Fluid Dynamics And Turbo Machines.
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Part D.
Module-1.
Lecture-1.
Viscous Flow.

Good morning, and welcome to the 4th week of this course on fluid dynamics and Turbo machines. This is the last week for the first module which deals with fluid dynamics part. In the last 3 weeks we have looked that introduction to fluid mechanics and then in the 2nd week we looked at the integral approach, 3rd week we look at the differential approach of fluid dynamics. In this week we will look at some applications of these approaches and also we will deal with viscous flow. So we have taken 3 cases to study, the we will be dealing with the first case which is flow over a flat plate, a viscous flow over a flat plate to begin with. So let us go into the slide.

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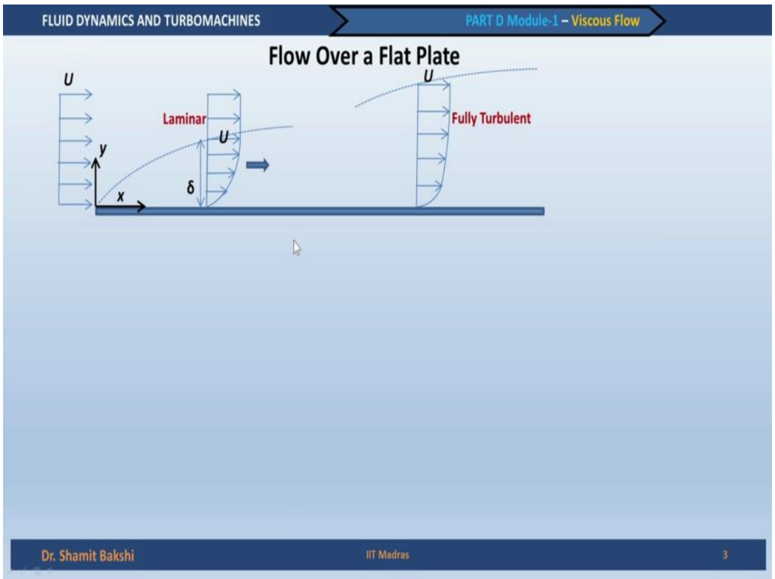

So this is the first lecture of the 4th week, we will begin with, like I said we will begin with the flow over a flat plate. We will look at how we can apply the concepts we learned before to analyse the flow over a flat plate and we will see certain characteristics of viscous flow over a flat plate. So let us consider a uniform flow. The flow is directed from left to right, the uniform flow means that the flow does not change spatially as it is impinging as it is directed towards the plate which is also aligned in the same direction of the flow. So basically the velocity vectors are parallel to the plate. So this is a very basic application of the viscous flow but we will begin with it so that we understand the basic concepts of viscous flow. So as the flow comes towards the plate, as it moves on the top of the plate what happens?

So what happens is, the velocity profile, that means the how the velocity changes along the spatial direction, what is shown here is the Y direction, how the velocity changes in the Y direction, that is the velocity profile, how it changes as it moves on the top of the plate. So what happens here is, on the top of the plate the flow, this being a continuum flow, it has to satisfy the no-slip condition and then the velocity, so the velocity of the plate is zero and in both the direction, so it satisfies that condition and as we move into the fluid, the velocity increases. Now what is happening here? So what is happening is if you look at the velocity vector here, as it moves on to the plate on top of the plate, it decelerates, its velocity reduces, that is the main thing which happens. Now this reduction is also in velocity is also transmitted in the lateral direction of the plate that means the Y direction.

So not only that, the fluid layers adjacent to the plate gets retarded in velocity, the fluid in direct contact with the plate comes to, becomes stationary from the continuum viewpoint. Whereas the fluid next to that is not stationary but it moves at a lesser velocity than it was moving in the uniform flow. So there is a reduction in the flow velocity. Now if we see this information of the presence of a plate is transmitted in the lateral direction. That means the direction perpendicular to the plate, to the surface of the plate. So as it is transmitted, so more and more fluid comes to a lesser velocity than the uniform flow. And as the flow moves on the top of the plate more is the region which is affected by this presence of the plate. So more region gets informed about the presence of the plate. The region outside this is actually independent of the presence of the plate because they keep continuing in the same velocity as the uniform flow.

So this layer of fluid at this into the plate which is affected by the presence of the plate is called boundary layer. This is the region where the viscosity is important, this is the region where the viscosity plays an important role. Now what we are seeing here, if this incident flow is laminar so by laminar we mean that if we, this flow takes place in laminas or it takes place in layers, it is not very random. So if it is very organised flow like what it is demonstrated in this velocity profile, then it remains to be laminar in the initial part of its, initial part of its existence on the surface of the plate. So this initial part remains laminar, that means it is very organised flow. Although the velocity reduces, entire flow will remain laminar but it will not remain laminar as the flow proceeds, if the plate is long enough and the flow proceeds in this direction, eventually there will be a turbulent flow.

So the velocity profile which we look here is basically of that of a turbulent flow. How we say that because if you see there is some, although the velocity changes from the no slip to the free stream velocity condition here so if you see, so this is basically, this dashed line actually shows the turbulent boundary layer and this is the region which is fully turbulent. That means if you introduce a die into this region, it will take a fluctuating path, it will take a random path not flow in lamina, like we demonstrated in the first week of this course. So this becomes fully turbulent and the velocity profile here, if you see it gradually changes to the free stream velocity in the laminar part. In the fully turbulent part the velocity quickly changes from the no slip condition to a higher velocity close to, not equal to, close to the free stream velocity.

So this initial transition is very, initial change is very fast, where it is more gradual in the case of laminar flow. This is because of certain characteristics of the turbulent flow. We will not go into the details of why it happens like that but we make an observation that the velocity profile is characteristically different in the case of a fully turbulent flow. Now if you see here we have looked at 2 possibilities so initially a laminar flow on top of the plate remains to be laminar with the growth of a laminar boundary layer. Later on at, by after travelling further on top of the plate it becomes fully turbulent. But something happens in between this fully laminar and fully turbulent region and that is called the transition region. So this is basically a region where the flow is neither fully laminar, nor fully turbulent.

