# Fluid Dynamics And Turbo Machines.

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**Part B.**
**Module-1.**
**Lecture-2.**
**Integral Analysis.**

So, let us begin with the 2nd lecture on the integral analysis. In the last lecture we have introduced the Reynolds transport theorem which converts the time derivative of any quantity for a system in terms of that of a control volume.

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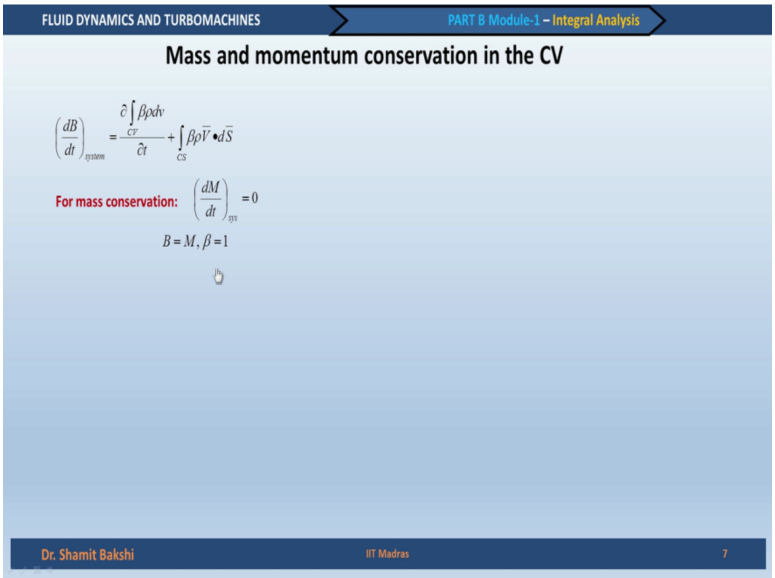


So, in this lecture we will apply the Reynolds transport theorem to mass and momentum conservation in the control volume. So, just to remind ourselves the time derivative of any quantity B in the system is given as the sum of these 2 terms, the first term is basically the time derivative of that quantity, this is basically the sum of the quantity in that control volume plus the surface integral of that quantity on the entire control surface for the control volume. For mass conservation, as we have mentioned before, for the system, the mass conservation equation is dM by dt is equal to 0. So, B, the parameter B is basically M and beta is basically M by M which is one. So, beta is directly 1 here.

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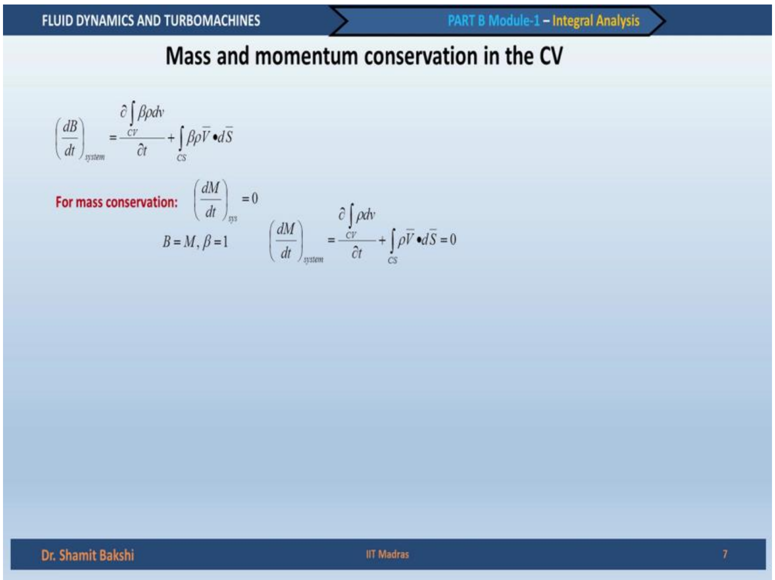


So, if we plug-in this into this equation, what we get is dM by dt for a system is equal to dell dell T of this class surface integral of rho V bar dot ds is equal to 0. So, let us, physically what it signifies, which we also saw in the last class, the first part signifies the rate of change of mass of the control volume and the 2nd term signifies the net rate of mass exiting the control volume. So, as we mentioned in the last lecture, if the net rate of mass exiting the control volume is positive, then the rate of change of mass of the control volume will be negative which is also visible from this expression. So, rate of change of mass time plus a positive quantity is zero, so rate of change of mass will be negative if this quantity is positive. If this is negative, the rate of change of mass will be positive. That means if mass is coming into the control volume, the rate of change of mass will be positive, which is understandable but this mathematical equation also reproduces that.

