

Fluid Dynamics And Turbo Machines.
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Part A.
Module-2.
Tutorial -1.
Week 5.

Good morning, I welcome you all for today's lecture in the week 5 of fluid dynamics and Machines course. Today we will talk about the tutorials about on the problems that we can do based on the lectures we have got in this week. So essentially what we will do is we will solve some problems based on thermodynamics as well as the non-dimensional approach that were followed in case of Turbo machines.

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FLUID DYNAMICS AND TURBOMACHINES
PART-A Module-02 - Tutorial 1

Problem on Thermodynamics

1) Air leaves a heat exchanger and enters a turbine at a temperature of 800 °C and at a speed of 30 m/s. The temperature of the air at the exit of turbine is 650 °C and its speed is 60 m/s. Mass flowrate of air is 2 kg/s. Calculate the power output from the turbine assuming no heat transfer. Take $C_p=1005$ J/kg-K for air. Assume the turbine to be 100% efficient.

Considering steady state steady flow energy equation for turbine, we can write:

$$Q' - W' = \dot{m} \left((h_{out} - h_{in}) + \frac{V_{out}^2 - V_{in}^2}{2} + g(z_{out} - z_{in}) \right)$$

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So in the first problem we are talking about the problem of thermodynamics where we will apply the first law of thermodynamics. The problem statement is like this, air leaves a heat exchanger and enters a turbine at a temperature of 800 degrees centigrade and at a speed of 30 metres per second. The temperature of the air at the exit of the turbine is 650 degrees centigrade and its speed is 60 metres per second, mass product of the air is given as 2 kg per second, we need to calculate the power output from the turbine assuming no heat transfer. You can consider that the CP for air is 1005 joules per kg Kelvin, you also assume for this problem the turbine to be 100 percent efficient.

So whenever we come across these problems, I suggest that you draw a picture like this and write down the values that are available, like for example we are having a flow from a flow from inlet to the outlet of the turbine. There is an expansion process, so just make a schematic for expansion, then we write down the inlet temperatures and velocity or pressure, whatever the information is given. In this case the temperature and velocity are given, you also note that the temperature is given in 800 degree centigrade, please convert it into absolute scale and write it in Kelvin and similarly the write down the quantities which are available in the outlet.

And if there is pressure information also available, please write it down. In this case we have only temperature and the velocity. So then we can apply and we also know that there is a work done and that is given by W_{Prime} . So when we apply the steady-state steady flow energy equation for the turbine we can write that $Q_{\text{prime}} - W_{\text{Prime}} = \dot{M} (h_{\text{out}} - h_{\text{in}} + \frac{V_{\text{out}}^2}{2} - \frac{V_{\text{in}}^2}{2} + g(Z_{\text{out}} - Z_{\text{in}}))$. And we know that the problem statement says that there is no heat transfer. So for under this condition of no heat transfer we can say that $Q_{\text{prime}} = 0$.

And we have also discussed while talking about the steady-state steady flow for gas turbines and steam turbines applications or for that matter compressors, we have talked about that the changes in potential energy can often be neglected in comparison to the other 2 terms and the changes. So we will take make this assumption and we will say that the changes in the potential energy is not significant. And then we can simplify this equation and write that $W_{\text{T prime}}$, please recollect that we have used to the notation T for the turbine, $W_{\text{T prime}}$ is equal to $\dot{M} (h_{\text{in}} - h_{\text{out}} + \frac{V_{\text{in}}^2}{2} - \frac{V_{\text{out}}^2}{2})$.

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FLUID DYNAMICS AND TURBOMACHINES PART-A Module-02 – Tutorial 1

Problem on Thermodynamics (contd)

$$W_t' = \dot{m} \left(h_{in} - h_{out} \right) + \frac{V_{in}^2 - V_{out}^2}{2}$$

$$W_t' = \dot{m} \left(C_p (T_{in} - T_{out}) + \frac{V_{in}^2 - V_{out}^2}{2} \right) \quad \text{Assuming constant } C_p$$

$$W_t' = 2 \left(1005 (1073 - 923) + \left(\frac{30^2 - 60^2}{2} \right) \right)$$

$$= 298.8 \text{ kW}$$

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And we can assume that CP is constant for air which is given with here and then we can write the WT prime is M dot CP T in - T out + V in square - V out square whole divided by 2. And thus when we substitute the values we get that WT prime is 298.8 kilowatts. Please note the unit and then if you take care of the calculation, if you have to divide the quantity that is given here by 1000, you will get this answer. So what we learnt here is we learnt to use the straightforward application of the first law of thermodynamics with respect to the steady-state steady flow.

