

Fluid Dynamics And Turbo Machines.
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Part C.
Module-2.
Tutorial 3.
Week 7.

Good afternoon, I welcome you all for this week 7's discussion on fluid dynamics and Turbo machines. So far in this week 7 we have studied pumps, we have talked about different types of hydraulic turbines and in the last lecture we have talked about cavitation in Hydro Turbo machines, that is cavitation in pumps and turbines. Today what we will do is we will take up some example problems that you can come across in these topics. We will do step-by-step and we will give you also the tutorials which you can solve to get a feel for different aspects of the theory covered in this week.

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1) A pump impeller with an inner radius of 50 mm and an outer radius of 200 mm rotates at 900 rpm. The inlet and outlet blade angles measured from the tangential direction are 20° and 10° respectively. The blade height is uniform and is 50 mm. Assume no pre-whirl (inlet whirl) and neglect slip. Hydraulic efficiency of the pump is 80%. Calculate:

- Volume flow rate
- Stagnation pressure rise across the pump
- Input power to the pump

$\beta_1 = 20^\circ$ $\beta_2 = 10^\circ$
 $N = 900$ rpm
 $U_1 = 4.71$ m/s
 $C_{1m} = U_1 \tan \beta_1$
 $= 1.72$ m/s

$C_1 = C_{1m}$

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So let us look at the first problem. The first problem states that we have a pump impeller whose inner radius is 50 MM, the outer radius is 200 MM and it rotates at 900 rpm. Please note we have not said which one is the inlet, which one is the exit, we have simply said inner radius and outer radius. But you should not get confused because you know in case of a pump the flow takes place from the inner radius to the outer radius and hence the inner radius in case of a pump corresponds to the pump inlet and the outer radius corresponds to the pump outlet.

The inlet and outlet blade angles measured from the tangential direction are 20 degree and 10 degrees respectively. The blade height is uniform and is 50 millimetres, assume that there is no pre-whirl or inlet whirl as we call it and neglect slip. The hydraulic efficiency of the pump is given as 80 percent, we need to calculate the volume flow rate, stagnation pressure rise across the pump and input power to the pump. As I have told you while discussing the theory, the most important part in solving these problems is drawing a velocity triangle.

The velocity triangles vectors need not be absolutely to the scale but it should give you a feel for the numbers. So let us look at the situations we have. We have the inlet whirl to be zero, hence C_1 equal to C_{1M} and C_{1U} is zero. Beta 11 is 20 degrees and beta 2 is 10 degrees, though it does not look really so nicely here but this as I told you is a representative and we have a C_{2U} value. And beta 1 and beta 2 values are given, we are neglecting slip, that means we will talk about W_{BL} infinity as equal to W_{BL} . We have given the N as rpm as 900.

So we have talked about U_1 as 41.7 metres per seconds and we get C_{1M} from the inlet velocity triangles $C_1 M$ by U_1 is nothing but $\tan \beta_1$ and we get $C_1 M$ to be 1.72 metres per seconds.

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$$\dot{V} = \pi D_1 b_1 C_{1m}$$

$$= 0.027 \text{ m}^3/\text{s}$$

$$\pi D_1 b_1 C_{1m} = \pi D_2 b_2 C_{2m} \quad C_{2m} = \frac{D_1}{D_2} C_{1m} = 0.43 \text{ m/s}$$

$$U_2 = 18.85 \text{ m/s}$$

$$C_{2u} = U_2 - C_{2m} \cot \beta_2 = 16.41 \text{ m/s}$$

$$W_{bl\infty} = (U_2 C_{2u} - U_1 C_{1u}) \quad W_{bl\infty} = 309.35 \text{ m}^2/\text{s}^2$$

$$s = \frac{W_{bl}}{W_{bl\infty}} = 1 \Rightarrow W_{bl} = W_{bl\infty}$$

$$\eta_h = \frac{W}{W_{bl}} \Rightarrow W = 247.48 \text{ m}^2/\text{s}^2$$

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We continue with the velocity triangles in the discussion so we can find out \dot{V} which is $\pi D_1 B_1$ multiplied by C_{1M} , please note that in this problem we do not have any information about the area blockage and hence we can neglect it. So we can write that letter \dot{V} equal to $\pi D_1 B_1 C_1 M$.