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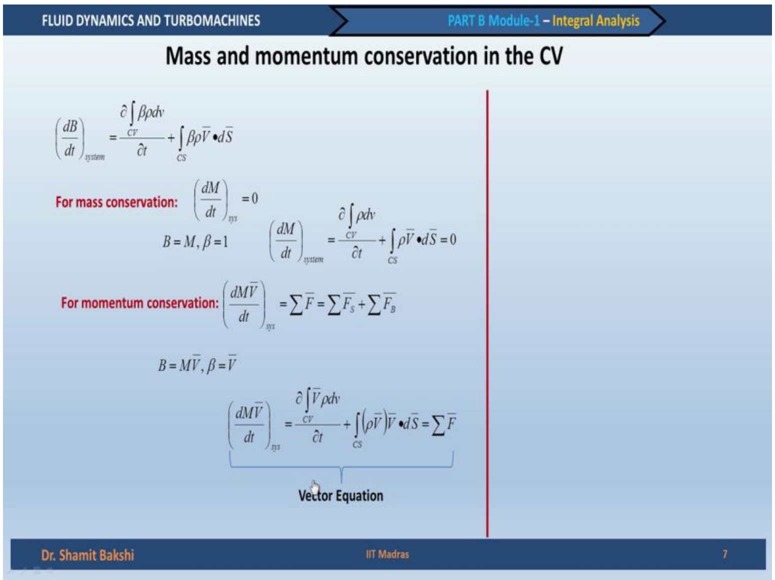


For momentum conservation, B, okay, for the system it is given as dM V by this dt for the system is sum of all the forces. So, rate of change of momentum for the system is basically sum of all the forces which has the surface forces and the body forces. Surface forces are the forces acting on the surface of the control volume and body force is the force acting on the bulk. Now B in this case is basically M into V bar and this parameter, so beta is basically V bar, that is the velocity vector directly. Let us see what it translates to in terms of the Reynolds transport theorem. So, if we plug-in these values into this equation, which is the statement or the mathematical expression for the Reynolds transport theorem, what we get is dM V by dt for the system is given by this.

So, we have just lived in the value of beta which is V bar, velocity vector into this equation. So, this is basically, so it looks mathematically complicated but we will see for special situations we can make, write it in a very simple way. And the statement is actually quite simple. In case of momentum renovation, it means, of course in the case of a system it is rate of change of momentum is summation of force and the rate of change of momentum of the system is given as the rate of change of momentum of the control volume, this is basically the rate of change of control volume plus the momentum which is exiting the control volume subtracted to the momentum which is coming into the control volume. Net rate of momentum of fluid is exiting the control volume.

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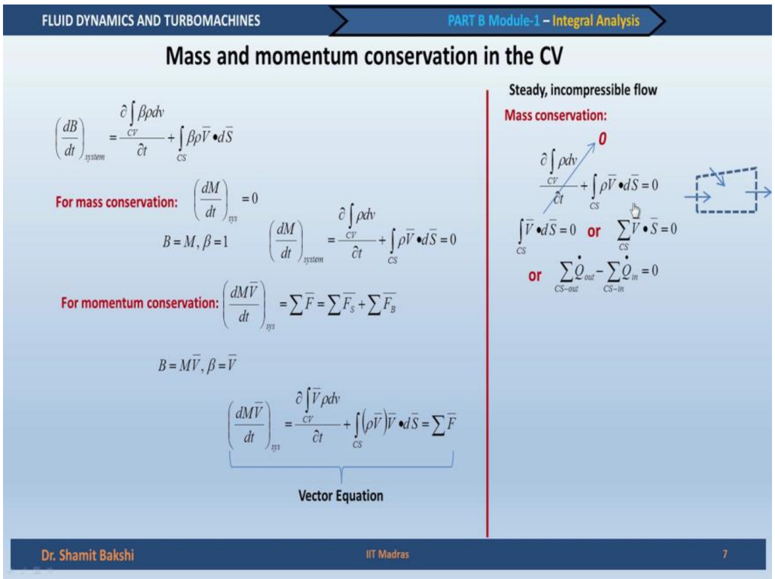


Now we have to remember the 2nd equation is a vector equation, that means, this is, this is basically a representative of a three-dimensional system, it is a representative of 3 equations, and each equation is for a particular component of the velocity vector. So, this is a vector equation, so basically this represents 3 equations for a three-dimensional system. So, you have to keep that in mind. Now let us look at a simplified case so that we can write this equation in a simple way for simple cases. The simplicity which we consider near is a steady and incompressible flow. So, if we consider that, what happens to the mass conservation equation? So, this is the general mass conservation equation which is directly taken in from here and for this situation of steady, what happens is this term cancels out, this term becomes zero because there is no time variation of anything, so mass cannot change with time if the case is steady.