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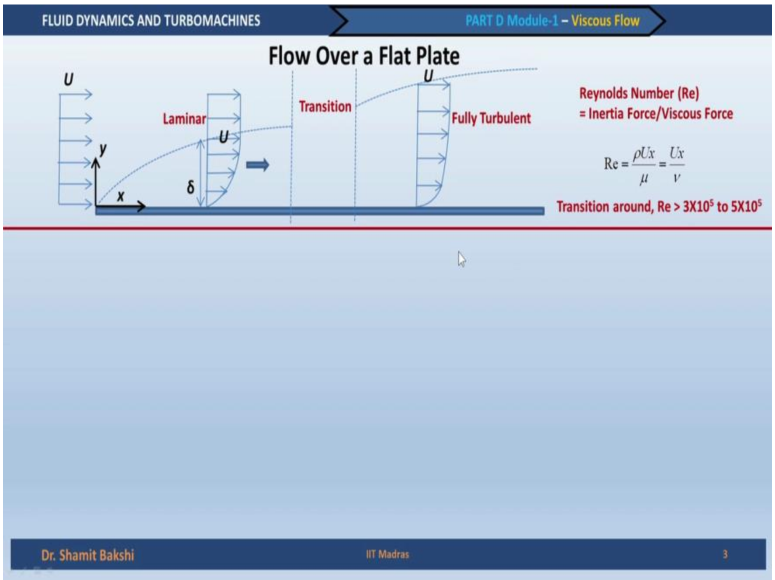

So it is transiting from a laminar flow to a turbulent flow, of course anything, any natural phenomena happens like that, it goes through a transition period. So we have this transition region for this situation also. Now more or less if you look at the flow over a flat plate, the velocity profile and the growth of the boundary layer on top of the plate will look something like this. Now let us see how to locate these lines. That means whether transition takes place and where the transition from transition region it becomes a fully turbulent flow. So **for** for that the number which is very useful is known as the Reynolds number. We got introduced to this number previously also, basically this is the ratio of the inertia force and the viscous force.

So this pertains to basically fact that inertia force is something which makes the flow disturbed. That means it makes the flow random, this is the characteristic, this is the force which makes, which tends to make the flow turbulent, whereas viscous flow is more dissipative and it tries to make the keep the flow laminar. So from that phenomenological description we can say that the ratio of these 2 forces will characterise how, whether the flow is laminar or turbulent. So a lower value will mean a higher viscous force and a lesser inertia force, so it will be laminar. So that is how this region is laminar. Now how is the Reynolds number is defined in this case, in the flow over a flat plate? So it is defined something like this, it is defined, if you do, if you try to do a dimensional analysis and try to find out a nondimensional number representing this ratio of these 2 forces, it will come out to be rho UX divided by mu. What is X, X is basically this position where we are trying to find the Reynolds number.

So that mean if you move along the surface of the plate at a different position, the Reynolds number is different. This is how the Reynolds number is defined in the case of a flow over a flat plate. So that means the Reynolds number is constantly changing as you move along the plate and it hits a value here for which the flow will now start transiting to a turbulent flow and in this region it hits a value where it becomes fully turbulent flow. So the flow over a flat plate, as it moves along the plate, the Reynolds number which is a function of the X position changes and the flow becomes goes through a transition and becomes fully turbulent.

Now can be also represented as UX by nu, so we will not very frequently use this symbol because this might confuse us with the velocity which is represented as V but just to introduce this particular thing we this nu is actually the kinematic viscosity, which is the ratio of the dynamic viscosity mu and the density. So dynamic viscosity we already know from Newton's law of viscosity for Newtonian fluid, so this ratio of mu to rho, that is the density of the fluid is basically the kinematic viscosity. Of course it has a different unit which is in SI unit it is, it has a unit of metre square per seconds. Whereas dynamic viscosity as we have seen in SI system, it has a unit of Pascal seconds or Newton seconds per metre square.

So this is useful representation in many cases because it makes the expression for the Reynolds number more compact by combining 2 parameters. Now the transition, let us now look at some numbers, the transition on this actually happens around a value of 3 to 10 to the power 5 or 5 to 10 to the power, so around this value, this is not very fixed value, so around this value the transition actually takes place. So about 300,000 or 500,000 of that, around that value the transition takes place. So this of course depends on whether the plate is smooth plate or what is the degree of roughness of the plate and so on so forth. For a very rough plate, of course the transition can take place even at a Reynolds number smaller than this.

Now this has given us a broad picture about how the flow over a flat plate takes place and how the boundary layer grows on top of the flat plate. So essentially as we see here that the velocity as it enters in this region it decelerates and gradually as it moves on the plate it becomes, it grows from laminar to a fully turbulent flow when the flow which is coming in is a laminar flow. Now let us look at what is this boundary layer thickness.

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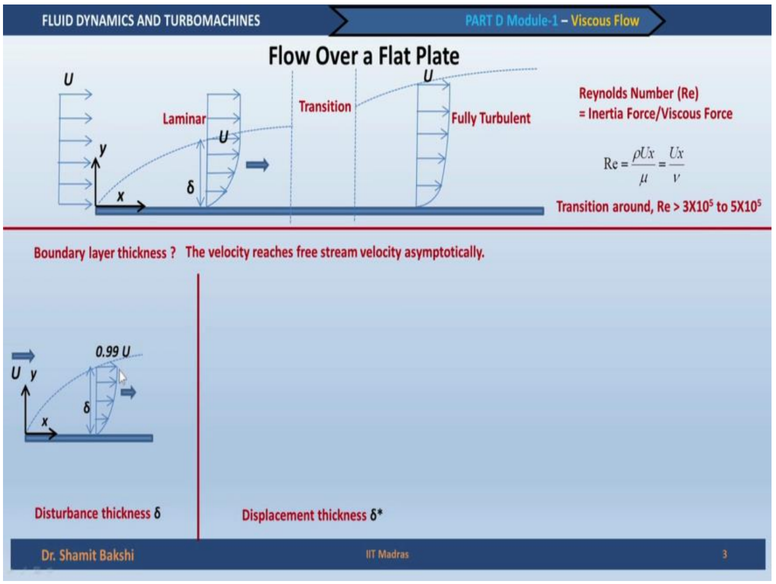