If you remember we had an expression with ϕ to talk about the blockage factor, but in this case blockages are not being formed, so we take ϕ equal to 1. And we get \dot{V} equal to 0.027 metre cube per second. This is the first part of the problem, we continue with it and since $\pi D_1 B_1 C_1 M$ equal to $\pi D_2 B_2 C_2 M$. Why do we get it, it is because of continuity. Since pump is handling water and we are talking about an incompressible flow, so the volume flow rate through the inlet and the outlet of the impellers are same and hence we can equate the volume flow rate at the inlet which is $\pi D_1 B_1 C_1 M$ with the volume flow rate at the outlet which is $\pi D_2 B_2 C_2 M$.

And hence we can write that B_1 equal to B_2 which is given or $C_2 M$ can be obtained because we know the $C_1 M$ and D_1 and D_2 . So we find that $C_2 M$ is 0.43 metres per second and we also know U_2 is 18.85 metres per second, so $C_2 U$ is equal to, this portion $C_2 U$ is equal to $U_2 - C_2 M \cot \beta_2$ which is 16.41 metres per second. And we get the $W_{BL \infty}$ is $U_2 C_2 U - U_1 C_1 U$. But $C_1 U$ has been given to be zero, so we get the $W_{BL \infty}$ is $U_2 C_2 U$, we know U_2 , we know $C_2 U$, so we can find out $W_{BL \infty}$ as 309.35 metres per second square.

And since slip is neglected or slip is 1, so W_{BL} is equal to $W_{BL \infty}$ and hydraulic efficiency expression in case of pump, please remember that in case of pump, the energy is added by the blade on the fluid. So the fluid effective energy, specific energy work as we find out from the difference of the useful energies of the fluid across the pump is W . So $\eta_{\text{hydraulic}}$ should be W by W_{BL} and hence W is equal to W_{BL} multiplied by $\eta_{\text{hydraulic}}$ and we get value for W as 247.48 metre square per second square.

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$$H = \frac{(P_2 - P_1)}{\rho g} + \frac{(C_2^2 - C_1^2)}{2g} + (z_2 - z_1)$$

$\Delta P_0 = \rho g H = \rho W = 247.48 \text{ kPa}$

Assuming no other losses (like, disc friction, etc.)

$$P_c = P_{bl} = \rho \dot{V} W_{bl} \quad P_c = 8.35 \text{ kW}$$

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The question for the 2nd part if you remember, it will show you once again, the 2nd part question that we were asking is stagnation pressure rise across the pump. When we are trying to find out the stagnation pressure rise across the pump, we need to find out the specific work. Why is it so, let us look into the expression for the specific work. The specific work or the head developed by the pump, if you multiply this head developed by the pump by multiplied by g, you get specific work.

So whether we express in terms of specific work or head developed by the pump, we can write that it is equal to P2 - P1, let us say the head developed by the pump we are talking about, so P2 - P1 by rho g + C2 square - C1 square by 2g + Z2 - Z1. Since no information is given about the elevation, we will neglect the elevation change for the time being. So what are we landing up, we say that H is nothing but equal to P2 by rho g + C2 square by 2g - P1 buy rho g - C1 square by 2g. So if you rearrange these terms, what you get is nothing but the stagnation pressure change.

You take these terms P2 with C2 and P1, let us put 2 ticks with C1 square by 2g, then what happens, if you take these 2 terms together, what you get is the total pressure rise across the impeller. So we can say the Delta P0 is equal to, we can take the rho g to the other side multiplied by H or rho W and we can find out the total pressure rise or the stagnation pressure rise in the impeller. Assuming no other losses because no information is given like disc friction loss, return flow loss, etc. we can say that the coupling power is nothing but the power which is in the blade and we, it is nothing but rho V dot which is the mass flow rate multiplied by the blade specific power.

