Fluid Dynamics And Turbo Machines.
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Part D.
Module-1.
Lecture-3.
Viscous Flow.

Good morning and welcome to the 3rd lecture of the 4th week of this course on fluid dynamics and Turbo machines. In the last lecture we, last 2 lectures actually we have dealt with flow over a flat plate, we have used 2 techniques, integral and differential approach to look at the flow over a flat plate and give a description of the viscous flow in the boundary layer over a flat plate. In today's lecture we are going to look at the viscous flow over again a solid surface but under a little different circumstances, particularly pertaining to the pressure gradient.

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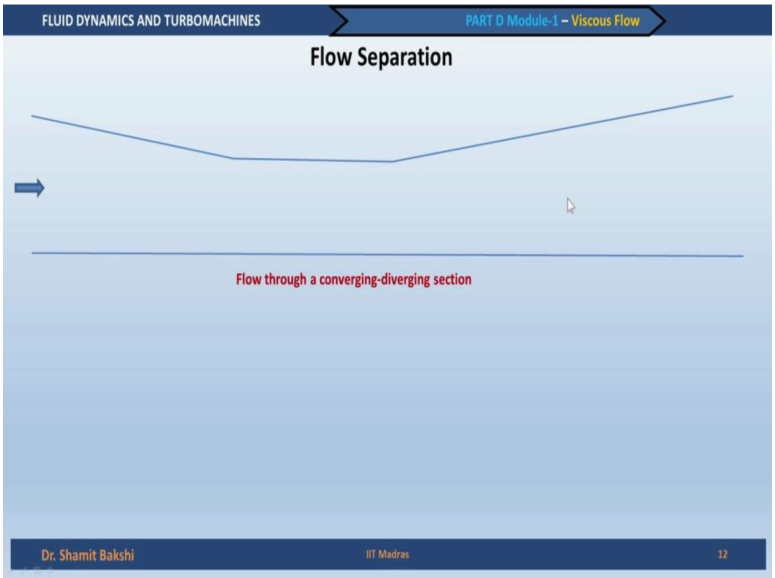

So we will look at the, we will introduce a particular thing known as flow separation in this particular, in this lecture. So, let us go to the slide. So here we have started with flow separation but actually we want to introduce the concept of the viscous flow where the pressure gradient outside the boundary layer, we have taken an example of flow or flow through a converging diverging section. So if you look at this, the lower edge of this looks like a flat plate but the upper edge, there is a converging region, there is a flat region parallel to the lower region in the intermediate portion and the diverging region.

So what happens is if we just apply the continuity equation for this flow we see that the flow velocities will increase as it approach this Central region which is the throat of this converging diverging section. And as it goes out through the diverging section, the flow velocities will reduce. So if we apply Bernoulli's equation we can also say that the in this region as we go towards the centre, as the velocity increases the pressure will reduce and as you go to the diverging section as the velocity reduces, the pressure will increase.

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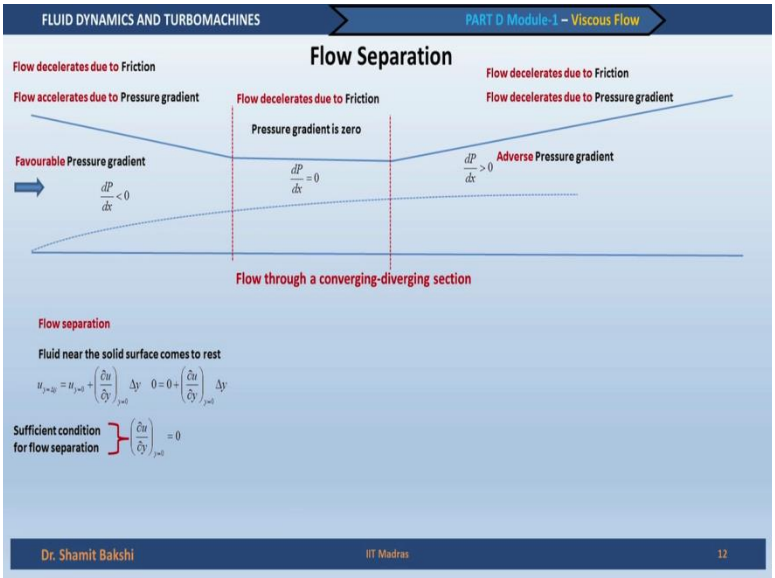

Now we consider a flow like this, let us say this is a kind of boundary layer, very approximate and we are looking at the flow inside this boundary layer. But outside the boundary layer right now, because of this converging diverging geometry the pressure gradient is not zero. So what is the pressure gradient there? So within this region, that means in the converging portion of this section the pressure gradient is negative, DP by DX is less than zero because as you go along this, the pressure is falling, the pressure is reducing.

So this we have introduced through a tutorial in the last chapter also, this is known as favourable pressure gradient. Why is it called a favourable pressure gradient, because this pressure as the, if the, if we have this kind of gradient, that means if you go along X direction, the pressure is reducing, the pressure is falling, so if that is the case, that means the flow is directed from high-pressure to low-pressure. So this pressure is actually helping the flow to take place, is actually helping the flow.

So this is called a favourable pressure gradient, that is why it is named as a favourable pressure gradient, it is favourable to the flow. In the throat region if you look at, that means between these 2 lines if you look at, then we get, it is of course like a flat plate, that is no change of pressure outside the boundary layer, so DP by DX is 0, the pressure gradient is zero in the throat region, in the Central region. In the diverging section, so the pressure gradient is zero here, so in the diverging section the pressure increases, the velocity reduces so the pressure increases according to Bernoulli's equation and you have a pressure gradient which is positive.

So what does it mean, this means an adverse pressure gradient, that means if you go along this X, if you travel along this X, pressure is increasing, so if you are facing more and more pressure or the flow is facing more and more pressure, so it is opposing the flow. It is not helping the flow, it is opposing the flow, so that is why it is called adverse pressure gradient. Now let us look back again at this favourable pressure gradient region, that is the converging region, we say, we see that the flow actually decelerates due to friction and the flow accelerates due to pressure gradient.

So effectively the flow will decelerates but or accelerate whatever but basically what we see is these are the 2 roles played by 2 things, one is friction and other is the pressure gradient. Friction tries to decelerates the flow even if the flow actually accelerates within this region, so in the, near the boundary, in the boundary layer region, the shear stresses or the friction, skin friction will try to decelerate the flow and the pressure gradient helps the flow. In the intermediate region again role played by the friction is the same, it decelerates the flow but pressure gradient has no role to play, it is basically zero, it neither help the play, nor retards the, it neither helps the flow nor retards the flow.