So basically we see that certain region within the fluid is disturbed or informed about the presence of the plate. So can we actually estimate this thickness? This thickness which is through which the effect of the plate is felt or the thickness within which the viscous forces are dominant, so can we make an estimate of this estimate of this particular thickness. The important thing to notice there is that the velocity reaches the free stream velocity asymptotically. That means although we have drawn the velocity profile in this way like the velocity varying from 0 to the free stream velocity U within the boundary layer thickness but if we when we get a more accurate solutions which we will introduce in a latter part of this during this week when we get a more accurate representation of the velocity profile we will see that the velocity does not actually become equal to the free stream velocity within the boundary layer thickness. It comes very close to the bound free stream velocity.

It reaches the free stream velocity at a very large distance from the plate, so it becomes almost asymptotic, that means it takes it to, you have to travel through a distance of infinitely long distance to reach the free stream velocity. But of course this within the laminar thickness the boundary layer, the flow comes to very close to the free stream velocity. So for example that is why we because of this fact we need to clearly define what is the, what do we mean by boundary layer thickness. So the first thing which we are, which we have already introduced here but we do it more specifically again is known as the, this thing is known as the disturbance thickness. That means the region within the fluid which is basically disturbed by the presence of the plate. So this disturbance thickness is the thickness within which the flow starting from a zero velocity on the top of the plate reaches to 99 percentage of the free stream velocity or 0.99 times of the free stream velocity.

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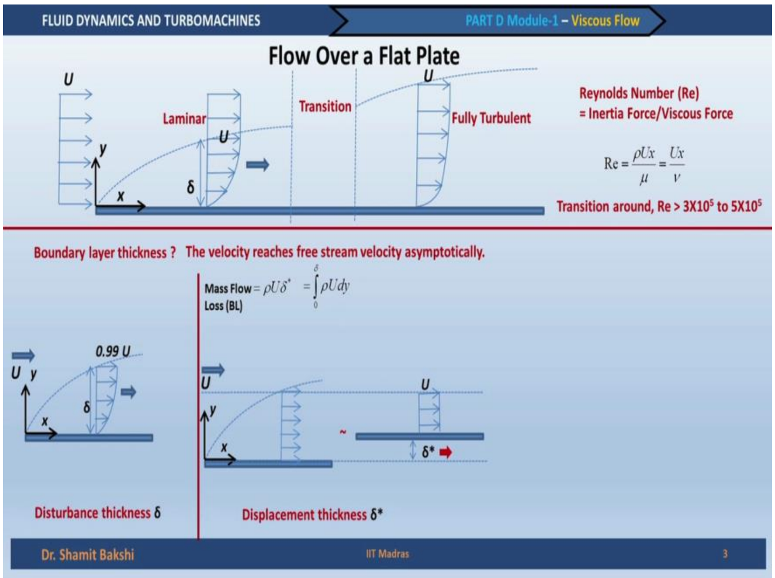

So this is basically the disturbance thickness, but not to connect more or to relate more phenomenologically to the thickness of the boundary layer we can bring in more conceptual definitions of the boundary layer thickness. So the next one which we will define in case of the boundary layer thickness is called a displacement thickness. Now what is a displacement thickness? Like we said the disturbance thickness actually relates to how the velocity varies in the boundary layer. The displacement thickness is connected more phenomenologically. How is it connected? So let us say we consider the same plate with and the flow approaching the plate, the leading edge of the plate and there is a growth of the boundary layer. Now if you look at this, as we began this lecture we said that the flow actually decelerates on the surface of the plate.

That means the velocity of the fluid near to the plate reduces as the flow, as the fluid flows on the surface of the, this is actually a very important concept because if we keep this in mind, this simple thing in mind we can understand the difficult concepts introduced in boundary layer very easily. That the fluid actually loses its momentum and it decelerates as it moves on the top of the plate. So let us see how we can find this displacement thickness. Now if this is the situation, you can actually create an equivalent situation. So what we can do, we can draw a horizontal line which is aligned with the top surface of the plate, another line which is aligned with the edge of the boundary layer. So what is the edge of the boundary layer?

The edge of the boundary layer as defined by the disturbance thickness, so as defined by this, the velocity reaching the 99 percentage of the free stream velocity. So after defining this, now what we can do, we can think of this as a equivalent situation, that means let us say the plate has not changed the velocity profile, it has kept the uniform flow as uniform flow, that means the flow velocity while it is approaching the leading edge of the plate was U, now also it is U Everywhere, that is the flow velocity in the X direction. It is U everywhere but actually the flow has decelerated. So if you see when the, if you let us say consider the streamlines between the surface of the plate and the streamline which passes through this edge of the boundary layer, you see, okay we will go to that little later, so if you see that, then between these 2, of course it is difficult to draw such a streamline, so we will come to that later.

So if you see this surface of the plate and the edge of the boundary layer and we consider the same region here, we will see that while the flow was approaching the plate, there was free stream velocity everywhere. There was free stream velocity everywhere throughout this length delta there was free stream velocity. Now you have reduced velocity near to the plate and it gradually becomes Delta, so basically it says that the flow rate has reduced. So this reduction in, for this reduction in flow rate can also be visualised as the plate moving, the plate displaced a little bit by a distance delta star which is basically a displacement thickness. So after you remove this, then the flow rate here, the volume flow rate here is same as the flow rate in this case.