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2) Cavitation starts in a pump, developing a head of 36 m at a flow rate of 0.05 m³/s, when the total (sum of pressure and velocity) head at pump inlet is reduced to 3.5 m. Atmospheric pressure is 750 mm Hg and vapour pressure of water is 1800 Pa.

The same pump will now operate at a place where atmospheric pressure is 620 mm Hg and vapour pressure is 830 Pa. What is the $NPSH_a$ when the pump develops the same head and flowrate? Is it necessary to reduce the height of the pump from the sump and by how much? Specific gravity of Hg is 13.56.

$$\frac{P_1}{\rho g} + \frac{C_1^2}{2g} = 3.5 \text{ m} \quad NPSH_a = \frac{P_1}{\rho g} + \frac{C_1^2}{2g} - \frac{P_v}{\rho g} = 3.32 \text{ m}$$

Critical condition $NPSH_r = 3.32 \text{ m}$

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And we get that coupling power is 8.35 kilowatts. So what we have done in this problem is we have tried to find out with the help of the values given using the velocity triangles and using the concept of hydraulic efficiency. The slip can also be accommodated if the slip was given, then we know that W BL will be related with W BL infinity by the slip term. In this problem of course you can think that slip value, slip factor has been given as 1.

The 2nd problem that we talk about is about cavitation in a pump. Cavitation starts in a pump developing a head of 36 metres at a flow rate of 0.05 metre cube per seconds, when the total, I explain it here as a sum of pressure and velocity head is reduced to 3.5 metres. That is we have a pump which produces 36 metres of head at a flow rate of 0.05 metre cube per second when the total head at the pump inlet is reduced to 3.5 metres. It is given that the atmospheric pressure is 750 millimetres of mercury and the vapour pressure is 1800 Pascals.

Now what is given as we have the same pump which will now operate at a different place where atmospheric pressure is 620 millimetre of mercury and the vapour pressure is 830 pascals. This is very interesting. See when we have talked about cavitation in pumps, I told you that as a user you know where the pump will be used what will be the atmospheric pressure, what will be the vapour pressure. So here we have a problem where a pump which was earlier used with an atmospheric pressure of 750 millimetres of mercury and vapour pressure of 1800 pascals is now used in a location where atmospheric pressure 620 millimetre of mercury and the vapour pressure is 830 pascals.

So if this pump is developing the same head and flow rate, we have to find out what is the NPSH available under the change conditions and then should we need to reduce the height of the pump from the sump and by how much. Of course the specific gravity of mercury you all know but it is given here as 13.56. So when we solve this problem, we have to keep one thing in mind that is if the pump is used, if a given pump is used, its NPSHR or NPSH required is fixed by the manufacturer. It does not depend on which location, what piping, etc. a user uses.

So from the first part of the problem, from the first paragraph of the problem, what we will try to find out is the NPSH required condition. How can we do that, we say that the cavitation starts, so that means that is a critical condition, so at that critical condition whatever is the NPSH available is the NPSH required because we know at critical condition NPSHA equal to NPSHR. So let us first find out how to get the value for NPSHA.

So we know that it is given that at the pump inlet, on the in the pipe, inside the pipe at the pump inlet, the total pressure is 3.5 metres of head. So we note that $P_1 + \rho g + C_1^2 / 2g$ is 3.5 metres. And we can write that NPSH available, we have already derived it in the class on cavitation in Hydro turbo machines that $P_1 + \rho g + C_1^2 / 2g - P_v / \rho g$ is 3.32 metres. This calculation you can do it and satisfy that we are getting the right values. So this is a critical condition because cavitation just starts.

