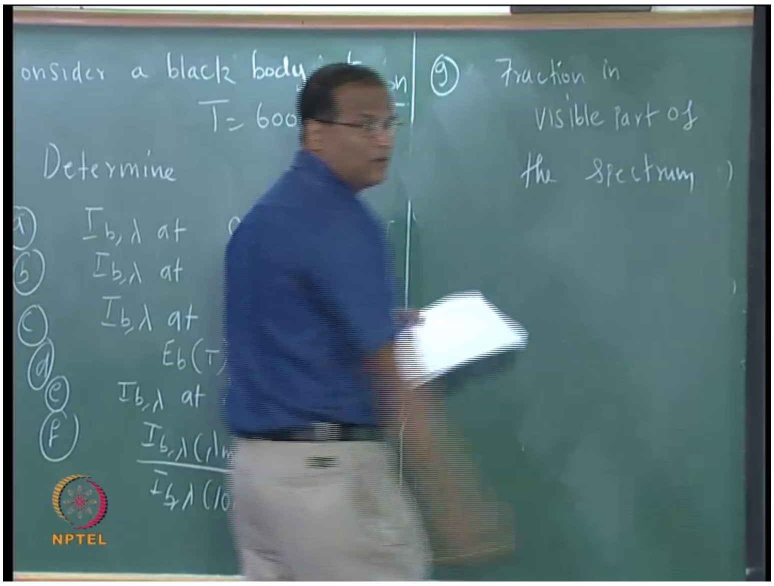

Then, raise at e to the power of that minus 1 and first figure out the denominator, I want to use this, I want to retain this, because the next half shall I erase this.

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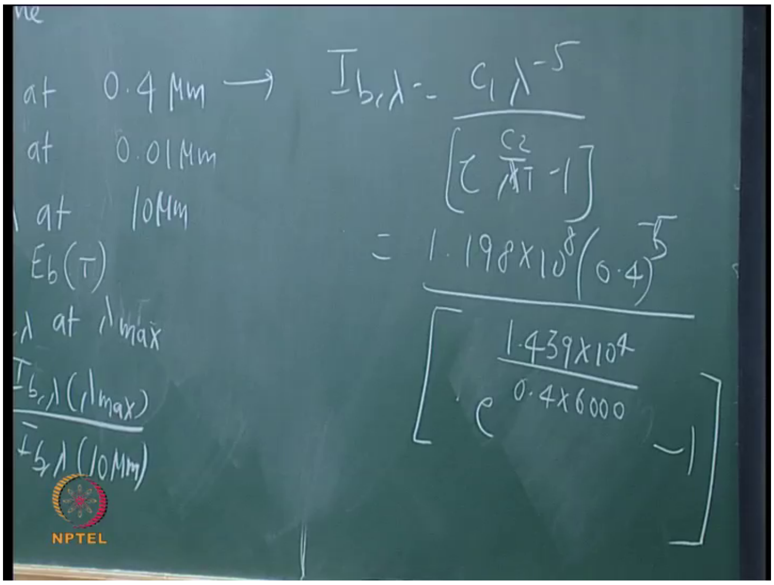

Now, that minus 1 is not on C 2 by lambda T minus 1 is on e to the power of C 2 by lambda T. We, can use this no problem then, sigma T to the power of 5. You have to multiply divided by pi. So, many things are there. That is E b T know, it is not pranay what is the matter no calculator finished (( )).

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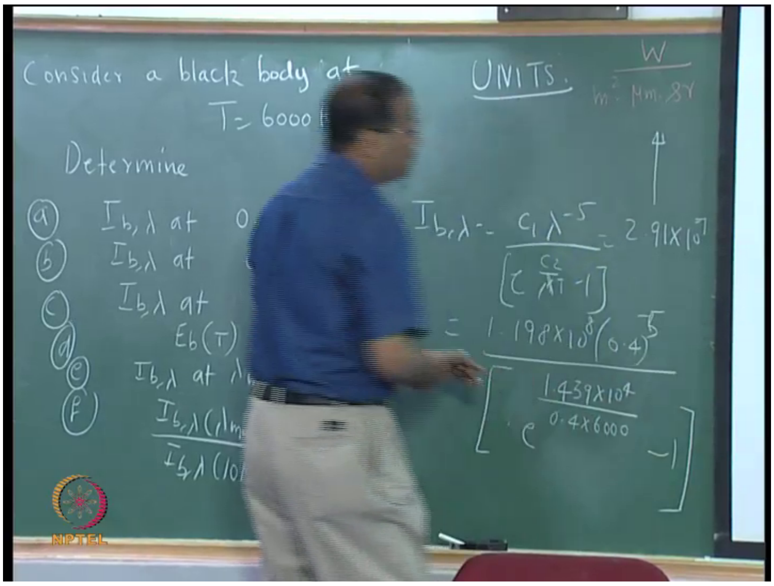