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FLUID DYNAMICS AND TURBOMACHINES PART-A Module-02 – Tutorial 1

Problem on Thermodynamics (contd)

2) Continuing from previous problem, let us assume that the exhaust from the turbine flows through a nozzle. It is given that the speed of air at the exit of the nozzle is 544 m/s. Calculate the temperature at the exit of the nozzle assuming no heat transfer and no friction.

Considering steady state steady flow energy equation for nozzle, we can write:

$$\cancel{Q'} - \cancel{W'} = \dot{m} \left((h_{out} - h_{in}) + \frac{V_{out}^2 - V_{in}^2}{2} + g (z_{out} - z_{in}) \right)$$

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Please note that it is not mandatory that Q prime is always zero, there could be some problem, some instances where heat transfer rate is given and in that case you have to consider it. Similarly the case of change of potential energy. Now we will take this problem further, we will assume that the exhaust from the turbine flow through a nozzle, it is given that the speed at the of the air at the exit of the nozzle is 544 metres per second, so what do we have, we have the air coming out of the heat exchanger, flows into the turbine and from the turbine the air enters the nozzle and at the exit of the nozzle it is 544 metres per second. So we will take whatever we have learned from the last example and we will start from there.

We need to calculate the temperature at the exit of the nozzle assuming no heat transfer and no friction. So what we will get here is that we have a nozzle again we have got the schematic of the nozzle, the flow in this case is from right to left, we have, we know from the previous problem that T_{in} is given and V_{in} is also given. At the exit however the T_{out} is not given and V_{out} is given as 544 metres per second. So we can apply the steady-state steady flow energy equation for nozzle and we can write the same equation. But here we have to take care of the fact that not only Q prime is zero because there is no heat transfer, we also should know that since the nozzle does no work or the flow inside the nozzle does no work, then in that case W Prime also goes to 0.

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FLUID DYNAMICS AND TURBOMACHINES
PART-A Module-02 – Tutorial 1

Problem on Thermodynamics

$$(h_{out} - h_{in}) = \frac{V_{in}^2 - V_{out}^2}{2}$$

$$h_{out} = h_{in} + \frac{V_{in}^2 - V_{out}^2}{2}$$

$$T_{out} = T_{in} + \frac{V_{in}^2 - V_{out}^2}{2C_p} \quad \text{Assuming constant } C_p$$

$$T_{out} = 923 + \frac{60^2 - 544^2}{2 \times 1005}$$

$$= 777.6 \text{ K} = 504.6 \text{ } ^\circ\text{C}$$

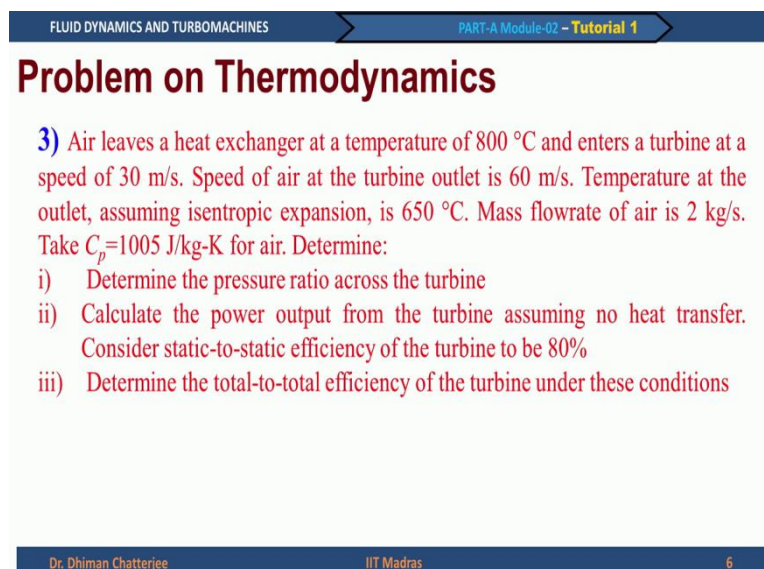
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And from our assumption, we continue from the last example, that the changes in the potential energy is zero or Negligible rather and then we can get the $h_{out} - h_{in}$ is $V_{in}^2 - V_{out}^2$ the whole divided by 2. Or in other words, h_{out} is equal to $h_{in} + V_{in}^2 - V_{out}^2$ the whole divided by 2.

- V out square by whole divided by 2. Assuming the constant CP as we have done in the last problem, we can say that T out is equal to T in + V in square - V out square by 2 CP. When you are using this value of CP here, please take care of the units recall sometimes we can say it in as 1.005 kilo joules per kg Kelvin. So you have to convert the unit accordingly.