In the diverging section the flow decelerates due to friction and it also decelerates due to pressure gradient. So the in this region we see the flow opposes a obstruction both in terms of friction and pressure gradient, both will oppose the flow. And what is the consequence of that? So to find the consequence of that, the consequence, the one consequence could be flow separation. What is flow separation, now let us introduce what is flow separation. It means the fluid near the solid surface comes to rest, so this is not the fluid on the fluid surface.

The fluid on the solid surface is always at rest by no-slip condition, it is always at rest. But now when you consider the layer next to the no-slip layer, that is also at rest, so that is basically what is meant by saying that fluid near the solid surface comes to rest. Not only the fluid on the solid surface but fluid near the solid surface comes to rest. Now, what does it actually mean? So if we write the velocity, that is U at Y is equal to Delta Y, Delta Y means at a distance, at a very small distance from Y is equal to 0.

So this is Y is equal to 0, this bottom plate is that Y is equal to 0, at a small distance from the bottom plate is actually at a distance Y is equal to Delta Y, 0+ Delta Y, that is Delta Y. So at that position if we just expand it using a Taylor's series, we see that it is, it will be equal to Y at Y is, U at Y is equal to 0, velocity at Y is equal to 0+ Dell U by Dell Y at Y is equal to 0 into Delta Y. Now what is this part, this is already zero, this part is already zero by no-slip condition. And now by the definition of flow separation that this fluid near the solid surface comes to rest, we say this is also zero.

So if the separation has to take place, then Y at the on the plate is zero, near the plate is both the velocities are zero. So we put zeros in both the cases, what does it mean, it means that Dell U by Dell Y at Y is equal to 0 is zero. So this is actually a sufficient condition for flow separation. If this occurs, you can say that the flow separation has taken place. So this is basically, this is the first point from here, this is the condition which can be applied for flow separation.

Because if the fluid has come to rest at a particular point, the fluid near the wall has come to rest at a particular point, the next point it will move in a backward direction, it will not follow the plate because it is already at rest, so it will not follow the plate and it will get separated from the surface, so this is basically flow separation and this is basically a sufficient condition. If you can say Dell U by Dell Y is equal to 0, this is a sufficient condition for finding out the condition for flow separation.

Now like I we were seeing this flow through this converging diverging section, let us look at what, the region where the flow is more likely to separate. Now why is the flow separating, it is because the flow is decelerating, as you if you see on the surface of the plate the flow or the fluid near to the plate loses its momentum. We saw that while defining momentum thickness also, so it loses its momentum, now if that is the situation then why it loses its momentum, one is due to friction, another is basically could be due to pressure gradient.

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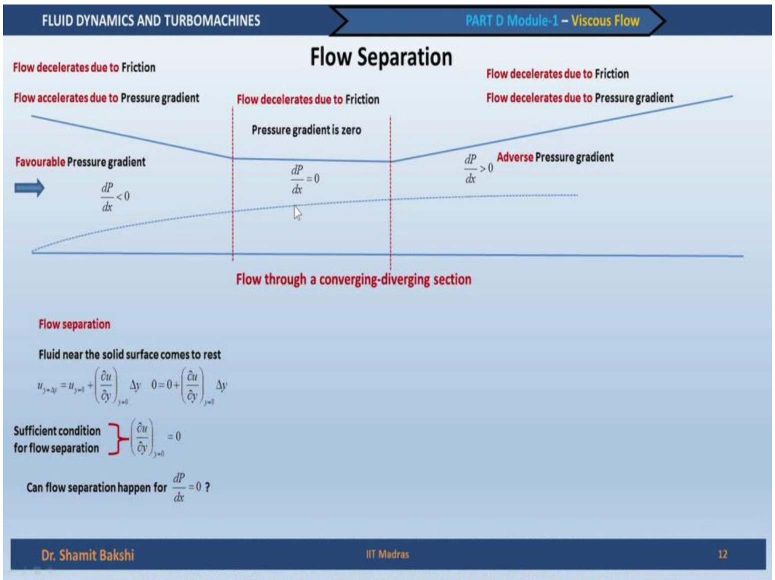

So this, so which is the most acute condition if you consider the deceleration of the fluid on the surface of the plate, near to the surface of the plate. So that condition is the most adverse conditions is this because here both fiction and pressure gradient is trying to retard the flow. The 2^{nd} severe condition is this that the friction is trying to oppose the flow and the pressure gradient has is not playing any role. The least severe condition is this because friction is of course opposing the flow but the pressure gradient is trying to accelerate, compensate for the deceleration caused by the friction, it is trying to compensate for friction.

So this is least likely for a flow to separate. So let us start with the intermediate condition that means the pressure gradient is zero. So can you have a separation, so can flow separation happen for DP by DX is equal to 0. So let us see if we can answer for this. Why do we start from here, because we have from our last 2 lectures we have already learned something about this kind of a flow where the pressure gradient outside the boundary layer is zero, flow over a flat plate.

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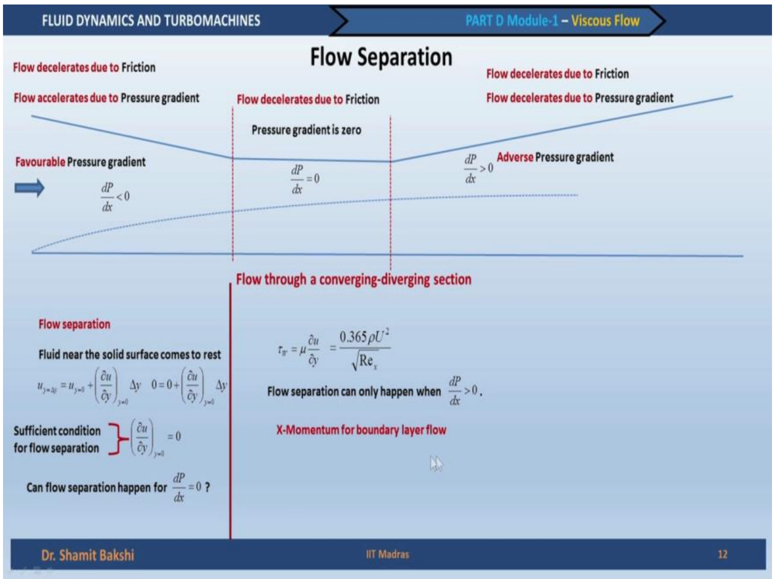

So let us try to look at this situation, can we have a flow separation or can be as more specifically a condition like this might like Dell U by Dell Y at Y is equal to 0 or on the surface of the plate is equal to 0. So this is the in terms of the likeliness of the flow to separate as far as our discussion has taken place so far, this is intermediate. So this is the most adverse, so let us see in this intermediate condition of likeliness of flow separation whether the flow can separate here.