What is the answer for the first 1 (( )) 10 the 7 correct may be it correct 2 0.9 1 into units watts per meter square micrometer multiplied by sardine, because Ib lambda is the flux around d lambda above lambda around elemental solid angle d omega about omega in the theta phi direction. The elemental solid angle d omega in the theta phi direction, we we do arrows and all that the spherical co-ordinate system. We did all that fine. So, 2 0.9 1 into 10 to the power of minus 7.

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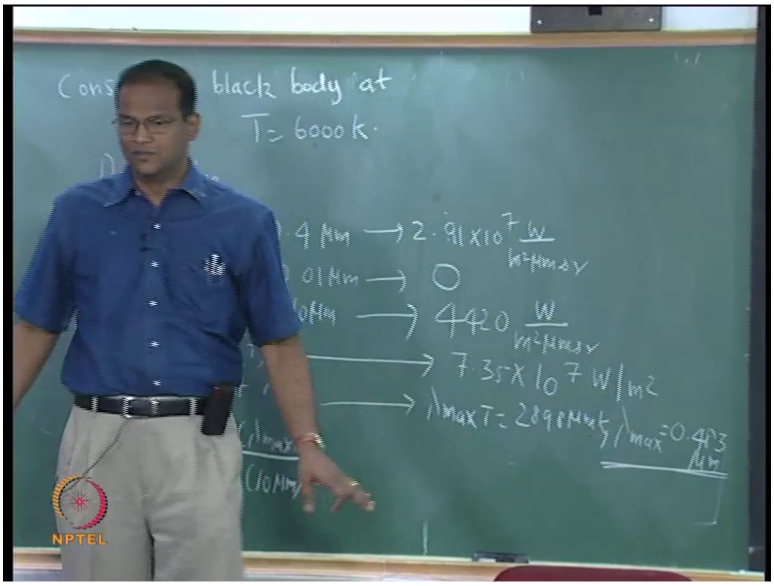

Similarly, we can work out the other things. We just start writing the answers it is 2 point pranay 2 point (( )) into 10 the 7 this fellow is 0 know e is 0 the calculator shows error that means, it is too small you do not have to worry. So, there is hardly anything that part of the spectrum from the sun 10 micrometer infrared, what is this point naught 1 is ultraviolet, from the ultraviolet part, there is nothing much actually it will be insightful. If, you look at all days and worked out and you can lean a lot of things when we working out problems 10 micrometer (( )) 4 2.

Eb of T, what is Eb of T sigma T to the power of 4 shrikanth. So, the power has come back. Eb is how much it will be high (( )) 5.35 (( )) 7.35 10 the 7 10 the (( )) watts per meter square.

What is the lambda max point; what is the beauty; what is what does 0.4 8 micrometer correspond to (( )) visual is (( )) blue; it corresponds to blue light, so the fundal is maximum in the solar energy of course, there will be a small change for if you all do solar radiation you want do 6000 kelvin. You may use 5 1800, but there also,because blue is 0.4 5 to 4 9.

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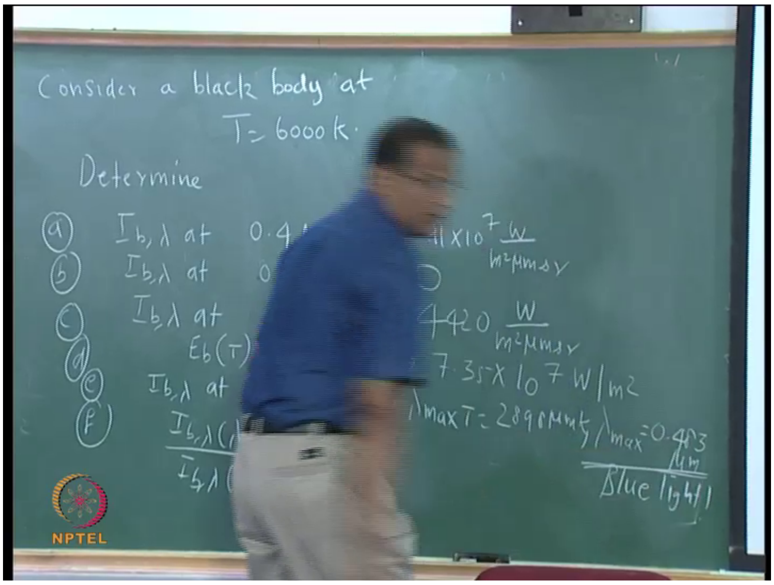

