Fluid Dynamics And Turbo Machines.
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Part D.
Module-2.
Lecture-13.
Steam and Gas Turbines: H-S Plots and Velocity Triangle.

Good afternoon, I welcome you to this discussion on steam and gas turbine. This is essentially the continuation of what we have discussed in the last class. In the last class as you can recollect that we have doubt about the construction of the steam and gas turbines, in particular the steam turbines and now we will talk about their performance. So to understand this we will take resort to the velocity triangles that we have already discussed in the earlier part of this module and then we will apply it for the case of steam turbines, of course you have to keep in mind what I talked about in the last class that we are talking about the steam as a superheated steam and not really talking about the wet phase.

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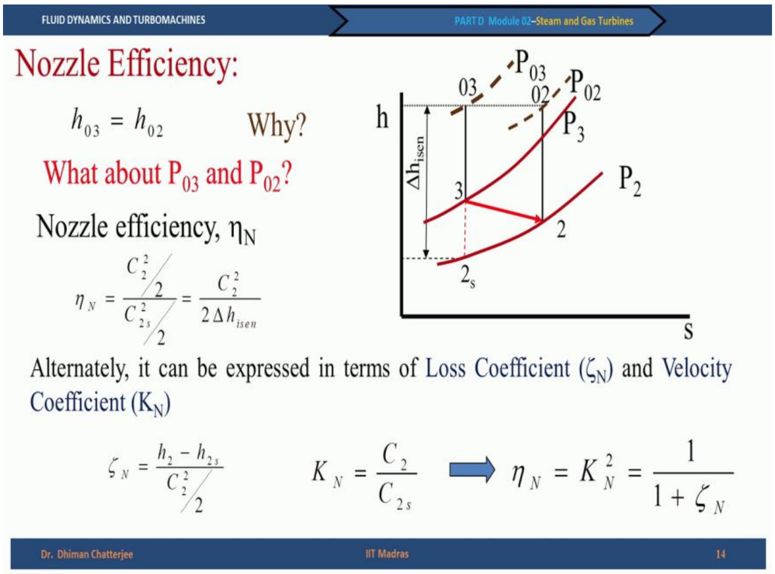

So we will talk about the HS plots and connect the 2 and most importantly we will talk about the degree of reaction or the reaction ratio. Now we will define few terminologies which will help us to also understand these turbines and solve the problem that we have given in the tutorial. The 1st is the nozzle efficiency. We have already talked about that the nozzle has a higher pressure, higher enthalpy at the inlet and after the expansion, the enthalpy and the pressure should drop.

And if we collect our terminology, we have said higher the value of pressure enthalpy, we will give a higher number. So in this case if you remember the simple impulse turbine for example, we have first the nozzle and then the rotor. So if I start from the rotor exit as 1, then rotor inlet becomes 2 which is also nothing but the nozzle exit. So the nozzle inlet should have a number 3. So please note that these numbers 3, 2 may differ in different books but we are following the unified notation that we have discussed towards the beginning of these lectures on Turbo machines.

So 3 to 2 is an expansion process and 3 to 2S is an idealised process where 2S refers to the isentropic expansion. 03 corresponds to the total pressure, P03 corresponds to the total pressure and 03 corresponds to the stagnation state. And 02 corresponds to the stagnation state at the exit of the nozzle. The corresponding pressure, please note is P0 2. In the last lecture on compressible flows, this aspect we have already discussed. We have shown that H 03 should be equal to H02, why is that? You recollect that nozzle does no work and there is no heat transfer, so from the 1^{st} law of thermodynamics, neglecting potential energy changes, we can say that H 03 is equal to H02.

But what about pressure? That also we have discussed in the last lecture that P0 3 is not equal to P0 2 and there is a pressure drop to overcome the friction. So this is depicted here. And that brings to us the concept of nozzle efficiency and we can say that nozzle efficiency Eta N is given as C2 square by 2 divided by C2 S square by 2. What we are essentially saying is this velocity, that is which takes from here to here as a actual velocity that is obtained that the exit of the nozzle to the ideal velocity that you can get from the exit of the nozzle.

And of course we know that ideal velocity is obtained if you say that from the isentropic expansion and Delta H isentropic or Delta H isen refers to the total drop in enthalpy from the State 03 to 2S. So Eta N then can be represented as C2 square by C2 S square or C2 square by 2 Delta H isen. Alternately it can also be expressed in terms of loss coefficient zeta N or velocity coefficient, the nozzle velocity coefficient for example we have discussed also in case of Pelton turbine. So we can say zeta N is nothing but H2 minus H2S, the loss in enthalpy divided by C2 square, the what fraction of it is lost. And then we can also define the velocity coefficient as C2 by C2 S.

If you manipulate with these terms and I would ask you to do it yourself, to show that that eta N is equal to K N square, of course this is obvious, is equal to 1, whole divided by 1+ zeta N.

So Eta N equal to K N square equal to 1 divided 1 plus zeta N. And this is, this is a way of expressing the nozzle efficiency.

