

**Fluid Dynamics And Turbo Machines.**  
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**Part D.**  
**Module-2.**  
**Tutorial-4.**  
**Week 8.**

Good afternoon, I welcome you for today's discussion on tutorials for week 8. In week 8 we have talked about the compressible flows and steam and gas turbines. So I will do 2 problems today to bring out some of the intricacies of this week's discussion. And of course as our practice is we will give you the tutorial problems which I encourage you to work it out.

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FLUID DYNAMICS AND TURBOMACHINES
PART D Module-02 – Tutorial 4

1. At a certain location in a flow of air static pressure has been measured to be  $P = 2.4$  bar and stagnation pressure,  $P_0 = 3$  bar. Measurement of the total temperature shows it to be  $T_0 = 468$  K. Find the (a) Mach number and (b) mass flow rate per unit area.

**Solution:** Static temperature can be determined from:

$$T = T_0 \left( \frac{P}{P_0} \right)^{\frac{\gamma-1}{\gamma}} \Rightarrow T = 468 \left( \frac{2.4}{3} \right)^{1.4-1/1.4} = 439.1 \text{ K}$$

In the absence of work and heat interaction, it was shown earlier that

$$\frac{T_0}{T} = \left( 1 + \frac{\gamma-1}{2} Ma^2 \right) \Rightarrow \frac{468}{439.1} = \left( 1 + \frac{1.4-1}{2} Ma^2 \right)$$

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So the 1<sup>st</sup> problem states that it is certain location in a flow of air, static pressure has been measured to be 2.4 bar and the stagnation pressure is 3 bar. So the measurement of the total temperature shows it to be that it is 468 Kelvin. We need to find out the Mach number and the mass flow rate per unit area. So let us start with what we have. We can say that the static temperature can be determined by the relationships of temperature and total stagnation temperature and the static temperature connected with the stagnation pressure on the static pressure.

In the problem that we have been given, we know both the static pressure  $P$  and the stagnation pressure  $P_0$  and we are also assuming that  $\gamma$  to be 1.4 for air. And if we take it, then we can find out that the static temperature is stagnation temperature times  $P/P_0$

whole to the power gamma -1 by gamma. And that gives me the static temperature to be 439.1 Kelvin. In the absence of work and heat interaction, we have shown you throughout this week as well as the earlier discussion on thermodynamics that T0 by T can be related with the Mach number as 1+ gamma -1 by 2 times Mach number square.

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On solving, we get  $Ma = 0.574$

Flow rate per unit area,  $\frac{\dot{m}}{A} = \rho C$

Velocity can be determined from,  $C = Ma \times C_s$

$$C_s = \sqrt{\gamma RT} = \sqrt{1.4 \times 287 \times 439.1} = 420 \text{ m / s}$$

Thus,  $C = Ma \times C_s = 0.574 \times 420 = 241.08 \text{ m / s}$

Static density is given by:  $\rho = \frac{P}{RT} = \frac{2.4 \times 10^5}{287 \times 439.1} = 1.904 \text{ kg / m}^3$

$$\frac{\dot{m}}{A} = \rho C = 1.904 \times 241.08 = 459.12 \text{ kg / m}^2 \cdot \text{s}$$

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And from here, since we know both T0 and T, we know gamma, we can find out Mach number easily and we can get the Mach number from this relationship as 0.574. So it also shows the justification for using a compressible flow relationship because you recollect that Mach number greater than 0.3 is considered to be a compressible flow. Now flow rate unit area M dot is given by the rho times C because we know that M dot is nothing but rho times C times A, where C is our usual nomenclature for velocity.

So this velocity can be determined in terms of Mach number we know, so Mach number is nothing but C by CS where CS is the speed of sound, which means that if I know the speed of sound CS, I can find out C. And CS can be found out by using the relationship gamma R T and that gives me for the temperature we have 439.1 Kelvin as 420 metres per second. So CS is 420 metres per second, which gives me C to be 241.08 metres per seconds. So our next target will be to find out rho and then we can find out the mass flow rate per unit area.