So, blue light is very important. So, this is blue light, that is very important blue light maximizing the blue light.

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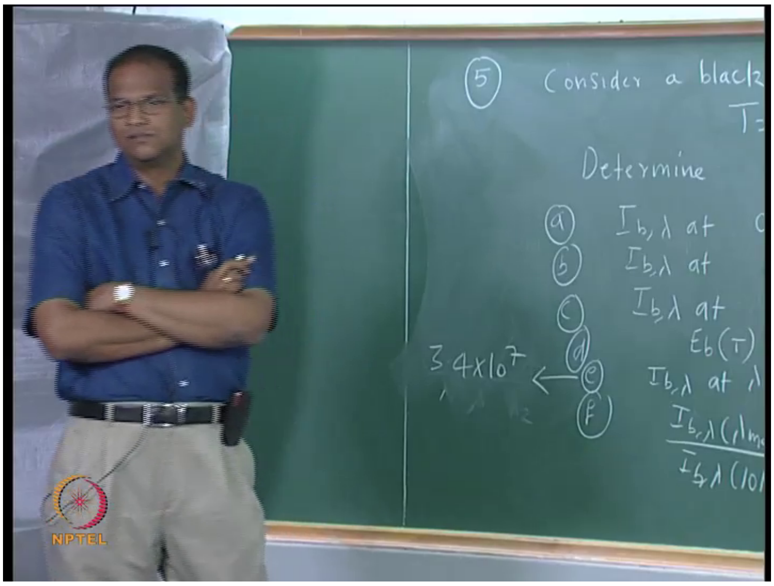

Now, what is Ib lambda corresponding to the you already have that fraction in the table, it will just above this so, in the first if they ask question like this how long it will take for 7 or 8 parts, you want such questions it was simple.

First question, I will give something like this, but not 6000 kelvin. So, it will test your basic fun days, either you understood all these formulae some, 5 marks you can give or 8 marks. What do you say ganesh samarjeet tell me 3 into 10 to the power of (( )) fine 3 0.4 into 10 to the power of 7. What is this no Ib lambda corresponding to lambda max divided by Ib lambda 10 micrometer huge number correct rohan what is that (( )) 7476.

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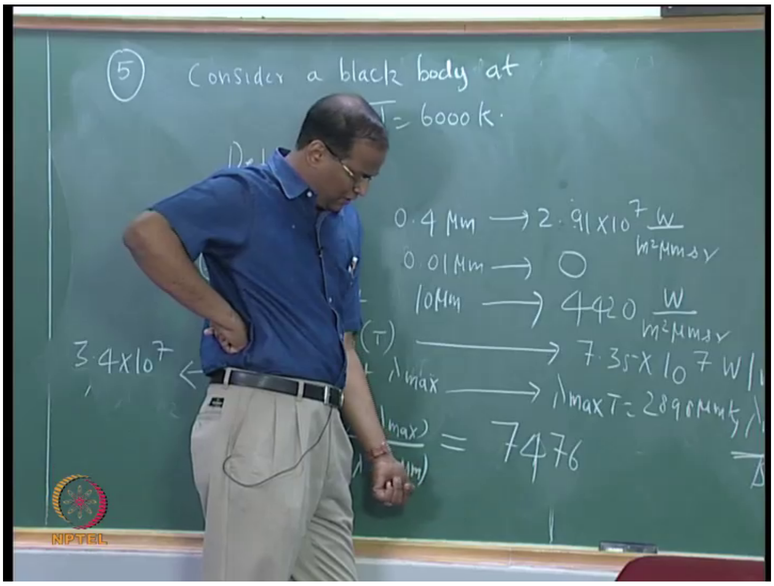

So, it is very important result, the Ib lambda corresponding to visible blue light divided by the Ib lambda corresponding to infrared at 10 micrometer, for the temperature of 6000 kelvin. Which corresponds to the outer temperature of the sun, you can see that visible to infrared the ratio is 7476.