So now we can also define this delta star from the mass flow loss due to the presence of the boundary layer. Like within this thickness delta there is a loss in the mass flow, so what is that loss? So that loss can be defined from this. So whatever was flowing through this region, if we consider the incident profile, this region at, there was flow with the velocity, uniform flow velocity U and that is not present here, now this flow is absent, so that is basically the loss, what is present in this region only. So now you can define this loss as rho U into Delta Star, of course we are we have assumed that the thickness, the play the width of the plate perpendicular to the slide is unity. Okay, so basically you can read it as rho U Delta Star multiplied by 1, so per unit width perpendicular to the slide. So for that we can say this is basically the mass flow loss rho U Delta Star.

While it was incident what was the flow rate, the flow rate was rho U into Delta, now this is a loss, only this part is allowed, with this is equal to this mass flow rate. We can represent this further as, say whatever was incident, let us see suppose the plate, see this velocity profile we have changed here. So just to emphasise, suppose this plate was present and there was, it did not change, there was a slip condition on top of the plate, that means did not change the uniform flow, if such was the case, then the velocity profile would have remained like this, uniform flow throughout the distance delta.

So then the flow rate in absence of the plate or presence of the plate but it did not influence the flow under that condition the flow rate would have been zero to delta integrated, integration of rho U dy integrated between 0 to Delta. So basically this would have been the flow rate, again for the unit width of the plate. So this is the and but actually it is this profile, so this loss can be defined as what would have been without the plate and what is it with the plate, that is the difference between that should give me the loss, the mass flow loss. So we can subtract is and we can actually define more specifically define this displacement thickness. This being a incompressible flow rho definitely consult out from both sides and we get this definition of the displacement thickness.

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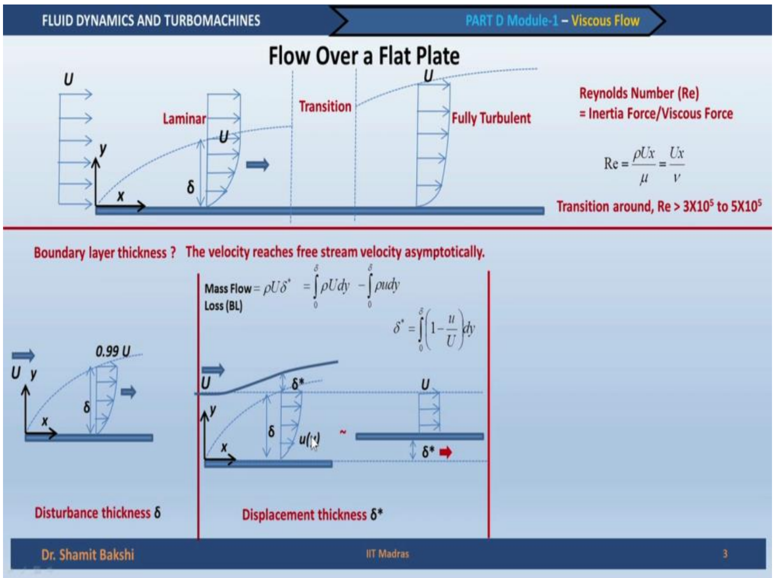

Now there is another way of actually looking at this displacement thickness and which is very useful. So what is that, so now what we see is we can actually consider a streamline here so this is what I started explaining but now let us get into that before because before it was, making this explanation was would have been difficult. So now what we see is, suppose you have a streamline on the surface of the plate and you have, you consider this streamline here, see this streamline cannot pass along this dashed line, why is that, because between 2 streamlines the flow rate has to remain constant. Between 2 streamlines because, why is that because perpendicular to the streamline there is no flow. So perpendicular to the streamline if there is no flow, so then between the 2 streamlines you should have the same flow rate, that means here and here you should have same flow rate.

But the flow rate has definitely reduced, if you draw a streamline parallel to the plate, the flow rate has definitely reduced at this section. So how can we explain the situation, definitely the streamline would have been displaced. So you can visualize the displacement thickness as the displacement in the outer streamline. So basically the streamline just outside the boundary layer, how much it is displaced, that will also explain what is a displacement thickness. And this happens because the flow decelerates, see this all happens because the flow decelerates as it moves on the top of the plate. But this is a more phenomenological description than with the disturbance thickness.

There is another way of defining the thickness of the boundary layer which is again phenomenological but it is related to a little, it is related to some other concept, what is that, see it is not that there is a mass flow loss, when the fluid moves on top of the plate, there is a momentum loss. So you can consider that momentum loss, if you consider that, considering that you can define thickness which is called the momentum thickness. Now let us look at how we can define the momentum thickness.

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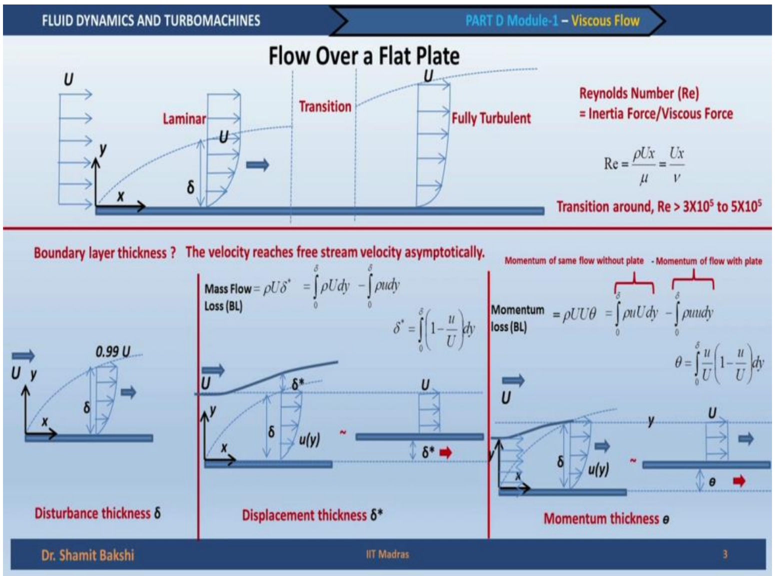

So this is the situation now, this is the plate, the same thing, same condition, U is the free stream velocity and this is the boundary layer and the velocity profile within the boundary layer. So now if we, like we made an equivalence in this case, we can make an equivalence here also. So we can make an equivalence, again we draw 2 lines, one at the edge of the boundary layer and other on the top surface of the plate, so now we can basically think of this as a situation in which the plate has been moved by a distance theta. And what is that distance theta, it has been moved in such a way that now this flow has remained uniform as it was when it was approaching the plate at it has the same momentum, the same momentum as this layer.