So, what remains is only and it is incompressible, so density can be taken out and density is not equal to 0, so we can just write this as V bar dot ds bar, integral of that over the control surface is zero. So, what it means is if you take the dot product of velocity and the area vector across the entire control surface of the control volume, the result will be zero. So, now let us look at a situation where the velocity is not continuously varying along the surface. So the derivations which we have done and the expression which we have written for mass conservation as well as for momentum conservation, they are more generalised, that means there applicable to a general situation. In many cases we can make very simple assumptions.

One assumption is let us say my control volume is of this particular shape and it has 2 inlets and one outlet and one surface in which there is no velocity, velocity is zero. So, and the velocity across the control surface is constant, velocity across this control surface is also constant, same here. So, if that is the situation, now we can write this equation in a more simple way, it will come out as V bar dot S, on the control surface will be equal to 0. So, the dot product of velocity and the area vector on individual surfaces. So, summation on the control surface means here there are 4 surfaces, 1st surface, 2nd, 3rd and 4th, of course on the 4th surface the velocity itself is zero and it need not be considered. So, we can just write this equation in this form. So this becomes very simple here.

This can be written in terms of more physically understandable quantities like we know V bar dot S bar, S being the surface area, vector form of the surface area, so V bar dot S bar is basically the volume flow rate. So, we can write, so if you say Q dot is the volume flow rate, this is essentially Q dot out Sigma of U dot out on the control surface through which the fluid is exiting the control volume minus Sigma of Q dot in, that through the control surface through which the fluid is coming into the control volume. If you multiply density here, you get M dot out, that is the mass flow rate out and mass flow rate in. This is the well-known continuity equation, so this becomes very simple for a specific situation. And in most of the cases we will deal with the simple forms of this equation. This equation, the generalised equation represents all the cases, that is why it looks complicated.

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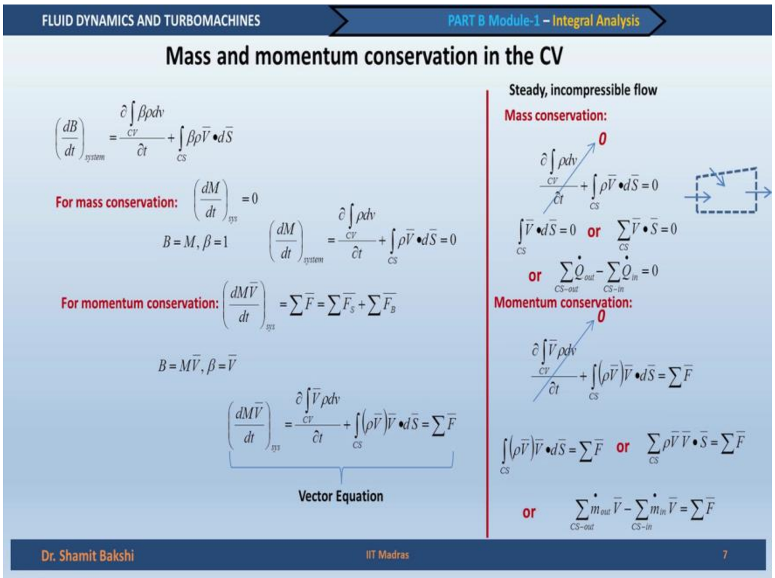


The 2nd one is the momentum conservation, now for the momentum conservation, we can again write it for the study and incompressible flow situation. So, if you write it for the steady case, the first term disappears, this becomes zero and the 2nd term becomes equal to the sum of all forces. So, this is basically for a incompressible steady flow. So density can also be taken out because the flow is, the density variation is not important, it is a incompressible flow. If we consider this kind of situation, again we can write the question in a more simple way, that this is summation over the control surface of this quantity of rho V bar and multiplied by V bar dot ds is equal to the force vector.

Again we have to remind ourselves that this is a vector equation. Although this is a scalar product, dot product, so V bar dot S bar is a scalar quantity which is actually the flow rate as we saw here. So, this is the volume flow rate, so this is the volume flow rate, that is a scalar quantity, but this rho into V bar, V bar is a vector quantity, so this returns the vector nature of this equation so that right-hand side is also force vector. If we want to write this in more form, in a form which is more physically understandable, we can write it in this way, that is we can write it as the net momentum going out of the system, that is M dot out multiplied by the velocity vector minus M dot in multiplied by velocity vector is the net force.

So, it simply means, this statement makes it quite simple to understand what is the statement of the conservation equation for a control volume. It only says the net force acting on the control volume for a steady situation is basically the net momentum of, the net momentum exiting the control volume. That is the difference between the momentum of the fluid which is going out of the control volume and the momentum of the fluid which is coming into the control volume, so that is basically the physical statement of this situation. And it will be very useful to use this form of the equation is that case is as simple as given, which will be useful for solving problem which will be discussed for this, in this chapter.