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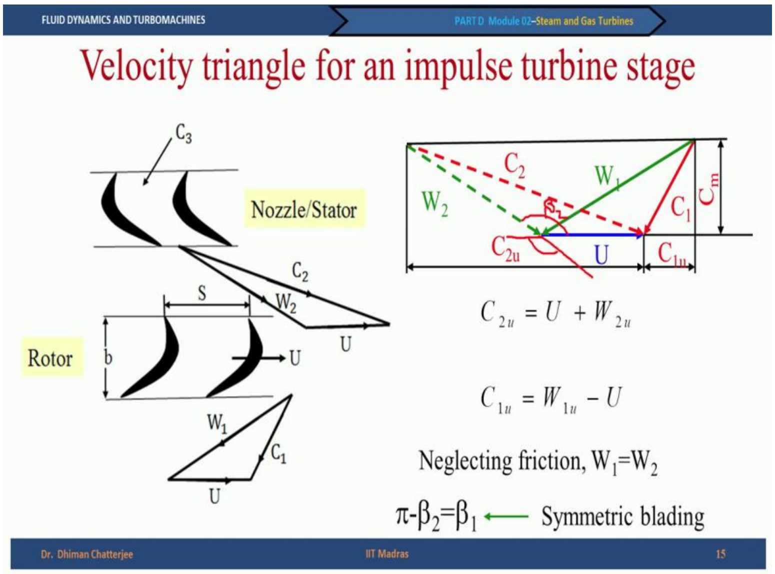

So let us look at the velocity triangles for an impulse turbine stage. So this is an impulse turbine stage and we are talking about the velocity triangle. So the velocity that enters here is C3, which is at the inlet of the nozzle, leaves with an absolute velocity C2, you see that C3 has increased by several orders, which is expected because nozzle is supposed to increase it. Then you know the pressure and the suction surfaces, we can get the direction of rotation as shown here, we can construct the velocity triangle at the inlet of the rotor and similarly at the exit of the rotor.

And you can also once again verify that had there been no blade, rotor blade, the velocity would have continued in the direction of W2 but now it is forced to go in W1 and hence the deflection angle is large. So let us draw the velocity triangles together. When we talk about axial flow turbines, many times what we do, we take advantage of the fact that this blade velocity, Blade peripheral velocity U is same as the rotor inlet and outlet. And we can draw it in terms of common base.

So this U serves as a common base and the direct method of comparing the inlet and outlet velocity triangles. We can mark C2U and C1U and please note that we have earlier written when we talked about Euler's energy equation or Euler's turbine equation that it is C2U minus C1U but that is because they were on the same side. Now if you look at this graphical representation, you can see that C2U is on one side and C1 U is on the other side. So what we are essentially trying to do is find out the distance between this tip and the other one.

So we are essentially trying to talk about the summation of the total distance C2U and C1U. So in case they are on the same side, then what will happen is it is going to be C2U minus C1U, in this case it will be C2U minus of minus C1U, because C1U is on the opposite side. This has to be borne in mind when we follow this discussion today. So let us look at it. We are talking about C2U as U plus W2U and C1U is W1U minus U and hence neglecting friction where we can consider the W1 equal to W2, the same as we have done in case of Pelton turbine, we can find out that pie minus beta 2 is beta 1, I will come back to this point again.

And this is known as symmetric blading. Please note the relationship that pie minus beta 2 is beta 1. This is valid because of the nomenclature that we have used, the sign convention of beta we have used. You can get a similar symmetric blading condition in other way if you use some other way of denoting the blade angles. But for the way we have denoted the blade angles, that is blade angle in our case is denoted by the angle between the positive direction of W for example let us say I am talking about positive direction of W and the negative direction of U which is this angle.

So this angle is my beta which means the vertically opposite angle, this also should be my beta. So this should be my beta 2. So please note that this is the sign convention we have used and hence because of the sign convention, the symmetric blade condition gives me pie minus beta 2 equal to beta 1. If you had followed, I repeat, if you had followed any other convention of the blade angles, then this relationship will be changed but still you should get the symmetric blading because that is important because velocity triangles here we are talking about is the same, the angle representation is different.

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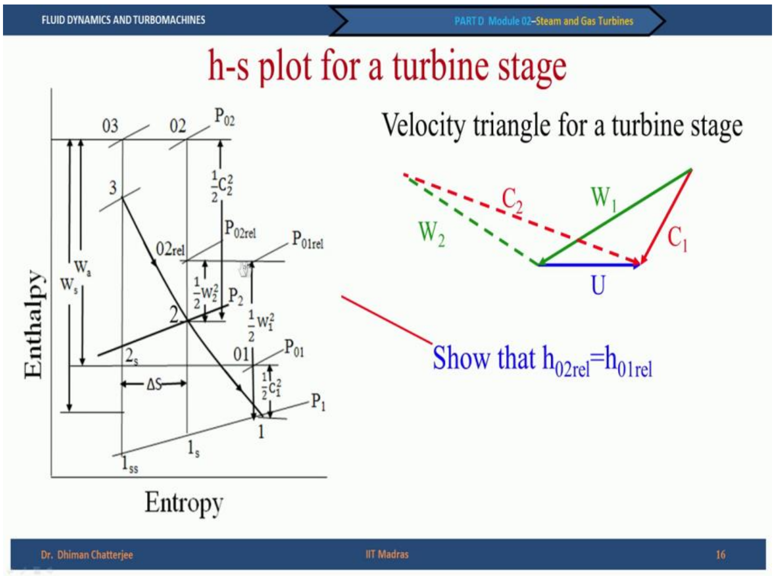

So we can look at the HS plot for a turbine stage, so 3 to 2 is an expansion in the nozzle, 2 to 1 is the expansion in the rotor. And we are talking about 3 to 2S as the isentropic expansion in the nozzle and 2 to 1S is the isentropic expansion in the rotor and 1 SS talks about the isentropic expansion from 3 at the pressure P3 to the pressure P1. We can also write that this is P02 and this line is corresponds to P0 3, so H 03 should be equal to H0 2 as we have discussed and then this distance is the velocity rise which is half C2 square.