So to get an answer to this question we use the expressions of which we had derived in previous lectures. So one thing is of course the wall shear stress is mu Dell U by Dell Y on the surface of the plate, Newton’s law of viscosity, we know that and we got an expression for this if you remember from the flow over a flat plate, we got an expression for skin friction coefficient, CF was tao wall, wall shear stress divided by half rho U square.

And that was from momentum integral approach we obtained an expression for the boundary layer thickness and from that we got an expression for the wall shear stress. So this is true, I mean this is a question by using Von Carmen approach which we found is quite accurate even with a very rigourous approach for getting the velocity profile and getting the expression for shear stress. Okay. So now if we have this, let us look back at the sufficient condition for flow separation, sufficient condition for flow separation is Dell U by Delta Y at Y is equal to 0 is zero.

So then what it means is this has to be zero, Dell U by Dell Y is zero means mu cannot be zero, so this is zero. So if this is zero, it means this expression is zero, nothing can be zero here. See for a finite velocity this Reynolds number have to be infinity for this has to be zero. So from this simple expression we can get some understanding about the likeliness of flow separation over a flat plate or over for a case where the pressure gradient is zero, we conclude that it cannot happen here for DP by DX is equal to 0, flows cannot separate.

We can also extend this because this case DP by DX is less than zero is less likely than this one to separate because here the pressure gradient is actually helping to compensate for the effect of deceleration due to friction. So here also we can extend this argument here and say this here also you cannot have flow separation because it is less likely to separate, then DP by DX is equal to 0. So these 2 case we can exclude, so only likelihood of flow separation is this DP by DX greater than zero. So DP by DX greater than zero where both the friction and pressure gradient opposes the flow has a likelihood of separation.

But see this is not sufficient condition, okay, but this is a necessary condition, I am sorry, this is a, this is not a sufficient condition because you can have pressure gradient is greater than zero, that means flow decelerates due to friction as well as for pressure gradient but need not separate, still it need not separate because nothing we have shown here, we have only said that this is the condition which has more likelihood of separation. And in the less likely situation the flow will not separate, that is what we have shown.

So here there is a likelihood of separation, so we say this is a necessary but not sufficient condition. Sufficient condition is that the wall shear stress should be zero or Dell U by Dell Y should be equal to 0. So in this region, in the adverse pressure gradient region the flow separation can take place. The reason for which we are so much interested in looking at the likelihood of flow separation is because it has a very important influence on the forces acting on the on a object, on a flow past any object.

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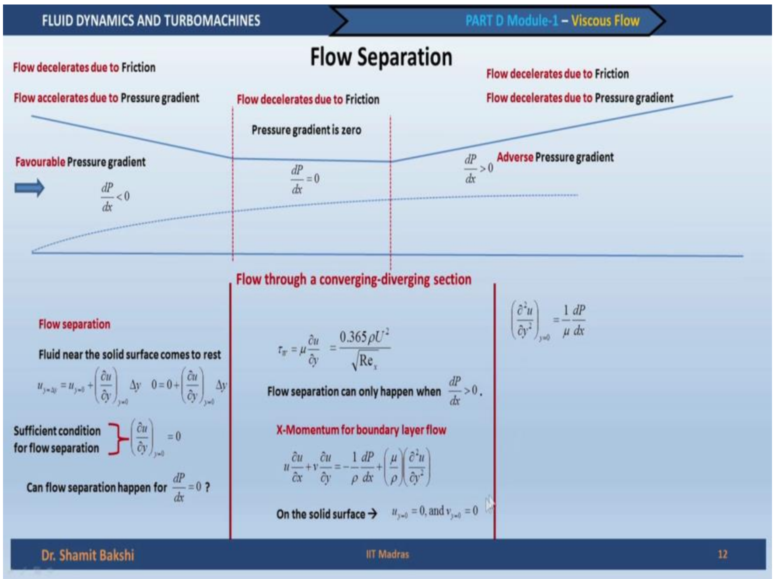

We will see that in the next part of this lecture. So another thing we can get here, if we look at X momentum for the boundary layer flow, X momentum equation for the boundary layer flow. If you remember we derived this equation, it came out like this. So for a boundary layer flow, we can drop out the viscous term written in form of Dell 2U Dell X 2 because 1 Reynolds number was coming here, so we can remove that term. See this equation is written for a general flow graph the pressure gradient is not zero. For the special case of the flow over a flat plate where the pressure gradient is zero, we have removed this term.

But for a general case we are retaining this and try to see what influence, how does this influence the velocity profile, that is what we can see by doing this simple analysis. So this is the X momentum for the boundary layer flow which is applicable within this boundary layer. Now if we move onto the solid surface, that means on this surface here, we see that U is zero and V is zero. You plug-in these 2 values here, U is equal to 0, V is equal to 0, so you will be left out with only the right-hand side, the left-hand side disappears. And you will be left out with this expression, this is a very important expression.

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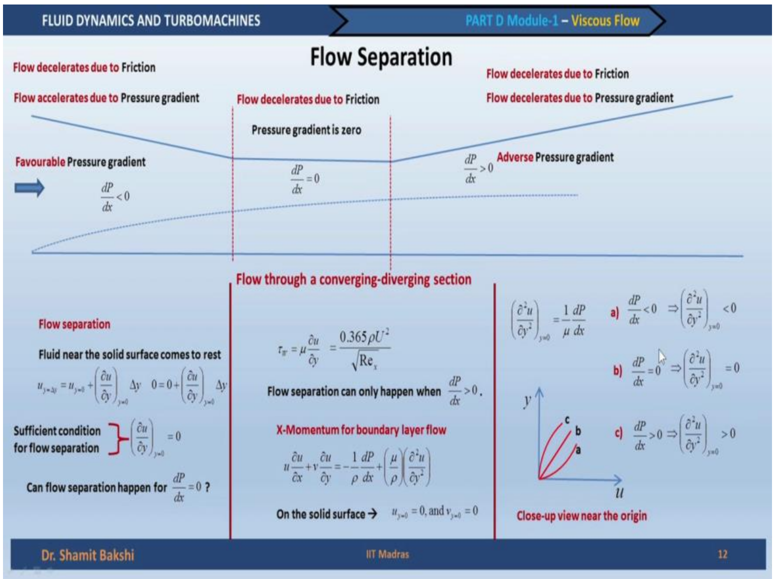

So we get to know, of course we have applied, we have applied this governing equation on the solid surface only, so from here we get the value of Dell 2U Dell Y 2 on the solid surface, 2nd gradient of velocity for the solid surface. This is a very important expression because this will let us know important things about the velocity profile, while the flow separates and while the flow do not separate, so we get this expression. And we take the first case where DP by DX is less than zero, that means the favourable pressure gradient.