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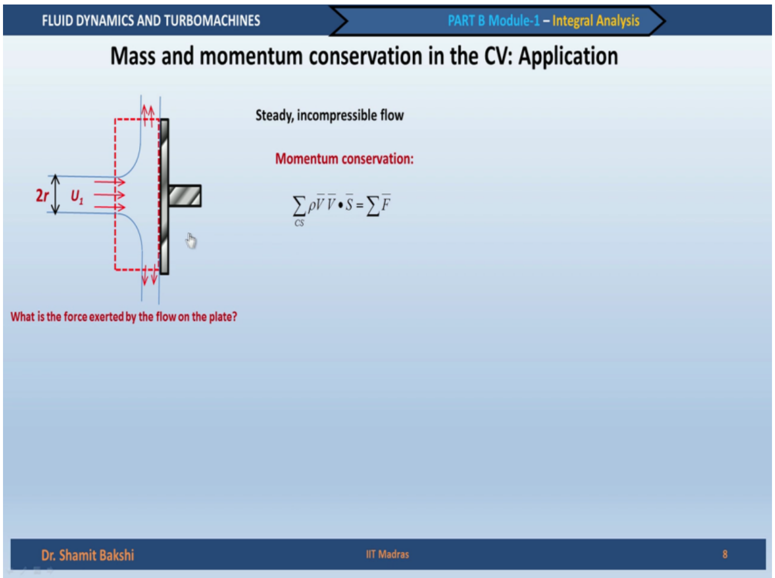

So, we take one application but when dealing with this application, what we will do, we will not directly use the simplified form but we will try to use the general form so that we can reduce the general form to a very simple form. So that we can we know how to apply the general form this equation, of the Reynolds transport theorem or the mass and momentum conservation equation to a given situation. So that is our objective. So, we take this example of force acting on the plate. There is a jet and this jet has a diameter 2R, comes in with a uniform velocity U1, uniform means the velocity across the cross-section of the jet is not changing. So, it comes on like this and then it hits the plate and goes out in this way and the plate is kept stationary, for this part of the problem we assume that the plate is kept stationary.

What we have to find out is what is the force exerted by the flow on the plate. So, how much force does the flow exerts on the plate. Of course we have seen if we do not hold this plate, it will start moving but we do not allow it to happen, we just allow the flow to exert the force let us say we measure that force. So, the this problem will show us to estimate the magnitude of that force. We take a control volume like this, so to begin with we will start with this kind of a control volume but for the same problem we will actually use a different control volume and show, demonstrate the efficacy in selecting the control volume, so that you can get a, get the solution in a easier way.

So the first choice is this, first choice in our demonstration in our lecture is this. The situation is again steady and incompressible flow, the flow is this velocity is, this velocity is not changing with time and it is and incompressible flow, so we can write the momentum conservation equation in this form, that is Sigma of rho V V bar dot ds, V bar dot S is equal to Sigma of F. Instead of writing directly as momentum out minus momentum in, we write it in this form and plug-in these values into this equation.

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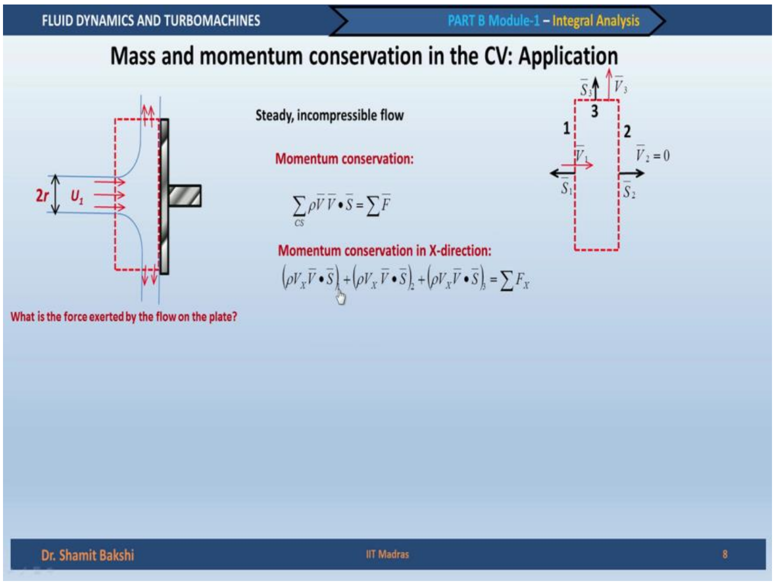


Now, what is the, this is the conservation overall, that means the vector equation, let us see what is the momentum conservation in X direction which is the only important which is the only part of this problem because we have to find out the force exerted by the flow on the plate and the slow here only exerts force in the X direction. So, let us take out this control volume and look at the different velocity components. So, we consider 3 control surfaces, the first surface is the surface here, shown here, the 2nd surface is the surface which is sticking to the plate and the 3rd surface is basically this surface through which the flow is exiting the control volume. So, this is basically a cylindrical part of a cylinder.