We can also show that this pressure is called P0 2 relative. Please note the distinction between P02 and P0 2 relative. When I say P0 2, I mean P2 plus half, basically it is connected with the 2 square, we cannot write with half rho C2 square, I am sorry, we cannot write it but we have to connect it with C2 square. For example, I can write the T0 2 is equal to T2 plus C2 square by 2 CP and we can connect the pressure also using the relationship we have derived earlier in terms of Mach number.

Now P02 relative is connected with the velocity of W2 instead of W 2. Similarly P0 1 relative is connected with the velocity W1, relative velocity W1 instead of C1. Whereas P01 is with the, connected with the absolute velocity C1. So let us look at it again. P1 and P01 is connected with the absolute velocity at the exit of the rotor C1. P01 relative is the stagnation pressure based on the relative velocity condition and that is given, related with W1 square. Similarly with P02 and P02 relative.

So now velocity triangle for a turbine stage, if I look at the velocity triangle for the turbine stage which we have already drawn in the last time, we can get that H02 relative equal to H01 relative. I am not showing in today's lecture, I will give a separate handout but I suggest that you 1st derive it yourself. If you are really not in a position to do it, then you look at the handout and then we can have the discretion if required. But please try to prove that for the condition given H02 relative equal to H01 relative, I have already shown it of course by this horizontal line.

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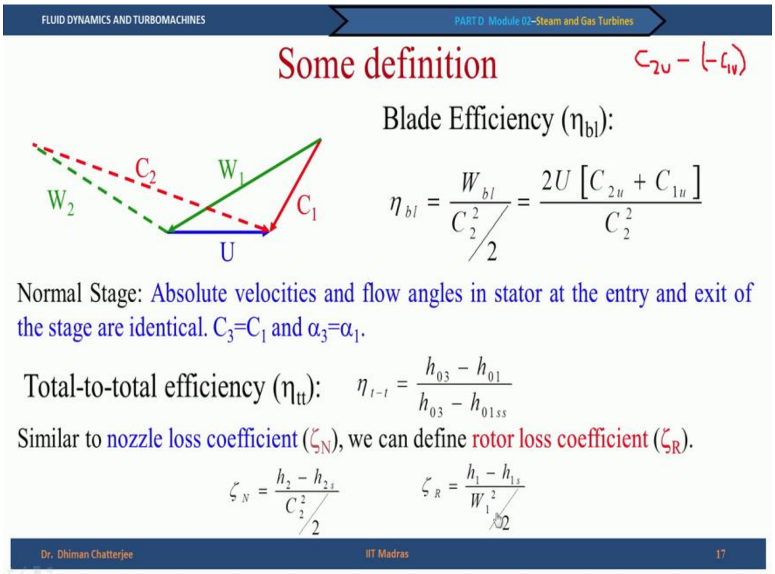

So some more definitions we need, we can talk about the blade efficiency Eta BL as W BL by C2 square by 2. What it is saying that how much specific work is obtained from the blade when the kinetic energy input is C2 square. So that means how much of the kinetic energy that is being converted into useful blade specific work. And this is exactly the same but we have talked about in case of Pelton turbine. Please keep the similarity between the Pelton turbine definitions and the steam turbine, exact expressions will little bit vary but the philosophy is same.

In case of Pelton turbine we talked about the jet velocity CJ which is our C2, which comes out with a high kinetic energy. In this case the scheme comes out of the nozzle with C2 square by 2 as the kinetic energy per unit mass. So we can write now 2U C2U plus C1U. I again reemphasise this point that you can think about this C2U plus C1U in this way. You can say that C2U minus of minus C1U, so this is what I am saying. C2U minus of minus C1U, this 2nd minus inside the bracket actually denotes that C1U is in the direction opposite to C2U and we are talking about the distance, total distance in the graphical scheme.

So we can say that C2U plus C1U by C2 square. Again to remind you we are talking about this distance of that from this part, from this tip of U we are going in the left side to get C2U and we are going on the right side to get C1U and has the total distance is C2U plus C1U. This has to be borne in mind. And we should not use just C2U minus C1U without having the velocity triangle. In fact let me also stress at this point that whenever you are solving problems where we need to find out velocity or the blade angles, it is always a good habit to draw the velocity triangle as accurately as you can and then work it out. That will always give the confidence that your values are becoming realistic.