So if DP by DX is less than zero, from this expression we can say Dell 2U Dell Y 2 at Y is equal to 0, in this we are near to the plate is less than zero, it is negative. What it means, so for that we take a closer look near the very near to this plate, so basically if we draw the origin here, very very near to the origin where this expression is applicable we take a look at that region. And if you look at the velocity profile there, from this expression we can say it will, of course while drawing the velocity profile, this axis is given as velocity axis and this axis is given as Y axis.

So basically this is how we plot velocity profile, not X here, only velocity here. Okay. Now we apply this condition that Dell 2U Dell Y 2 is less than zero, is negative. So if we apply that condition to the velocity profile, what it means is the velocity profile will look like this. Okay. So it means, see the velocity is actually Dell U by Dell Y is actually positive because if you move along Y, the velocity, it was zero here, it is increasing but it is increasing by this profile, you go to this Y, it has this velocity, you go to higher value of Y, the velocity is more.

So Dell U by Dell Y is greater than zero but see it increases at a decreasing rate. Suppose you draw a straight line here, it would have increased more, so if it increase at a decreasing rate, that means the 2nd derivative of the velocity gradient or any gradient in general should be negative, less than zero. So that is what it means in terms of the velocity profile. So if you look close to the surface, you will see the velocity profile will have this kind of a characteristics, this kind of nature. The 2nd, so that means the velocity increases with Y but the rate of increase of velocity decreases with Y, that is why finally it merges with the free stream velocity.

The case B is DP by DX is equal to 0, that means pressure gradient is zero, we are looking at the 2nd portion of the converging diverging section or even for a flow over a flat plate. So this region if you look at, what we get is D 2U DY 2 is equal to 0. So if you draw this kind of a velocity profile, you will get this is basically a straight line. That means okay velocity is increasing along Y direction and the rate at which the velocity is increasing in the Y direction is fixed, it is same, it does not change with Y.

If you go to a different Y also, the rate at which velocity changes with Y is the same. Okay, so this is the condition which is met, so this is not the total velocity profile, this is if we magnify very close to the origin or very close to the, because this expression is only valid at the surface, at Y is equal to 0, that is what we have to keep in mind. So this portion very close to the origin, it is a straight. What happens for the 3rd situation, that means adverse pressure gradient?

So adverse pressure gradient DP by DX is greater than zero, so it means Dell 2U Dell Y 2 is also greater than zero. Now we can draw the velocity profile for this, that is the case C, it will look like this. That means if you go along Y, velocity is increasing and the rate at which the velocity is increasing is also increasing. So Dell 2U by Dell Y 2 is greater than zero. But see you cannot continue all throughout the boundary layer like this, finally it has to come to this kind of shape for merging to the free stream flow.

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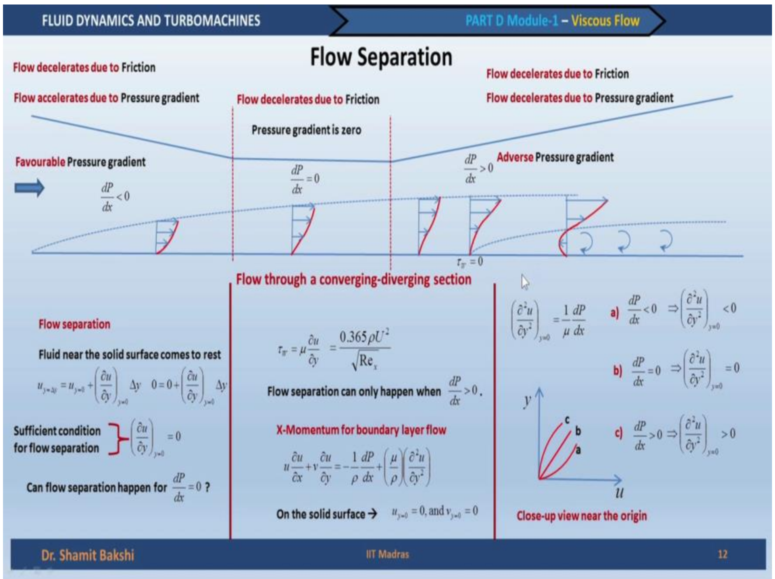

We will look at the full velocity profile a little later but at least very near to the surface of the plate the velocity profile will be distinctly different for different pressure gradients, for pressure gradient less than zero, for a pressure gradient equal to 0 and for a pressure gradient greater than zero. So now let us look at a, take a view of the total velocity profile. So in this region if you look at, this is the velocity, this will be the velocity profile where the pressure gradient is negative or favourable pressure gradient. If you look at very close to the wall, it will be something like this.

In a region like here, this will be almost straight and then it will constantly go. So this profile will again not be a linear profile or a linearly increasing, velocity will not linearly increasing away from the surface, solid surface but it will be something like this. On the top of that solid surface Dell 2U by Dell Y 2 will be zero signifying a little difference in the velocity profile than the region with favourable pressure gradient. But if you look at a velocity profile in the adverse pressure gradient region, it will be very distinctly different.

So how distinctly it is different, you see, start seeing here, it has, initially it has a region where Dell 2U by Dell Y 2 is greater than zero in this part. And then you have a region where Dell 2U by Dell Y 2 will be equal to 0 and then you have a region here where Dell 2U by Dell Y 2 will be finally less than zero because that is a velocity profile which will smoothly merge it into the boundary layer, into the free stream, out of the boundary layer, that means into the free stream. This region, mathematically, this region where you have Dell 2U Dell Y 2 equal to 0 is called the point of inflection where basically the gradient change sign.