And then when we substitute the values we get the T out equal to $923+60$ square - 544 square, both are given divided by 2 into 1005. And this gives rise to the temperature of 504.6 degrees centigrade or 777.6 Kelvin. We can extend this problem of expansion in the turbine by now considering a more realistic case where deficiency of the turbine is not 100 percent. And that will solve in the 3rd problem.

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FLUID DYNAMICS AND TURBOMACHINES

PART-A Module-02 - Tutorial 1

Problem on Thermodynamics

3) Air leaves a heat exchanger at a temperature of 800 °C and enters a turbine at a speed of 30 m/s. Speed of air at the turbine outlet is 60 m/s. Temperature at the outlet, assuming isentropic expansion, is 650 °C. Mass flowrate of air is 2 kg/s. Take $C_p=1005$ J/kg-K for air. Determine:

- Determine the pressure ratio across the turbine
- Calculate the power output from the turbine assuming no heat transfer. Consider static-to-static efficiency of the turbine to be 80%
- Determine the total-to-total efficiency of the turbine under these conditions

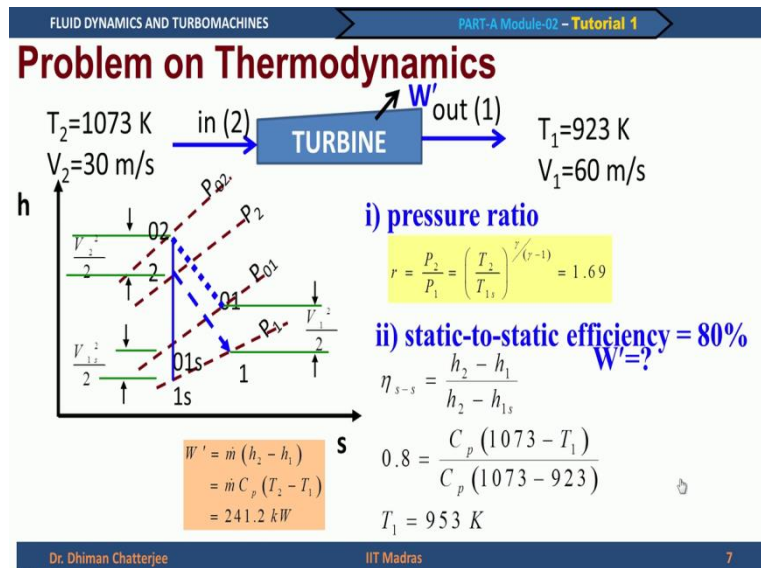
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In this problem, let us look at it more carefully. We see that air leaves the heat exchanger at a temperature of 800 degrees centigrade, the same problem statement that we have given in the first one and enters a turbine at a speed of 30 metres per second. Speed of air at the turbine outlet is 60 metres per seconds, temperature of the outlet assuming isentropic expansion is given as 650 degrees centigrade as we have given in the first problem.

But this 60 metres per second corresponds to a real case because in this case we will consider the efficiency. So the mass flow rate of air is 2 kg per second, we need to determine the pressure the issue across the turbine, we calculate the power output from the turbine assuming no heat transfer, we also consider the static to static efficiency of the turbine to be 80 percent.

So whatever the conditions have been given for the velocity corresponds to the actual case of a static to static efficiency of 80 percent. And then we need to determine also the total to total efficiency of the turbine under these conditions. Again, while solving such a problem, we should do 2 things.

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First we should draw the layout and write down what all are given. So we can say that at the inlet the temperature is given T_1 I am using the notation 2 and 1 as I have done in the graph, that is because writing in and out will be little more cumbersome in the graph, so I am writing in for 2 and out for 1, I will give a justification in the next week's lecture why I am using such a nomenclature because in most of the textbooks you will see that in is either given with 1 or written as in.

Here I am using in with 2 and out with 1, that is because just to remind ourselves that at the inlet the pressure and temperature of a turbine or the flow inside the turbine, that is going inside the turbine is actually higher than compared to the, when compared with the flow that is leaving the turbine.

So these 2 and 1 actually reminds us about these notations. So when we look at the process in a HS curve, which we have already done while discussing, let us revisit this graph again. So 2 is the point here which is my inlet, had it been an isentropic expansion, I would have reached the point 1s, however the static to static efficiency of 80 percent, so we reach a point 1. The corresponding stagnation states are 02, 01s and 01. 02 and 2 are related with this difference of V square by 2 terms like in 01s and 1s and 01 and 1.