So, these 3 surfaces is a surface which is, which represents both the bottom and the top one because we have considered this plate as a circular plate, this is not a rectangular plate, this is a circular plate and the jet is also a circular jet. Now the velocity at the inlet of the control volume which is this one, this is the velocity vector, this is the direction of the velocity vector V1 bar and the direction of the area vector is opposite to that. Okay. So, it is S1 bar V 2 bar, that is the velocity vector in the 2nd control surface is essentially zero because there could be no velocity normal to the surface and the flow also stagnates here, so the velocity comes to 0 velocity.

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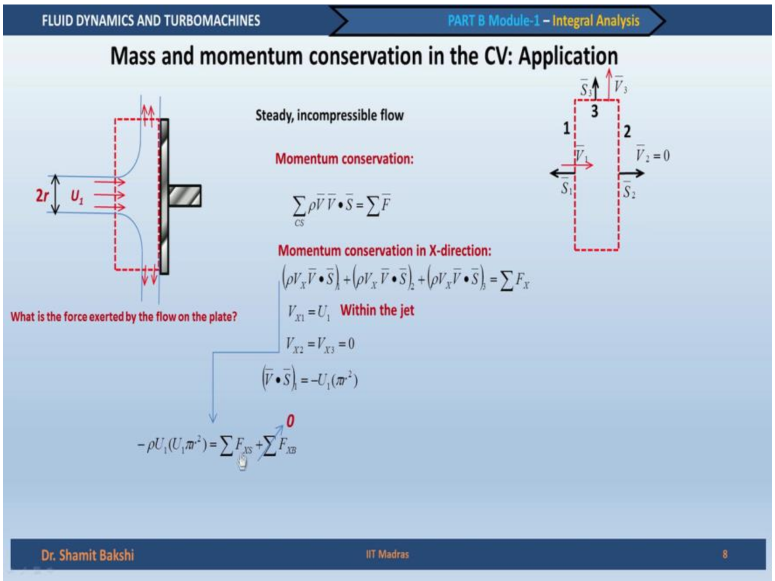


The direction of the area vector is shown like this. Now, the 3rd surface, the area vector is shown like this and the velocity vector is shown like this, V3 bar. Let us break down this equation into the different control surfaces. So, there are 3 control surfaces here, 1, 2 and 3, so we write it for this momentum conservation in X direction for these 3 surfaces. So, it becomes like, so this is now a equation for a particular direction, so in a letter X direction, so this equation becomes rho V X V bar dot S for 1 and rho V X V bar dot S for 2 and 3 and the summation of that is basically the force in the, net force acting in the X direction. VX1, that means the velocity component in X direction on the control surface 1, that is V X1 is a really U1, as given in this problem.

But you have to remember this is only within the jet, only within this region, the control surface actually extends beyond the jet, the control surface is this entire region, the velocity here is zero. Okay, so in this region is zero, so we only consider the region where the velocity is nonzero. So, in that region it is U1, VX2 and VX3, that means the X component of velocity in the 2nd control surface, this one is zero and the X component of velocity in the 3rd control surface is also zero. Okay, that is how the problem is given here. So, now for this V dot S, so these 2 parameters, if VX 2, VX 3 are 0, then these 2 particular terms will become zero, so we are only left with the first term. We know V X1 which is U1 within the jet, and we can find out what is V bar dot S for the control surface 1. So, V bar dot S for the control surface 1, we have to notice very minutely that the velocity vector and the surface S are oriented opposite to each other.

So this will come out to be minus of U1 into pie R Square. So, U1 is the magnitude of velocity and pie R Square is the magnitude of area and because they are oriented opposite to each other, there will be a negative sign. Now, we can if we plug-in here, we can write this equation as minus rho U1 multiplied by U1 pie R Square which is basically V dot S is equal to sum of all the surface forces and the body forces. The body force is of course is zero in this case, we are writing it in a general way so that we know how to apply this equation for any situation. The next task is to find out what is the surface force.