Please note this term just starts, cavitation starts, so this is very important. So this start is related with this 3.5 metres that we are talking about and hence the value that we obtained here 3.32 metres also corresponds to the critical condition. And we can say since it is critical condition, NPSH required is equal to NPSH available is equal to 3.32 metres. And this will not change even if you take the pump or some other place, this will remain constant. So what we now need to find out is under the changed condition, what happens to NPSH available, is it more than the required or less than the required?

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Applying Bernoulli's equation between the sump and pump inlet for the first case we get,

$$\frac{P_0}{\rho g} = \frac{P_1}{\rho g} + \frac{C_1^2}{2g} + H_s + H_{loss}$$

$$H_s + H_{loss} = 6.67 \text{ m}$$

For the second case the pump develops the same head and the same flow rate, at the new location.

Thus, the suction pipe loss is going to remain unchanged. Further retaining old height of the pump above the sump, we get

$$\frac{P_1}{\rho g} + \frac{C_1^2}{2g} = \frac{P_0}{\rho g} - (H_s + H_{loss}) = 1.74 \text{ m}$$

$$NPSH_a = \frac{P_1}{\rho g} + \frac{C_1^2}{2g} - \frac{P_v}{\rho g}$$

$$= \frac{P_0}{\rho g} - (H_s + H_{loss}) - \frac{P_v}{\rho g} = 1.66 \text{ m}$$

Now applying Bernoulli's equation between the sump and the pump inlet for the first case, that is on the first paragraph of the problem. So applying Bernoulli's equation between the sump and the pump inlet for the first case we get that P_0 by ρg is equal to P_1 by $\rho g + C_1^2$ by $2g + H_s + H_{loss}$. Or we get that $H_s + H_{loss}$ is 6.67 metres. Now let us look at the 2nd problem very carefully. The problem states that the pump in the new condition, new ambient condition delivers the same head at the same flow rate.

Now when you have the same flow rate, what happens to H_s loss? H_s loss we know is going to be proportional to V dot square. If V dot remains constant, then H_s loss will also remain constant. So we can say that if it is developing the same head and the same flow rate at the new location, we can say that the suction pipe loss is going to remain unchanged. Which means H_s loss is going to remain unchanged. So let us first say that we keep the same old height of the pump of the sump, that is whatever the height we had in the first case H_s , we retain the same H_s above the sump and let us try to find out whether $NPSH$ available is more or less than the $NPSH$ required in this case.

If we find that $NPSH$ available is less than the $NPSH$ required in this case, then we have to think about lowering the pump. Otherwise we can be happy with what we have got. So let us find out this. So then we can write that P_1 by $\rho g + C_1^2$ by $2g$ is equal to P_0 by $\rho g - H_s + H_{loss}$. So you see basically we are using the same expression, only difference comes is that P_0 by ρg has now changed. P_0 by ρg has changed because the atmospheric

condition values are given to be different. And we get a value of P_1 by $\rho g + C_1^2$ by $2g$ as 1.74 metres.

So this when we put it with the NPSH available, we find that this becomes 1.66 metres. Please recollect that NPSH required we have obtained from the earlier part of the problem is more than the NPSH available now. And you also know when NPSH available falls below the NPSH required, you have cavitation. So hence we cannot run the pump at the same elevation, we need to reduce the pump height above the sump level. So the answer will be yes we need to reduce the pump height above the sump level because NPSH available in the new setup is only 1.66 metres.

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Thus, the new $NPSH_a < NPSH_r$. Hence the pump will cavitate.

So the pump should be lowered by $NPSH_r - NPSH_a = 1.66$ m.

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And then we can find out that we have to, find out that the pump should be lowered by NPSH required which we have obtained in the first part of the problem - NPSH available in the 2nd part of the problem which is equal to 1.66 metres. So the final answer is that the pump in the 2nd case has to be lowered by height of 1.66 metres in order to just avoid cavitation or in order to pump to, pump to just get into cavitation. So I hope you get an idea about how to use NPSH with the pump data given and use it to determine whether pump will cavitate or not.