So this layer has of course a lower momentum than the approaching flow but now if you consider that the plate is displaced to account for the loss in momentum. So the loss in momentum is accounted by, accounted for by moving the plate a little bit and but keeping the flow to be uniform. So now you can define the momentum loss. So there is actually a momentum loss of the fluid as it moves on the top of the plate. Momentum thickness accounts for that momentum loss. So there is a mass flow loss, there is a momentum loss. These are the basic things which are happening when the fluid moves on top of the plate. So now how to, how to find the magnitude of this momentum loss.

So the momentum loss in the boundary layer, we can again define it as the momentum of the fluid which is moving through this region which in the approaching flow or the incident flow, so that will be density multiplied by U into U, that is the free stream velocity multiplied by the displacement thickness theta, rho U U theta. Again this can be defined like we did here in the case of mass flow, in the case of displacement, this can be defined as momentum of the same flow without the plate that means what would have been the momentum without the plate and same flow means now this momentum loss should only account for the momentum loss and not for the mass flow loss, we will feel that very soon.

So momentum of the same flow without the plate, suppose you remove the plate, then what was the momentum, what would have been the momentum of the fluid and subtracted to momentum of flow with the plate. So this, difference between these 2 should give me the momentum loss. So this is the expression now we will try to, this is the description, now we will try to give an expression to both these descriptions. So momentum of the same flow without the plate, how can you find it out? So consider this streamline, so basically we consider a streamline like we saw here that the streamline which was passing through the line drawn along the edge of the boundary layer, it will get displaced but if we have to draw a streamline through this point, that means to the edge of the boundary layer itself, I want to draw a streamline. So how will you draw it?

So you draw it like this, you pass a streamline through the edge of the boundary layer and this streamline will not be parallel because it has to satisfy the mass flow equivalence, so it has to be bent like this. So like the streamline, the outer streamline has bent like this, the streamline passing through the edge of the boundary layer also should bend, so if we consider this now, this is basically the meaning of the description that momentum of same flow without the plate. Now without this plate this flow would have a momentum like this. So this momentum of the fluid coming in with the uniform velocity. So please note here that this momentum pertains only to this region, so and not to the entire thickness, disturbance thickness delta, not through the entire disturbance thickness.

So the same flow rate, if it passes on the surface of the plate, what is the loss of momentum? Okay, if the same flow rate passes on the top of the plate, what is the loss in the momentum? So that, that is only accounted, so it only purely considers, see this definition purely considers the momentum loss excluding the mass flow rate loss. Okay, so let us see how it can be described, so this can be described as, so what is this? What is the momentum of this fluid which is coming in? So we will come to that later but what is the mass flow rate of the fluid which is coming in, it is 0 to Delta rho U dy, how do we get that, see this U is actually the velocity here, see the problem of finding the mass flow rate for this flow, for this region is that we do not know the thickness here.

We know the thickness of the boundary layer Delta at the location where we are trying to find out the momentum thickness. But we do not know how much it has bent unless we bring in the definition of the displacement thickness, we do not want to bring in that definition here. So without bringing in that you can say find out this mass flow rate, how do you find out, because this mass flow rate is equal to this mass flow rate because they are bounded by 2 same, bounded by the same streamlines, hence this mass flow rate is actually 0 to Delta, that means this mass flow rate 0 to Delta rho U dy, so basically this mass flow rate. This mass flow rate but not at this velocity, this mass flow rate, both the mass flow rates are same, this mass flow rate and what is the momentum per unit mass flow rate for this flow?

That is U, so you multiply this with U, you get the momentum of the flow, same flow without the plate. So there is basically the momentum of this incoming flow subtracted to, so this subtracted to the momentum of flow with the plate. What is the momentum of the flow with the plate? If the plate is present, then it is not very difficult to write because it is 0 to Delta rho U U dy. So small u of course is the velocity at the location where we are trying to find out the momentum thickness. So this difference will give me the momentum loss in the boundary layer.

So see that is how in the case of the mass, the displacement thickness, the concept which is connected with it is a mass flow loss. In the case of the momentum thickness the concept which is connected with it is the momentum loss of the fluid on the top of the surface.So now we can define from here, if we cancel, it is an incompressible flow, by cancelling rho we can write this as a definition of the momentum thickness. This will be very useful for us in the next part of the lecture.

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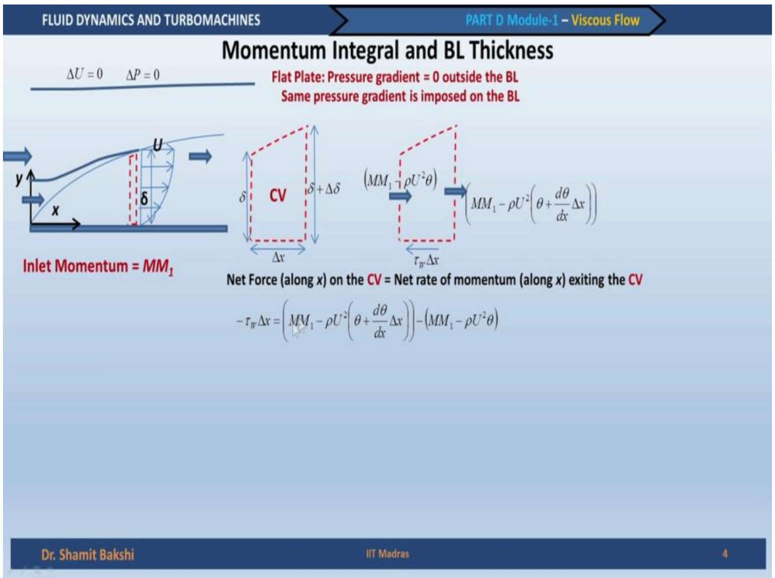