So at that point where the gradient where it changes sign, gradient of the quantity changes sign, it becomes negative to positive or positive to negative is the point that where the gradient of the gradient, the double derivative becomes zero. So as you see here what is happening, so this velocity profile has no point of inflection, it is actually all throughout the velocity profile if you see Dell 2U by Dell Y 2 is less than zero is negative. If you see here, this has the point of inflection on the surface of the plate, this in this velocity profile, adverse pressure gradient the point of inflection has moved into the fluid. Okay, it is somewhere here.

So it the gradient actually changes sign from here to this. If we will go further, we meet this condition, see here the flow has not separated even though the pressure gradient is adverse, the flow has not separated. This, because this is just a necessary but not a sufficient condition, the sufficient condition is met little later where what happens Dell U by Dell Y becomes zero, that means wall shear stress becomes zero. So this is the region where actually flow has now started separating and the velocity profile looks something like this.

The point of inflection of course is inside the fluid and if you follow this further you will get a flow reversal, that means now what happens, what has happened is you you see the flow has actually started decelerating, moving in the opposite direction recirculation. So this, this kind of recirculation is a very important feature of many different types of flows which we experience in many practical applications. So this region where you have this flow separated, the flow reversal or flow separated, so as if the flow came up to here and it is now following this separation line and within this region there is a recirculation vortex.

So there is a circular flow or a recirculation present and this region is basically the wake of the boundary layer. So basically the boundary layer separates out here because of the adverse pressure gradient and you have a recirculating region here. So if you have, if you have a sufficiently long adverse pressure gradient region in a converging diverging section, you will certainly see this region recirculating region within the flow. So this slide actually gives us an overview of how the flow characteristics within the boundary layer changes in presence of different types of pressure gradient, that is negative pressure gradient or favourable pressure gradient which is also called as favourable pressure gradient, zero pressure gradient which we already saw in the case of flow over a flat plate and a adverse pressure gradient, that means a positive value of pressure gradient.

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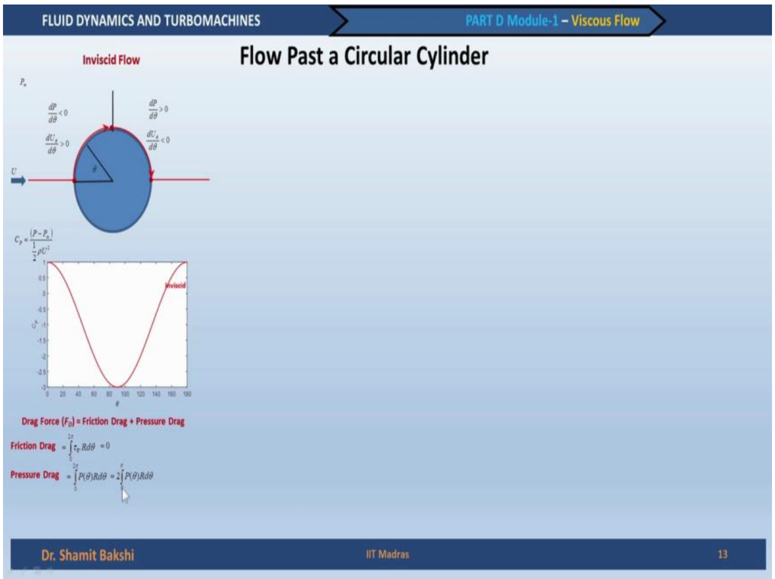

So we saw that in the case of a adverse pressure gradient or positive pressure gradient you can have flow separation, you can have a recirculating region like this and we also saw how the velocity profile looks like for these 3 different regions of the flow. We will use this information now to study more closely flow past a circular cylinder. We will introduce a little bit of it and get the most important concept related to flow past take a circular cylinder.

So let us before going into the viscous flow, also in this chapter we are looking at the viscous flow, for the context of the circular cylinder, it is important to actually start with an inviscid flow, then we get a clear understanding of what is happening in the case of viscous flow. So let us look at the very quickly look at what happens if there is a inviscid, inviscid flow incident on a circular cylinder. So this is basically our cylinder and there is a flow which is approaching the cylinder like this.

This is again a uniform flow approaching the cylinder in this way. So if we see these 2 there are 2 red lines which are appearing in this figure which signifies the stagnation streamline. So basically if you follow the flow here, the flow stagnates finally at the this first point. So the flow velocity becomes zero. So this is basically the streamline which ends at the stagnation point and it is called the stagnation streamline. Similarly there is very stagnation point also in this flow. The flow moves in this direction but at that particular point you cannot have any flow velocity.

Now if you consider a fluid element here moving you know on the surface of the plate, it will move something like this and what happens as it moves here, so from a no velocity, it goes to a higher velocity region because it is an inviscid flow, so you can have velocity even on the surface of the solid surface, exactly on the solid surface also. And then the flow has to, then this fluid particle is actually accelerating, it is moving, it is going to a higher velocity, from a stagnant situation to a dynamic situation, to a moving situation.

So if we see DU Theta, U Theta being the tangential component of the velocity, that means if you draw a tangent to this circle, then the velocity along the tangential component is the tangential velocity. So basically dU Theta by dTheta, Theta is basically the angle made, subtended by this radius drawn from the first stagnation point to the Centre to any other radius. So basically that is our reference, the first, the reference is the radial line joining the first stagnation point and the centre of this circle. Okay I think it will come up little later in the presentation also.

So if you have dU Theta by dTheta greater than zero, that means the flow is accelerating, that means this is basically a dP by dTheta less than zero, that means it is a favourable pressure gradient region. So this region within the industry leader is actually a favourable pressure gradient region like in the case of converging diverging duct we saw up to this point. Beyond this point what happens, beyond this point actually if you see dU Theta by dTheta, I am sorry dU Theta by dTheta is less than zero because it has to again stagnate at this point, right.

So it decelerates, so it the fluid actually accelerates and then decelerates, so this deceleration is associated with again a positive pressure gradient. By using of course Bernoulli's equation or in the differential form the Euler equation we can say if the velocity reduces, so the pressure has to increase. So the pressure gradient, DP by dTheta is positive here, that means this is a, if you, as you move the pressure increases, that means it is a adverse pressure gradient. Okay, so positive pressure gradient is adverse pressure gradient.

But right now we are considering, we are just making this observation but right now this is not important because we are considering an inviscid flow. We define a parameter called CP, so coefficient of pressure. So we define this parameter as P - P infinity by half rho U square. So in this expression P infinity is actually the free stream pressure. Like U is the free stream velocity, this is the free stream pressure. And P is the pressure at any point. So **so** at any point on the surface of this particular cylinder. So now okay, so now we can actually plot this CP with respect to Theta.