So we can say that the static density is given by rho by P by RT and we get it as 1.904 KG per cubic meter when we substitute the value of P which is 2.4 bar and temperature which we have obtained as 439.1 Kelvin. So now if we substitute this value of rho and the value of C we have obtained just now 241.08, we get that the mass flux or the mass flow rate per unit

area is equal to 459.12 KG per metre square second. So from this example problem, what we found is we used the relationship of temperature, static and stagnation and we talked about static and stagnation pressures and their connection.

We talked about the velocity of sound as we have obtained earlier in the theoretical discussions and connecting with Mach number.

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2. At the nozzle exit of the certain stage in a steam turbine, absolute velocity is 300 m/s. Rotor speed is 150 m/s and the nozzle angle is 18°. The sum of rotor blade inlet angle and rotor outlet blade angle is 176.5°. Determine the power output from the stage for a steam flow rate of 8.5 kg/s. Assume  $W_1 = W_2$ . Also, determine utilization factor.

**Given**

- $C_2 = 300 \text{ m/s}$
- $U = 150 \text{ m/s}$
- $\alpha_2 = 18^\circ$
- $\beta_2 + \beta_1 = 176.5^\circ$
- $\dot{m} = 8.5 \text{ kg/s}$
- $W_1 = W_2$

Inlet velocity triangle      Outlet velocity triangle

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In the next problem that we are going to talk about is we will take a steam turbine, the problem statement is this. At the nozzle exit of a certain stage in a steam turbine, absolute velocity is 300 metres per second, which means in our notations,  $C_2$  which is the velocity with which the flow approaches the rotor, absolute velocity with which the flow approaches the rotor is 300 metres per second. The rotor speed, that is the blade peripheral speed is 150 metres per second and the nozzle angle is 18 degrees. They remember that the nozzle's purpose is to direct the absolute velocity at a defined angle.

So this 18 degree corresponds to our  $\alpha_2$ . If the sum of the rotor blade angle, inlet angle and rotor outlet blade angle is 176.5 degrees, we need to find out the power output from the stage for a steam flow. The sum of the rotor blade inlet angle and the rotor outlet blade angle is 176.5 degrees, this is given but we neither know rotor blade inlet angle, nor rotor blade outlet angle and we have to find out the power output from the stage for a steam flow rate of 8.5 KG per second. It is further given that we can assume  $W_1$  equal to  $W_2$ , also we need to find out utilisation factor.

I will introduce the utilisation factor through this example. It is given that  $C_2$  is 300 metres per second,  $U$  is 150 meters per second,  $\alpha_2$  is 18 degree. We do not know individually  $\beta_1$  and  $\beta_2$  but together  $\beta_1 + \beta_2$  is equal to 176.5 degree, we know the mass flow rate  $\dot{M}$  8.5 KG per second and do we do not know the values of  $W_1$  and  $W_2$ , but it is given that  $W_1$  is equal to  $W_2$ . That is the relative velocity at the inlet and exit of the rotor are same.

And we can draw the velocity triangle where  $C_2$ ,  $W_2$  and  $U$ , please note that  $U$  is same for both inlet and outlet because we are talking about an axial flow machine and we have the other component  $C_{2u}$ ,  $C_{2m}$  and the corresponding component  $C_{1u}$  and  $C_{1m}$  are shown. As we have discussed in the last lecture that we can use the common base method and draw the velocity triangle one on top of the other with the common base.

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From inlet velocity triangle,

$$C_{2u} = C_2 \cos \alpha_2 = 300 \times \cos 18 = 285.3 \text{ m/s}$$

$$C_{2m} = C_2 \sin \alpha_2 = 300 \times \sin 18 = 92.7 \text{ m/s}$$

$$W_2 = \sqrt{C_{2m}^2 + (C_{2u} - U)^2} = \sqrt{92.7^2 + (285.3 - 150)^2} = 164.01 \text{ m/s}$$

$$W_1 = W_2 = 164.01 \text{ m/s}$$

$$\sin(180 - \beta_2) = \frac{C_{2m}}{W_2} = \frac{92.7}{164.01} = 0.565$$

$$\beta_2 = 145.58^\circ$$

From outlet velocity triangle,

$$\beta_1 = (176.5^\circ - \beta_2) = 30.92^\circ$$

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So let us do the same thing here. And once we do it, we can find out that this distance is  $C_{2u}$  and please note that in this case  $C_{1u}$  is also in the same direction. This is important, in the theory class we talked about the opposite sides but here in the problem given, it is on the same side. So from the inlet velocity triangle, I can get that  $C_{2u}$  is  $C_2 \cos \alpha_2$  which is equal to 285.3 metres per second. And  $C_{2m}$  is  $C_2 \sin \alpha_2$  which gives me 92.7 metres per second.