So now let us look at another important concept on regarding how to find out the boundary layer thickness. So this is related to the momentum integration, momentum integral within the boundary layer. So this is the boundary layer as described before, what we can do is we can apply the concepts of integral approach within the boundary layer. So how do we do that, we take a control volume. So this is my control volume showed by the red boundary, so if we take this control volume we can do the force balance here, like we did in the case of integral approach. So we take this control volume, see another thing is, this control volume as a very small length and we consider this as the streamline which is passing through the top edge of the control volume, so basically the top edge of this control volume, it merges with the streamline, with the upper streamline.

And from our last slide we know that the streamline will bend like this. This we had also introduced while doing the problem in the 2nd week of this course. Okay so the bending of the streamline is understood, now this is my control volume, so this is a control volume, this is a magnified view of the control volume and the thickness of the control volume is very small, Delta X. So this is very small, we have just magnified it so that we can clearly write the terms appearing here. The thickness of the boundary layer at one end of the control volume is Delta, as it is shown here and on the other end it is Delta plus Delta Delta, so this is basically a small increase in the thickness of the control volume. So we realise that this thickness actually changes as we move along the control volume, the disturbance thickness.

This Delta is the disturbance thickness, whenever we used delta it will mean it is a disturbance thickness, Delta Star is a displacement thickness and Theta will mean momentum thickness. So we will keep these terminologies in our mind while going through this chapter. So this is disturbance thickness and all these thicknesses like the momentum thickness and the displacement thickness like the disturbance thickness vary along the length of the plate. So at different position of the plate the disturbance thickness is different naturally as we can see through this figure, the displacement thickness will also be different and the momentum thickness will also be different. So we draw this control volume and try to apply the integral, the Reynolds transport theorem basically, integral approach of dealing with momentum conservation.

So we take out the control volume here and see what is flowing into the control volume, basically the Reynolds carport theorem tells us that the net force along X, this is this is the X direction along X along this control volume is net rate of momentum exiting the control volume, of course along the X direction. So the net force that is basically Sigma F, Sigma FX, so the net force acting on this control volume is basically equal to the momentum exiting the control volume subtracted to what is coming into the control volume. So let us see what is the momentum which is coming, going out and coming in to the control volume and what are the forces acting on this control volume. So for going into that, what we first say is the inlet momentum, let us say is MM1, so what is meant by this inlet momentum, this inlet momentum is the momentum of the at the between the of the fluid bounded by the same 2 streamlines.

As we say this control volume, the top edge of the control volume is merged with the streamline with a particular streamline and if you, and it is possible to do that because the Delta X, the thickness of the control volume is very small. So now if you extend this streamline, it will become like this, it will have a small distance from the plate in the uniform flow and let us say the inlet momentum is MM 1, so the inlet momentum of the fluid which is passing through these 2 particular streamlines is MM 1. What happens as it approaches the front edge of the control volume, the inlet momentum reduces. So what is that, how much it reduces? It reduces now we know what is momentum thickness, see the advantage of defining a thickness based on the momentum loss, now we can easily write an expression for this momentum.

So this is basically MM 1 and this is the loss in momentum, rho U square into Theta, so this is basically the loss in the momentum. So the momentum of the fluid which is coming into this control volume is this MM 1 minus rho U square Theta. The momentum what is going out of the control volume, again we can write like this MM 1 minus rho U square multiplied by this, what is this, basically Theta is also, Theta is the momentum thickness, this is also changing, I am sorry, this is also changing along the length X. So this can be now expanded in a Taylor series and we can write it as Theta plus d Theta by dx into Delta X. So basically this is the exiting momentum. Is there any momentum from this surface? No, there is no, because this is the surface of the plate and on the top surface again no because this is the streamline.

So perpendicular to the streamline there cannot be any flow and perpendicular to the plate there cannot be any flow. So these are the 2 momentum which we consider. So if we subtract this from this, we get the net of, net rate of momentum exiting along X direction. Now, what are the forces? The force, the first force to consider would be the shear stress on the plate. See the plate is actually applying a shear stress which will be wall shear stress tao W is basically, we will keep using this during this chapter tao W is basically wall stress coming from the wall.

So in this case on the surface of the plate multiplied by Delta X. So shear, stress into area, of course again we have taken unit length, so this multiplied by 1 actually. So the unit length perpendicular to the slide. So that is unit width, not unit length, actually unit width of the plate, so length we say in the X direction, width is a perpendicular to the slide. So tao 1 into Delta X is one force definitely acting on this control volume, now what is the pressure force? We know this Sigma Fx brings about the pressure force, what are the pressure acting on this entire control volume.

So to look at that, we look at a streamline far from this plate. If we look at a streamline far from this plate, you can say that this Delta U along the streamline is zero. That means if you move along the streamline, the velocity is not changing, it is uniform flow because this is outside the boundary line. So it also means if you now apply the Euler equation for this, you will get Delta P is equal to 0. Okay, there is no change in height of this or if you can neglect gravity also, even if you include gravity, so if it is parallel to the horizontal, it is if it is almost horizontal then we can say Delta P is also zero.

So Delta U is zero, Delta P is zero, so what does it mean, that in the case of a flat plate, the pressure gradient is actually zero outside the boundary layer. So this is a boundary layer, outside the boundary layer the pressure gradient is zero, that is easily established from here. Actually the same pressure gradient is also imposed on the boundary layer, that means if you move into the boundary layer also, the pressure gradient is zero. So we just accept it for the time being, we will actually prove this that the pressure gradient within the boundary layer for a flat plate is actually zero.