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3) The gross head for a Pelton turbine is 600 m and the approach losses in the penstock is 48 m. Determine the net head.

The buckets deflect the jet through an angle of 170° , while the relative velocity is reduced by 15% due to friction. Calculate the efficiency of the turbine assuming bucket-jet speed ratio is 0.47.

The diameter of the wheel is 900 mm and there are two jets in this turbine. The nozzle velocity coefficient is 0.98. Find the speed of rotation of the wheel and the diameter of the nozzles if the power output from the turbine is 1250 kW.

The net head for a Pelton turbine is gross head – losses.
So, net head (H) = (600-48) m = **552 m**

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More along these problems will be given in the tutorial. So we now talk about the 3rd problem in this tutorial on the Hydro turbo machines and this is on a Pelton turbine. The gross head for a Pelton turbine is 600 metres and the approach losses in the penstock is 48 metres. As we have discussed in the theory, the net head that is available to the turbine is nothing but the gross head - the approach losses and hence in this case we can say that the net head for the Pelton turbine is going to be 552 metres. Okay, the 2nd part of the problem says that the buckets deflect the jet through an angle of 170 degree, while the relative velocity is reduced by 15 percent due to friction.

We need to calculate the efficiency of the turbine assuming the bucket jet speed of 0.47. It is also given that the diameter of the wheel is 900 millimetres and there are 2 jets in this turbine. The nozzle velocity coefficient given for this problem is 0.98, we also need to find out the speed of rotation of the wheel and the diameter of the nozzles if the power output from the turbine is 1250 kilowatts. We look into now the 2nd part of the problem which talks about the deflection of the jet due to the bucket in the next slide.

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No information is given about splitter angle (β_s). Assuming, $\beta_s = 0 \rightarrow \beta_2 = 180^\circ$

$$\beta_1 = (180 - \delta) = 10^\circ \quad C_j = k_N \sqrt{2gH} = 0.98 \sqrt{2 \times 9.81 \times 552} = 101.99 \text{ m/s}$$

$$C_2 = C_j = 101.99 \text{ m/s}$$

$$\frac{U}{C_j} = 0.47 \Rightarrow U = 42.84 \text{ m/s} \quad \rightarrow W_2 = C_j - U = 59.15 \text{ m/s} \quad \rightarrow W_1 = 0.85 * W_2 = 50.28 \text{ m/s}$$

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The direction of the flow and had there been no bucket, the flow would have continued, in the presence of the bucket there is a deflection of angle Delta. This angle Delta in the present problem is 170 degrees. And hence beta 1 which is the direction of the velocity, relative velocity that makes with the tangential direction in the outlet of the turbine is equal to 180 degree mind is the angle of reflection. And we have to consider the splitter blade. In this case the splitter blade is not mentioned, splitter angle is not mentioned and we talk about the beta S to be 0 and we get that this triangle degenerates into a straight line.

And we get that U W-2 and C2, all are along one line, why, because this beta S, the splitter angle has become zero. That is our assumption and we take that beta 2 equal to 180 degree. So beta 1 is by the problem given is 180 degrees -170 degrees or 10 degrees and beta2 is 180 degree, so we can find out the jet velocity using the nozzle velocity coefficient which is equal to the 101.99 metres per second. You remember that we have talked about that this nozzle velocity coefficient KN deals with how much of the energy that is available in the potential energy is converted into kinetic energy.

So this value is 0.98 as it is very close to 1, as it ideally should be, that we have discussed the theory and we use the head as 552 metres as we have obtained as the net head. So we get that C2 is nothing but CJ is 101.99 metres per second. We also know that U by CJ is 0.47, this is given in the problem and hence we can find out U to be 42.84 metres per second. Let us look at this relationship. This entire line which is red is C2, this blue portion is U, so what is remaining is W-2 and hence W2 is nothing but C2 or CJ - U which is 59.15 metres per

second. So W_1 is equal to 0.85 times W_2 because we have said that, because of friction there is 15 percent reduction in the relative velocity which is given as 50.28 metres per second.