So next definition that we need to know is the normal stage definition. See we are talking about multiple stages of a steam or gasturbine, so in this case the normal stage is one in which the absolute velocities and flow angles in stator at the entry and exit of the stage are identical. That is C 3 is equal to C1 and Alpha 3 is equal to Alpha 1. Why do we need such a normal stage definition? That is because the flow that leaves from one stage actually goes to the next stage. So if we are trying to talk about the similar constructions, then we are saying that C 3 is equal to C1 and Alpha 3 is equal to Alpha 1.

So unless otherwise mentioned, we can always assume that this is normal stage assumption is valid. The next thing is our total to total efficiency. We have talked about the total to total efficiency earlier also and we have told that time that total to total efficiency is nothing but H03 minus H01 divided by H03 - H01 SS. Please recollect that H03 minus H 01 SS is isentropic enthalpy, isentropic process by which the enthalpy drop is calculated and then H03 minus H01 is the actual process. Similar to nozzle loss coefficient zeta N, we can also define rotor loss coefficient zeta R as follows. We can say that if zeta N is nothing but H2 minus H 2S by C2 square by 2, then zeta R will be H1 minus H1 S.

H1 minus H1 S is actually reflecting how much of the enthalpy is lost and then divided by the corresponding velocity, in case the rotary rotating, so we can talk in terms of the relative velocity W1 square by 2 which comes out of the rotor exit.

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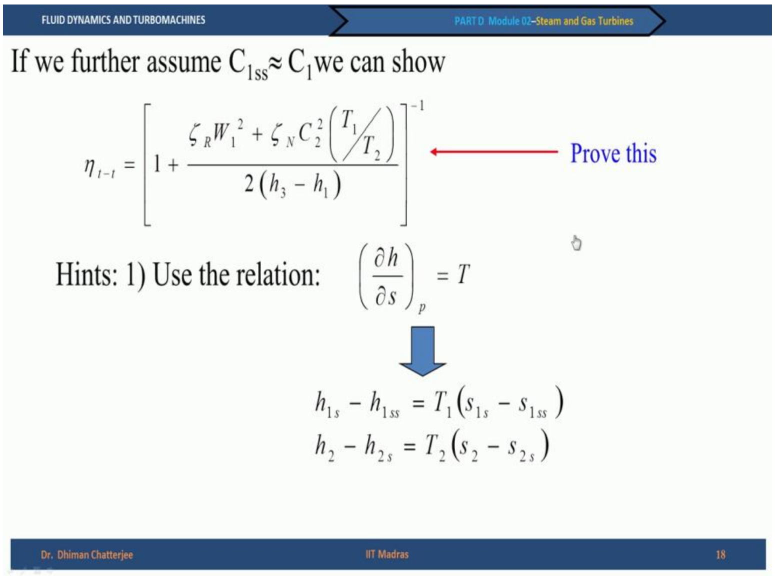

And if we further assume that C1 SS is almost equal to C1, you know that they are not exactly same. Why, because densities are not same and if you have to say the same mass flow rate as we have discussed earlier, then C1 SS is not equal to C1.

But if we assume that the variation between C1 SS and C1, the difference between C1 SS and C1 is not significant, then we can say that Eta TT is can be given as 1+ zeta R times W1 square plus zeta N times C2 square multiplied by T1 by T2, whole divided by 2 H3 minus H1 to the power -1. You can prove this, I will also leave this in the handout but you try do it yourself and I am giving you a few hints. You can use the relationship that Dell H Dell S at constant pressure is T and then using that relationship you can write that H1 S minus H1 SS is equal to T1 times S1 S minus S1 SS and H2 minus H 2 S is T2 times S2 to minus S2S.

That is coming directly from this relationship. And once you use the definitions of zeta R, zeta N, you should be able to get an expression of efficiency, total total efficiency in terms of zeta R and zeta N. And as you can see that this relationship will require little bit of manipulation of the expressions you have given, so I will leave it for you to prove it, I will also give you handout which you can refer, I would suggest only if you are not able to do it yourself. Try it out, I hope you will be able to derive it.

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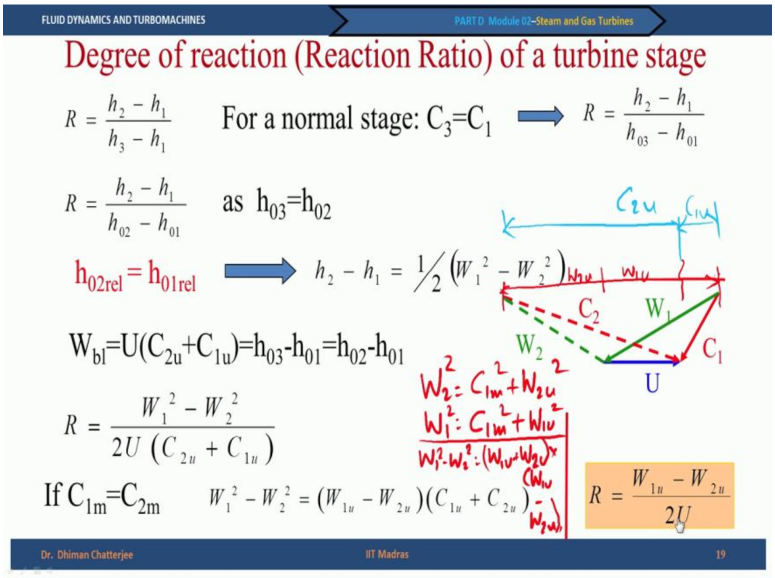

So the next thing that we have to talk about is degree of reaction or sometimes called reaction ratio in different textbooks and literature of a turbine stage. We know that R equal to degree of reaction is equal to H2 minus H1 divided by H3 minus H1. And for a normal stage, we know that C3 equal to C1, just now we have talked about it, so we can add in the, all the terms, the C3 and the C1 respectively and then we can write that R equal to H2 minus H1 whole divided by H03 minus H01. What we have done, we have written H03 as H3 plus C3 square by 2. And H1 as, H01 as H1 plus C1 square by 2. And since C3 and C1 are same so H3 minus H1 is equal to H03 minus H01.