Okay, outside we have already shown from the streamline that the pressure gradient is zero outside the boundary layer, inside the boundary layer, let us for the time being take that it is the same pressure gradient is imposed within the boundary layer, the same, the external pressure gradient outside the boundary layer is imposed into the boundary layer. Actually this is true for all boundary layer flows, not only over a flat plate. That whatever is outside the boundary layer, that pressure gradient is imposed into the boundary layer. Of course we will give a more rigourous proof of this little later.

So that means, what it means is the pressure gradient if it is zero, that means pressure is same everywhere around this. So it does not does not make any net effect on the forces acting on the control volume. So that is it, that means you have shear stress and you have these 2 momentum exiting, 2 momentum terms, one exiting and another in coming into the control volume. So now we can write this as Tao 1 into Delta X, just keeping in mind the sign and we will get this expression, so this MM 1 will cancel out, of course from both these terms and what we will be left out with is basically this term MM 1 and rho U square Theta, both will cancel out actually from both these terms and what we are left out with is the wall shear stress is equal to rho U square d Theta by dx.

This is actually called momentum integral theorem which is a very important theorem in boundary layer analysis. And we will see how powerful this theorem is. This can be actually extended to the case where the pressure gradient outside the boundary layer is nonzero. So there is a nonzero pressure gradient outside the boundary layer but we are not doing it in that kind of a general for such a general case, we are just doing it for the flat plate. So the wall shear stress is related to the gradient of the momentum thickness. This is understandable because the shear stress actually arises because there is a momentum loss. This we have seen while solving a problem in the 2nd chapter.

And so how the momentum thickness changes along the X direction will definitely influence the shear stress. So this is how this expression is derived. Or this is the description, the physical description of this expression. So now let us say, see we cannot proceed further with this unless we assume some profile of the velocity because this profile is still unknown. Let us say we assume a quadratic profile, so if we do that, then we can find an expression for Theta, Theta is the, Theta is dependent on the velocity, small u by capital U, we saw in the last slide that it is defined by the velocity profile.

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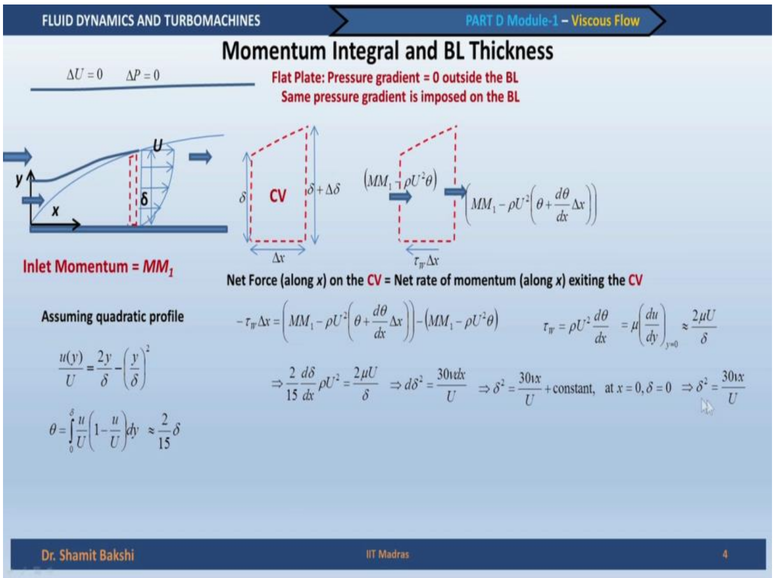

Basically let say we assume this kind of profile and we know that this will actually represent the boundary layer, we have seen in the first chapter that this satisfy, this kind of a velocity profile satisfy the no-slip condition on the top of the plate the free stream velocity at the other edge of the boundary and the zero gradient condition, again at the edge of the boundary layer because there is no shear stress transmitted from the boundary layer to outside the boundary layer. Why is that because if it is transmitted then the velocities adjacent to the edge of the boundary layer will also change because the shear stress is the mechanism through which the velocity gets reduced or the flow decelerates.

Fine, so this is basically our velocity profile. Now if we use that in the definition of the Theta, Theta is basically our momentum thickness. So if we use that we can actually integrate this expression now and if we do that we get this as 2 by 15 Delta for a quadratic profile. We can plug this in here, before doing that we can also write this, see the wall shear stress, just from Newton's law of viscosity that this is mu into dU by dy at Y is equal to 0. So if you do that, then and we know the velocity profile, we get an expression like this, twice mu U by Delta.

So with assumed quadratic profile, this will be the wall shear stress, with assumed quadratic profile, this will be the momentum thickness. Now we can plug-in this Theta into here and see what happens. So now once we write Theta in terms of Delta, this is d delta by dx 2 by 15 and rho U square twice mu U by Delta. This expression can now be integrated because we can take in delta from here and write this as 2 d delta square and if we rearrange the terms here, so we can take dx here, this becomes 30 nu dx by U.

So integrate this, you get this expression that nu square is basically 30 nu X by U plus a constant. Constant can be easily obtained because we know that at X is equal to 0 the Delta is zero. Right so Delta square is also zero, because the boundary layer thickness, the disturbance thickness is zero at X is equal to 0. So that means the constant is also zero and okay, we apply this condition at X is equal to 0 delta is zero, so Dell square is basically this. So now we can rewrite this expression as delta by X is equal to square root of 30 nu by UX. What is UX by nu?