So we have got  $C_{2m}$  and  $C_{2u}$ , I can find out  $W_2$ . How do I find out? If I use this right angled velocity, right angled triangle and then I can find out  $C_{2u} - U$ , that is this component, this is  $C_{2u} - U$  which is nothing but  $285.3 - 150$  and this is  $C_{2m}^2$ . So  $W_2^2$  square is

nothing but  $C_2^2 M^2 + C_2 U - U$  whole square. And that gives me  $W_2$  to be equal to 164.01 metre per second. And from the problem we know that  $W_2$  equal to  $W_1$  and hence  $W_1$  also is equal to 164.01 metre per seconds.

And we can get the angle  $\sin 180 \text{ degree} - \beta_2$ , please note that in our case  $\beta_2$  is this angle and hence this angle is  $\beta_2$ . Let us write down the  $\beta_2$ , this is our angle  $\beta_2$ . So if this is my  $\beta_2$ , then this angle is  $180 \text{ degree} - \beta_2$  and hence I can find out  $\sin$  of  $180 \text{ degree} - \beta_2$  is giving me  $C_2 M$  divided by  $W_2$  and hence I get that  $\beta_2$  as  $145.58$  degrees. And also we know that  $\beta_1 + \beta_2$  is  $176.5$  degrees which gives me that  $\beta_1$  is  $30.92$  degrees.

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$$C_{1u} = U - W_1 \cos \beta_1 = 150 - 164.01 \times \cos(30.92) = 9.3 \text{ m/s}$$

$$C_{1m} = W_1 \sin \beta_1 = 164.01 \times \sin(30.92) = 84.28 \text{ m/s}$$

$$C_1 = \sqrt{C_{1u}^2 + C_{1m}^2} = \sqrt{9.3^2 + 84.28^2} = 84.79 \text{ m/s}$$

$$P = \dot{m} U (C_{2u} - C_{1u}) = 8.5 \times 150 (285.3 - 9.3) = 351900 \text{ W} = 351.9 \text{ kW}$$

**Utilization factor,**

$$\epsilon = \frac{W_{bl}}{W_{bl} + \frac{C_1^2}{2}} = \frac{\frac{1}{2} \{ (C_2^2 - C_1^2) + (U^2 - U_1^2) + (W_1^2 - W_2^2) \}}{\frac{1}{2} \{ (C_2^2 - C_1^2) + (U^2 - U_1^2) + (W_1^2 - W_2^2) \} + \frac{C_1^2}{2}}$$

$$\epsilon = \frac{C_2^2 - C_1^2}{C_2^2} = \frac{300^2 - 84.79^2}{300^2} = 0.92$$

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Continuing further we can say that  $C_{1U}$  is  $U$ ,  $C_{1U}$  is this portion,  $C_{1U}$  is nothing but  $U - W_1 \cos \beta_1$  and we get  $C_{1U}$  to be 9.3 metres per second.  $C_1 M$  is  $W_1 \sin \beta_1$  and we get 84.28 metres per second. And we see that  $C_{1U}$ ,  $C_1$  is equal to 84.79 metre per second. We get power equal to  $\dot{m} U$  times  $C_{2U} - C_{1U}$  which gives me 351.9 kilowatts.

Now, utilisation factor, let us try to understand what does the utilisation factor try to tell you. It tells us that how much of the exit kinetic energy has been utilised. We do not want the exit kinetic energy to be at all present, we talked about it but in reality, exit kinetic energy is inevitable, however small it is, it is not 0. So utilisation factor actually talks about how much of it we have been able to use it. And utilisation factor  $\epsilon$  is given as  $W_{BL}$ , the blade specific work, divided by  $W_{BL} + C_1^2$  by 2.