And we can write R equal to H2 minus H1 by H0 2 minus H01. How can we write it, because H03 is equal to H02. We have already shown that in the nozzle the stagnation enthalpy remains constant. We also know that H0 2 relative is H01 relative, I am giving it in red to remind you that you have to prove it and then we get that H2 minus H1 is half W1 square minus W2 square. That is I can write H0 2 relative as H2 plus half W2 square and H1 plus half W1 square will give me H0 1 relative.

If I write the H2 minus H1, because that is in the numerator in terms of the velocity differences or the kinetic energy differences, then H2 minus H1 can be related with W1 and W2. So we can also know that WB L is U times C2U plus C1U which gives me H0 3 minus H01. And which is nothing but H0 2 minus H0 1. So now we have an expression for H0 2 minus H0 1 as well as H2 minus H1 in terms of velocity components. And we can write in either way R, that is in terms of the enthalpies or in terms of the velocities.

And we can write that W1 square minus W2 square by 2U times C2U plus C1U. Let us look at the velocity triangle once again and we can say that if C1 M equal to C2 M, if you make this further assumption, then we should be able to say that W1 square minus W2 square can be written as W1U minus W2U multiplied by C1U plus C2U. How do you get it? Let us look at how you get this. You can write for example W2 square equal to C1 M square plus W2U square. And W1 square as C1 M square because I was assumed that C1 M equal to C2 M plus W1U square.

So if you subtract it out, what do get it is W1 square minus W2 square will be W1 U plus W1 W 2U multiplied by W1U minus W 2U. So now what happens is, this is we obtained, so what happens is now W1U minus W2U is nothing but we are talking about this distance. We are talking about this distance which is exactly same as C1U plus C2U. We are essentially talking about let me mark it, this distance. So this distance is nothing but W1U, this much is W1U, sorry this much is W2U and this much is W1U. You can alternatively think that this one is C2U and this portion is C1U.

That is you can say it as either in terms of W2U and W1U or you can say this portion is C2U and this portion is C1U. So I guess you now understand that what is the reason behind this derivation. So you see that W1 square minus W2 square is W1U minus W2U times W1U plus W2U and W1U plus W2U is nothing but C1U plus C2U. So we can say that the degree of reaction R then becomes W1U minus W2U whole divided by 2U.

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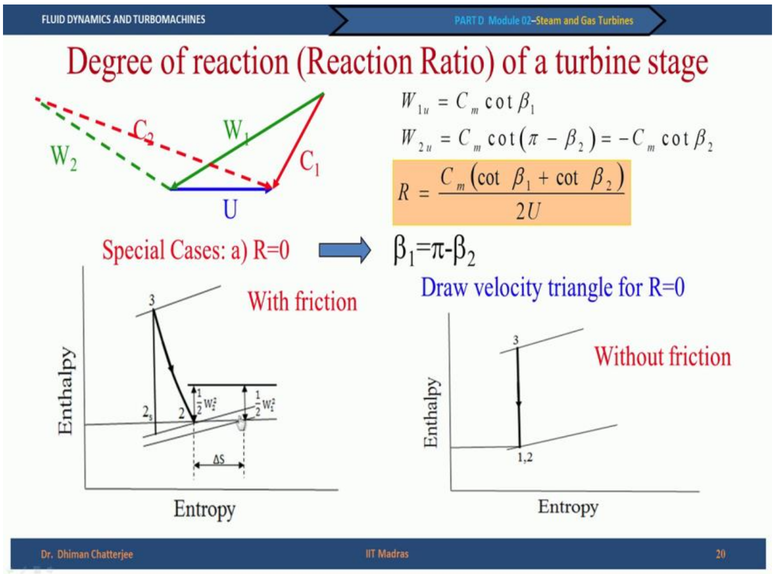

So if we continue the discussion further on the degree of reaction and we can write that W1U is CM cot beta 1, so W1U is CM cot beta 1 and W2U is CM cot pie minus beta 2 because my beta 2 is this angle, hence we are talking about this angle which is pie minus beta 2 or minus CM cot beta 2. So I can write also in terms of the blade angles and hence I write that R equal to CM time cot beta 1+ cot beta 2, whole divided by 2U. And now let us take some special cases.

Let us say R equal to 0, which means the numerator has to be 0. Now CM cannot be 0 because in that case there is no mass flow rate. If there is no mass flow rate, then there is no question of the turbine performance. So the only way R can be 0 is if this bracketed term is 0, which means that beta 1 is pie minus beta 2. So we see that in case of the impulse turbine R equal to 0, we get back the symmetric blade condition which we have already discussed earlier. So we get back and you can draw the velocity triangle which we have discussed earlier.