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$$\begin{aligned}
 W_{bl} &= U W_2 (-\cos \beta_2 + K \cos \beta_1) \\
 &= U (C_2 - U)(1 + K \cos \beta_1) \quad W_1 = K W_2, K=0.85 \\
 &= 4655.15 \text{ m}^2 \text{ s}^{-2}
 \end{aligned}$$

Efficiency: $\eta = \frac{\rho V W_{bl}}{\rho V g H} = 0.86$

Rotational speed: $U = \frac{\pi N D}{60} \Rightarrow N = \frac{60 U}{\pi D} = 1017 \text{ rpm}$

$P = \dot{m} W_{bl} \Rightarrow \dot{m} = 268.52 \text{ kg/s}$ $\dot{m} = 2 \times \rho \frac{\pi}{4} d_j^2 C_j$

$$d_j = \sqrt{\frac{2 \dot{m}}{\rho \pi C_j}} = 41 \text{ mm}$$

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And then we can write W_{BL} which is we have already derived in the theory class that is equal to U times $W_2 - \cos \beta_2 + K \cos \beta_1$ which will give me U times $C_2 - U$ times $1 + K \cos \beta_1$, remind you that K is 0.85 and we get the W_{BL} is 4655.15 metre square per seconds square when we substitute the values of C_2 , K and β_1 .

Now efficiency is nothing but, since other efficiency, the losses are not mentioned, we can write it as $\rho V \dot{W}_{BL}$ by $\rho V \dot{g}H$ which is 0.86 or 86 percent. And the rotational speed can be found out by the relationship U equal to πND by 60, U already were found out from the relationship of the ratio of the bucket speed with the jet speed and hence we can find out N to be 1017 rpm. And the power out we have found out, we know is 1200 kilowatts. So we can find out \dot{M} by dividing by W_{BL} .

W_{BL} already we have found out, so we get \dot{M} as 268.52 KG per second. And now the problem says that there are 2 jets. If there are 2 jets, then this mass flow rate 268.52 or \dot{M} dot will be divided into 2 jets, to nozzles, so each nozzle will handle a velocity C_j and the diameter of the nozzle will be d_j . We need to find out the diameter of the jet which is d_j . So we can write that \dot{M} dot is equal to 2 times the mass flow rate through each of the jets, each of the nozzle.

So we can say $\rho \pi 4 D_J^2 C_J$. And hence we can get the D_J , we can write the D_J is nothing but $2M \dot{\rho} \pi C_J$ and which is equal to 41 millimetre. And you can also check what is the ratio of capital D by D_J which is called the ratio of the wheel diameter to the jet diameter and as we have discussed in the theory, it should be between 11 and 14 but need not be in all the cases. So ideal cases it should be between 11 and 14 for a well-designed turbine. Other problem that we have taken here need not give you that range.

So we have now talked about in this Pelton turbine problem how to assimilate the data, construct the velocity triangles and then apply the different coefficients for example nozzle velocity coefficient, what does it mean, what it connects with, that is the connection between the jet velocity with the head, net head available, we have connected with the, called the nozzle velocity, velocity coefficient, we can talk about the ratio of the bucket speed to the jet speed, we have found out the rotational speed using the relationship $\pi N D$ by 60. And since in this problem we had 2 jets, we have found out the jet diameter by considering that there are 2 nozzles and identical nozzles.

So with this I come to a conclusion for the tutorials on the hydro Turbo machines, namely pumps, turbine and cavitation in hydro Turbo machines. In the next week we will talk about a little bit of introduction of compressible flows because we will talk about steam and gas turbines. And as we have already talked about in the introduction to Turbo machines that we for steam and gas turbines, there will be a large change in the density and the flow is going to be compressible. So in the next week we will start with a discussion on compressible flow, a one-dimensional compressible flow and then from there we will take up steam and gas turbine. Thank you.