So this Theta we already defined, Theta is basically the angle between this reference line, reference line is the line joining the forward stagnation point with the Centre of the cylinder and any other radial line. So this is Theta, so as we go along this Theta, go from Theta is equal to 0 to 180 degree, so from the forward stagnation point to the rear stagnation point, that is Theta is equal to 0 to 180 degree and if we plot CP, okay, so this particular, it will essentially give me the variation of pressure. Okay, because of the quantities P infinity, rho and U all are constant.

This is a nondimensional way of plotting this particular variable. If we plot this, it will from the inviscid flow theory or the potential flow theory, okay, it will, we can get this kind of a variation of pressure. So what is happening is, basically this region if you go through the pressure is reducing and then the pressure is increasing. And in an inviscid flow you will see that the pressure recovery is complete, that means pressure reduces as it comes to 90 degrees. That means Theta is equal to 90 degrees that means this top of the plate, sorry the top of the cylinder but it again recovers to the same value of pressure which was here as it, as it approach the forward stagnation point.

So forward stagnation point and rear stagnation point, the pressures are same. So there is a complete pressure recovery. Now we can use this pressure information to get the drag force on this particular cylinder. So this drag actually results from 2 things, one is the friction drag, that means due to viscosity the flow decelerates, of course this is an inviscid flow, so that is not important here, the friction drag. Another is the pressure drag, what with pressure drag, pressure drag is, we know that if we have to find out the net force acting on a control volume we have to find out what is the pressure around that control volume.

Like that if you have to find out the force on this particular cylinder we should know what is the pressure at different points within the cylinder and then find out what is the force acting. To write a exact expression for this, friction drag can be written as 0 to 2 pie integrated over 0 to 2 pie, so the entire cylinder, wall shear stress multiplied by R into dTheta. So R dTheta is basically the arc length multiplied by 1 will be the area. So R is basically the radius of this cylinder. Now this is the friction drag which will be zero in this case because this is an inviscid flow. Now what is the pressure drag?

So pressure drag is basically equal to, again 0 to 2 pie P Theta R dTheta. With the same reasoning, R dTheta is arc length multiplied by 1 is the area on which the pressure is acting. And how is the pressure variation? So we can write this as from the symmetry because the pressure recovery as we see, it is symmetric about treatise equal to 90 degree. So we can write this as 2 into 0 to pie P Theta R dTheta.

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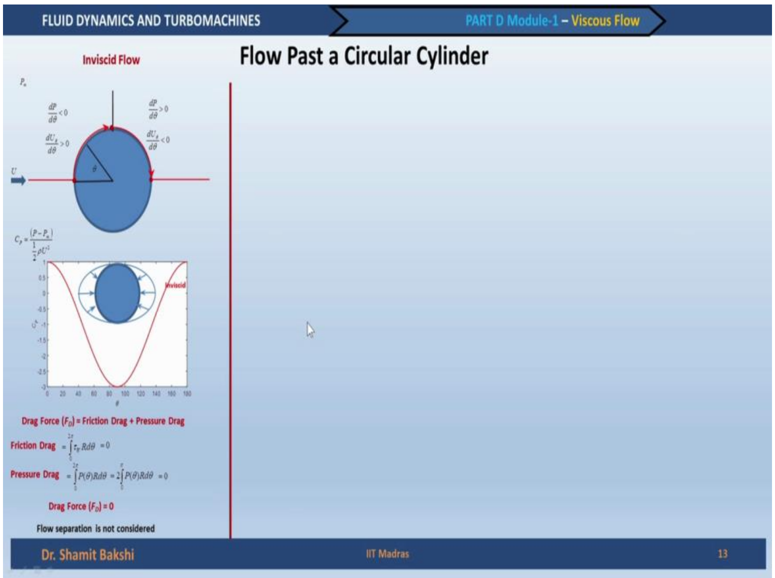

Now this is basically our cylinder. If you consider this as the cylinder, this is how the pressure is acting at the forward and the rear part of the, front and the rear part of the cylinder. So this integration from the symmetry of this pressure, as we see from this CP variation of from this particular representation of the pressure on the surface of the cylinder we can say this is also zero. So what it means is the drag force on this particular cylinder is zero. This is of course contradictory to our understanding that is a we know that if we place a cylinder on a flow, it will experience some drag.

But what this inviscid flow theory is telling me is that there is no drag because there is a full pressure recovery. So this is a, this is due to the inappropriateness of the inviscid flow theory and this is also said as the D'Alembert's paradox, this is the paradox and this paradox is named after D'Alembert because he first pointed out that this paradoxical observation from the inviscid flow theory, from the potential flow theory. Because this, this was actually resolved when Ludwick Pandal proposed the boundary layer theory in the beginning of the 20^{th} century.

Otherwise the inviscid flow theory from Euler was already there from 1755, so a long time back but it was not able to explain the drag on a cylinder. So the drag on the cylinder could be only explained by looking at the viscous flow theory. That is why here we want to now relook at situation with our understanding of the viscous flow understanding which we have gained from the flow over a flat plate and flow over converging diverging section. So this, actually this happens due to the fact that flow separation is not considered here.

The reason for that is that the flow separation which can be only present in a viscous flow, that has not been considered here. So what happens if you consider that?

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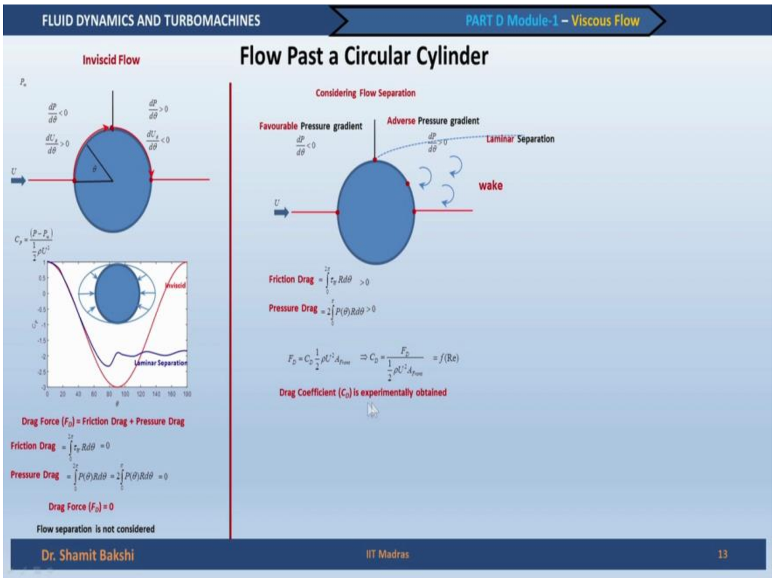

So for going into that let us look at the flow separation, how, what happens to flow separation. We know from our previous slide that the flow separation, the necessary conditions for flow separation is adverse pressure gradient and the sufficient condition is wall shear stress becoming zero, we saw that already. Let us apply those criterias to this flow over a circular cylinder. So this is a cylinder again, redrawn and the flow is approaching the cylinder with the uniform velocity capital U. This region is negative pressure gradient, that means it is a favourable pressure gradient region and this is positive pressure gradient, that is adverse pressure gradient region.