Now let us look at 2 cases of R equal to 0, 1 with friction and other without friction. Let us start without friction. So in case of without friction we see that the expansion takes place in the nozzle from 3 to 2. Since R equal to 0, we know that H2 equals to H1 and there is no pressure drop, so 2 and 1 becomes coincident points. In case of the frictional case, we know that the pressures cannot be same, however R equal to 0 forces me to say that H2 equals to H1. So 3 to 2 is an expansion in the nozzle, then this is a horizontal line and this point is your point one.

So what we find that there is an entropy increase because there is a friction and the pressure has reduced. So in this case strictly speaking, even though we get R equal to 0 from the enthalpy definitions, there is a pressure change.

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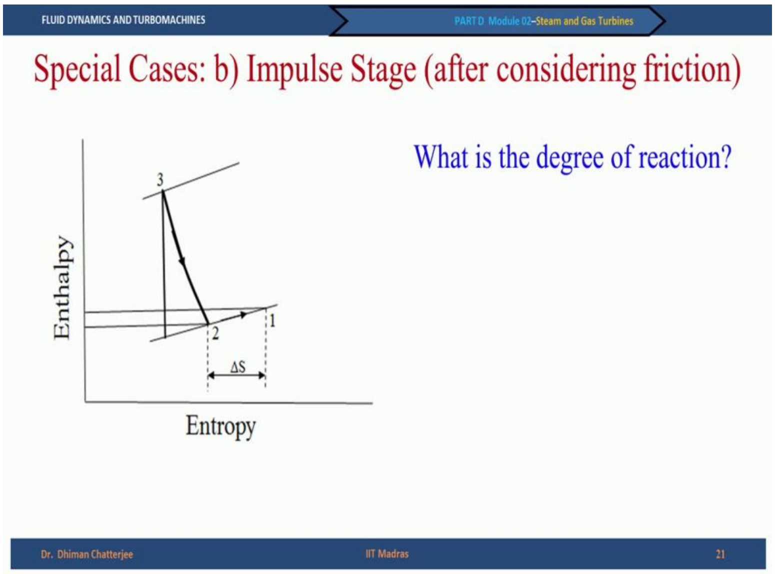

And if we consider that, if we want to insist that we want an impulse stage, that is we want the pressure not to change, then what happens, the 2 and 1, the points 2 and 1 should lie on the same pressure curve. So in that case what happens is 3 to 2 is an expansion in the nozzle and then it goes from 2 to 1 because there will be an increase in entropy because friction is present and we find that H1 has become more than H2. Recollect what is the definition of R, let us go back and write, see the definition of R once again.

We will see that R is H2 minus H1. Now in the case of friction and impulse stage, we find that in the case of friction with impulse stage, we find that H1 is more than H2, which means H2 minus H1 is negative. So what happens to the degree of reaction, you can find out now. So it is actually going to be negative. So let us try to get these points back in our minds again. So 1st is when we say R equal to 0, in the idealised case of no friction, there is no issues. Because points 2 and 1 are coincident points, so H2 minus H1 is 0, the pressure is, now there is no pressure drop, P 2 to equal to P1.

The moment we insist that R is equal to 0 and there is friction, then what happens, the pressure should drop though enthalpy should remain same. So H2 equal to H1 is honoured but not pressure P2 equal to P1. So in that case we find that strictly speaking, definition of impulse stage, that is no pressure drop is not valid. Whereas if we want to insist that there is no pressure drop but there is friction, then we find the degree of reaction becomes negative.

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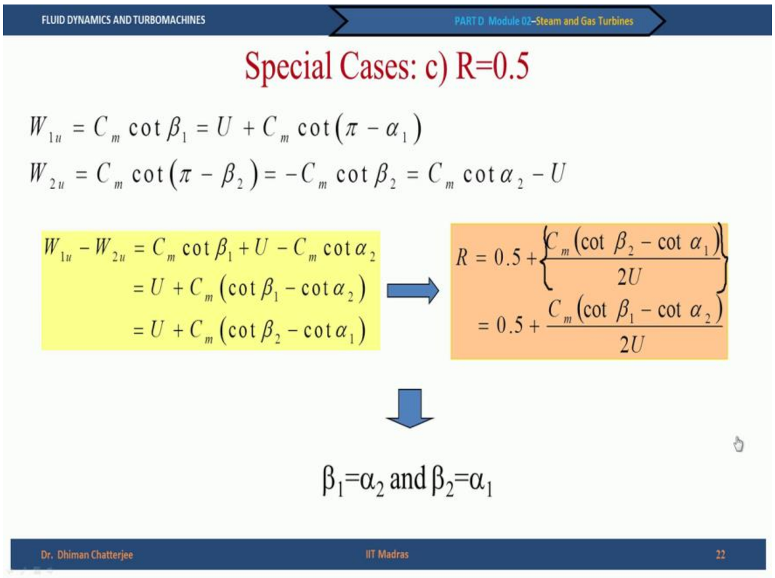

So this has to be kept in mind and another important special case that I will consider now is R equal to 0.5. So I write in the same way that W1U equal to CM cot beta 1 is equal to U plus CM cot pie minus Alpha 1 and W2U is CM cot pie minus beta 2 is equal to minus CM cot beta 2 and I write it in terms of CM cot alpha 2 and U. So basically I am trying to use the blade angles which is beta 1 and beta 2 and the nozzle directed angles of the absolute velocities alpha1 and alpha2.