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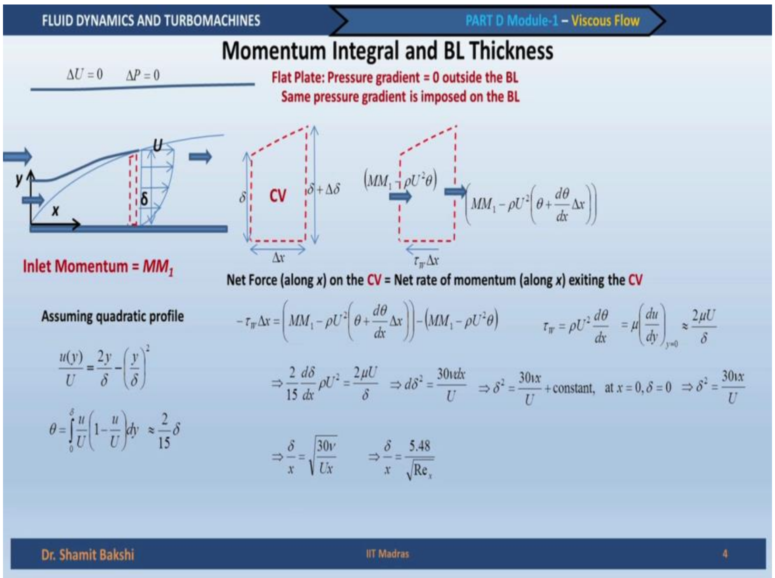

So now you see we have made use of kinematic viscosity that is mu by rho here and this right here we have done that, we have written mu by rho as nu. So now UX by nu, what with that, that is basically Reynolds number, what we defined before. So Delta by X is equal to 5.48 square root of that, square root of 30 divided by square root of Reynolds number. Now see we have used a suffix X here just emphasise the fact that this Reynolds number is independent on the station where or the location, X location of that particular point.

So that is how our boundary layer thickness is also related to X. In fact you can see another important thing here that if you see how the, if you want to draw this curve, you can easily draw if you have, of course with a assumed velocity profile which is quadratic and which is not very inaccurate, we will come to that. So we can see that Delta versus X, how does it vary, it is delta square varies, varying as X, proportional to X, so this is basically the variation of the, this is the actual curve, so this is the actual curve which represents the boundary layer.

Now this is a variation of boundary layer thickness, delta by X is basically 5.48 by square root of Rex. And this is for a laminar flow of course because this profile is more representative of a laminar flow. The velocity profile which we have seen in the first slide, the slide before, that can be more represented using this kind of a assumption. See other than that there is no assumption for laminar or turbulent flow in this entire derivation. So if you change the profile and repeat this calculation, you can get a relation for turbulent flow also.

This expression does not hold for turbulent flow because of the fact that the velocity profile is not very good representative of a turbulent flow situation. Okay, so now this is basically my variation and this is quite accurate, because this was actually derived by Von Carmen and this was found to be quite accurate even after making very more robust solutions which we will discuss in the next lecture of boundary layer flows. So this solution is actually the accurate value of this or more accurate value of this is 5.

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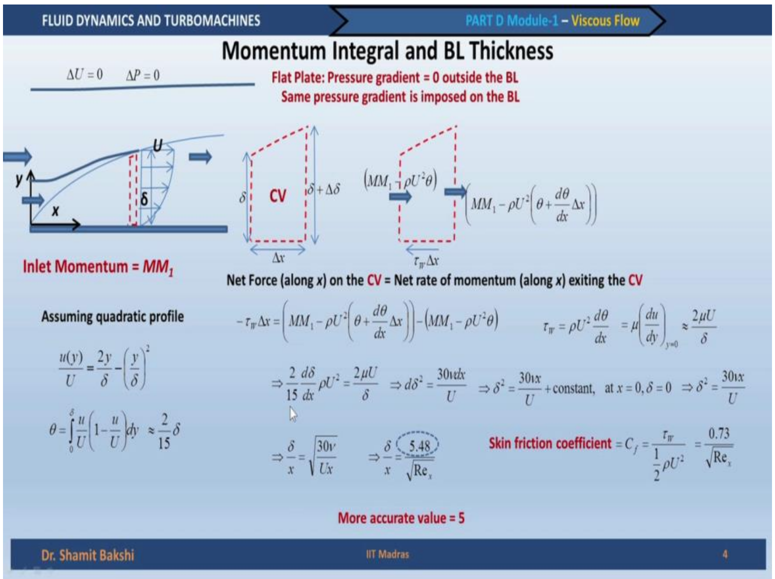

So you get almost within 10 percent accuracy that by assuming very simple quadratic profile. It is actually not quadratic profile, so we will see in the next lecture how to get a velocity profile within the boundary layer, that is more rigourous analysis of the velocity profile without using this. But this itself is quite accurate as we can see here. Now, now that you have this boundary layer you can also get the expression for the wall shear stress. So how to get the wall shear stress, or in other words skin friction coefficient.

So what we have, that is basically, so what generally is done is these are related through some coefficient. This is basically the ratio of the wall shear stress and half rho U square. So if you do experiment you find out skin friction coefficient which can be related to Reynolds number and things like that. So we will see that in the case of drag coefficient also, that will come up later but now from this solution we can get an expression for skin friction coefficient.

So if you plug-in that tao W, wall shear stress value from here now noting and from here twice mu U by Delta and Delta is already known in terms of Reynolds number, so if you do that, then you get that the skin friction coefficient is 0.73 by square root of Rex. This is a very useful expression to find out the resistance given by a plate to a flow.

So basically this brings us to the end of the first lecture and just to repeat what we have discussed in this particular lecture is we have started with the basic structure of the boundary layer over a plate and showed that the incident laminar flow goes from a laminar boundary layer, from say laminar boundary layer go through a transition and becomes fully turbulent and that is this transition to turbulence can be expressed in terms of Reynolds number. We also looked at different ways of defining thickness of a boundary layer, the disturbance thickness, the momentum thickness and the displacement thickness.

We also derived the momentum integral theorem for a case where, that means the momentum integral relation which is basically an application of Reynolds transport theorem for the case of the flat plate boundary layer flow and we obtained an expression for the boundary layer thickness in terms of the Reynolds number. We can now say how basically the boundary layer thickness changes as you move onto the different part of the plate. Thank you, we will take up this from the viewpoint of Naviar-Stokes equation in the next lecture.