So now we can say if there is a separation, the separation has to happen in this region, in the adverse pressure gradient region. And actually it happens, you can have a laminar separation line like this, so if you, this is the separation part, actually it happens very close to 80 to 90 degree or 85 to 90 degree depending on the Reynolds number of course. For a turbulent flow it is even latter, so this laminar separation now brings in a new picture of the flow around the cylinder.

And what is that, we see you have a weight behind the cylinder, so the pressure when we see the rear part of the cylinder, the recovery is not happening because there is a flow recirculation and in this region the pressure is very small. So if you look at from this point of view, now you let us look at the total drag acting on the cylinder, so the friction drag will be given by this because we consider this as a viscous flow, this wall shear stress will not be zero now.

So this is greater than zero, more importantly, which is the major component of the drag force for most reasonable Reynolds number or high Reynolds number flows is the pressure drag. The pressure drag, the expression for this is borrowed from here, so 2 times 0 to pie P Theta R dTheta, this will be nonzero. How do we know that, let us look at the coefficient of pressure, so if you look at the coefficient of pressure, here is the plot of that, then you see it follows the inviscid curve up to some point but after that there is a adverse pressure gradient even before treatise equal to 90 degree.

The reason for that is basically the influence of the wake actually changes the shape of the body and makes it to actually separate or the adverse pressure gradient to exist even before Theta is equal to 90 degree. So without going into too much of details into that, we make a note of the fact that for a laminar separation the CP, that the coefficient of pressure varies something like this. Okay so this rear part, the pressure recovery path does not exist. So the pressure is very low in the wake region.

So now if we find the pressure drag, in this region you have a positive pressure, in this region you have a I am in pressure force, positive pressure force, force arising due to the pressure acting in the positive X direction. In this region you have a force acting due to pressure in the negative X direction but the force arising due to pressure in the negative X direction is much less because the pressure itself is very small as compared to the pressure on the front side. So this negative force is less and the net force will be in the positive X direction.

And so you have a pressure drag which is greater than zero which is also called form drag because it depends on the form of this particular geometry. So in this case it is a circular cylinder. This is, it is not so easy actually to get the expression for this pressure variation for an inviscid flow, so what people resort to is experimentation. So in connection with the experiment, like I was talking about the same friction coefficient, it is better to introduce coefficients like this which can be obtained experimentally.

So with respect to drag force also we introduce a coefficient known as drag coefficient and what it does is, it is actually the, the product of the track coefficient and the half rho U square will give you the force per unit area. So multiplied by the frontal area will give the drag force. So basically this is the expression for the drag coefficient. And this can be experimentally determined. Using dimensional analysis you can show that this drag coefficient actually depends on the Reynolds number of the flow and if you plot this drag coefficient, once you know the drag coefficient you can find out, knowing the Reynolds number you can find out what is the drag force acting on the, on a particular object, in this case a circular cylinder.

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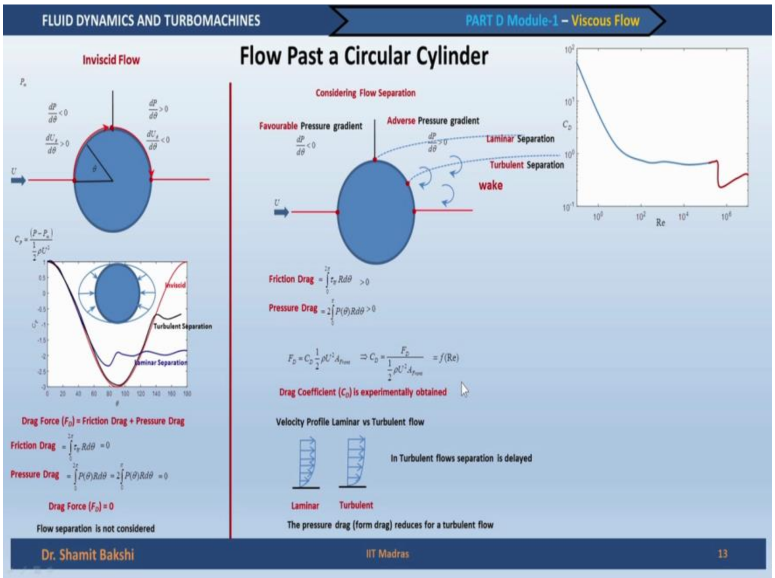

So basically this approach was able to resolve the D'Alembert's paradox by considering the viscous flow, boundary layer flow around the cylinder. So it is a small region around the cylinder which is actually viscous but it plays a very important role in finding out the drag force acting on the object. So like we mentioned that this drag coefficient is actually experimentally obtained and if you look at the plot of this drag coefficient with respect to Reynolds number, it looks something like this. So this is basically, so the drag coefficient actually reduces as the Reynolds number increases, so this is the laminar part of the flow.

Till the flow remains laminar, this is basically the variation of the drag coefficient. Without going into there are a lot of correlations for different bodies, different types of bodies, for example right circular cylinder and you know sphere, aerofoils and so on so forth. The coefficients, the drag coefficients as a function of Reynolds number are available in open literature. So we do not go into the discussion of that, that is basically aerodynamics, more description on the effect of the shape of the object on the drag coefficient.

But without going into that we can now look at what happens afterwards or at higher Reynolds number. So that means we are going into a turbulent region. Before going into that we can take a look at the velocity profile for the laminar versus the turbulent flow. This will be useful in explaining the drag coefficient. So the velocity profile for laminar is like this, the velocity from the no-slip condition to free stream velocity, it, the change is gradual. But whereas in the case of turbulent flow initial change is very rapid, it is very quick.