So please note, I am reminding you again that 2 corresponds to the inlet, 1 corresponds to the outlet, anything with 0 in the beginning like 01, 02, 01s, etc. corresponds to the stagnation states, s denotes the isentropic expansion process for the inside the turbine. So we have to find out the pressure ratio P2 by P1. By doing it we can invoke the pressure temperature relationship, we can write the pressure ratio R is given by P2 by P1 and it is equal to T2 by T1s whole to the power gamma by gamma -1.

When we substitute the values we will get that the pressure ratio is 1.69. So P1 is the isobar on which both 1s and 1 lies. And since this is an isentropic process, this relationship we can use. Now the 2nd part of the problem says that we have a static to static efficiency of 80 percent, we need to find out what is the actual work output from the turbine or actual power output from the turbine W Prime.

To do that we can invoke the relationship that we have for static to static efficiency, we know that it is h2 - h1, that is the actual enthalpy drop divided by h2 - h1s, the ideal enthalpy drop and when we substitute the values, CP comes out because it is constant like we have done before and then we can find out that the temperature at the outlet that is 1 is 953 Kelvin. And then we can find out W prime as M dot h2 - h1 which is M dot CP T2 - T1 and we get 241.2 kilowatt. Again please note the unit. Whenever you are writing solving problems, you have to give the appropriate unit.

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PART-A Module-02 – Tutorial 1

Problem on Thermodynamics

iii) total-to-total efficiency =?

$$h_{02} = h_2 + \frac{V_2^2}{2}$$

$$T_{02} = T_2 + \frac{V_2^2}{2C_p} = 1073.45K$$

$$h_{01} = h_1 + \frac{V_1^2}{2}$$

$$T_{01} = T_1 + \frac{V_1^2}{2C_p} = 954.79K$$

Is $V_1 = V_{1s}$? **NO! Why?** Densities at 1 and 1s are not same

$$\dot{m} = \rho_1 V_1$$

$$= \rho_{1s} V_{1s}$$

$$\Rightarrow \rho_1 V_1 = \rho_{1s} V_{1s}$$

$$\Rightarrow V_{1s} = V_1 \frac{\rho_1}{\rho_{1s}}$$

$$= V_1 \frac{P_1 / RT_1}{P_1 / RT_{1s}}$$

$$= V_1 \frac{T_{1s}}{T_1}$$

$$= 58.11 \text{ m/s}$$

$$T_{01s} = T_{1s} + \frac{V_{1s}^2}{2C_p} = 924.68K$$

$$\eta_{t-t} = \frac{h_{02} - h_{01}}{h_{02} - h_{01s}}$$

$$= 79.76\%$$

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And next we come to the 3rd part of the problem which is total to total efficiency. Let us go back and look at this graph again, so we are talking about total to total efficiency, means we

have to deal with T_2 , T_{2s} and T_1 , none of these are given in the problem. We have to then first find out these states T_2 , T_1 and T_{2s} , let us see how we do it. So we know that the stagnation enthalpy h_{02} is nothing but $h_2 + \frac{V_2^2}{2}$ and then we can write the T_{02} is $T_2 + \frac{V_2^2}{2 c_p}$.

And in this case of course we note that the change because the kinetic energy being very small from 1073 the value of T_2 to T_{02} is only 1073.45 Kelvin. In the same way we can find out h_{01} equal to $h_1 + \frac{V_1^2}{2}$ and then we can find out the T_{01} is $T_1 + \frac{V_1^2}{2 c_p}$ which gives me 954.79 Kelvin. How do we find out the T_{01s} or h_{01s} ? Then we have to ask ourselves that is V_1 equal to V_{1s} ? I go back to the curve and you can see that $\frac{V_1^2}{2}$ is this distance and $\frac{V_{1s}^2}{2}$ is this difference.

So from the graph what does appear, does it appear same, if not, why? So let us look at the reason. Is V_1 equal to V_{1s} , we ask ourselves this question and we see that it is no. Why? This is because the densities at 1 and $1s$ are not same, why it is not same? See the it is on the same isobar, so pressures are same but its temperature same? We just found out that the temperature at 1 that is T_1 is not the same as T_{1s} and hence the densities will not be same. So let us look at it. So we are talking about the mass flow rate \dot{M} equal to $\rho_1 A_1 V_1$ which even in the ideal case should have been $\rho_{1s} A_{1s} V_{1s}$.

Now what is A_1 , letter A_1 is the area at the exit and V_1 is the corresponding velocity at the same location. Now area of the turbine does not change depending on the process. Can we say that the area of the turbine will change just because the process is not isentropic? No. So what does it mean, it means that A_1 and A_{1s} are same, so we get from this relationship that $\rho_1 V_1$ equal to $\rho_{1s} V_{1s}$ and since we are interested in finding out V_{1s} , we can write the V_{1s} is V_1 times $\frac{\rho_1}{\rho_{1s}}$.