Let us try to understand physically what it means. The denominator talks about that  $W_{BL} + C_1^2$  by 2 is the total energy that has been either utilised or leaves with the exit fluid. So if we say that  $W_{BL}$  is converted in the useful work, then  $C_1^2$  is lost, so the total energy is  $W_{BL} + C_1^2$  by 2 and of which  $W_{BL}$  is utilised. And hence when we substitute it, we can write  $W_{BL}$  as  $\frac{1}{2} C_2^2 - C_1^2 + U_2^2 - U_1^2 + W_1^2 - W_2^2$ .

It is a general expression I am writing, I will apply for the particular case of axial flow turbine now. But let us first right down the general expression. So divided by half, the same expression of  $W_{BL}$  continues  $+ C_1^2$  by 2. Since we are talking about an axial flow turbine, so we can say that  $U_1 = U_2 = U$  and these terms drop-off. In this particular problem it is not generally true, in this particular problem we have said that  $W_1 = W_2$  and then this term also drops off.

And finally we have here one  $C_1^2$  square term with the - sign and here one  $C_1^2$  square term with the + sign, these 2 also gets cancelled and this half also gets cancelled. So in our problem, this utilisation factor reduces as  $C_2^2 - C_1^2$  which is the numerator and divided by  $C_2^2$ . And hence we get an utilisation factor of 0.92. So in this problem what we tried to do is we tried to connect the velocity triangles with the power output and also introduced a terminology called utilisation factor.

In the problems that are given in the tutorials for this week, you will come across similar problems on steam turbines where also the enthalpy relationships that we have used you have to attempt. And I hope that this tutorial will put you in a firm footing for starting those tutorial problems that are given for this week. This brings us to the end of this course on fluid dynamics and Turbo machines. This course as you have gone through was in 2 modules, in the 1<sup>st</sup> module Dr Shamit Bakshi talked about the fluid dynamics, the different aspects of fluid dynamics like introduction to fluid flows.

He talked about the integral and differential approaches, talked about the mass and momentum conservation equations and also talked about the viscous flows inside the duct all over a flat plate. In the 2<sup>nd</sup> module we talked about the thermodynamics related with Turbo machines, it was not a full-fledged discussion of thermodynamics but we talked mainly about what is relevant for the Turbo machine course, but we touched up on the 1<sup>st</sup> and 2<sup>nd</sup> laws of thermodynamics and introduced the concept of efficiency.

At that time we did not really talk about the efficiency of the machine but we appreciated that efficiency of a real machine will not be 100 percent. We continued the discussion with the different types of Turbo machines, we introduce how Turbo machines can be different from the aspects of positive displacement machines and then we talked about classification of Turbo machines and also we in week 1 we talked about the dimensional analysis. We talked about also these velocity components leading to Euler's energy equation.

We understood why the real fluid are not going to perform the same way as a vane congruent or the ideal flow and we talked about the different losses and the causes behind the deviations from the vane congruent flow. In the week 7 we talked about the pumps and cavitation and we talked about hydraulic turbines and in the last week, that is in the week ending today we talked about the compressible flows, steam turbines and we also talked about what are the effects on the classification of the turbines as well as on the degree of reaction and velocity triangles.

We believe that after doing this course, you will be able to connect the fundamentals of fluid dynamics that is covered by Dr Bakshi, the mass and momentum conservation equations. For example, you can easily see now that the Pelton turbine bucket which is having a large deflection or the impulse steam turbine blades which are having a large deflection can be explained from the momentum conservation principle, the linear momentum conservation principle. You can appreciate that Euler's energy equation is nothing but an outcome of the angular momentum conservation principles.

And this course also talked about little bit about the compressible flows and the details about the incompressible flows. So this course we believe that will put you in a strong foundation to take an advanced course in fluid dynamics or learn more about the Turbo machines and if some of you are practising engineers, then you will also like to use these ideas and concepts of Turbo machines in your regular work where you are trying to design or make a design improvement of pumps and turbines or compressors. And with this I come to the conclusion for this course on fluid dynamics and Turbo machines. Thank you.