Means that fluid particles near the surface, very near to the surface are at higher velocity in case of a turbulent flow as compared to the laminar flow. The deceleration is actually less in the case of turbulent flow. So it continues at a higher velocity. So as the deceleration or retardation of the fluid particle near to the surface is less, now what happens as a consequence of that, the flow separation is delayed. See the separation will only happen when the velocity near the plate becomes zero, the play velocity of the fluid on the plate is already zero, the velocity of the particle of fluid particle near the plate becomes zero.

But from this velocity profile we can see as the velocities or fluid particles near the plate are more or much more as compared to or more closer to the uniform flow velocity or the free stream velocity in case of a turbulent flow. So this layer to come to 0 velocity interval take more time or the fluid can travel through more distance, so the flow separation is delayed and if you try to look at that delayed separation in this particular figure you will see the separation for a turbulent flow will happen at a later stage, so it will separate here.

So the wakes of, wake also gets shifted. As a result of this what happens is now we can look at the CP variation. Flow separation gets delayed so the S CP variation now is like this, adverse pressure gradient and the separation happens at the rear part of the cylinder. Now if you try to use this particular pressure coefficient of pressure in the expression of the pressure drag then you naturally you will find that the pressure drag will be lesser in the case of a turbulent flow because this portion of the cylinder will have very similar pressure is to this portion of the cylinder.

So the effectively you see this region on the top half of the cylinder or this entire region in the entire cylinder will have a much lower pressure as compared to the other side. So effectively they will contribute to the drag force. And because of this delayed transition, what happens is that drag force reduces and if you look at the drag coefficient, it drastically falls at the point where the flow becomes turbulent. So there is a drastic fall in the drag coefficient, of course later on it increases with increasing Reynolds number, but this fall is very easily observable even in the experimental variation of drag coefficient with respect to Reynolds number.

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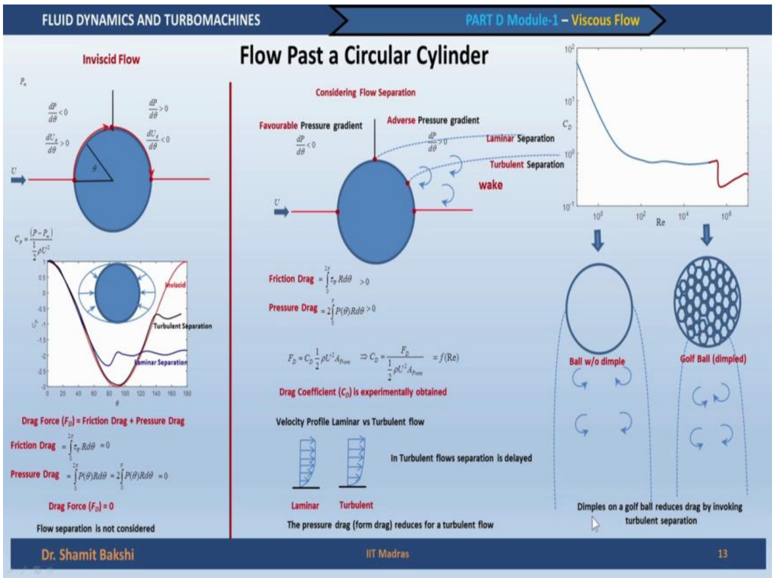

So the pressure drag or the form drag significantly reduces for a turbulent flow. In many cases we use this particular idea that by making a flow turbulent you can delay the transition. One such example is the golf balls. So if you have seen a golf ball you will see there are dimples on the surface of the balls, okay, it is not a smooth surface. So why is it so that you have to Dimple the surface of a golf ball? There are other examples of this also but we are just taking this example to demonstrate the delayed transition in a turbulent flow.

So let us say this is the ball without the Dimple, without any Dimple on the surface, so the flow is approaching from the top and the separation will take place close to the 90 degree or 85 degree to 90 degree depending on the Reynolds number. So this is how the wake will look like. So now basically, when we look at the golf ball, this golf ball is actually moving, the flow is stationary but it is similar to the situation where we make the golf ball stationary and allow the flow to move. So without Dimple this is the situation, and the drag force effectively will be due to this entire projected area of this ball.

On the other hand if you see the Dimpled golf ball, you will see the, because of the presence of this roughness on the surface of the golf ball the transition at the same velocity, the transition at the same velocity of the flow will be, transition to turbulent will be earlier, and you will have a turbulent flow. You will have a turbulent flow and due to this turbulent flow the transition will be delayed, like what happened here, the Turbo separation was delayed. So here also the separation is delayed and as a result of that knowledge you will have to bother about this part of the projected area which will be acting in terms of the drag force.

Because this which is the drag force resulting from the pressure which is a major part of the drag force for the kind of Reynolds number for high Reynolds number or the kind of Reynolds number which is of interest here. And so the golf ball, because of the presence of these dimples can travel through a long distance. It is very difficult to actually hit a non-dimpled golf ball and make it travel through a long distance. So that is basically an idea which borrowed from fluid mechanics or fluid dynamics to our daily life.

There are many other examples of this flow separation also, for example, swing of a cricket ball, that can also be explained by using this delayed transition in case of a turbulent flow. So basically in this example the dimples on a golf ball reduces drag by invoking turbulent separation. So instead of a laminar separation, when you want to reduce drag, on technique of reducing drag, in the beginning of this course we were talking about reducing drag force and increasing the lift force, so one way of reducing drag is actually to invoke turbulent separation.

Of course there are other ways also of reducing drag by boundary layer suction and things like that where again, there also we avoid separation of the flow. So this is one way of invoking, one way of reducing the drag force by invoking turbulent separation, as opposed, as compared to a laminar separation. This brings us to the end of the 3rd lecture of this week where we talked about the flow separation and flow past, flow in presence of in a boundary layer, in presence of pressure gradient. Particularly important is a positive pressure gradient or adverse pressure gradient which actually results in, which can result in flow separation.

We looked at the condition of flow separation, the necessary and sufficient condition for flow separation, we looked at how the flow, how in the case of an inviscid flow, the drag force is predicted as you which resulted in the D'Alembert's paradox and how it this, the paradox can be resolved by considering a viscous flow and considering the boundary layer structure. At the end of this lecture we gave an example of a application of boundary layer separation, **,** in a turbulent flow situation to explain the dimples given or on a golf ball to reduce the drag force on them. This brings us to the end of this lecture, thank you very much.