Now, let us do the last part is this fine. So, visible part of the spectrum is 0.4 to 0.7 T is 6000 kelvin.

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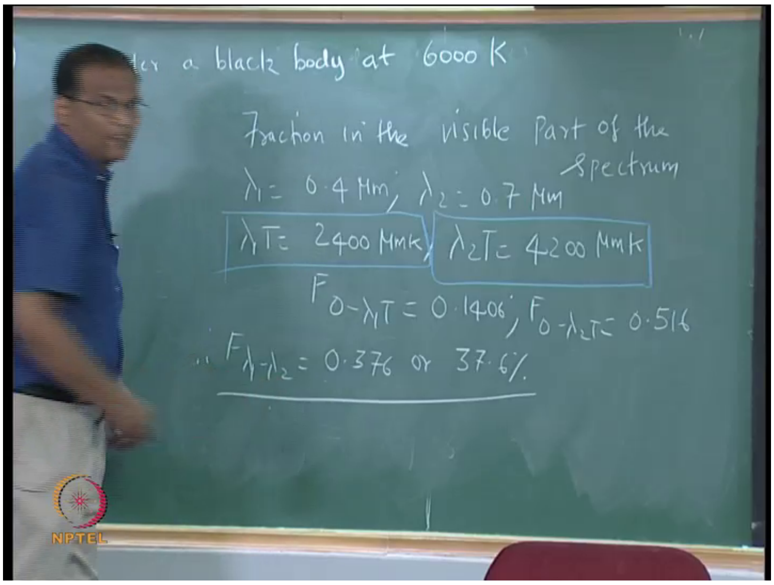

Now, no that is not correct can you use the table, can you focus on this now. So, we have 2 so corresponding to we use have a so 2400. So, corresponding to 2000 no 0.14 point 1 4. So, that is corresponding to 2400. Now, corresponding to 4 1200 5 1 6 correct it is 5 1 6, what are the 2 numbers 0.24 (( )) 0.5163 (( ))

Please do not underestimate the importance of this result. I come again please do not underestimate the importance of this result,because 0 to infinity is so huge 0.4 to 0.7 is so small, but F of 0.4 to 0.7 is nearly 40 percent of 0 to infinity. So, 40 percent of the solar radiation is concentrated in a very small part of the spectrum. The visible part, that is why we are all happy. We are having daylight and this thing if, we want to reproduce all these lights even during night, we want to have this white light then, this effect we want to produce we produce.

That is why because of this only, because of this particular temperature, because of this particular temperature it is so habitable dark and this thing and or if the temperature is such that if the temperature of the sun have to be different then, you may have the peak at some other part of the spectrum. We may not get enough radiation (( )) what come again (( )) I do not know, whether it was differently or so, it is comes back to that so it makes the question, if know we have this if we have a temperature of the solar system. Now, to be 6000 kelvin, which makes the earth. So, habitable therefore, when the big bang theory, it is started with an basically, it is all cooling problem or a heating problem whatever. So, it is started with some initial conditions.

So, if the initial condition were to be different then, the temperature of the sun today could have outer temperature would be different. Which means, fraction of the radiation falling would be different therefore, we may not be there may not know nptel course. I may not be there, I do not know why was initial condition chosen that, way who chose the initial condition is it the head of god. It could have been different, what is that (( )) so vocalizes-noise you did not get it, they are saying that the act of the robot, which robot therefore, you can link it our basic question, why is a temperature like this and so it follows a certain cooling rate.

Whatever, we can write equations, but who will give initial condition. There was an initial condition suppose, why was this initial condition given, who gave this initial condition? We do not know fine.

Now, this is an important result. So, about 40 percent is in very narrow range of 0.4 to 0.7 micrometer and the peak is also embedded in this 0.4 8 or 8 9 deepak has a question are the ratio of Ib lambda and the F same, they would not be the same. But F is a integrated quantity correct between 2 limits 0 to that particular value of lambda T.

So, for we are hung up on this blackbody, but we know that there is no blackbody and the blackbody is the concept, because there is no perfect blackbody. Which absorbs, all the incident radiation now, by virtual in which it will be the perfect emitter real bodies are neither perfect absorbers nor perfect emitters. So, there is a departure or there is a deviational departure from blackbody behavior. However, as engineers we have to live with real surfaces.