So then if I write W1U minus W2U, I can get an expression in terms of the velocities U and CM and Alpha and beta. So we get it that CM cot beta 1 which we get from here minus CM cot alpha2 plus U. Why am I getting it, because I am talking about minus W2U. So minus W2 will give me minus CM cot alpha-2 and plus U. So then I tried U plus CM common cot beta 1 minus cot alpha-2 or I did not terms of cot beta to minus cot alpha-1. What does it mean, it means that I can write R equal to 0.5+ CM within bracket cot beta 2 minus cot alpha-1 divided by 2U, because this definition of R W1U minus W 2U by 2U is already discussed.

So then we get, in case I want to insist that R has to be 0.5, then this entire expression which is shown here, this entire expression which is shown here has to go to 0. So this term, this bracketed term as to go to 0. And it can go to 0 only if following the same logic case beta 2 equal to Alpha 1 or from the 2^{nd} expression, beta 1 equal to Alpha 2. So what we get in case of degree of reaction or reaction ratio of 0.5 beta 1 equal to alpha-2 and beta 2 equal to alpha-1.

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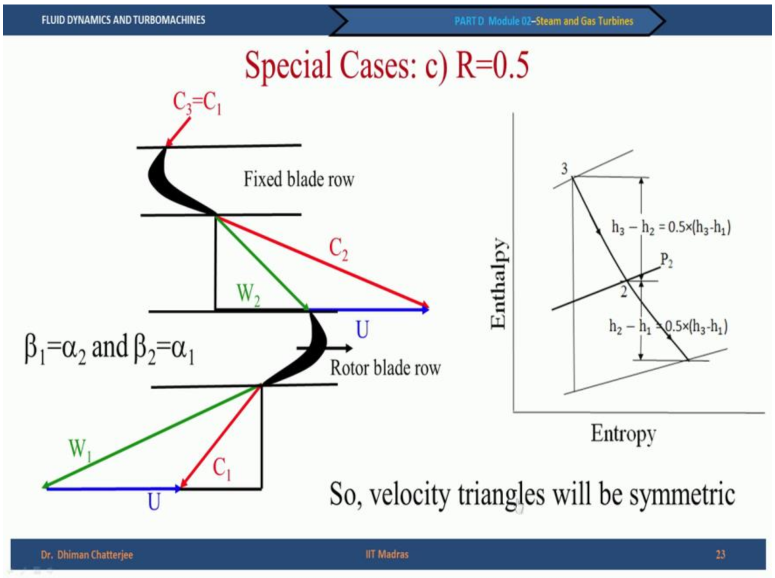

And we can draw the velocity triangles and the enthalpy and entropy diagram. First let us see the enthalpy entropy diagram, we see that H3 minus H2, this is exactly same as H2 minus H1 so that the total enthalpy drop H3 minus H1 is half of, is double of this H3 minus H2 or H2 minus H1 giving me reaction ratio or degree of reaction and 0.5. And you see that C3 equal to C1, the normal stage has been assumed and we get C3 equal to C1 which comes out with C1, which comes in at a stage is C3.

And beta 1 equal to Alpha 2 and beta 2 equal to Alpha 1 and hence velocity triangles will be symmetric. Please do not confuse this velocity triangles will be symmetric with the symmetric blading assumption that we have talked about earlier.

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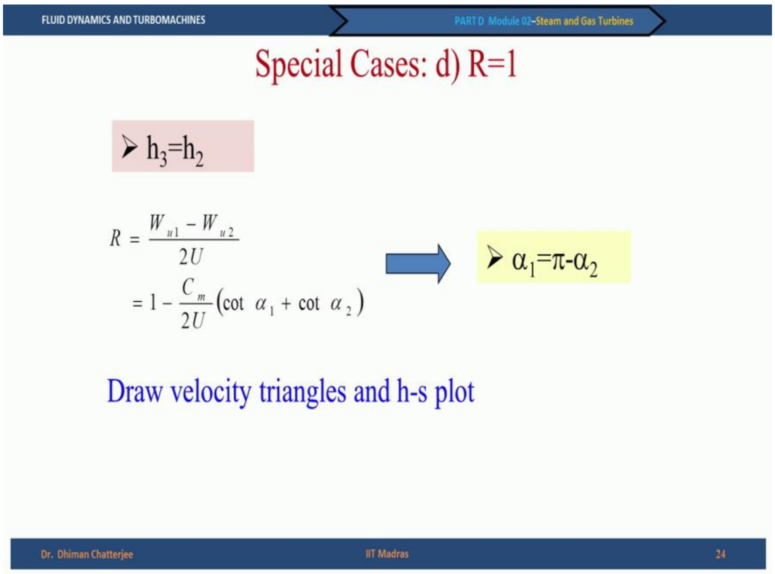

And finally the special case which we are talking about is R equal to 1. In this case what we get it is H3 equal to H2 and we can say that R equal to 1 with this expression in terms of velocity triangles is WU1 minus WU 2 by 2U will give me 1 minus CM by to you cot alpha-1 plus cot alpha-2 and hence we get alpha-1 equal to pie minus alpha-2. I suggest that to get a practice, you can draw the velocity triangles and the HS plot for this case as well. Please note we are talking about R equal to1.