And now we can write density in terms of pressure and temperature and then we can write that V_{1s} is essentially V_1 multiplied by $\frac{P_1}{P_{1s}}$ by $\frac{T_1}{T_{1s}}$ the whole divided by $\frac{P_1}{P_{1s}}$ by $\frac{T_1}{T_{1s}}$ and that leads to that V_{1s} is V_1 multiplied by $\frac{T_{1s}}{T_1}$ and we get V_{1s} is slightly less than V_1 . V_1 you remember was 60 metres per second, V_{1s} 58.11 metres per second particularly for this problem. And then we can find out T_{01s} in the same way as we have done earlier for T_{02} and T_{01} and we can get that T_{01s} is equal to $T_{1s} + \frac{V_{1s}^2}{2 c_p}$, which gives me 924.68 Kelvin.

So we got T01s at 924.68 Kelvin and then we can substitute in the relationship for Eta tt which is h02 - h01 which is the drop in the stagnation enthalpy between the inlet and the outlet condition at the exit, hold divided by h02 - h01s corresponding to the ideal stagnation enthalpy drop and that gives me 79.76 percent. So you learned that with the help of of course the example of the turbine how to apply these definitions of the static to static and total to total efficiencies, you can also explain these ideas to find out the other terms like pressure if it is required.

Because we already know pressure ratios, if in the problem any one of the pressures are given we can find out all other pressures. That type of problem I will set it in the tutorial and I hope you will all able to solve it. So this brings us to the last problem that we are going to discuss for today and that is on dimensional analysis. The problem that I have chosen is based on the Turbo machine applications but in the tutorial that I have given corresponding to this week 5 we will also talk about the dimensional analysis problems related with fluid flow. Let us look at this problem.

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FLUID DYNAMICS AND TURBOMACHINES
PART-A Module-02 – Tutorial 1

Problem on Dimensional Analysis

4) Prototype of a hydraulic turbine having 6 m diameter produces 55 MW when running at a rotational speed of 94.7 rpm under a net head of 25 m. If a model of diameter 300 mm is made which also operates under a head of 25 m. Determine:

- i) The rotational speed of the model turbine
- ii) The power produced by this model turbine

$$\frac{gH_m}{N_m^2 D_m^2} = \frac{gH_p}{N_p^2 D_p^2}$$

$$N_m = N_p \frac{D_p}{D_m}$$

$$= 1894 \text{ rpm}$$

$$\frac{P_m}{\rho_m N_m^3 D_m^5} = \frac{P_p}{\rho_p N_p^3 D_p^5}$$

$$P_m = P_p \left(\frac{D_m}{D_p} \right)^5 \left(\frac{N_m}{N_p} \right)^3$$

$$= 137.5 \text{ kW}$$

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It is said that we want to make up prototype of a hydraulic turbine which has 6 metres diameter and which will produce 55 megawatt of power when it is running at a rotational speed of 94.7 rpm under a net head of 25 metres. So this is a turbine we want to make and as I have said that one of the motivations for doing nondimensional analysis is to make a model, test it and then fabricate the prototype. So we want to make a laboratory scale model, in this case whose diameter is 300 millimetre which will also operate under a head of 25 metres. We

need to determine the rotational speed of the model turbine and the power produced by this model turbine.

So we can use the relationship connecting the specific work or gh if you remember I have talked about that in case of hydraulic machines like hydraulic turbines or in pumps we often replace the specific work with the product of g and h and so we can write the gh for the model is divided by $N^2 D^5$ is equal to gh for the prototype divided by $N^2 D^5$. That comes from the relationship that $gh / N^2 D^5$ is constant. So then we get that N is nothing but N_p multiplied by D_p / D_m and we get that N is 1894 rpm, that is 20 times.

And then I should have also mentioned that H_m is equal to H_p is given in the problem. Then we come to the power produced by the model turbine. We know that $P_m / \rho M^3 D^5$ is constant so we can write $P_m / \rho M^3 D^5$ is equal to $P_p / \rho P^3 D_p^5$ which gives me P_m equal to P_p multiplied by D_m^5 / D_p^5 multiplied by N^3 / N_p^3 . If you substitute the values you should get 137.5 kilowatt.

This brings us to an end of dimensional analysis, in the next week, that is week 6 we will start with the description of Turbo machines in the sense that we will talk how to represent the Turbo machines in a graphically, we will talk about the velocities that are present inside the Turbo machines and we will take you Euler's equation and we will talk about the losses. So with this idea include this discussion on the tutorials corresponding to week 5. Thank you.