So, if I already know upfront, that real surfaces are not blackbodies. I need to characterize their behavior, that is characterize their behavior, with respect to radiation. Therefore, you have to introduce a concept of radiation surface properties.

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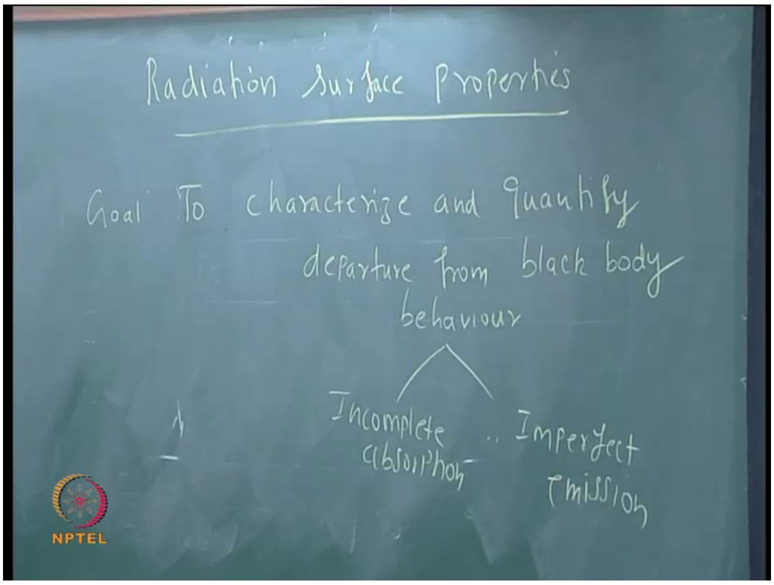

So, what is a goal to characterize and quantify departure from blackbody behavior. So, this departure from blackbody behavior, manifests itself as incomplete absorption. So, there could be there is imperfect absorption by virtual, which there is imperfect emission. Now, we have to characterize.

Now, let us look at some surfaces, which departs from blackbody behavior and project this also onto the screen. I think this discursions cannot be completed.

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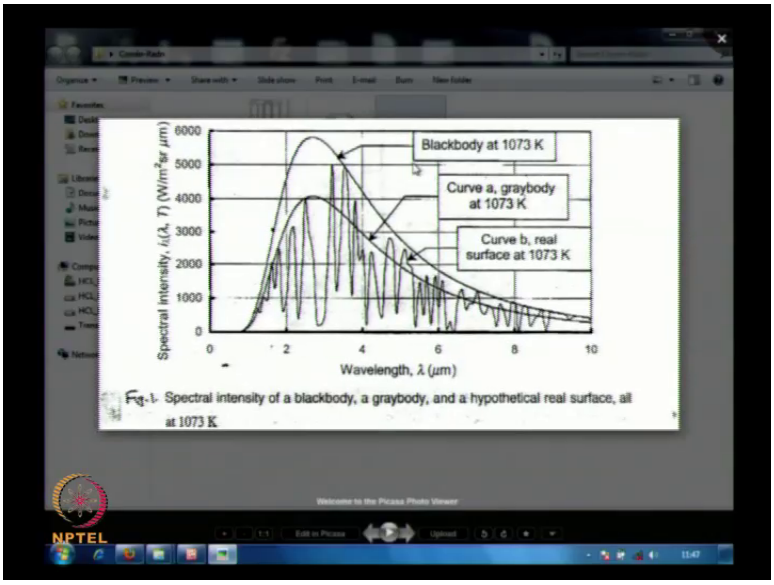

Is it, I say with this or I open with this fine figure 2 is not good man. So, first we start with figure one. Please look at the figure 1 and figure 2, back to back did it correct tejas it is not over. I have enough here pradeep here everybody got it.

Let us, look at figure 1. So, this is spectral intensity, you worry about this later weather it is Ib lambda Eb lambda by sigma 2, whatever versus wavelength. So, this is for a blackbody at 1073 kelvin. Which is about 800 degree centigrade. Which is temperature normally, uncounted in engineering 800 degree centigrade, it is not a temperature. Which is impossible in engineering practice.