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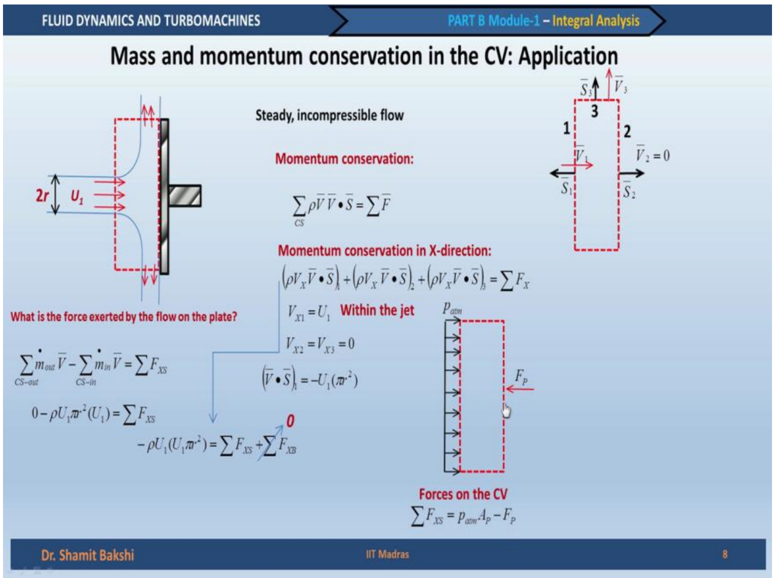


Now instead of going in this way, writing it in this form, we could have solved this problem by writing it out in the in this form which will give us a very direct answer that momentum of the fluoride going out of the control volume subtracted to the momentum of the fluid coming into the control volume is equal to the sum of all the forces acting in that direction. So, momentum out in X direction minus momentum in in X direction is equal to sum of all the forces in the X direction which is only the surface forces here. So if you see momentum out of this control volume in X direction is essentially zero, momentum in to this control volume is given as rho U1 pie R Square U1. Of course it gives the same expression as we got here.

So this can be directly obtained by applying this force balance in terms of force acting is equal to momentum of the fluid is exiting the control volume through the control surface and the momentum of the flow rate coming into the control volume through the control surface. Now to find out what is the value of the force, okay this force is actually not the force which is acting on the plate, so to find out this, what we do is we find out what are the forces acting on the control volume. So, if you see the forces acting on, so, if you look at the control volume boundary, that is the control surface, this surface is exposed to the atmosphere, so it has atmospheric pressure acting along this central surface. On the other hand, on the surface of the plate, there is a force exerted by the plate onto the control volume. How to get the value of this particular force?

For that we need to draw a free body diagram of the plate. That means we have to make the plate free of all the object, that is the jet here in this direction and also this path and then support it with a force. Replace those things with a force. So, we will go to that, before going into that, we look at the forces on the control volume, it is given as the net forces on the control volume is given as the force due to the atmospheric pressure, the atmospheric pressure into AP, AP is basically the area of the plate. So, this is the force acting in the positive direction minus FP, FP is basically the force coming from the plate onto the control volume.

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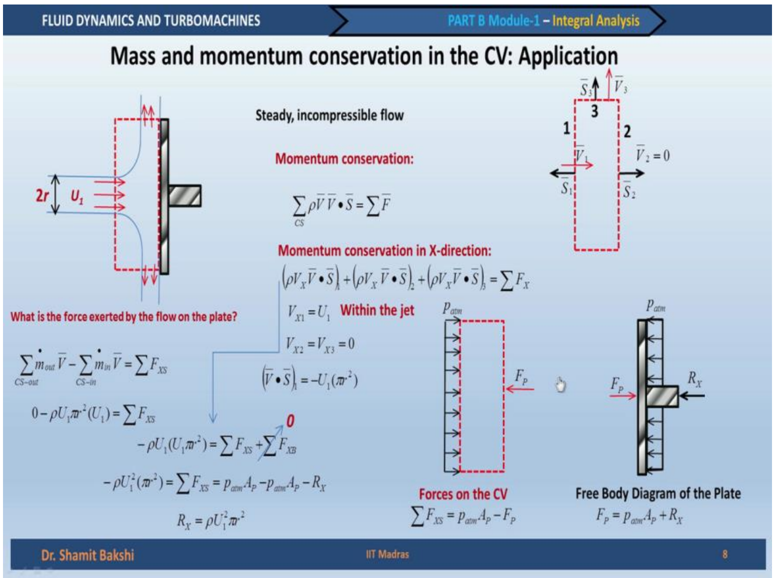


So now we draw the free body diagram of the plate to estimate the value of FP. So, let FP will be, F P can be obtained, if we look at that this is the force acting on the plate, same force in opposite direction is actually acting on the plate as on the control volume and on the other side of the plate, you have a atmospheric pressure acting supported by this reaction coming from the support of the plate. So, we can easily write from this force balance that FP is equal to P Atmosphere atmospheric pressure multiplied by the area of the plate plus the reaction force. So, if you plug-in this here, what you get, you can get directly the value of FXS which is basically minus of RX.