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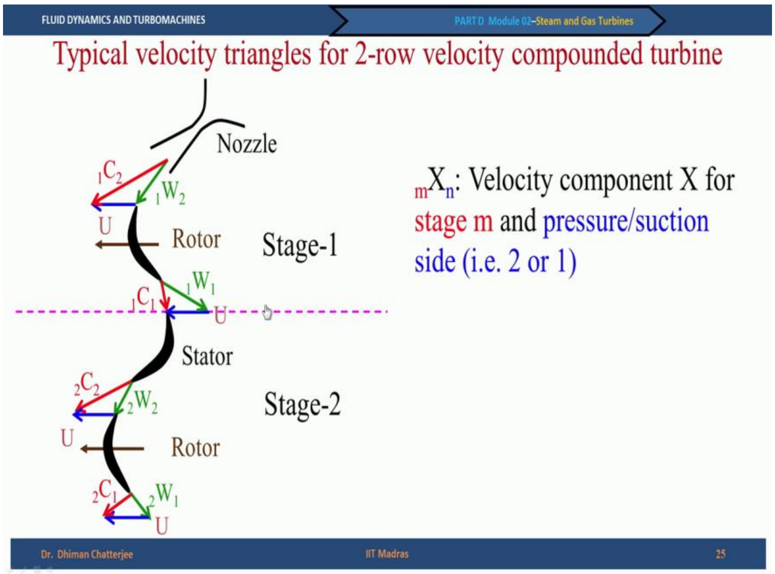

I will, we can extend this discussion from a single stage to multiple stages, I will show you a simple example of typical velocity triangles for 2 row velocity compound a turbine and I leave it to you to do the other parts. So first you have to recollect that a velocity compounded turbine, the first stage has a nozzle and the 2nd stage has a state or the guide blade or the fixed blade. So the flow is directed from the nozzle to the 1st stage rotor and the flow leave the 1st stage rotor and goes into the 2nd stage which is having a stator and then from the stator it goes to the 2nd stage rotor and leaves.

So we can say that the velocity is defined in terms of MXN, we are now using 2 subscripts, the 1st subscript of M talks about Mth stage, for example if it is stage 1, all of these quantities C, W, etc. will have 1 as the M. And N refers to the pressure and suction sides which is talking about 2 and 1. So that means when it intends the rotor, it is 2 and when it leaves the rotor, it is 1. Thus we have for the 1st stage, for the rotor, velocity can be given at 1W2 at the inlet, relative velocity and the relative velocity at the exit of the 1st stage rotor can be given as 1W1. And similarly we can talk about 2W2 and 2W1 for the 2nd stage rotor respectively. And this is the stage one, this pink line shows and the stage 2.

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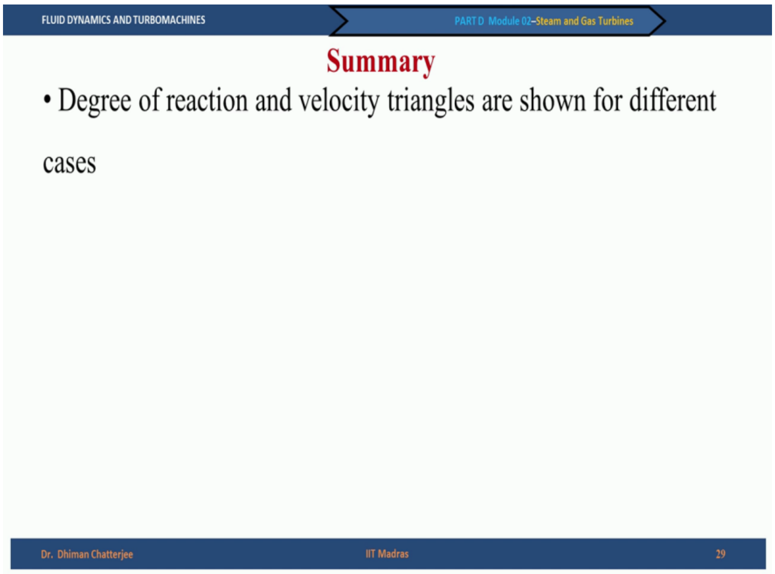

And with this I come to the conclusion of today's discussion which we had on the degree of reactions, we talked about different special cases starting from the generalised description of degree of reaction in terms of enthalpy and the velocity components. And then we connected and showed what will be the HS plot for different degrees of reaction in particular for R equal to 0 or the impulse stage or R equal to 0.5. We have talked about the requirements of the velocity triangles and from there we talked about the requirements of the blades.

We talked about the symmetric blading when we talked about that beta 1 and beta 2 are connected with the relationship that 180 degree minus beta 1 equal to beta 2 and we also talked about the symmetric velocity triangles when we talked about, when we talked about R equal to 0.5. With this I stop and in the next class we will talk about the few tutorial problems we want to discuss to bring clarity to do discussion we had done so far. Thank you.