So, from you can draw the Ib lambda, you also know the peak with 1073 maximum will be around 2898 divided by 1073 about 2.8 micrometer. It is consistent with our so fundal 1 this blackbody distribution is correct. So, that is the first 1 blackbody at 1073, but you could have a body corresponding to curve a which is called a gray body at 1073 kelvin.

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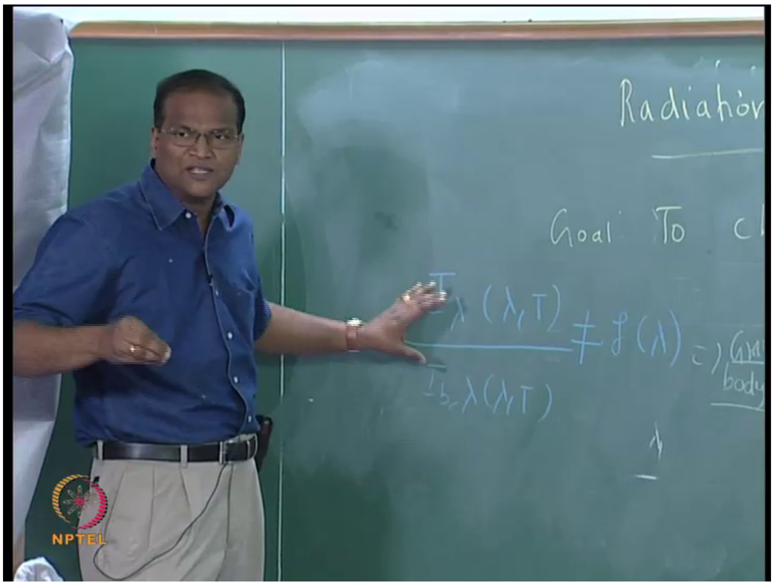

A gray body, at 1073 kelvin is a body the ratio of its emission. At a particular wavelength, the temperature of the body is fixed. Do not worry about temperature. There is a body at 1073 kelvin, Ib lambda will varies with lambda according to planck's distribution. So, the I lambda also varies with lambda if, somebody measures I lambda and you for that temperature corresponding, Ib lambda from the theoretical planck's distribution. It is a dimensionless ratio, which is varying between 0 and 1 because no body can exceed the emission of the blackbody, but I am saying that there could be a body where this ratio is independent of the emissivity independent of the wavelength.

So, such a body is called a gray body. That gray body is an idealization, but still we want to use it, because it helps us simplify calculations in radiative heat transfer. Otherwise, if you want to consider this ratio as a function of lambda the analysis becomes, more and more TDS.

Go to curve b, the curve b is indeed the behavior of most real surfaces. It will be like this. The curve b is like this. So, you can figure out that radiative analysis of a surface which follows b is a lot more difficult than, a surface which follows a behavior a,but still the area under the curve for a and b be more or less the same, but if you use the gray body assumption, that is you have the smooth curve. It may lead to some errors in the local thing for a particular value of lambda. When, you are finding out this I lambda it may differ there may be some error, but if you average out, but if you integrating from 0 to infinity it may be or from lambda 1 to lambda 2. It may be in the error may not be much.

Sir, why do you want to use this,because for most surfaces I lambda versus lambda is not known for many surfaces that is this ratio is called as emissivity. I we will come to that I do not want to formally, introduce in today's class. This ratio is called emissivity, if it is function of wavelength then, called spectral emissivity. Spectral emissivity of most surfaces are known therefore, we may go for a gray body behavior point number 1, point number 2 spectral emissivity is known, but you do not know the fundal to put it in a analysis, that is generally the case somewhere in some reference, it may be there some date of books. Somewhere is there, but do you have the confidence to put this spectral information and do your transfer calculation, very few people in world can do that.

Therefore, because of these 2 points either, because the spectral information is not known or you do not have the competence to handle the spectral information. People going in a simple assumption. Where I lambda by Ib lambda is not a function of lambda that is called a gray body.

In tomorrow's class if, I lambda is not a function of the angle then, it is called a diffuse surface that is figure 2. We will discuss that diffuse surface, tomorrow and formally introduce all these properties 1 by 1 we will get into the math and then, subsequently we will solve lots of problems, with epsilon lambda, epsilon theta all that and logically it will come to finally, that global emissivity called epsilon. Which you abused,left and center in basic courses epsilon sigma T to the power of 4. I will tell you that story epsilon, got a long and and you have to do, so many integrations to get that epsilon we will stop.