So, if we plug-in these values then, we directly get the value of the reaction force acting on the plate and that is the force, RX is the force which is exerted by the flow on the plate. RX is essentially the force, the reaction force experienced by the plate, that means this is the force which is exerted by the fluid on the plate. Now, we can actually demonstrate solving the same problem using a little different control volume which will be real to solve and we will not need to utilise the free body diagram like we did here.

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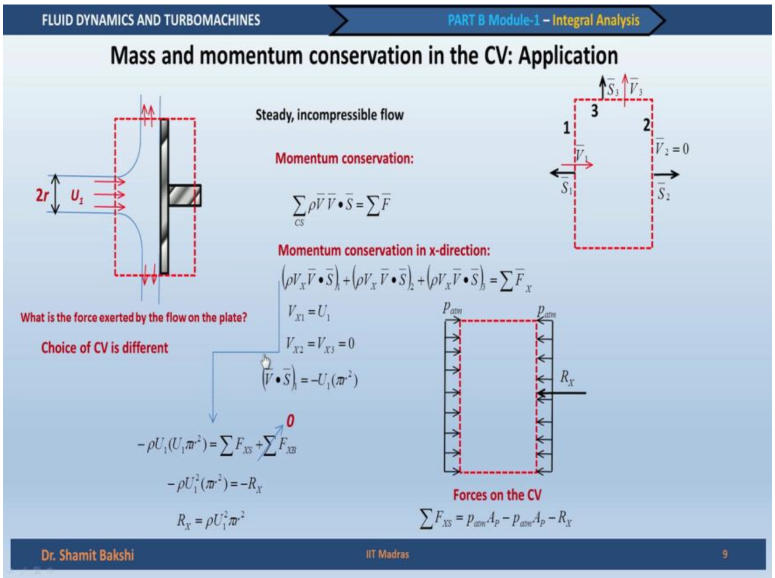


So let us take the same situation but and we want to find this same thing, that means the force exerted by the flow on the plate. We take the control volume in a different way. So, the control volume now instead of ending on the plate surface, it extends, it extends beyond the plate surface and it is shown like this. So, the choice of control volume is different. If we do this, of course the other part of the problem remains the same and we do not want to repeat it, so we can come up to this part where we can see that the left-hand side is basically the net momentum exiting the control volume is equal to sum of the forces. So, to this point for this control volume, the situation is same because if you look at the velocity is at the control surfaces, they are the same.

So, it is the same up to this point. The way it differs is the when we try to find out the force FXS, that is the X directional force, surface force, when we try to do that, we can see now across the control volume boundary in X direction, in this the boundary 1 and boundary 2, in both these boundaries are exposed to atmospheric pressure and so because of this, the atmospheric, the same atmospheric pressure and the atmospheric almost the same atmospheric force acts on both the boundary. Of course we neglect the small area of the support for the plate. And by doing this now we can easily analyse the forces on this control volume, we can write FXS is equal to basically these 2 forces cancel each other, the pressure forces on one side of the control volume and the other side of the control volume, they cancel each other and we are left out with minus RX only. That is what we got in the previous case also.

So, we can plug it in here and get the reaction force directly like this. So, this actually shows us that the choice of control volume is actually quite critical because a correct choice of control volume can help us to avoid complicacy in solving the problem. Here is basically the complicacy in the previous case arose to do the fact that for, if we take the control surface year, the 3rd or the 2nd control surface along the plate, the force is directly not known and it cannot be directly equated to the force acting on the, neither the pressure on the surface is known. So that is how this choice of control volume is better.

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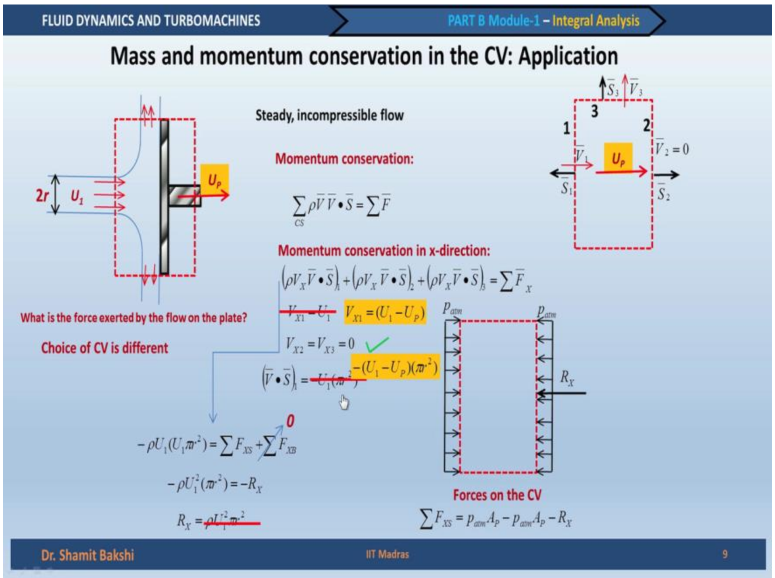


Now the centre analysis which we did can be extended to a case where the plate is moving with a constant velocity, we will demonstrate this before ending this lecture. So, in this case now let us say the plate is moving with a velocity UP. So, if we, we can very easily imagine the situation that the forces acting on the plate will definitely depend on the value of UP. For example, if the plate is moving at the same velocity as the fluid, as the velocity of the jet, that means UP is equal to U1, then no force will be exerted on the plate. If the velocity of the plate is more than that of this velocity of the jet, then the fluid will not able to even touch the plate because the plate will go away before it comes in contact with the liquid. So, naturally when UP is less than U1 but more than zero, than the magnitude of the force acting on the plate will also change.

Now the question is how it changes. So, if you look at the situation, the control volume itself now as a velocity because as the plate moves, we have to also translate the control volume so that we can keep, so that we can track the situation. So, the control volume if it moves as we told before that we have to write all the velocities on the control surfaces with respect to a reference frame fixed to the control volume. So, the reference frame will be fixed to this moving control volume. So now the velocity seen by an observer sitting on the moving control volume will be different from the velocity seen by an observer who is stationary or with respect to which the control volume is moving at a velocity UP. So, now this analysis, this part of the analysis will also change. How it changes, we will see this equation VX1 is no more is equal to U1.

U1 is the U1 is the velocity seen by a stationary observer, by an observer sitting on a moving control volume, V X1 will be U1 minus UP minus the velocity of the observer or the control volume. So, V X1 will be U1 minus UP, this equation will still hold good, V X2 and V X3, both will be zero because the observer of the reference frame is moving along with the control volume. So, with that, with respect to the observer, the control surface, the velocity of fluid on this control surface will still be zero. So, this is correct. Again this V dot S at control surface 1 will be different because V bar itself is different and it will be minus of, instead of being minus of U1 into pie R square, it will be minus of U1 minus UP into pie R square.

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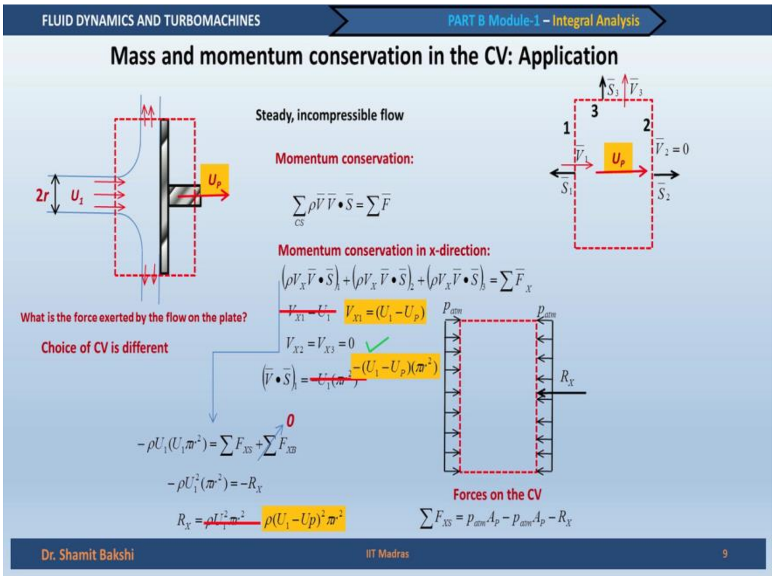

So, final force will also be different, it is rho into U1 minus UP whole square into pie R square. And we can easily see that as the value of UP increases, the force exerted by this jet on this surface, on the plate will reduce. Finally if UP becomes equal to U1, then the force exerted by the plate exerted by the fluid or the flow on the surface, on the plate will be zero. So, this can be demonstrated through this expression now. So, as we saw that we can write conservation equations for inertial control volume which is either stationary or moving at a constant speed. We will see how to use the conservation principles with respect to conservation of angular momentum in the next lecture.

So, in this lecture actually we started with the Reynolds transport theorem which was derived in the previous lecture and we saw how this can be used to write the conservation, mass conservation and momentum conservation equation for a control volume. We also demonstrated the application of mass and momentum conservation equation for a control volume by calculating the forces exerted on a plate by a fluid jet, thank you.